Resolução da ficha n.º 7 de Funções - 10.º ano

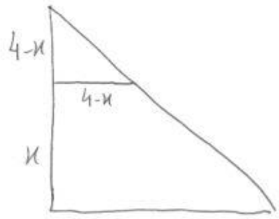
1.1 $V_{cubo} = 4^3 = 64$

$V_{pirâmide} = \frac{1}{3} A_b \times h = \frac{1}{3} \times \frac{4 \times 4}{2} \times 4 = \frac{32}{3}$

$V_0 = 64 - \frac{32}{3} = \frac{160}{3}$

1.2

a)



b) $A_{quadrado} - A_{triângulo} =$

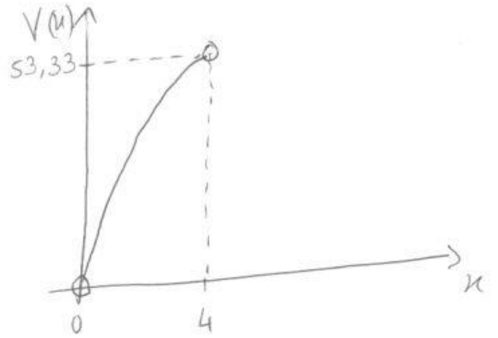
$= 4 \times 4 - \frac{x \times x}{2} = 16 - \frac{x^2}{2}$

1.3 a) $V_{pretendido} = V_{paralelipípedo[ABCDISLM]} - V_{pirâmide[BSJK]} =$

$= 4 \times 4 \times x - \frac{1}{3} \times A_b \times h =$

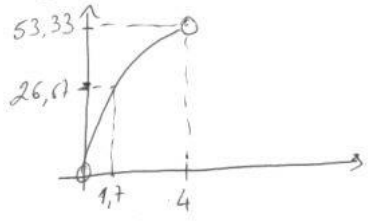
$= 16x - \frac{1}{3} \times \frac{x \times x}{2} \times x = 16x - \frac{1}{6}x^3$

b)



c) $V_0 = \frac{160}{3} = 53,33 \Rightarrow \frac{1}{2} V_0 = 26,67$

$\exists \alpha \in]0,4[\; : \; V(\alpha) = 26,67 \Leftrightarrow \alpha = 1,7$



outro modo,

$V(x) = \frac{1}{2} V_0 \Leftrightarrow$

$\Leftrightarrow 16x - \frac{1}{6}x^3 = \frac{1}{2} \times \frac{160}{3} \Leftrightarrow$

$\Leftrightarrow \underset{(6)}{16x} - \frac{x^3}{6_{(1)}} = \frac{160}{6_{(1)}} \Leftrightarrow$

$\Leftrightarrow 96x - x^3 = 160 \Leftrightarrow$

$\Leftrightarrow x^3 - 96x + 160 = 0$

d) Como as medidas são em cm, fazer uma aproximação a menos de 1mm é fazer uma aproximação com 1 casa decimal (cm — mm)

$\alpha \simeq 1,719$

$1,7 < \alpha < 1,8$

② a) $l(x) < 0 \Leftrightarrow -x^3 + 34x^2 - 220x < 0 \Leftrightarrow x(-x^2 + 34x - 220) < 0$

Raízes:

$-x^2 + 34x - 220 = 0 \Leftrightarrow$

$\Leftrightarrow x = \dfrac{-34 \pm \sqrt{34^2 - 4(-1)(-220)}}{2(-1)}$

$\Leftrightarrow x \simeq \boxed{8,7} \vee x \simeq \boxed{25,3}$

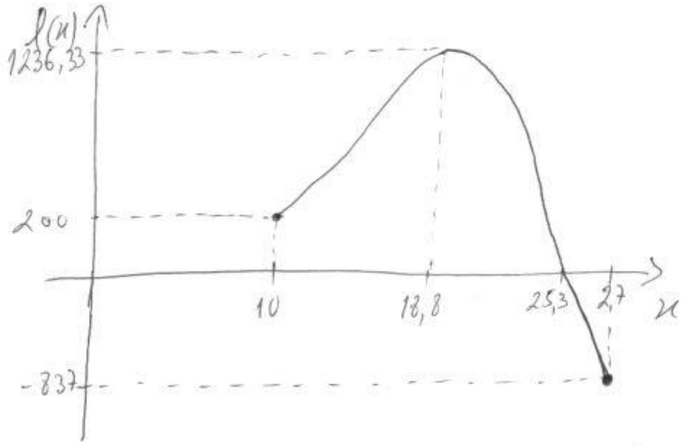
$x = \boxed{0}$

	10		25,3		27
x		+	+	+	
$-x^2+34x-220$		+	0	−	
$l(x)$		+	0	−	

$S = \left]25,3 \, ; \, 27\right]$

R: O lucro é negativo se o nº de estatuetas for 26 ou 27.

b)



R: Como o n.º de estatuetas é inteiro o lucro máximo é para 19 estatuetas, ou seja 1235 euros.

(3)

a) $A(0;-3)$ $\quad \overrightarrow{AB} = (6;3)$ $\quad y = \frac{1}{2}x + b$

$B(6,0)$ $\quad m = \frac{3}{6} = \frac{1}{2}$ $\quad b = -3$

$y = \frac{1}{2}x - 3$

b) $f(x) = a(x+2)(x-4)(x-6)$

$f(0) = -3 \Leftrightarrow a(2)(-4)(-6) = -3 \Leftrightarrow 48a = -3 \Leftrightarrow a = -\frac{1}{16}$

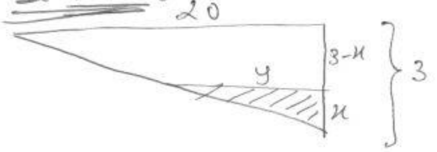
$f(x) = -\frac{1}{16}(x+2)(x-4)(x-6) = -\frac{1}{16}(x+2)(x^2-10x+24) =$

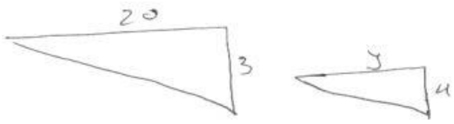
$= -\frac{1}{16}(x^3-10x^2+24x+2x^2-20x+48) = -\frac{1}{16}x^3+\frac{1}{2}x^2-\frac{1}{4}x-3$

c) $g(x) < f(x) \Leftrightarrow x \in [0;2] \cup [6;+\infty[$

d) Máx $\simeq 0,44$ para $x \simeq 3,07$

4) a) Se $x \leq 3$:





$20 — y$

$3 — x$

$y = \frac{20}{3}x$

$V(x) = V_{prisma} =$

$= A_b \times h =$

$= \dfrac{x \times \frac{20}{3}x}{2} \times 10 =$

$= \frac{20}{6}x^2 \times 10 = \frac{200}{6}x^2 = \frac{100}{3}x^2$

Se $x = 3$ $V = \frac{100}{3} \times 3^2 = 300$

Se $x > 3$:

$V_{paralelipípedo} = 20 \times 10 \times x = 200(x-3)$

Resumindo,

$$V(x) = \begin{cases} \frac{100}{3}x^2 & \text{se } 0 < x < 3 \\ 300 + 200(x-3) & \text{se } 3 < x < 4 \end{cases}$$

b) $V(1) = \frac{100}{3} \times 1^2 = \frac{100}{3}\ m^3$

c) $V(2) = \frac{100}{3} \times 2^2 = \frac{400}{3}\ m^2$

d) $V = 20 \Leftrightarrow$

$\Leftrightarrow \frac{100}{3}x^2 = 20 \Leftrightarrow$

$\Leftrightarrow 10\cancel{0}x^2 = 6\cancel{0} \Leftrightarrow$

$\Leftrightarrow x^2 = \frac{3}{5}$

$\Leftrightarrow x = \sqrt{\frac{3}{5}} \simeq 0,77\ m = 77\ cm$

$V = 100 \Leftrightarrow$

$\Leftrightarrow \frac{100}{3}x^2 = 100 \Leftrightarrow$

$\Leftrightarrow 1\cancel{00}x^2 = 3\cancel{00} \Leftrightarrow$

$\Leftrightarrow x^2 = 3 \Leftrightarrow$

$\Leftrightarrow x = \sqrt{3} \simeq 1,73\ m$

(5)

$C(q) = \frac{1}{36} q^3 - \frac{1}{6} q^2 + \frac{1}{3} q + 0,1$ q em milhares

$C(q)$ em milhares

a) $C(3) = \frac{1}{36} \times 3^3 - \frac{1}{6} \times 3^2 + \frac{1}{3} \times 3 + 0,1 = 0,35 \rightarrow 350$ euros

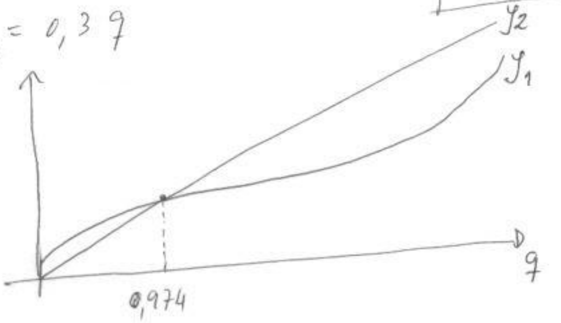
encargos fixos: $C(0) = 0,1 \rightarrow 100$ ~~no~~ euros

b.1) $V(q) = 0,3q$

$y_1 = \frac{1}{36} q^3 - \frac{1}{6} q^2 + \frac{1}{3} q + 0,1$

y_1 = custo
y_2 = venda

$y_2 = 0,3q$

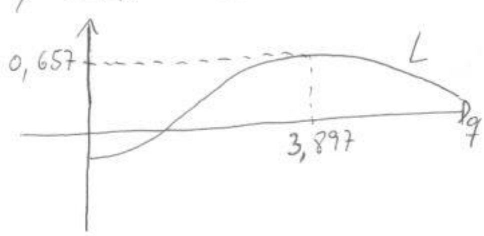


b.2) $q \simeq 0,974$

b.3) $V(q) \geq C(q) \Leftrightarrow q \in [0,974; 5]$

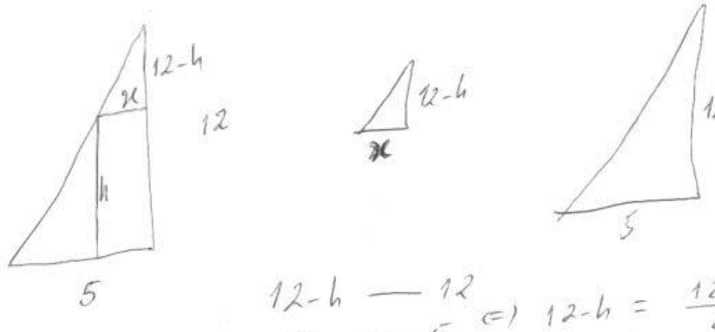
∴ A empresa tem lucro para vendas entre 975 e 5000.

c.1) $L(q) = V(q) - C(q) = 0,3q - \frac{1}{36} q^3 + \frac{1}{6} q^2 - \frac{1}{3} q - 0,1$



c.2) lucro máximo ≃ 657 euros para $q \simeq 3897$ unidades

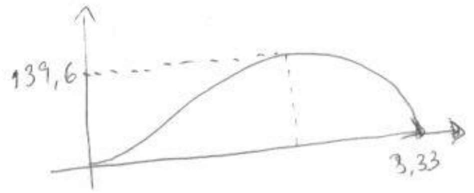
(6)



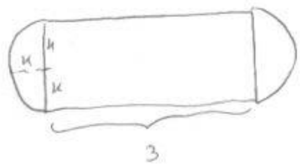
$$12-h \;—\; 12$$
$$x \;—\; 5 \iff 12-h = \frac{12x}{5} \iff h = 12 - \frac{12}{5}x$$

$$V_{cilindro} = Ab \times h = \pi r^2 \times h = \pi \times x^2 \times \left(12 - \frac{12}{5}x\right)$$

$V_{máx} = 139,6$ para $x = 3,3$.



(7)

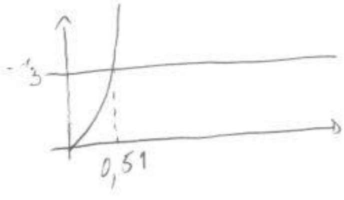


$$V_{esfera} = \frac{4\pi}{3} r^3 = \frac{4}{3}\pi x^3$$

$$V_{cilindro} = Ab \times h = \pi r^2 \times h = \pi x^2 \times 3$$

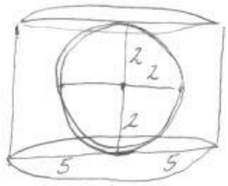
$$V_{total} = \frac{4}{3}\pi x^3 + 3\pi x^2$$

$V = 3$



R: 51 cm

8.a

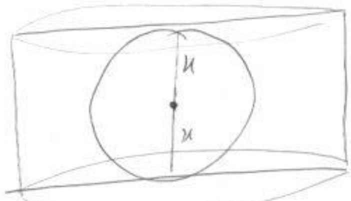


$$V_{esfera} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \times 2^3 = \frac{32\pi}{3}$$

$$V_{cilindro} = Ab \times h = \pi r^2 \times 4 =$$
$$= \pi \times 5^2 \times 4 = 100\pi$$

$$V_{pretendido} = 100\pi - \frac{32\pi}{3} = \frac{268\pi}{3}$$

b)



$$V_{esfera} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi x^3$$

$$V_{cilindro} = Ab \times h = \pi r^2 \times 2x = \pi \times 5^2 \times 2x =$$
$$= 50\pi x$$

$$V_{água} = \frac{268}{3}\pi$$

$$V_{esfera} + V_{água} = V_{cilindro} \Leftrightarrow$$
$$\Leftrightarrow \frac{4}{3}\pi x^3 + \frac{268}{3}\pi = \underset{(3)}{50\pi x} \Leftrightarrow 4\cancel{\pi} x^3 + 268\cancel{\pi} = 150\cancel{\pi} x \Leftrightarrow$$
$$\Leftrightarrow 4x^3 + 268 = 150x \Leftrightarrow 4x^3 - 150x + 268 = 0 \Leftrightarrow$$

2 é raiz (por 8.a)

	4	0	−150	268
2		8	16	−268
	4	8	−134	0

$$(x-2)(4x^2 + 8x - 134) = 0 \Leftrightarrow x = 2 \vee x = \frac{-8 \pm \sqrt{64 - 4 \times 4 \times (-134)}}{2 \times 4}$$

$$\Leftrightarrow x = 2 \vee x = 4{,}9 \quad \cancel{\vee\, x = -6{,}9}$$

R: O raio da nova esfera seria 4,9 cm.

(9)

9.a) $R \nearrow$ em $[0; 1500[$ $\quad R \searrow$ em $]1500; 2500]$

9.b) máx = 150.000 euros para 1500 toneladas

9.c) $C \nearrow$ em $[0; 2500]$ (mín = 75.000 para 0 toneladas)

9.d) $R = C$ para $q \simeq 300 \vee q = 2000$ (lucro nulo)

9.e) $q \in]300; 2000[$

(10) B

(11) B