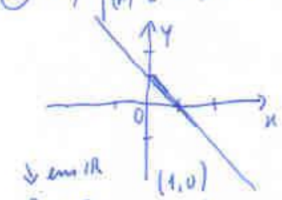
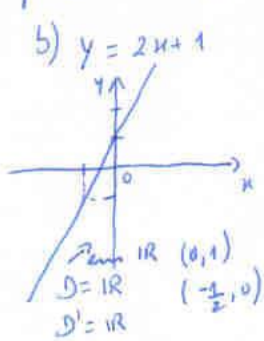
Resolução da ficha n.º 3 Funções - 10.º

(1) a) $f(x) = -x + 1$



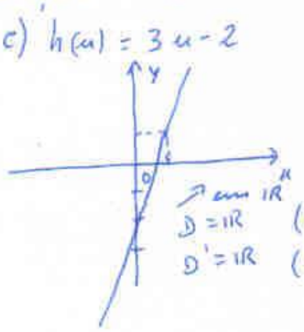
↘ em IR
$D = \mathbb{R}$
$D' = \mathbb{R}$
$(1,0)$
$(0,1)$

b) $y = 2x + 1$



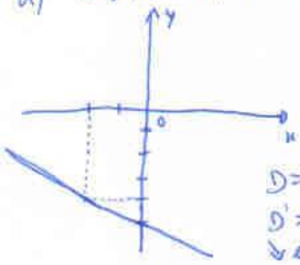
↗ em IR $(0,1)$
$D = \mathbb{R}$ $(-\frac{1}{2}, 0)$
$D' = \mathbb{R}$

c) $h(x) = 3x - 2$



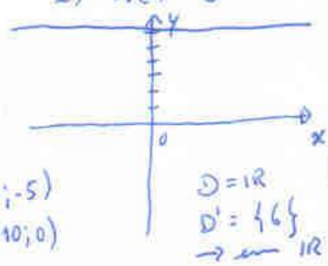
↗ em IR
$D = \mathbb{R}$ $(0; -2)$
$D' = \mathbb{R}$ $(\frac{2}{3}; 0)$

d) $v(t) = -0{,}5t - 5$



$D = \mathbb{R}$ $(0; -5)$
$D' = \mathbb{R}$ $(-10; 0)$
↘ em IR

e) $s(t) = 6$



$D = \mathbb{R}$ $(0; 6)$
$D' = \{6\}$
→ em IR

(2) → função superior esquerda:

$D = [-5; 5]$; $D' = [-1; 2]$; Não sobrej.; Não injet.;
⊕va em $]-5; -\frac{3}{2}[\cup]\frac{3}{2}; 5[$ ⊖va em $]-\frac{3}{2}; \frac{3}{2}[$
Zeros: $\{-5; -\frac{3}{2}; \frac{3}{2}; 5\}$
↗ em $[-5; -3[$ e em $]0; 2[$ ↘ em $]-2; 0[$ e em $]3; 5]$ → em $]-3; -2[$ e em $]2; 3[$

extremos
- max
  - relativos → 2 para $x \in [-3; -2]$ e $x \in [2; 3]$
  - absol. → 2 para $x \in [-3; -2]$ e $x \in [2; 3]$
- min
  - abs. → -1 para $x = 0$
  - relat. → 0 para $x = -5$ e $x = 5$, -1 para $x = 0$

→ função superior central:

$D = [-6; -1] \cup]0; +\infty[$; $D' = \mathbb{R}$; sobrej.; Não injet.;
⊕va em $[-6; -1] \setminus \{-4\} \cup]1; +\infty[$; ⊖va em $]0; 1[$; Zeros: $\{-4; 1\}$;
↗ em $]-4; -1]$ e em $]0; +\infty[$ ↘ em $[-6; -4[$

ext
- max
  - abs. → não existe
  - rel. → 2 para $x = -6$ e 3 para $x = -1$
- min
  - abs → não existe
  - relat. → 0 para $x = -4$

→ função superior direita:
$D = [-4,5; -1] \cup]0; 3] \cup \{4\} \cup \{5\}$; $D' = [-1; 4]$; Não sobrej.; Não inj.;
⊕va em $]-4; -2[\cup]0; 3] \cup \{4\} \cup \{5\}$; ⊖va em $[-4,5; -4[\cup]-2; -1]$
Zeros: $\{-4; -2\}$
↗ em $[-4,5; -3[$ e em $]0; 2[$ ↘ em $]-3; -1]$ e em $]2; 3[$

ext. — max — abs → 4 para $x = 2$
— rel. → 1 para $x = -3$; 4 para $x = 2$
— min — abs → -1 para $x = -1$
— rel → $-\frac{1}{2}$ para $x = -4,5$, -1 para $x = -1$; 2 para $x = 3; 4; 5$

→ função inferior esquerda:
$D = [-5; -2[\cup]-2; 0] \cup [1; 5]$ $D' = [-2; 0] \cup]1; 3]$; Não sobrej.; Injet.;
⊕va em $[-5; -2[$, ⊖va em $]-1; 0] \cup [1; 5[$ Zeros: $\{5\}$
↗ em $[1; 5]$ ↘ em $[-5; -2[$ e em $]-1; 0]$

ext. — max — abs → 3 para $x = -5$
— rel → 3 para $x = -5$, 0 para $x = 5$
— min — abs → -2 para $x = 0$
— rel. → -2 para $x = 0$; -1 para $x = 1$

→ função inferior central:
$D = \mathbb{R}$; $D' = [-4; +\infty[$; Não sobrej.; Não inject.; Zeros: $\{-5; 0\}$
⊕va em $]-\infty; -5[\cup]0; +\infty[$ ⊖va em $]-5; 0[$
↗ em $]-2; 3[$ em $]5; +\infty[$ ↘ em $]-\infty; -2[$ e em $]3; 5[$

ext — max — abs → não existe
— rel → 2 para $x = 3$
— min — abs → -4 para $x = -2$
— rel → -4 para $x = -2$; 1 para $x = 5$

→ função inferior direita:
$D =]-\infty; 2]$; $D' = [0; +\infty[$; Não sobrej.; Não inject.; Zeros: $\{-2; -1; 0; 1\}$
⊕va em $]-\infty; 2] \setminus \{-2; -1; 0; 1\}$;
↗ em $]-2; -1[$ e em $[-1; 0[$ e em $[0; 1[$ e em $[1; 2]$ ↘ em $]-\infty; -2[$

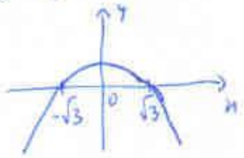
ext — max — abs → não existe
— rel → 1 para $x = 2$
— min — abs → 0 para $x = -2; -1; 0; 1$
— rel → 0 para $x = -2; -1; 0; 1$

③ a) $2x^2 - x + 4 \rightarrow D = \mathbb{R}$ (função quadrática)

b) $\dfrac{2x-3}{5x+2} \rightarrow 5x+2 \neq 0 \Leftrightarrow x \neq -\frac{2}{5}$ $\quad D = \mathbb{R} \setminus \{-\frac{2}{5}\}$

c) $\sqrt{3-x^2} \rightarrow 3 - x^2 \geq 0$

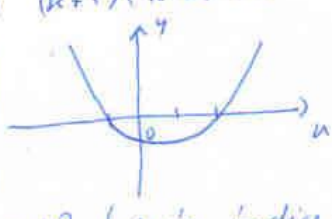
Raízes:
$3 - x^2 = 0$
$x^2 = 3$
$x = \pm\sqrt{3}$



$\rightarrow D = [-\sqrt{3}; \sqrt{3}]$

d) $\sqrt{(x+1)(x-2)} \rightarrow (x+1)(x-2) \geq 0$

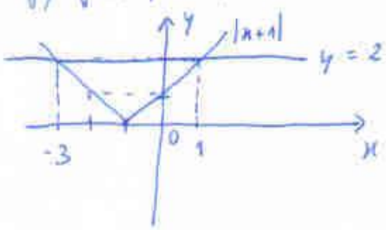
Raízes:
$x+1 = 0 \vee x-2 = 0$
$x = -1 \vee x = 2$



$\rightarrow D =]-\infty; -1] \cup [2; +\infty[$

e) $\sqrt[3]{5x-1} \rightarrow D = \mathbb{R}$ (raiz índice ímpar)

f) $\sqrt{2-|x+1|} \rightarrow 2 - |x+1| \geq 0 \Leftrightarrow |x+1| \leq 2$



$\rightarrow D = [-3; 1]$

g) $\dfrac{\sqrt{x-3}}{\sqrt{5-x}} \rightarrow x - 3 \geq 0 \wedge 5 - x > 0 \Leftrightarrow$
$\Leftrightarrow x \geq 3 \wedge x < 5$
$\hookrightarrow D = [3; 5[$

④ a) $y = 2(x-3)^2 - 1$
$V(3, -1)$; $x = 3$; $(2{,}29; 0)$; $(3{,}7; 0)$; $(0; 17)$

b) $y = -(x+3)^2 + 1$
$V(-3; 1)$; $x = -3$; $(-4; 0)$; $(-2; 0)$; $(0; -8)$

c) $y = x^2 - 2x + 1$
$V(1; 0)$; $x = 1$; $(1; 0)$; $(0; 1)$

d) $y = -x^2+6x+8$

$V(3;17)$; $x=3$; $(-1,12;0)$; $(7,12;0)$; $(0;8)$

e) $y = -2x^2+4x-5$

$V(1;-3)$; $x=1$; $(0;-5)$

6) a) $a=1$ $V(5,2)$

$y = a(x-h)^2+K$

$y = 1(x-5)^2+2$

b) $a=1$ $V(-2,3)$

$y = a(x-h)^2+K$

$y = 1(x+2)^2+3$

c) $a=2$ $(1,0)(3,0)$

$y = 2(x-h)^2+K$

$$\begin{cases} 0 = 2(1-h)^2+K \\ 0 = 2(3-h)^2+K \end{cases} \Leftrightarrow \begin{cases} 0 = 2(1-2h+h^2)+K \\ 0 = 2(9-6h+h^2)+K \end{cases} \Leftrightarrow \begin{cases} 0 = 2-4h+2h^2+K \\ 0 = 18-12h+2h^2+K \end{cases}$$

$$\Leftrightarrow \begin{cases} \overline{} \\ 2-4h+\cancel{2h^2}+\cancel{K} = 18-12h+\cancel{2h^2}+\cancel{K} \end{cases} \Leftrightarrow \begin{cases} \overline{} \\ 8h = 16 \end{cases} \begin{cases} K = -2 \\ h = 2 \end{cases}$$

↳ $y = 2(x-2)^2-2$

d) $V(1;-2)$ $(1,5;0)$

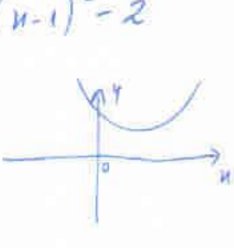
$y = a(x-1)^2-2$

$0 = a(1,5-1)^2-2 \Leftrightarrow 0 = 0,25a-2 \Leftrightarrow a = 8$

↳ $y = 8(x-1)^2-2$

7) a) x^2-2x+2

Raízes: Não tem

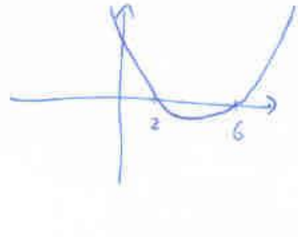


⊕ vc em IR

b) $x^2-8x+12$

Raízes:

$x=6 \lor x=2$

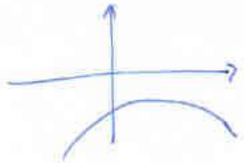


⊕ va em $]-\infty;2[\,\cup\,]6;+\infty[$

⊖ va em $]2;6[$

c) $-x^2-5x-8$

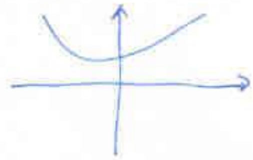
Não tem raízes



$\ominus$ va em $\mathbb{R}$

d) $5x^2-7x+8$

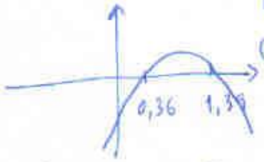
Não tem raízes



$\oplus$ vc em $\mathbb{R}$

e) $-4x^2+7x-2$

Raízes: $x \simeq 0,36 \vee x \simeq 1,39$

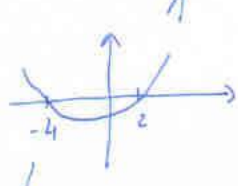


$\oplus$ va em $]0,36;\ 1,39[$

$\ominus$ va em $]-\infty;\ 0,36[\ \cup\ ]1,39;+\infty[$

f) $(x-2)(x+4)$

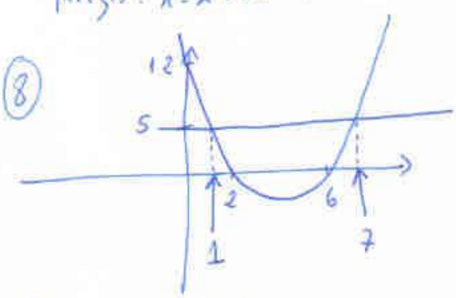
Raízes: $x=2 \vee x=-4$



$\oplus$ vc em $]-\infty;-4[\ \cup\ ]2;+\infty[$

$\ominus$ va em $]-4;\ 2[$

(8)

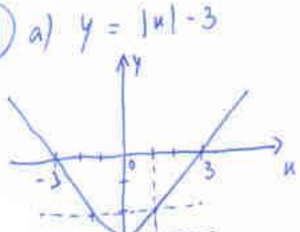


$C.S =]-\infty;\ 1[\ \cup\ ]7;\ +\infty[$

(9) a) $y = |x|-3$

$\oplus$ va em $]-\infty,-3[\ \cup\ ]3;+\infty[$

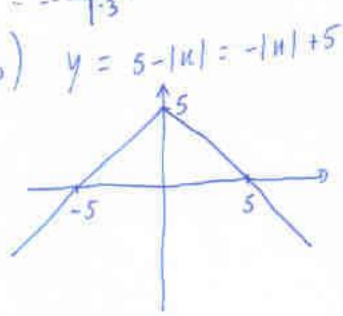
$\ominus$ va em $]-3;3[$



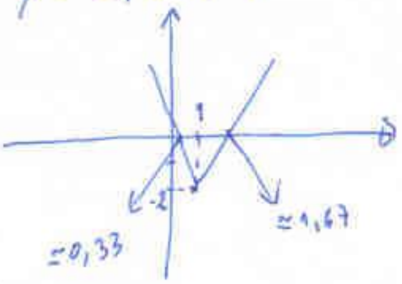
b) $y = 5-|x| = -|x|+5$

$\oplus$ vc em $]-5;5[$

$\ominus$ va em $]-\infty;-5[\ \cup\ ]5;+\infty[$



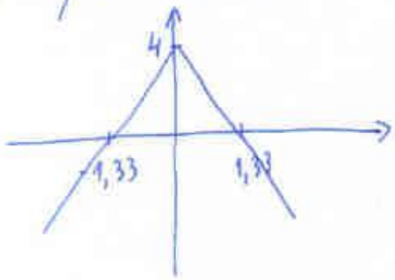
c) $y = 3|x-1| - 2$



$\oplus$vc em $]-\infty; 0{,}33[\cup]1{,}67; +\infty[$

$\ominus$va em $]0{,}33; 1{,}67[$

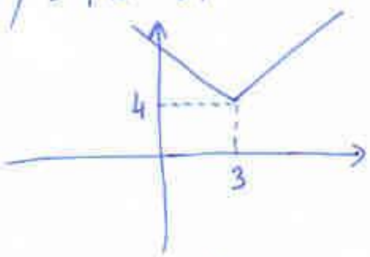
d) $y = 4 - 3|x|$



$\oplus$vc em $]-1{,}33; 1{,}33[$

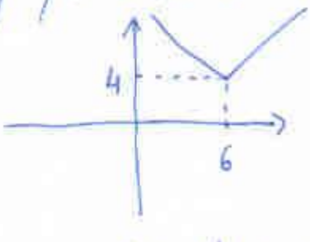
$\ominus$va em $]-\infty; -1{,}33[\cup]1{,}33; +\infty[$

e) $y = |2x-6| + 4$



$\oplus$vc em $\mathbb{R}$

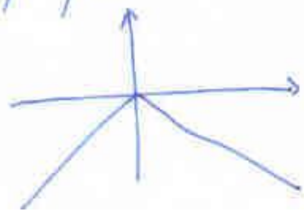
f) $y = 2|x-6| + 4$



$\oplus$vc em $\mathbb{R}$

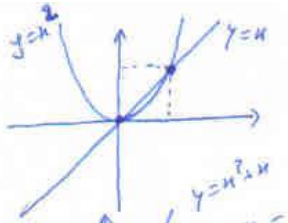
g) $y = 2|x-3| + 4 = |2(x-3)| + 4 = |2x-6| + 4 \Rightarrow e)$

h) $y = -|x|$



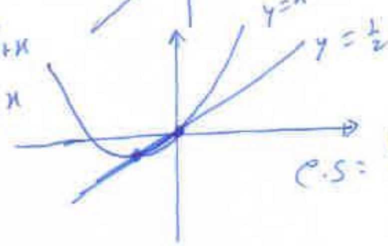
$\ominus$va em $\mathbb{R} \setminus \{0\}$

(10) a) $\begin{cases} y = x^2 \\ y = x \end{cases}$



$C.S = \{(0;0); (1,1)\}$

b) $\begin{cases} y = x^2+x \\ y = \frac{1}{2}x \end{cases}$



$C.S = \{(-\frac{1}{2}; -\frac{1}{4}); (0;0)\}$

(11) a) $f \mapsto A$
$g \mapsto C$
$h \mapsto B$

b) $A \to V(0,3)$
$B \to V(3,0)$
$C \to V(-1,1)$

c) Parábola A
$\nearrow$ em $]0,+\infty[$ $\searrow$ em $]-\infty;0[$ ⊕va em IR
Parábola B
$\nearrow$ em $]-\infty;3[$ $\searrow$ em $]3;+\infty[$ ⊕va em $IR\setminus\{3\}$ ⊖va em $]-\infty;-2[\cup]0;+\infty[$
Parábola C
$\nearrow$ em $]-\infty;-1[$ $\searrow$ em $]-1;+\infty[$ ⊕va em $]-2;0[$

d) $g(x) \geq 0 \Leftrightarrow x \in [-2;0]$

(12) a) $f(x) = Kx^2+5x+K$

$f(x) > 0, \forall x \in IR \Leftrightarrow K>0 \wedge$ "b^2-4ac" $<0 \Leftrightarrow$
$\Leftrightarrow K>0 \wedge 25-4K^2<0 \Leftrightarrow$
$\Leftrightarrow K>0 \wedge$ Raízes: $25-4K^2=0 \Leftrightarrow 4K^2=25 \Leftrightarrow K=\pm\frac{5}{2}$

$\Leftrightarrow K>0 \wedge \left(K<-\frac{5}{2} \vee K>\frac{5}{2}\right) \Leftrightarrow$
$\Leftrightarrow K>\frac{5}{2}$ //

b) $f(x) < 0 \,,\ \forall x \in \mathbb{R} \Leftrightarrow K < 0 \wedge \text{"} b^2 - 4ac \text{"} < 0 \Leftrightarrow$

$\Leftrightarrow K < 0 \wedge \left(K < -\frac{5}{2} \vee K > \frac{5}{2} \right) \Leftrightarrow$

$\Leftrightarrow K < -\frac{5}{2}$ //

(13) $f(x) = Kx^2 + 3x + 2K + 1$

$Kx^2 + 3x + 2K + 1 + 2 \geq 0 \Leftrightarrow$

$f(x) + 2 \geq 0 \Leftrightarrow Kx^2 + 3x + 2K + 3 \geq 0 \Leftrightarrow K > 0 \wedge \text{"} b^2 - 4ac < 0 \text{"} \Leftrightarrow$

$\Leftrightarrow K > 0 \wedge 9 - 4K(2K+3) < 0 \Leftrightarrow K > 0 \wedge 9 - 8K^2 - 12K < 0 \Leftrightarrow$

$\Leftrightarrow K > 0 \wedge -8K^2 - 12K + 9 < 0 \Leftrightarrow K > 0 \wedge 8K^2 + 12K - 9 > 0$

Raízes:

$8K^2 + 12K - 9 = 0$

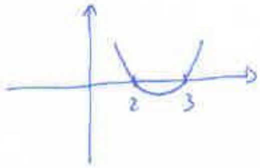
$K = \frac{-12 \pm \sqrt{144 + 32 \times 9}}{16}$

$K \simeq 0,549 \vee K \simeq -2,049$

$\Leftrightarrow K > 0 \wedge \left(K < -2,049 \vee K > 0,549 \right) \Leftrightarrow K > 0,549$ //

(14) a) $x^2 - 5x + 6 > 0$

Raízes: $x = 2 \vee x = 3$



$C.S =]-\infty; 2[\cup]3; +\infty[$

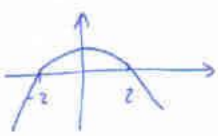
b) $2x^2 + 3 \geq 0$

Raízes: Não tem

$C.S = \mathbb{R}$

c) $-x^2 + 4 \leq 0$

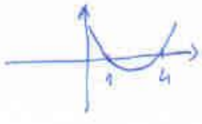
Raízes: $x = -2 \vee x = 2$



$C.S =]-\infty; -2] \cup [2; +\infty[$

d) $x^2 - 5x + 4 \leq 0$

Raízes: $x = 1 \vee x = 4$



$C.S = [1; 4]$

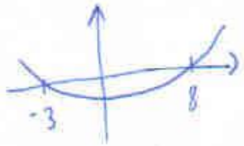
e) $x^2-4x+3>0$

Raízes: $x=1 \vee x=3$

$C.S =]-\infty; 1[\cup]3; +\infty[$

f) $x(x+3)-8(x+3) \leq 0 \Leftrightarrow (x+3)(x-8) \leq 0$

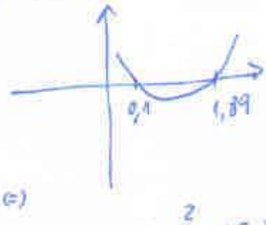
Raízes: $x=-3 \vee x=8$



$C.S = [-3; 8]$

g) $\underset{(1)}{\frac{5x^2-3}{4}} + \underset{(2)}{\frac{x}{2}} > \underset{(4)}{3x}-\underset{(4)}{1} \Leftrightarrow 5x^2-3+2x > 12x-4 \Leftrightarrow 5x^2-10x+1>0$

Raízes: $x \simeq 1,89 \vee x \simeq 0,1$



$C.S =]-\infty; 0,1[\cup]1,89; +\infty[$

h) $4x^3-3x^2-x \geq 4x(x^2-4)+1 \Leftrightarrow$

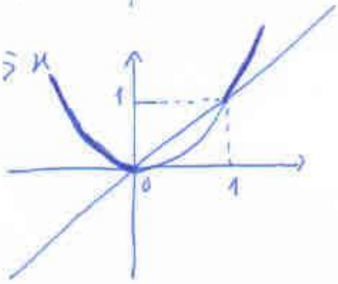
$\Leftrightarrow \cancel{4x^3}-3x^2-x \geq \cancel{4x^3}-16x+1 \Leftrightarrow -3x^2+15x-1 \geq 0$

Raízes:

$x \simeq 0,07 \vee x \simeq 4,9$

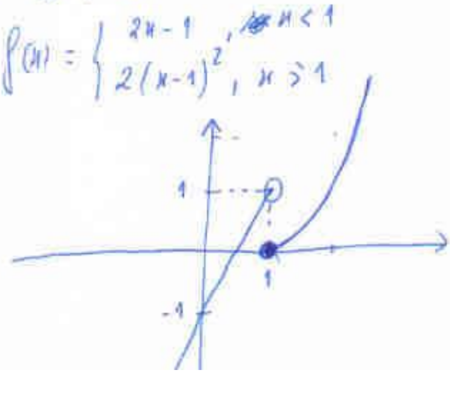
$C.S = [0,07; 4,9]$

(15) $y = x^2 \wedge y \geq x$

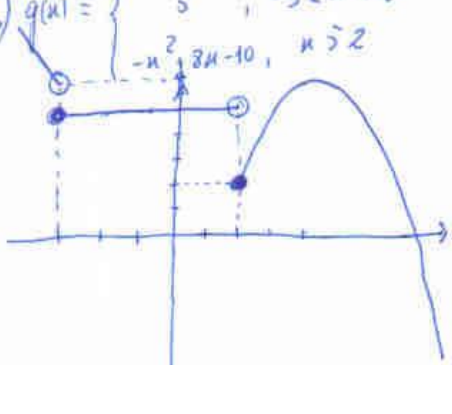


R: $]0; 1[$

(16) a) $f(x) = \begin{cases} 2x-1, & x<1 \\ 2(x-1)^2, & x \geq 1 \end{cases}$

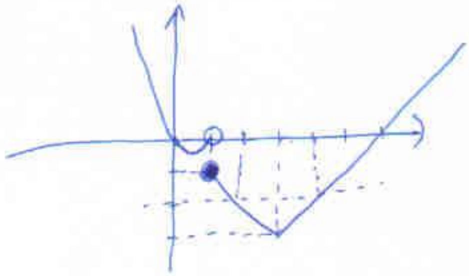


b) $g(x) = \begin{cases} -2x+1, & x<-3 \\ 5, & -3 \leq x < 2 \\ -x^2+8x-10, & x \geq 2 \end{cases}$



(17)

a) $m(x) = \begin{cases} x^2 - x & , x < 1 \\ -3 + |x-3| & , x \geq 1 \end{cases}$



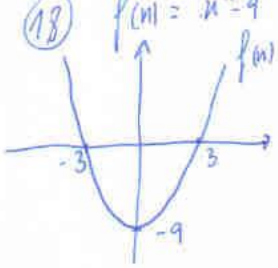
b) Não injet.

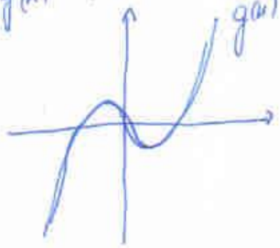
c) $D' = [-3; +\infty[$

d) $m(x) = 0 \Leftrightarrow x = 0 \vee x = 6$

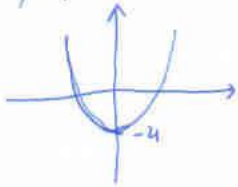
$m(x) < -2 \Leftrightarrow x \in]2;4[$

(18) $f(x) = x^2 - 9$ $g(x) = x^3 - 2x$



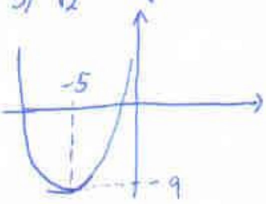


a) $f_1(x) = f(x) + 5$

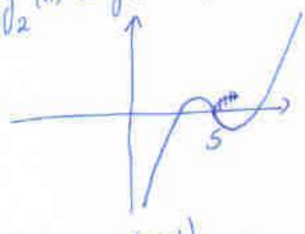


$g_1(x) = g(x) - 3$

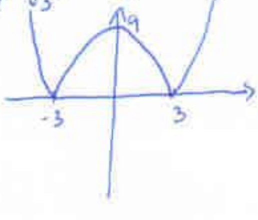
b) $f_2(x) = f(x+5)$



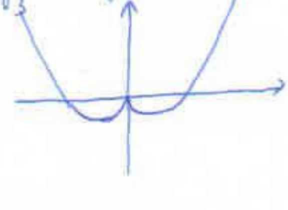
$g_2(x) = g(x-5)$



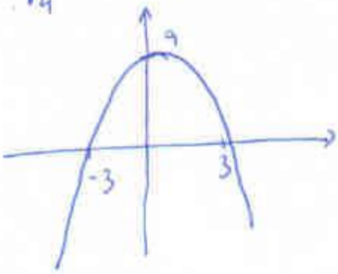
c) $f_3(x) = |f(x)|$



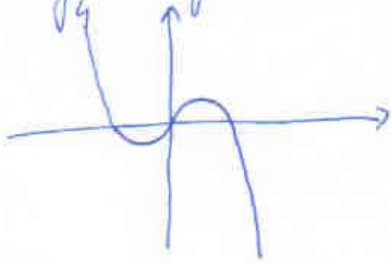
$g_3(x) = g(|x|)$



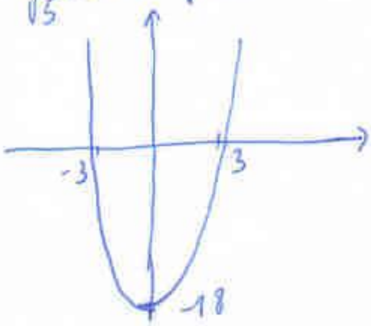
d) $f_4(x) = -f(x)$



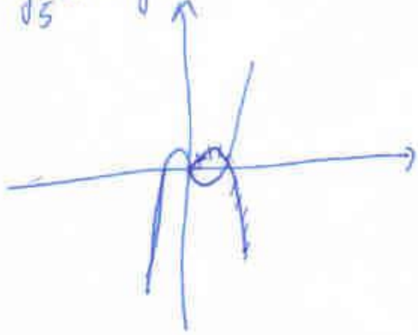
$g_4(x) = g(-x)$



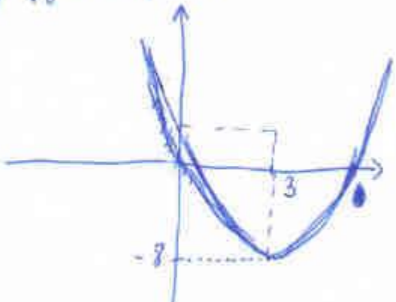
e) $f_5(x) = 2f(x)$



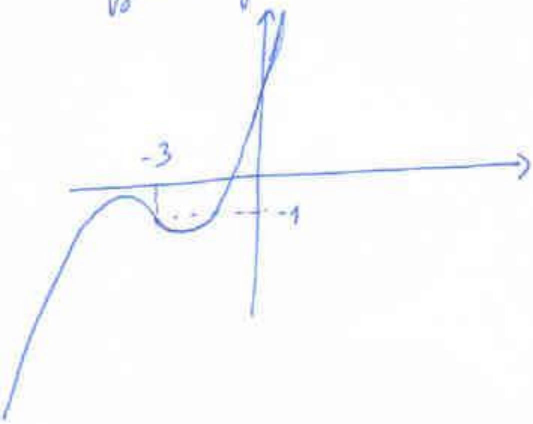
$g_5(x) = g(2x)$



f) $f_6(x) = f(x-3)+1$



$g_6(x) = g(x+3)-1$



(19) gráfico B

(20) $A(-5,0)$
$B(-3,2)$
$\vec{AB} = (2,2)$
$m = 1$
$y = x + b$
$(-5,0)\ 0 = -5 + b$
$b = 5$
$y = x + 5$

$A(3,2)$
$B(5,0)$
$\vec{AB} = (2,-2)$
$m = -1$
$y = -x + b$
$(5,0) \to 0 = -5 + b$
$b = 5$
$y = -x + 5$

$y = a(x-h)^2 + k \quad V(0,-1)$
$y = a(x-0)^2 - 1$
$y = a x^2 - 1$
$(2,2) \to 2 = 4a - 1$
$4a = 3$
$a = \frac{3}{4}$
$y = \frac{3}{4} x^2 - 1$

$$f(x) = \begin{cases} x+5 & , -5 \le x \le -3 \\ 2 & , -3 < x < -2 \\ \frac{3}{4}x^2 - 1 & , -2 \le x \le 2 \\ 2 & , 2 < x < 3 \\ -x+5 & , 3 \le x \le 5 \end{cases}$$