2.1. Radicais

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1. Seja l o comprimento do lado de um quadrado.

1.1. $l=\sqrt{16}=4$

$l = 4$ cm

1.2. $l=\sqrt{81}=9$

$l = 9$ cm

1.3. $l=\sqrt{0{,}01}$

$l = 0{,}1$ cm

2. Seja a o comprimento da aresta de um cubo.

2.1. $a=\sqrt[3]{8}=2$

$a = 2$ cm

2.2. $a=\sqrt[3]{1000}=10$

$a = 10$ cm

2.3. $a=\sqrt[3]{0{,}001}=0{,}1$

$a = 0{,}1$ cm

3.1. $\sqrt{324}=\sqrt{2^2\times3^2\times3^2}=$

$= 2\times3\times3 = 18$

$$\begin{array}{r|l} 324 & 2 \\ 162 & 2 \\ 81 & 3 \\ 27 & 3 \\ 9 & 3 \\ 3 & 3 \\ 1 & \end{array}$$

3.2. $\sqrt{2500}=\sqrt{25\times100}==5\times10=50$

3.3. $\sqrt{\dfrac{4}{0{,}25}}=\dfrac{\sqrt{4}}{\sqrt{0{,}25}}=\dfrac{2}{0{,}5}=4$

4.1. $\sqrt{2}\times\sqrt{8}=\sqrt{16}=4$

4.2. $2\sqrt{2}\times\sqrt{2}=2\left(\sqrt{2}\right)^2=2\times2=4$

4.3. $\sqrt{\dfrac{1}{2}}\times\sqrt{2}=\sqrt{\dfrac{1}{2}\times2}=\sqrt{1}=1$

4.4. $\dfrac{\sqrt{8}}{\sqrt{2}}=\sqrt{\dfrac{8}{2}}=\sqrt{4}=2$

4.5. $\dfrac{\sqrt{12}}{\sqrt{3}}=\sqrt{\dfrac{12}{3}}=\sqrt{4}=2$

4.6. $\dfrac{\sqrt{8}\times\sqrt{7}}{\sqrt{2}}=\sqrt{\dfrac{8\times7}{2}}=\sqrt{28}=\sqrt{4\times7}=2\sqrt{7}$

5.1. $\sqrt{12}=\sqrt{4\times3}=\sqrt{4}\times\sqrt{3}=2\sqrt{3}$

5.2. $\sqrt{27}=\sqrt{9\times3}=\sqrt{9}\times\sqrt{3}=3\sqrt{3}$

5.3. $\sqrt{48}=\sqrt{16\times3}=\sqrt{16}\times\sqrt{3}=4\sqrt{3}$

5.4. $\sqrt{80}=\sqrt{16\times5}=\sqrt{16}\times\sqrt{5}=4\sqrt{5}$

5.5. $\sqrt{600}=\sqrt{100\times6}=\sqrt{100}\times\sqrt{6}=10\sqrt{6}$

5.6. $\sqrt{2450}=\sqrt{2\times5^2\times49}=$

$=\sqrt{5^2}\times\sqrt{49}\times\sqrt{2}=$

$=5\times7\times\sqrt{2}=35\sqrt{2}$

$$\begin{array}{r|l} 2450 & 2 \\ 1225 & 5 \\ 245 & 5 \\ 49 & 49 \\ 1 & \end{array}$$

6.1. $3\sqrt{2}=\sqrt{9}\times\sqrt{2}=\sqrt{18}$

6.2. $4\sqrt{5}=\sqrt{16}\times\sqrt{5}=\sqrt{16\times5}=\sqrt{80}$

6.3. $a\sqrt{5}=\sqrt{a^2}\times\sqrt{5}=\sqrt{5a^2}$, $a>0$

6.4. $3x^2y\sqrt{z}=\sqrt{9}\times\sqrt{\left(x^2\right)^2}\times\sqrt{y^2}\times\sqrt{z}=\sqrt{9x^4y^2z}$;

$x\in\mathbb{R}$ e $y, z\in\mathbb{R}^+$

7.1. $\sqrt{2}+\sqrt{50}-\sqrt{98}+\sqrt{18}=$

$=\sqrt{2}+\sqrt{25\times2}-\sqrt{49\times2}+\sqrt{9\times2}=$

$=\sqrt{2}+\sqrt{25}\times\sqrt{2}-\sqrt{49\times2}+\sqrt{9\times2}=$

$=\sqrt{2}+5\sqrt{2}-7\sqrt{2}+3\sqrt{2}=$

$=(1+5-7+3)\sqrt{2}=2\sqrt{2}$

7.2. $\sqrt{20}-\sqrt{45}+\sqrt{5}-\sqrt{80}=$

$=\sqrt{4\times5}-\sqrt{9\times5}+\sqrt{5}-\sqrt{16\times5}=$

$=\sqrt{4}\times\sqrt{5}-\sqrt{9}\times\sqrt{5}+\sqrt{5}-\sqrt{16}\times\sqrt{5}=$

$=2\sqrt{5}-3\sqrt{5}+\sqrt{5}-4\sqrt{5}=$

$=(2-3+1-4)\sqrt{5}=$

$=-4\sqrt{5}$

7.3. $2\sqrt{48}+3\sqrt{27}-\sqrt{75}-\sqrt{3}=$

$=2\sqrt{16\times3}+3\sqrt{9\times3}-\sqrt{25\times3}-\sqrt{3}=$

$=2\sqrt{16}\times\sqrt{3}+3\sqrt{9}\times\sqrt{3}-\sqrt{25}\times\sqrt{3}-\sqrt{3}=$

$=2\times4\times\sqrt{3}+3\times3\times\sqrt{3}\times5\times\sqrt{3}\times\sqrt{3}=$

$=(8+9-5-1)\sqrt{3}=11\sqrt{3}$

7.4. $2-3\sqrt{2}+\sqrt{8}=$

$=2-3\sqrt{2}+\sqrt{4\times2}=$

$=2-3\sqrt{2}+2\sqrt{2}=$

$=2-\sqrt{2}$

7.5. $\sqrt{4a}+\sqrt{9a}+\dfrac{1}{2}\sqrt{a}=$

$=2\sqrt{a}+3\sqrt{a}+\dfrac{1}{2}\sqrt{a}=$

$=\left(2+3+\dfrac{1}{2}\right)\sqrt{a}=\dfrac{11}{2}\sqrt{a}$, $a>0$

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8.1. $\left(\dfrac{1}{2}\right)^3\times(-3)^{-2}:\left(-\dfrac{1}{2}\right)^{-1}=$

$=\dfrac{1^3}{2^3}\times\left(-\dfrac{1}{3}\right)^2:(-2)=$

$=\dfrac{1}{8}\times\dfrac{1}{9}\times\left(-\dfrac{1}{2}\right)=$

$=-\dfrac{1}{8\times9\times2}=-\dfrac{1}{144}$

8.2. $\dfrac{\left[\left(\dfrac{1}{2}\right)^2\times\left(\dfrac{3}{5}\right)\right]^{-2}}{\left(2-\dfrac{1}{2}\right)^{-1}}=\dfrac{\left(\dfrac{1}{4}\times\dfrac{3}{5}\right)^{-2}}{\left(\dfrac{4}{2}-\dfrac{1}{2}\right)^{-1}}=$

$=\dfrac{\left(\dfrac{3}{20}\right)^{-2}}{\left(\dfrac{3}{2}\right)^{-1}}=\dfrac{\left(\dfrac{20}{3}\right)^{2}}{\dfrac{2}{3}}=$

$=\dfrac{20^2}{3^2}\times\dfrac{3}{2}=\dfrac{400}{3\times2}=$

$=\dfrac{200}{3}$

8.3. $\dfrac{3^{-7}\times4^{-7}}{12^{-7}\times12^{-2}}=\dfrac{(3\times4)^{-7}}{12^{-7}\times12^{-2}}=\dfrac{12^{-7}}{12^{-7}\times12^{-2}}=\dfrac{1}{12^{-2}}=$

$=12^2=144$

8.4. $\left(7^{-2}\right)^{-5}\times\left[\left(-\dfrac{1}{7}\right)^2\right]^{-1}:\left(7^4\right)^3=$

$=7^{10}\times\left(-\dfrac{1}{7}\right)^{-2}:7^{12}=7^{10}\times7^2:7^{12}=$

$=7^{12}:7^{12}=1$

9.1. $(x-3)^2-2(x+1)^2-2(x-3)(x+3)=$

$=x^2-6x+9-2(x^2+2x+1)-2(x^2-9)=$

$=x^2-6x+9-2x^2-4x-2-2x^2+18=$

$=-3x^2-10x+25$

9.2. $\left(\frac{1}{2}x-3\right)^3=\left(\frac{1}{2}\times-3\right)^2\left(\frac{1}{2}x-3\right)=$

$=\left(\frac{1}{4}x^2-3x+9\right)\left(\frac{1}{2}x-3\right)=$

$=\frac{1}{8}x^3-\frac{3}{4}x^2-\frac{3}{2}x^2+9x+\frac{9}{2}x-27=$

$=\frac{1}{8}x^3-\frac{9}{4}x^2+\frac{27}{2}x-27$

9.3. $\left[\left(\sqrt{2}-x\right)\left(\sqrt{2}+x\right)\right]^2=\left(\left(\sqrt{2}\right)^2-x^2\right)^2=$

$=\left(2-x^2\right)^2=4-4x^2+x^4=$

$=x^4-4x^2+4$

10.1. $\left(x-\frac{1}{2}\right)(2x-1)=0\Leftrightarrow$

$\Leftrightarrow x-\frac{1}{2}=0\vee 2x-1=0\Leftrightarrow$

$\Leftrightarrow x=\frac{1}{2}\vee x=\frac{1}{2}\Leftrightarrow$

$\Leftrightarrow x=\frac{1}{2}$

$S=\left\{\frac{1}{2}\right\}$

10.2. $3x^2-5x+2=0\Leftrightarrow$

$\Leftrightarrow x=\frac{5\pm\sqrt{25-24}}{6}\Leftrightarrow$

$\Leftrightarrow x=\frac{5\pm1}{6}\Leftrightarrow$

$\Leftrightarrow x=\frac{4}{6}\vee x=1\Leftrightarrow$

$\Leftrightarrow x=\frac{2}{3}\vee x=1$

$S=\left\{1\,,\frac{2}{3}\right\}$

11. Por exemplo:

11.1. $(x+2)\left(x-\frac{1}{2}\right)=0$

11.2. $\left(x-\sqrt{2}\right)\left(x-\frac{1}{2}\right)=0$

12.1. $9x^2-1=(3x)^2-1^2=(3x-1)(3x+1)$

12.2. $(x-2)^2-2(x-2)=$

$=(x-2)(x-2-2)=$

$=(x-2)(x-4)$

12.3. $4x^2-5=(2x)^2-\left(\sqrt{5}\right)^2=$

$=\left(2x-\sqrt{5}\right)\left(2x+\sqrt{5}\right)$

12.4. $4x^2+12x+9=(2x)^2+2\times3\times2x+3^2=$

$=(2x+3)^2$

12.5. $25x^2-10x\sqrt{3}+3=\left(5x-\sqrt{3}\right)^2=\left(5x-\sqrt{3}\right)\left(5x-\sqrt{3}\right)$

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Atividade inicial 1

1.1. $\sqrt{2}<2\Leftrightarrow\left(\sqrt{2}\right)^2<2^2$

1.2. $\sqrt{2}<2\Leftrightarrow\left(\sqrt{2}\right)^3<2^3$

1.3. $\sqrt{2}<2\Leftrightarrow\left(\sqrt{2}\right)^4<2^4$

1.4. $\sqrt{2}<2\Leftrightarrow\left(\sqrt{2}\right)^5<2^5$

2.1. $-2<-\sqrt{2}\Leftrightarrow(-2)^2>\left(-\sqrt{2}\right)^2$

2.2. $-2<-\sqrt{2}\Leftrightarrow(-2)^3<\left(-\sqrt{2}\right)^3$

2.3. $-2<-\sqrt{2}\Leftrightarrow(-2)^4>\left(-\sqrt{2}\right)^4$

2.4. $-2<-\sqrt{2}\Leftrightarrow(-2)^5<\left(-\sqrt{2}\right)^5$

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1.1. $-7<-\sqrt{2}\Rightarrow(-7)^7<\left(-\sqrt{2}\right)^7$

1.2. $-7<-\sqrt{2}\Rightarrow(-7)^{10}>\left(-\sqrt{2}\right)^{10}$

1.3. $-4<1\Rightarrow(-4)^9<1^9$

1.4. $-4<1\Rightarrow(-4)^{16}>1^{16}$

1.5. $a<b<0\Rightarrow(-a)^4>(-b)^4$

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2.1. $\sqrt[4]{81}=\sqrt[4]{9^2}=\sqrt[4]{\left(3^2\right)^2}=\sqrt[4]{3^4}=3$ e $3>0$

2.2. $\sqrt[5]{10^5}=10$

2.3. $\sqrt[6]{-4}$ não está definido porque – 4 < 0 e 6 é par.

2.4. $\sqrt[20]{9^{20}}=9$ e $9>0$

2.5. $\sqrt[7]{-2^{14}}=-\sqrt[7]{\left(2^2\right)^7}=-\sqrt[7]{4^7}=-4$

2.6. $\sqrt[6]{(-2)^{12}}=\sqrt[6]{\left[(-2)^2\right]^6}=4$ e $4>0$

2.7. $\sqrt[7]{0}=0$

2.8. $\sqrt[16]{0}=0$

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3.1. $\sqrt[4]{2^2}=\sqrt[4]{2^{2:2}}=\sqrt{2}$

3.2. $\sqrt[4]{4}=\sqrt[4]{2^2}=\sqrt{2}$

3.3. $\sqrt[12]{3^4}=\sqrt[12:4]{3^{4:4}}=\sqrt[3]{3}$

3.4. $\sqrt[12]{3^9}=\sqrt[12:3]{3^{9:3}}=\sqrt[4]{3^3}$

3.5. $\sqrt[8]{2^6}=\sqrt[8:2]{2^{6:2}}=\sqrt[4]{2^3}$

3.6. $\sqrt[5]{5^{20}}=\sqrt[5]{\left(5^4\right)^5}=5^4$

3.7. $\sqrt[6]{2^2\times3^4}=\sqrt[3]{2\times3^2}$

3.8. $\sqrt[18]{a^2b^6c^{12}d^{14}}=\sqrt[9]{ab^3c^6d^7}$; $a,b,\ c,d\in\mathbb{R}^+$

3.9. $\sqrt{6^4}=6^2=36$

3.10. $\sqrt[3]{27a^6}=\sqrt[3]{3^3a^6}=3a^2$, $a>0$

3.11. $\sqrt[4]{81a^2}=\sqrt[4]{3^4}\times\sqrt[4]{a^2}=3\sqrt{a},\ a>0$

3.12. $\sqrt[5]{32b^{10}a^{15}}=\sqrt[5]{2^5b^{10}a^{15}}=2b^2a^3$, $a>0$ e $b>0$

4.1. $\sqrt{3}=\sqrt[3\times2]{3^3}=\sqrt[6]{3^3}$ e $\sqrt[3]{2}=\sqrt[2\times3]{2^2}=\sqrt[6]{2^2}$

4.2. m.m.c. (3 , 12) = 12

$\sqrt[3]{4}=\sqrt[4\times3]{4^4}=\sqrt[12]{4^4}$ e $\sqrt[12]{3}$

4.3. m.m.c. (1 , 3 , 4)

$\sqrt{a}=\sqrt[6\times2]{a^6}=\sqrt[12]{a^6}$, $\sqrt[3]{a}=\sqrt[4\times3]{a^4}=\sqrt[12]{a^4}$ e

$\sqrt[4]{a^3}=\sqrt[3\times4]{a^{3\times3}}=\sqrt[12]{a^9}$

4.4. m.m.c. (4 , 6) = 12

$\sqrt[4]{ab^2}=\sqrt[3\times4]{a^3b^{3\times2}}=\sqrt[12]{a^3b^6}$ e $\sqrt[6]{2ab}=\sqrt[2\times6]{2^2a^2b^2}=\sqrt[12]{4a^2b^2}$

4.5. m.m.c. (10 , 3 , 5) = 30

$\sqrt[10]{ab^2}=\sqrt[3\times10]{a^3b^{3\times2}}=\sqrt[30]{a^3b^6}$, $\sqrt[3]{ab^2}=\sqrt[10\times3]{a^{10}b^{10\times2}}=\sqrt[30]{a^{10}b^{20}}$

$\sqrt[5]{a^2b}=\sqrt[6\times5]{a^{6\times2}b^6}=\sqrt[30]{a^{12}b^6}$

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5.1. $\sqrt[5]{a}\times\sqrt[5]{ab}=\sqrt[5]{a\times a\times b}=\sqrt[5]{a^2b}$

5.2. m.m.c. (2 , 3) = 6

$^{(\times3)}\sqrt{5}\times{}^{(\times2)}\sqrt[3]{4}=\sqrt[6]{5^3}\times\sqrt[6]{4^2}=\sqrt[6]{5^3\times4^2}=\sqrt[6]{2000}$

5.3. m.m.c. (3 , 4) = 12

$^{(\times4)}\sqrt[3]{2}\times{}^{(\times3)}\sqrt[4]{3}=\sqrt[12]{2^4}\times\sqrt[12]{3^3}=\sqrt[12]{2^4\times3^3}=\sqrt[12]{432}$

5.4. m.m.c. (3 , 5) = 15

$^{(\times3)}\sqrt[3]{5}\times{}^{(\times3)}\sqrt[5]{2}=\sqrt[15]{5^5}\times\sqrt[15]{2^3}=\sqrt[15]{5^5\times2^3}=\sqrt[15]{25\,000}$

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6.1. $\left(\sqrt{2^2\times3^5}\right)^6=\sqrt{2^{12}\times3^{30}}=2^6\times3^{15}$

6.2. $\left(\sqrt[5]{a^{10}b^4c}\right)^3=\sqrt[5]{a^{30}\times b^{12}\times c^3}=\sqrt[5]{a^{30}\times b^{10}\times b^2\times c^3}=$

$=a^6b^2\sqrt[5]{b^2c^3}$

6.3. $\left(\sqrt[8]{a^5b^7c^{10}}\right)^4=\sqrt[8]{\left(a^5b^7c^{10}\right)^4}=\sqrt{a^5b^7c^{10}}=\sqrt{a^4a\times b^6bc^{10}}=$

$=a^2b^3c^5\sqrt{ab};\ a,\ b,\ c\in\mathbb{R}^+$

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7.1. $\sqrt[10]{\frac{1}{3}}:\sqrt[10]{\frac{1}{243}}=\sqrt[10]{\frac{1}{3}:\frac{1}{243}}=\sqrt[10]{\frac{1}{3}\times243}=$

$=\sqrt[10]{81}=\sqrt[10]{9^2}=\sqrt[5]{9}$

7.2. $\dfrac{\sqrt{5}\times\sqrt[3]{2}}{\sqrt[6]{100}}=\dfrac{\sqrt[2\times3]{5^3}\times\sqrt[3\times2]{2^2}}{\sqrt[6]{100}}=\sqrt[6]{\dfrac{5^3\times2^2}{100}}=\sqrt[6]{\dfrac{500}{100}}=\sqrt[6]{5}$

7.3. $\dfrac{\sqrt[3]{2}}{\sqrt[9]{8}:\sqrt[3]{4}}=\dfrac{\sqrt[3\times3]{2^3}}{\sqrt[9]{8}:\sqrt[3\times3]{4^3}}=\sqrt[9]{\dfrac{2^3}{8:4^3}}=\sqrt[9]{\dfrac{2^3\times4^3}{2^3}}=\sqrt[9]{4^3}=\sqrt[3]{4}$

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8.1. **a)** m.m.c. (10 , 3 , 5)

$\sqrt[3]{10}=\sqrt[3\times5]{10^5}=\sqrt[15]{100\,000}$

$\sqrt[5]{100}=\sqrt[5\times3]{100^3}=\sqrt[15]{1\,000\,000}$

$\sqrt[5]{5}=\sqrt[5\times3]{5^3}=\sqrt[15]{125}$

$\sqrt[3]{3}=\sqrt[3\times5]{3^5}=\sqrt[15]{243}$

$\sqrt[15]{125}<\sqrt[15]{243}<\sqrt[15]{100\,000}<\sqrt[15]{1\,000\,000}$

Logo, $\sqrt[15]{5}<\sqrt[3]{3}<\sqrt[3]{10}<\sqrt[5]{100}$.

b) m.m.c. (4 , 6 , 3 , 8)

$\sqrt[4]{4}=\sqrt[4\times6]{4^6}=\sqrt[24]{4096}$

$\sqrt[6]{6}=\sqrt[6\times4]{6^4}=\sqrt[24]{1296}$

$\sqrt[3]{3}=\sqrt[3\times8]{3^8}=\sqrt[24]{6561}$

$\sqrt[8]{8}=\sqrt[8\times3]{8^3}=\sqrt[24]{512}$

$\sqrt[24]{512}<\sqrt[24]{1296}<\sqrt[24]{4096}<\sqrt[24]{6561}$

Logo, $\sqrt[8]{8}<\sqrt[6]{6}<\sqrt[4]{4}<\sqrt[3]{3}$.

8.2. **a)** Proposição falsa

b) Proposição verdadeira

9.1. $\left(\sqrt[3]{\frac{1}{2}}\right)^{-4}=\sqrt[3]{\left(\frac{1}{2}\right)^{-4}}=\sqrt[3]{2^4}=\sqrt[3]{2^3\times2}=2\sqrt[3]{2}$

9.2. $\left(\sqrt[4]{0,2}\right)^{-5}=\sqrt[4]{\left(\frac{2}{10}\right)^{-5}}=\sqrt[4]{5^5}=\sqrt[4]{5^4\times5}=5\sqrt[4]{5}$

9.3. $\left(\sqrt[5]{a^{-2}}\right)^{-7}=\sqrt[5]{\left(a^{-2}\right)^{-7}}=\sqrt[5]{a^{14}}=\sqrt[5]{a^{10}\times a^4}=a^2\sqrt[5]{a^4}$

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10.1. $\sqrt{\sqrt[3]{3}}=\sqrt[6]{3}$

10.2. $\sqrt[5]{\sqrt{32}}=\sqrt[10]{2^5}=\sqrt{2}$

10.3. $\sqrt{2\sqrt{2}}=\sqrt{\sqrt{4\times2}}=\sqrt[4]{8}$

10.4. $\sqrt[4]{2\sqrt[3]{a^4}}=\sqrt[4]{2\sqrt[3]{a^4}}=\sqrt[4]{\sqrt[3]{2^3\times a^4}}=\sqrt[12]{2^3a^4}$, $a>0$

10.5. $\sqrt{\frac{1}{2}\sqrt[3]{2}}=\sqrt{\sqrt[3]{\left(\frac{1}{2}\right)^{3\times2}}}=\sqrt[6]{\left(\frac{1}{2}\right)^2}=\sqrt[3]{\frac{1}{2}}$

10.6. $\left(\sqrt{a^5\sqrt{a^3}}\right)^3=\left(\sqrt{\sqrt{a^{10}\times a^3}}\right)^3=\left(\sqrt[4]{a^{13}}\right)=\sqrt[4]{\left(a^{13}\right)^3}=$

$=\sqrt[4]{a^{39}}=\sqrt[4]{a^{36}\times a^3}=a^9\sqrt[4]{a^3}$, $a>0$

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11.1. $\dfrac{1}{3\sqrt{5}}=\dfrac{1\sqrt{5}}{3\sqrt{5}\times\sqrt{5}}=\dfrac{\sqrt{5}}{3\times5}=\dfrac{\sqrt{5}}{15}$

11.2. $\dfrac{3}{\sqrt[3]{5}}=\dfrac{3\times\sqrt[3]{5^2}}{\sqrt[3]{5}\sqrt[3]{5^2}}=\dfrac{3\sqrt[3]{25}}{\sqrt[3]{5^3}}=\dfrac{3\sqrt[3]{25}}{5}$

11.3. $\dfrac{3}{2\sqrt[4]{5}}=\dfrac{3\times\sqrt[4]{5^3}}{2\sqrt[4]{5}\sqrt[4]{5^3}}=\dfrac{3\sqrt[4]{5^3}}{2\sqrt[4]{5^4}}=\dfrac{3\sqrt[4]{5^3}}{2\times5}=\dfrac{3\sqrt[4]{125}}{10}$

11.4. $\dfrac{3}{\sqrt[7]{2^5}}=\dfrac{3\times\sqrt[7]{2^2}}{\sqrt[7]{2^5}\sqrt[7]{2^2}}=\dfrac{3\sqrt[7]{2^2}}{\sqrt[7]{2^7}}=\dfrac{3\sqrt[7]{4}}{2}$

11.5. $\dfrac{5}{\sqrt[10]{3^7}}=\dfrac{5\times\sqrt[10]{3^3}}{\sqrt[10]{3^7}\times\sqrt[10]{3^3}}=\dfrac{5\times\sqrt[10]{27}}{\sqrt[10]{3^{10}}}=\dfrac{5\times\sqrt[10]{27}}{3}$

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12.1. $\dfrac{\sqrt{3}}{\sqrt{2}-3}=\dfrac{\sqrt{3}\left(\sqrt{2}+3\right)}{\left(\sqrt{2}-3\right)\left(\sqrt{2}+3\right)}=\dfrac{\sqrt{6}+3\sqrt{3}}{2-9}=\dfrac{-3\sqrt{3}-\sqrt{6}}{7}$

12.2. $\dfrac{\sqrt{2}}{2\sqrt{3}+3\sqrt{2}}=\dfrac{\sqrt{2}\left(2\sqrt{3}-3\sqrt{2}\right)}{\left(2\sqrt{3}+3\sqrt{2}\right)}=\dfrac{2\sqrt{6}-3\times2}{4\times3-9\times2}=$

$=\dfrac{2\sqrt{6}-6}{-6}=\dfrac{3-\sqrt{6}}{3}=1-\dfrac{\sqrt{6}}{3}$

12.3. $\frac{1}{\sqrt{a+3}+\sqrt{a}} = \frac{\sqrt{a+3}-\sqrt{a}}{\left(\sqrt{a+3}+\sqrt{a}\right)\left(\sqrt{a+3}-\sqrt{a}\right)} =$

$= \frac{\sqrt{a+3}-\sqrt{a}}{a+3-a} = \frac{\sqrt{a+3}-\sqrt{a}}{3}, a \in \mathbb{N}$

12.4. $\frac{3-\sqrt{5}}{\sqrt{5}-1} = \frac{\left(3-\sqrt{5}\right)\left(\sqrt{5}+1\right)}{\left(\sqrt{5}-1\right)\left(\sqrt{5}+1\right)} = \frac{3\sqrt{5}+3-5-\sqrt{5}}{5-1} =$

$= \frac{2\sqrt{5}-2}{4} = \frac{\sqrt{5}-1}{2}$

12.5. $\frac{1}{\sqrt{3}-\sqrt{2}+1} = \frac{\sqrt{3}+\sqrt{2}-1}{\left[\sqrt{3}-\left(\sqrt{2}-1\right)\right]\left[\sqrt{3}+\sqrt{2}-1\right]} =$

$= \frac{\sqrt{3}+\sqrt{2}-1}{3-\left(\sqrt{2}-1\right)^2} = \frac{\sqrt{3}+\sqrt{2}-1}{3-\left(2-2\sqrt{2}+1\right)} =$

$= \frac{\sqrt{3}+\sqrt{2}-1}{2\sqrt{2}} = \frac{\left(\sqrt{3}+\sqrt{2}-1\right)\sqrt{2}}{2\sqrt{2}\times\sqrt{2}} =$

$= \frac{\sqrt{6}+2-\sqrt{2}}{4} = \frac{\sqrt{6}-\sqrt{2}+2}{4}$

12.6. $\frac{2}{\sqrt{1+\sqrt{2}}-1} = \frac{2\left(\sqrt{1+\sqrt{2}}+1\right)}{\left(\sqrt{1+\sqrt{2}}-1\right)\left(\sqrt{1+\sqrt{2}}+1\right)} =$

$= \frac{2\left(\sqrt{1+\sqrt{2}}+1\right)}{1+\sqrt{2}-1} = \frac{2\left(\sqrt{1+\sqrt{2}}+1\right)\sqrt{2}}{\sqrt{2}\sqrt{2}} =$

$= \frac{\cancel{2}\left(\sqrt{2}\sqrt{1+2}+\sqrt{2}\right)}{\cancel{2}} = \sqrt{2+2\sqrt{2}}+\sqrt{2}$

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13.1. $\frac{1}{\sqrt[3]{9}-2} =$

$A^3-B^3=(A-B)\left(A^2+AB+B^2\right)$

$A=\sqrt[3]{9}$ e $B=2$

$= \frac{\left(\sqrt[3]{9}\right)^2+\sqrt[3]{9}\times 2+2^2}{\left(\sqrt[3]{9}\right)^3-2^3} =$

$= \frac{\sqrt[3]{81}+2\sqrt[3]{9}+4}{9-8} =$

$= \sqrt[3]{3^3\times 3}+2\sqrt[3]{9}+4 =$

$= 3\sqrt[3]{3}+2\sqrt[3]{9}+4$

13.2. $\frac{2}{\sqrt[3]{3}+\sqrt[3]{2}} =$

$A^3-B^3=(A-B)\left(A^2+AB+B^2\right)$

$A=\sqrt[3]{3}$ e $B=-\sqrt[3]{2}$

$= \frac{2\left[\left(\sqrt[3]{3}\right)^2+\sqrt[3]{3}\times\left(-\sqrt[3]{2}\right)+\left(-\sqrt[3]{2}\right)^2\right]}{\left(\sqrt[3]{3}\right)^3\left(-\sqrt[3]{2}\right)^3} =$

$= \frac{2\left(\sqrt[3]{9}-\sqrt[3]{6}+\sqrt[3]{4}\right)}{3+2} =$

$= \frac{2\sqrt[3]{9}-2\sqrt[3]{6}+2\sqrt[3]{4}}{5}$

13.3. $\frac{5}{1+2\sqrt[3]{3}} =$

$A^3-B^3=(A-B)\left(A^2+AB+B^2\right)$

$A=\sqrt[3]{24}$ e $B=-1$

$= \frac{5}{\sqrt[3]{24}+1} =$

$= \frac{5\left[\left(\sqrt[3]{24}\right)^2+\sqrt[3]{24\times(-1)}+(-1)^2\right]}{\left(\sqrt[3]{24}\right)^3-(-1)^3} =$

$= \frac{5\left(\sqrt[3]{24^2}-\sqrt[3]{24}+1\right)}{24+1} = \frac{\sqrt[3]{(3\times 8)^2}-\sqrt[3]{24}+1}{5} =$

$= \frac{\sqrt[3]{9\times\left(2^3\right)^2}-\sqrt[3]{24}+1}{5} =$

$= \frac{4\sqrt[3]{9}-\sqrt[3]{24}+1}{5}$

13.4. $\frac{\sqrt[4]{2}+1}{\sqrt[4]{2}-1} =$

$A^4-B^4=(A-B)\left(A^3+A^2B+AB^2+B^3\right)$

$A=\sqrt[4]{2}$ e $B=1$

$= \frac{\left(\sqrt[4]{2+1}\right)\left[\left(\sqrt[4]{2}\right)^3+\left(\sqrt[4]{2}\right)^2-1+\sqrt[4]{2}\times 1^2+1^3\right]}{\left(\sqrt[4]{2}\right)^4-1^4} =$

$= \frac{\left(\sqrt[4]{2}+1\right)\left(\sqrt[4]{2^3}+\sqrt[4]{2^2}+\sqrt[4]{2}+1\right)}{2-1} =$

$= \sqrt[4]{2^4}+\sqrt[4]{2^3}+\sqrt[4]{2^2}+\sqrt[4]{2}+\sqrt[4]{2^3}+\sqrt[4]{2^2}+\sqrt[4]{2}+1 =$

$= 2+2\sqrt[4]{3^3}+2\sqrt{2}+2\sqrt[4]{2}+1 =$

$= 3+2\sqrt{2}+2\sqrt[4]{8}+2\sqrt[4]{2}$

13.5. $\frac{6}{\sqrt[4]{3}+\sqrt{3}} = \frac{6}{\sqrt[4]{3}+\sqrt[4]{3^2}} =$

$A^4-B^4=(A-B)\left(A^3+A^2B+AB^2+B^3\right)$

$A=\sqrt[4]{3}$ e $B=-\sqrt[4]{9}$

$= \frac{6}{\sqrt[4]{3}+\sqrt[4]{9}} =$

$= \frac{6\left[\left(\sqrt[4]{3}\right)^3+\left(\sqrt[4]{3}\right)^2\left(-\sqrt[4]{9}\right)+\sqrt[4]{3}\times\left(-\sqrt[4]{9}\right)^2+\left(-\sqrt[4]{9}\right)^3\right]}{\left(\sqrt[4]{3}\right)^4-\left(-\sqrt[4]{9}\right)^4} =$

$= \frac{6\left(\sqrt[4]{3^3}-\sqrt[4]{3^2}\sqrt[4]{3^2}+\sqrt[4]{3}\sqrt[4]{(3)^2}-\sqrt[4]{\left(3^2\right)^3}\right)}{3-9} =$

$= 3-\sqrt[4]{27}+3\sqrt[4]{9}-3\sqrt[4]{3}$

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14.1. $\left(\sqrt[3]{2}\right)^4-\frac{\sqrt{8}+\sqrt{8}}{\sqrt{\sqrt[3]{2}}} = \sqrt[3]{2^4}-\frac{2\sqrt{2^3}}{\sqrt[6]{2}} =$

$= \sqrt[3]{2^3\times 2}-\frac{2\times\sqrt{2^2\times 2}}{\sqrt[6]{2}} =$

$= 2\sqrt[3]{2}-\frac{2\times 2\sqrt{2}}{\sqrt[6]{2}} =$

$= 2\sqrt[3]{2}-4\frac{\sqrt[6]{2^3}}{\sqrt[6]{2}} =$

$= 2\sqrt[3]{2}-4\sqrt[6]{\frac{2^3}{2}} =$

$= 2\sqrt[3]{2}-4\sqrt[6]{2^2} =$

$= -2\sqrt[3]{2}$

14.2. $\dfrac{\sqrt{108}\times\sqrt[3]{3\sqrt{3}}}{\left(\sqrt[4]{3}\right)^3}=$

$$\begin{array}{r|l}108&2\\54&2\\27&3\\9&3\\3&3\\1&3\end{array}$$

$=\dfrac{\sqrt{2^2\times3\times3}\times\sqrt[4]{\sqrt{3^2\times3}}}{\sqrt[4]{3^3}}=$

$=\dfrac{2\times3\sqrt{3}\times\sqrt[8]{3^3}}{\sqrt[4]{3^3}}=$

$=\dfrac{6\times\sqrt[8]{3^4}\times\sqrt[8]{3^3}}{\sqrt[8]{3^6}}=$

$=6\sqrt[8]{\dfrac{3^4\times3^3}{3^6}}=6\sqrt[8]{3}$

14.3. $\dfrac{\sqrt[3]{24}-\sqrt[3]{81}}{\sqrt{32}\times\sqrt{2\sqrt[3]{9}}}=\dfrac{\sqrt[3]{8\times3}}{\sqrt{2^5}\times\sqrt{\sqrt[3]{2^3\times9}}}=$

$=\dfrac{\sqrt[3]{8}\times\sqrt[3]{3}-\sqrt[3]{3^3}\sqrt[3]{3}}{\sqrt{2^4}\times\sqrt{2}\times\sqrt[6]{2^3\times9}}=$

$=\dfrac{2\sqrt[3]{3}-3\sqrt[3]{3}}{4\sqrt{2}\sqrt[6]{2^3\times9}}=\dfrac{-\sqrt[3]{3}}{4\sqrt[6]{2^3}\times\sqrt[6]{2^3\times9}}=$

$=\dfrac{-\sqrt[3]{3}}{4\sqrt[6]{2^3\times2^3\times3^2}}=\dfrac{-\sqrt[3]{3}}{4\sqrt[6]{2^6}\times\sqrt[6]{3^2}}=$

$=\dfrac{-\sqrt[3]{3}}{4\times2\times\sqrt[3]{3}}=-\dfrac{1}{8}$

14.4. $\dfrac{\sqrt{\sqrt{3}+\sqrt[4]{144}}}{\sqrt[8]{3^2}}\times\dfrac{\sqrt{3}}{\sqrt[3]{3}}=$

$$\begin{array}{r|l}144&2\\72&2\\36&2\\18&2\\9&3\\3&3\\1&\end{array}$$

$=\dfrac{\sqrt{\sqrt{3}+\sqrt[4]{2^4\times3^2}}}{\sqrt[4]{3}}\times\dfrac{\sqrt[6]{3^3}}{\sqrt[6]{3^2}}=$

$=\dfrac{\sqrt{\sqrt{3}+2\sqrt[4]{3^2}}}{\sqrt[4]{3}}\times\sqrt[6]{\dfrac{3^3}{3^2}}=$

$=\dfrac{\sqrt{3\sqrt{3}+2\sqrt{3}}}{\sqrt[4]{3}}\times\sqrt[6]{3}=$

$=\dfrac{\sqrt{3\sqrt{3}}}{\sqrt[4]{3}}\times\sqrt[6]{3}=\dfrac{\sqrt{\sqrt{3^2\times3}}}{\sqrt[4]{3}}\times\sqrt[6]{3}=$

$=\dfrac{\sqrt[4]{3^3}}{\sqrt[4]{3}}\times\sqrt[6]{3}=\sqrt[4]{\dfrac{3^3}{3}}\times\sqrt[6]{3}=$

$=\sqrt[4]{3^2}\times\sqrt[6]{3}=\sqrt{3}\times\sqrt[6]{3}=$

$=\sqrt[6]{3^3}\times\sqrt[6]{3}=\sqrt[6]{3^4}=\sqrt[3]{3^2}$

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15.1. $3+2\sqrt{2}=3+2\times1\sqrt{2}=$

$1^2+\left(\sqrt{2}\right)^2=3$

$=\left(\sqrt{2}\right)^2+2\sqrt{2}+1^2=$

$=\left(\sqrt{2}+1\right)^2$

$\sqrt{3+2\sqrt{2}}=\sqrt{\left(1+\sqrt{2}\right)^2}=\sqrt{2}+1=1+\sqrt{2}$

15.2. $3-2\sqrt{2}=3-2\times1\sqrt{2}=$

$1^2+\left(\sqrt{2}\right)^2=3$

$=\left(\sqrt{2}\right)^2-2\times1\times\sqrt{2}+1^2=$

$=\left(\sqrt{2}-1\right)^2$

$3-2\sqrt{2}>0$

$\sqrt{2}-1>0$

$\sqrt{3-2\sqrt{2}}=\sqrt{\left(\sqrt{2}-1\right)^2}=\sqrt{2}-1$

15.3. $37-20\sqrt{3}=37-2\times10\sqrt{3}=$

$ab=10\sqrt{3}$

$10^2+\left(\sqrt{3}\right)^2=10^3$

$5^2+\left(2\sqrt{3}\right)^3=25+12=37$

$=5^2-2\times10\sqrt{3}+\left(2\sqrt{3}\right)^2=$

$=\left(5-2\sqrt{3}\right)^2$

$\sqrt{37-20\sqrt{3}}=\sqrt{\left(5-2\sqrt{3}\right)^2}=5-2\sqrt{3}$

15.4. $21-8\sqrt{5}=21-2\times4\sqrt{5}=$

$ab=4\sqrt{5}$

$4^2+\left(\sqrt{5}\right)^2=16+5=21$

$=4^2-2\times4\sqrt{5}+\left(\sqrt{5}\right)^2=$

$=\left(4-\sqrt{5}\right)^2$

$\sqrt{21-8\sqrt{5}}=\sqrt{\left(4-\sqrt{5}\right)^2}=4-\sqrt{5}$

15.5. $\dfrac{9}{4}-\sqrt{2}=\dfrac{9}{4}-2\times\dfrac{1}{2}\sqrt{2}=$

$ab=\dfrac{1}{2}\sqrt{2}$

$\left(\dfrac{1}{2}\right)^2+\left(\sqrt{2}\right)^2=\dfrac{1}{4}+2=\dfrac{9}{4}$

$=\left(\sqrt{2}\right)^2-2\times\dfrac{1}{2}\sqrt{2}+\left(\dfrac{1}{2}\right)^2=$

$=\left(2-\dfrac{1}{2}\right)^2$

$\sqrt{\dfrac{9}{4}-\sqrt{2}}=\sqrt{\left(\sqrt{2}-\dfrac{1}{2}\right)}=\sqrt{2}-\dfrac{1}{2}$

15.6. $5+2\sqrt{6}=5+2\times1\times\sqrt{6}=$

$ab=1\times\sqrt{6}=\sqrt{2}\times\sqrt{3}$

$\left(\sqrt{6}\right)^2+1^2=7$

$\left(\sqrt{3}\right)^2+\left(\sqrt{2}\right)^2=5$

$=\left(\sqrt{3}\right)^2+2\times1\times\sqrt{6}+\left(\sqrt{2}\right)^2=$

$=\left(\sqrt{3}+\sqrt{2}\right)^2$

$\sqrt{5+2\sqrt{6}}=\sqrt{\left(\sqrt{3}+\sqrt{2}\right)^2}=\sqrt{3}+\sqrt{2}$

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16.1. $51+14\sqrt{2}=51+2\times7\sqrt{2}=$

$ab\sqrt{c}=7\sqrt{2}$

$7^2+\left(\sqrt{2}\right)^2=51$

$=7^2+2\times7\sqrt{2}+\left(\sqrt{2}\right)^2=$

$=\left(7+\sqrt{2}\right)^2$

$\sqrt{51+14\sqrt{2}}=\sqrt{\left(7+\sqrt{2}\right)^2}=7+\sqrt{2}$

16.2. $22-8\sqrt{6}=22-2\times4\sqrt{6}=$

$ab\sqrt{c}=4\sqrt{6}$

$4^2+\left(\sqrt{6}\right)^2=22$

$=4^2-2\times4\sqrt{6}+\left(\sqrt{6}\right)^2=$

$=\left(4-\sqrt{6}\right)^2$

$4-\sqrt{6}>0$

$\sqrt{22-8\sqrt{6}}=\sqrt{\left(4-\sqrt{6}\right)^2}=4-\sqrt{6}$

16.3. $33+20\sqrt{2}=33+2\times10\sqrt{2}=$

$ab\sqrt{c}=10\sqrt{2}$

$=5\times2\sqrt{2}$

$10^2+\left(\sqrt{2}\right)^2=10^2$

$5^2+\left(2\sqrt{2}\right)^2=33$

$=5^2+2\times10\sqrt{2}+\left(2\sqrt{2}\right)^2=$

$=\left(5+2\sqrt{2}\right)^2$

$\sqrt{33+20\sqrt{2}}=\sqrt{\left(5+2\sqrt{2}\right)^2}=5+2\sqrt{2}$

16.4. $\sqrt{\dfrac{9+\sqrt{32}}{4}}=\dfrac{\sqrt{9+\sqrt{32}}}{2}=$

$9+\sqrt{32}=9+\sqrt{16\times2}=9+4\sqrt{2}=$

$= 9 + 2 \times 2\sqrt{2} =$

$= \left(2\sqrt{2}\right)^2 + 2 \times 2\sqrt{2} + 1^2 =$

$= \left(2\sqrt{2} + 1\right)^2$

$ab\sqrt{c} = 2\sqrt{2}$
$2^2 + \left(\sqrt{2}\right)^2 = 6 \neq 9$
$1 + \left(2\sqrt{2}\right)^2 = 9$

$$\sqrt{\frac{9+\sqrt{32}}{4}} = \frac{\sqrt{9+\sqrt{32}}}{2} =$$

$$= \frac{\sqrt{\left(2\sqrt{2}+1\right)^2}}{4} = \frac{2\sqrt{2}+1}{2} = \sqrt{2} + \frac{1}{2}$$

17.1. $7 - 2\sqrt{6} = 7 - 2 \times 1\sqrt{6} = (a+b)^2$

$= \left(\sqrt{6}^2 - 2\sqrt{6} + 1^2\right) =$

$= \left(\sqrt{6} - 1\right)^2$

$ab = 1\sqrt{6}$
$\left(\sqrt{6}\right)^2 + 1^2 = 7$

$15 - 6\sqrt{6} = 15 - 2 \times 3\sqrt{6} = (a+b)^2$

$= 3^2 - 2 \times 3\sqrt{6} + \left(\sqrt{6}\right)^2 =$

$= \left(3 - \sqrt{6}\right)^2$

$ab = 3\sqrt{6}$
$3^2 + \left(\sqrt{6}\right)^2 = 15$

$\sqrt{7 - 2\sqrt{6}} - \sqrt{15 - 6\sqrt{6}} =$

$= \sqrt{\left(\sqrt{6}-1\right)^2} - \sqrt{\left(3-\sqrt{6}\right)^2} =$

$= \sqrt{6} - 1 - \left(3 - \sqrt{6}\right) =$

$= 2\sqrt{6} - 4$

$\sqrt{6} - 1 > 0$
$3 - \sqrt{6} > 0$

$2\sqrt{6} - 4$ é um número irracional.

17.2. $9 + 4\sqrt{5} = 9 + 2 \times 2\sqrt{5} = (a+b)^2$

$= \left(\sqrt{5}\right)^2 + 2 \times 2\sqrt{5} + 2^2 =$

$= \left(\sqrt{5} + 2\right)^2$

$ab = 2\sqrt{5}$
$2^2 + \left(\sqrt{5}\right)^2 = 9$

$9 - 4\sqrt{5} = \left(\sqrt{5}\right)^2 - 2 \times 2\sqrt{5} + 2^2 =$

$= \left(\sqrt{5} - 2\right)^2$

$\sqrt{9 + 4\sqrt{5}} - \sqrt{9 - 4\sqrt{5}} =$

$= \sqrt{\left(\sqrt{5}+2\right)^2} \sqrt{\left(\sqrt{5}-2\right)^2} =$

$\sqrt{5} - 2 > 0$

$= \sqrt{5} + 2 - \left(\sqrt{5} - 2\right) = 4$

4 é um número natural.

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18.1. $\sqrt{5}x + \sqrt{2}x - 3 = 0 \Leftrightarrow$

$\Leftrightarrow x\left(\sqrt{5} + \sqrt{2}\right) = 3 \Leftrightarrow$

$$\Leftrightarrow x = \frac{3}{\sqrt{5}+\sqrt{2}} \Leftrightarrow$$

$$\Leftrightarrow x = \frac{3\left(\sqrt{5}-\sqrt{2}\right)}{\left(\sqrt{5}+\sqrt{2}\right)\left(\sqrt{5}-\sqrt{2}\right)} \Leftrightarrow$$

$$\Leftrightarrow x = \frac{3\left(\sqrt{5}-\sqrt{2}\right)}{5-2} \Leftrightarrow$$

$\Leftrightarrow x = \sqrt{5} - \sqrt{2}$

$S = \left\{\sqrt{5} - \sqrt{2}\right\}$

18.2. $\sqrt{2} = \sqrt[3]{2}x \Leftrightarrow$

$$\Leftrightarrow x = \frac{\sqrt{2}}{\sqrt[3]{2}} \Leftrightarrow$$

$$\Leftrightarrow x = \frac{\sqrt[6]{2^3}}{\sqrt[6]{2^2}} \Leftrightarrow$$

$$\Leftrightarrow x = \sqrt[6]{\frac{2^3}{2^2}} \Leftrightarrow x = \sqrt[6]{2}$$

$S = \left\{\sqrt[6]{2}\right\}$

18.3. $\left(2 - \sqrt{2}\right)x = \sqrt{2} \Leftrightarrow x = \dfrac{\sqrt{2}}{2-\sqrt{2}} \Leftrightarrow$

$$\Leftrightarrow x = \frac{\sqrt{2}\left(2+\sqrt{2}\right)}{\left(2-\sqrt{2}\right)\left(2+\sqrt{2}\right)} \Leftrightarrow x = \frac{2\sqrt{2}+2}{4-2} \Leftrightarrow$$

$\Leftrightarrow x = \sqrt{2} + 1$

$S = \left\{\sqrt{2} + 1\right\}$

18.4. $x^2 - 2\sqrt{2}x - 1 = 0 \Leftrightarrow$

$$\Leftrightarrow x = \frac{2\sqrt{2} \pm \sqrt{\left(2\sqrt{2}\right)^2 + 4}}{2} \Leftrightarrow$$

$$\Leftrightarrow x = \frac{2\sqrt{2} \pm \sqrt{12}}{2} \Leftrightarrow$$

$$\Leftrightarrow x = \frac{2\sqrt{2} \pm \sqrt{4 \times 3}}{2} \Leftrightarrow$$

$$\Leftrightarrow x = \frac{2\sqrt{2} \pm 2\sqrt{3}}{2} \Leftrightarrow$$

$\Leftrightarrow x = \sqrt{2} + \sqrt{3} \vee x = \sqrt{2} - \sqrt{3}$

$S = \left\{\sqrt{2} + \sqrt{3}\ ,\ \sqrt{2} - \sqrt{3}\right\}$

18.5. $\sqrt[3]{2}x - x - 1 = 0 \Leftrightarrow$

$\Leftrightarrow x\left(\sqrt[3]{2} - 1\right) = 1 \Leftrightarrow$

$$\Leftrightarrow x = \frac{1}{\sqrt[3]{2} - 1} \Leftrightarrow$$

$$\Leftrightarrow x = \frac{\left(\sqrt[3]{2}\right)^2 + \sqrt[3]{2} + 1}{\left(\sqrt[3]{2}\right) - 1} \Leftrightarrow$$

$A^3 - B^3 = (A - B)\left(A^2 + AB + B^2\right)$
$A = \sqrt[3]{2}\ ,\ B = 1$

$$\Leftrightarrow x = \frac{\sqrt[3]{2^2} + \sqrt[3]{2} + 1}{2 - 1} \Leftrightarrow$$

$\Leftrightarrow x = \sqrt[3]{4} + \sqrt[3]{2} + 1$

$S = \left\{\sqrt[3]{4} + \sqrt[3]{2} + 1\right\}$

19. Área de cada face do icosaedro:

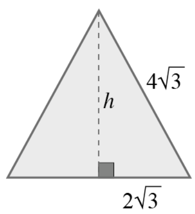
$h^2 + \left(2\sqrt{3}\right)^2 = \left(4\sqrt{3}\right)^2 \Leftrightarrow$

$\Leftrightarrow h^2 = 48 - 12 \Leftrightarrow$

$\Leftrightarrow h^2 = 48 - 12 \Leftrightarrow$

$\Leftrightarrow h^2 = 36 \quad (h > 0)$

$\Leftrightarrow h = 6$



Área de cada face: $\dfrac{4\sqrt{3} \times 6}{2} = 12\sqrt{3}$

cm²

Área lateral:

$\left(20 \times 12\sqrt{3}\right)$ cm² = $240\sqrt{3}$ cm²

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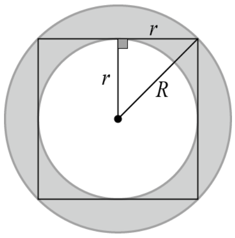
20. Área da coroa circular: A_C

$A_C = \pi R^2 - \pi r^2$



Pelo Teorema de Pitágoras:

$r^2 + r^2 = R^2 \Leftrightarrow R^2 = 2r^2$

Então:

$A_C = \pi \times 2r^2 - \pi r^2 = \pi r^2$

Como $A_C = \pi a$, vem:

$\pi r^2 = \pi a \Leftrightarrow r^2 = a \Leftrightarrow r = \sqrt{a}$

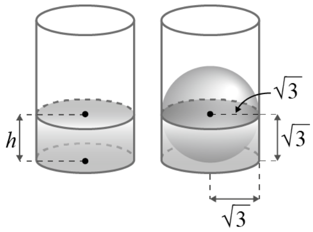
Como o lado do quadrado é igual a $2r$, o perímetro é:

$P = 4 \times 2r = 8\sqrt{a}$

Logo, $P = 8\sqrt{a}$ cm.

21. $V_{\text{esfera}} = \dfrac{4}{3}\pi\left(\sqrt{3}\right)^2 \times \sqrt{3}$

$= 4\pi\sqrt{3}$



$V_{\text{semiesfera}} = 2\pi\sqrt{3}$

Volume do cilindro de altura $\sqrt{3}$:

$V_{\text{clinidro}} = \quad = \pi\left(\sqrt{3}\right)^2 \times \sqrt{3} = 3\pi\sqrt{3}$

$V_{\text{água}} = 3\pi\sqrt{3} - 2\pi\sqrt{3} = \pi\sqrt{3}$

$3\pi h = \pi\sqrt{3} \Leftrightarrow h = \dfrac{\sqrt{3}}{3}$

$h = \dfrac{\sqrt{3}}{3}$ cm

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Atividades complementares

22.1. $-\dfrac{1}{2} < -\dfrac{1}{3} \Leftrightarrow \left(-\dfrac{1}{2}\right)^3 < \left(-\dfrac{1}{3}\right)^3$

22.2. $-\dfrac{1}{4} > -\dfrac{1}{2} \Leftrightarrow \left(-\dfrac{1}{4}\right)^4 < \left(-\dfrac{1}{2}\right)^4$

22.3. $-\dfrac{\sqrt{3}}{3} > -2 \Leftrightarrow \left(-\dfrac{\sqrt{3}}{3}\right)^2 < (-2)^2$

22.4. $-\dfrac{1}{3} > -1 \Leftrightarrow \left(-\dfrac{1}{3}\right)^2 < (-1)^2$

23.1. $-\sqrt{5} < -\sqrt{3} \Leftrightarrow \left(-\sqrt{5}\right)^5 < \left(-\sqrt{3}\right)^5$

23.2. $\dfrac{-\sqrt{2}}{2} < \dfrac{-\sqrt{3}}{3} \Leftrightarrow \left(-\dfrac{\sqrt{2}}{2}\right)^6 > \left(\dfrac{-\sqrt{3}}{3}\right)^6$

23.3. $\dfrac{1}{3} > \dfrac{1}{4} \Leftrightarrow \left(\dfrac{1}{3}\right)^8 > \left(\dfrac{1}{4}\right)^8$

23.4. $\dfrac{-7}{8} > \dfrac{-8}{7} \Leftrightarrow \left(-\dfrac{7}{8}\right)^7 > \left(-\dfrac{8}{7}\right)^7$

24.1. $x^3 = 8 \Leftrightarrow x = \sqrt[3]{8} \Leftrightarrow x = 2$

$S = \{2\}$

24.2. $x^3 = -27 \Leftrightarrow x = \sqrt[3]{-27} \Leftrightarrow x = -3$

$S = \{-3\}$

24.3. $x^4 = -3$ é impossível

$S = \{\ \}$

24.4. $x^4 = 3 \Leftrightarrow x = -\sqrt[4]{3} \vee x = \sqrt[4]{3}$

$S = \left\{-\sqrt[4]{3}\ ,\ \sqrt[4]{3}\right\}$

24.5. $x^4 = 10\,000 \Leftrightarrow x^4 = 10^4 \Leftrightarrow x = \sqrt[4]{10^4} \vee x = \sqrt[4]{10^4} \Leftrightarrow$

$\Leftrightarrow x = -10 \vee x = 10$

$S = \{-10\ ,\ 10\}$

24.6. $x^6 = -4$, é impossível

$S = \{\ \}$

24.7. $x^7 = -1 \Leftrightarrow x = \sqrt[7]{-1} \Leftrightarrow x = -1$

$S = \{-1\}$

24.8. $x^5 = -32 \Leftrightarrow x = \sqrt[5]{-32} \Leftrightarrow x = -2$ $\quad (2^5 = 32)$

$S = \{-2\}$

24.9. $x^8 = 0 \Leftrightarrow x = 0$

$S = \{0\}$

24.10. $x^3 = -\dfrac{1}{64} \Leftrightarrow x = \sqrt[3]{\dfrac{1}{64}} \Leftrightarrow x = -\dfrac{1}{4}$ $\quad (4^3 = 64)$

$S = \left\{-\dfrac{1}{4}\right\}$

25.1. $\sqrt[3]{\dfrac{1}{27}} = \dfrac{1}{3}$

25.2. $\sqrt[4]{7^4} = 7$

25.3. $\sqrt[8]{|-1|} = \sqrt[8]{1} = 1$

25.4. $\sqrt[7]{-1} = -1$

25.5. $\sqrt[7]{2^7} = 2$

25.6. $\sqrt[6]{2^{12}} = \sqrt[6]{\left(2^2\right)^6} = 4$

25.7. $\sqrt[8]{(-8)^8} = \sqrt[8]{8^8} = 8$

25.8. $\sqrt[7]{(-5)^7} = -5$

25.9. $\sqrt[5]{\dfrac{1}{32}} = \dfrac{1}{2}$ $\quad (2^5 = 32)$

25.10. $\sqrt[15]{0} = 0$

25.11. $\sqrt[4]{0,0001} = \sqrt[4]{(0,1)^4} = 0,1$

25.12. $\sqrt[3]{0,000\,000\,008} = \sqrt[3]{(0,002)^3} = 0,002$

26.1. $\sqrt{3} = \sqrt[2\times 2]{3^2} = \sqrt[4]{9}$

26.2. $\sqrt[6]{7^2} = \sqrt[6:2]{7^{2:2}} = \sqrt[3]{7}$

26.3. $\sqrt{3a} = \sqrt[2\times 2]{(3a)^2} = \sqrt[4]{9a^2}$, $a > 0$

26.4. $\sqrt[5]{a^2 b} = \sqrt[2\times 5]{\left(a^2 b\right)^2} = \sqrt[10]{a^4 b^2}$

26.5. $\sqrt[15]{a^5 b^{10}} = \sqrt[15:5]{a^1 b^2} = \sqrt[3]{ab^2}$

26.6. $\sqrt{ab^3} = \sqrt[6]{a^3 b^9}$, $a > 0$ e $b > 0$

27. Por exemplo:

27.1. $\sqrt{2} = \sqrt[4]{2^2} = \sqrt[6]{2^3} = \sqrt[8]{2^4}$

27.2. $\sqrt[3]{5} = \sqrt[6]{5^2} = \sqrt[9]{5^3} = \sqrt[12]{5^4}$

27.3. $\sqrt[9]{3^3} = \sqrt[3]{3} = \sqrt[6]{3^2} = \sqrt[12]{3^4}$

27.4. $\sqrt[3]{2x^2} = \sqrt[6]{2^2 x^4} = \sqrt[9]{2^3 x^6} = \sqrt[12]{2^4 x^8}$, $x > 0$

27.5. $\sqrt[4]{2x^2} = \sqrt[8]{2^2 x^4} = \sqrt[12]{2^3 x^6} = \sqrt[16]{2^4 x^8}$, $x > 0$

27.6. $\sqrt[5]{\dfrac{xy^2}{z^3}} = \sqrt[10]{\dfrac{x^2 y^4}{z^6}} = \sqrt[15]{\dfrac{x^3 y^6}{z^9}} = \sqrt[20]{\dfrac{x^4 y^8}{z^{12}}}$; $x > 0$, $y > 0$ e $z > 0$

28.1. m.m.c. (3 , 2) = 6

$\sqrt[3]{5}=\sqrt[6]{5^2}=\sqrt[6]{25}$

$\sqrt{2}=\sqrt[6]{2^3}=\sqrt[6]{8}$

28.2. m.m.c. (4 , 8) = 8

$\sqrt[4]{12}=\sqrt[8]{144}=\sqrt[8]{144}$

$\sqrt[8]{144}=\sqrt[8]{5}$

28.3. m.m.c. (3 , 4) = 12

$\sqrt[3]{7}=\sqrt[12]{7^4}=\sqrt[12]{2401}$

$\sqrt[4]{2}=\sqrt[12]{2^3}=\sqrt[12]{8}$

28.4. m.m.c. (2 , 4) = 4

$\sqrt{2a^2}=\sqrt[4]{\left(2a^2\right)^2}=\sqrt[4]{4a^4}$, $a>0$

$\sqrt[4]{\frac{1}{2}b}$, $b>0$

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29.1. m.m.c. (3 , 5 , 15) = 15

$\sqrt[3]{2}=\sqrt[15]{2^5}=\sqrt[15]{32}$

$\sqrt[5]{2^3}=\sqrt[15]{2^9}=\sqrt[15]{512}$

$\sqrt[15]{7^2}=\sqrt[15]{49}$

$\sqrt[15]{32}<\sqrt[15]{49}<\sqrt[15]{512}$

Logo, $\sqrt[3]{2}<\sqrt[15]{7^2}<\sqrt[5]{2^3}$.

29.2. m.m.c. (3 , 6 , 9) = 18

$\sqrt[3]{2}=\sqrt[18]{2^6}=\sqrt[18]{64}$

$\sqrt[6]{3^2}=\sqrt[18]{3^6}=\sqrt[18]{729}$

$\sqrt[9]{5^2}=\sqrt[18]{5^4}=\sqrt[18]{625}$

$\sqrt[18]{64}<\sqrt[18]{625}<\sqrt[18]{729}$

Logo, $\sqrt[3]{2}<\sqrt[9]{5^2}<\sqrt[6]{3^2}$.

29.3. m.m.c (4 , 6 , 8) = 24

$\sqrt[4]{2}=\sqrt[24]{2^6}=\sqrt[24]{64}$

$\sqrt[6]{2^2}=\sqrt[24]{2^8}=\sqrt[24]{256}$

$\sqrt[8]{5}=\sqrt[24]{5^3}=\sqrt[24]{125}$

$\sqrt[24]{64}<\sqrt[24]{125}<\sqrt[24]{256}$

Logo, $\sqrt[4]{2}<\sqrt[8]{5}<\sqrt[6]{2^2}$.

29.4. m.m.c. (2 , 3 , 4) = 12

$\sqrt{3}=\sqrt[12]{3^6}=\sqrt[12]{729}$

$\sqrt[3]{3^2}=\sqrt[12]{3^8}=\sqrt[12]{6561}$

$\sqrt[4]{2^2}=\sqrt[12]{2^6}=\sqrt[12]{64}$

$\sqrt[12]{64}<\sqrt[12]{729}<\sqrt[12]{6561}$

Logo, $\sqrt[4]{2^2}<\sqrt{3}<\sqrt[3]{3^2}$.

30.1. $\sqrt{2}\times\sqrt{32}=\sqrt{64}=8$

30.2. $\sqrt{3}\times\sqrt{4}=2\sqrt{3}$

30.3. $\sqrt{2}\times\sqrt{8}=\sqrt{16}=4$

30.4. $\sqrt[3]{2}\times\sqrt[3]{5}=\sqrt[3]{10}$

30.5. $\sqrt[4]{2}\times\sqrt[4]{8}=\sqrt[4]{16}=2$

30.6. $\sqrt{2}\times\sqrt[3]{32}=\sqrt[6]{2^3}\times\sqrt[6]{\left(2^5\right)^2}=$

$=\sqrt[6]{2^3\times 2^{10}}=\sqrt[6]{2^{12}\times 2}=$

$=\sqrt[6]{2^{12}}\times\sqrt[6]{2}=4\sqrt[6]{2}$

30.7. $\sqrt[3]{2}\times\sqrt[4]{8}=\sqrt[12]{2^4}\times\sqrt[12]{\left(2^3\right)^3}=$

$=\sqrt[12]{2^4\times 2^9}=\sqrt[12]{2^{12}\times 2}=2\sqrt[12]{2}$

30.8. $\sqrt[4]{x}\times\sqrt[8]{x^6}=\sqrt[4]{x}\times\sqrt[4]{x^3}=$

$=\sqrt[4]{x\times x^3}=\sqrt[4]{x^4}=x,\ x>0$

30.9. $2\sqrt[3]{5}\times 3\sqrt{2}=6\times\sqrt[6]{5^2}\times\sqrt[6]{2^3}=6\sqrt[6]{5^2\times 2^3}=$

$=6\sqrt[6]{25\times 8}=6\sqrt[6]{200}$

30.10. $\sqrt[4]{x}\times\sqrt[5]{x}=\sqrt[4]{x}\times\sqrt[4]{x^{10}}=\sqrt[4]{x\times x^{10}}=$

$=\sqrt[4]{x^{11}}=\sqrt[4]{x^8\times x^3}=x^2\sqrt[4]{x^3}$, $x>0$

30.11. $\sqrt[6]{2^5}\times\sqrt[4]{2^3}=\sqrt[12]{2^{10}}\times\sqrt[12]{2^9}=\sqrt[12]{2^{19}}=$

$=\sqrt[12]{2^{12}\times 2^7}=2\sqrt[12]{128}$

30.12. $\sqrt[5]{128}\times\frac{1}{4}\sqrt[3]{16}=\sqrt[5]{2^7}\times\frac{1}{4}\times\sqrt[3]{2^4}=$

$=2\times\sqrt[5]{2^2}\times\frac{1}{4}\times 2\sqrt[3]{2}=$

$=2\times\sqrt[5]{2^2}\times\frac{1}{4}\times 2\sqrt[3]{2}=$

$=\sqrt[15]{2^6}\times\sqrt[15]{2^5}=\sqrt[15]{2^{11}}=$

$=\sqrt[15]{2048}$

31.1. $\left(2-\sqrt{3}\right)\left(2+\sqrt{3}\right)=2^2-\left(\sqrt{3}\right)^2=4-3=1$

31.2. $\left(\sqrt{2}+\sqrt{3}\right)^2=2+2\sqrt{2}\sqrt{3}+3=5+2\sqrt{6}$

31.3. $\left(2\sqrt{2}-5\sqrt{3}\right)^2=8-2\times 2\sqrt{2}\times 5\sqrt{3}+75=$

$=83-20\sqrt{6}$

31.4. $\left(2\sqrt{3}-\sqrt{2}\right)\left(\sqrt{3}-\sqrt{2}\right)=$

$=2\sqrt{3}\sqrt{3}-2\sqrt{3}\sqrt{2}-\sqrt{2}\sqrt{3}+\sqrt{2}\sqrt{2}=$

$=2\times 3-2\sqrt{6}-\sqrt{6}+2=$

$=8-3\sqrt{6}$

31.5. $\sqrt[4]{8}\left(\sqrt[4]{2}-\sqrt{2}\right)=\sqrt[4]{8}\left(\sqrt[4]{2}-\sqrt[4]{4}\right)=$

$=\sqrt[4]{16}-\sqrt[4]{32}=$

$=\sqrt[4]{2^4}-\sqrt[4]{2^5}=$

$=2-\sqrt[4]{2^4\times 2}=$

$=2-2\sqrt[4]{2}$

31.6. $\left(2\sqrt[3]{81}-3\sqrt[3]{3}\right)^2=$

$=\left(2\sqrt[3]{3^4}-3\sqrt[3]{3}\right)^2=$

$=\left(2\times 3\times\sqrt[3]{3}-3\sqrt[3]{3}\right)^2=$

$=\left(3\sqrt[3]{3}\right)^2=9\left(\sqrt[3]{3}\right)^2=$

$=9\sqrt[3]{9}$

32.1. $\sqrt{63}-5\sqrt{28}+\sqrt{112}-7\sqrt{252}+2\sqrt{448}=$

$=\sqrt{9\times 7}-5\sqrt{4\times 7}+\sqrt{16\times 7}-4\sqrt{36\times 7}+2\sqrt{64\times 7}=$

$=3\sqrt{7}-5\times 2\sqrt{7}-4\sqrt{7}-7\times 6\sqrt{7}+2\times 8\sqrt{7}=$

$=\left(3-10+4-42+16\right)\sqrt{7}=$

$=-29\sqrt{7}$

32.2. $5\sqrt{12}+7\sqrt{48}-2\sqrt{108}-\frac{1}{2}\sqrt{192}=$

$=5\sqrt{4\times 3}+7\sqrt{16\times 3}-2\sqrt{36\times 3}\times\frac{1}{2}\sqrt{64\times 3}=$

$=5\times 2\sqrt{3}+7\times 4\sqrt{3}-2\times 6\sqrt{3}-\frac{1}{2}\times 8\sqrt{3}=$

$=\left(10+28-12-4\right)\sqrt{3}\ =22\sqrt{3}$

32.3. $3\sqrt{125}+\frac{1}{2}\sqrt{80}+\frac{1}{3}\sqrt{245}-\frac{1}{3}\sqrt{20}=$

$=3\sqrt{25\times 5}+\frac{1}{2}\sqrt{16\times 5}-\frac{1}{3}\sqrt{49\times 5}-\frac{1}{3}\sqrt{4\times 5}=$

$=3\times 5\sqrt{5}+\frac{1}{2}\times 4\sqrt{5}-\frac{1}{3}\times 7\sqrt{5}-\frac{1}{3}\times 2\sqrt{5}=$

$=\left(15+2-\frac{7}{3}-\frac{2}{3}\right)\sqrt{5}=$

$=14\sqrt{5}$

33.1. $\left(\sqrt{3}\right)^4=\left[\left(\sqrt{3}\right)^2\right]^2=3^2=9$

33.2. $\left(\sqrt[3]{2}\right)^5=\left(\sqrt[3]{2}\right)^3\times\left(\sqrt[3]{2}\right)^2=2\sqrt[3]{4}$

33.3. $\left(\frac{1}{2}\sqrt[3]{2}\right)^6=\left(\frac{1}{2}\right)^6\times\left[\left(\sqrt[3]{2}\right)^3\right]^2=\frac{1}{2^6}\times 2^2=\frac{1}{16}$

33.4. $\left(\sqrt[3]{2^4ab^5}\right)^2=\sqrt[3]{\left(2^4ab\times 5\right)^2}=\sqrt[3]{2^8a^2b^{10}}=$

$=\sqrt[3]{2^6\times b^9\times 2^2a^2b}=\sqrt[3]{b^9}\times\sqrt[3]{4a^2b}=$

$=4b^3\sqrt[3]{4a^2b}$

34. $2x^2+2xy+y^2$

34.1. Para $x=\sqrt{2}$ e $y=2\sqrt{5}$:

$2\left(\sqrt{2}\right)^2-2\sqrt{2}\times 2\sqrt{5}+\left(2\sqrt{5}\right)^2=$

$=2\times 2-4\sqrt{10}+20=$

$=24-4\sqrt{10}$

34.2. Para $x=\sqrt[6]{8}=\sqrt[6]{2^3}=\sqrt{2}$ e $y=\frac{\sqrt{2}}{2}$:

$2\left(\sqrt{2}\right)^2-2\sqrt{2}\times\frac{\sqrt{2}}{2}+\left(\frac{\sqrt{2}}{2}\right)^2=$

$=4-2+\frac{1}{2}=\frac{5}{2}$

35.1. $\sqrt[5]{4}:\sqrt[5]{2}=\sqrt[5]{4:2}=\sqrt[5]{2}$

35.2. $\sqrt[4]{10}:\sqrt[4]{15}=\sqrt[4]{10:15}=\sqrt[4]{\frac{2}{3}}$

35.3. $\sqrt[3]{5}:\sqrt[6]{3}=\sqrt[6]{5^2}:\sqrt[6]{3}=\sqrt[6]{\frac{25}{3}}$

35.4. $\sqrt[4]{2}:\sqrt[6]{3}=\sqrt[12]{2^3}:\sqrt[12]{3^2}=\sqrt[12]{\frac{8}{9}}$

35.5. $\frac{\sqrt[3]{2}\times\sqrt[3]{8}}{\sqrt[3]{4}}=\sqrt[3]{\frac{2\times 8}{4}}=\sqrt[3]{4}$

35.6. $\sqrt{2}\times\sqrt[3]{3}:\sqrt[4]{4}=$

$=\sqrt[12]{2^6}\times\sqrt[12]{3^4}:\sqrt[12]{4^3}=$

$=\sqrt[12]{2^6\times 3^4:\left(2^2\right)^3}=\sqrt[12]{\frac{2^6\times 3^4}{2^6}}=$

$=\sqrt[12]{3^4}=\sqrt[3]{3}$

35.7. $\frac{\sqrt[4]{16}:\sqrt[6]{3}}{\sqrt{2}:\sqrt[3]{4}}=\frac{\sqrt[6]{2^6}:\sqrt[6]{3}}{\sqrt[6]{2^3}:\sqrt[6]{2^4}}=\sqrt[6]{\frac{2^6:3}{2^3:2^4}}=$

$=\sqrt[6]{\frac{\frac{2^6}{3}}{\frac{2^3}{2^4}}}=\sqrt[6]{\frac{2^6\times 2^4}{3\times 2^3}}=\sqrt[6]{\frac{2^7}{3}}=$

$=\sqrt[6]{2^6\times\frac{2}{3}}=2\times\sqrt[6]{\frac{2}{3}}$

35.8. $\frac{\sqrt[3]{3}\times\sqrt[4]{4}}{\sqrt[3]{6}:\sqrt[6]{1}}=\frac{\sqrt[12]{3^4}\times\sqrt[12]{4^3}}{\sqrt[12]{6^4}:1}=\sqrt[12]{\frac{3^4\times 4^3}{6^4}}=$

$=\sqrt[12]{\frac{3^4}{6^4}\times 4^3}=\sqrt[12]{\left(\frac{3}{6}\right)^4\times\left(2^2\right)^3}=$

$=\sqrt[12]{\left(\frac{1}{2}\right)^4\times 2^6}=\sqrt[12]{\frac{1}{2^4}\times 2^6}=\sqrt[12]{2^2}=\sqrt[6]{2}$

36.1. $\left(\sqrt[5]{\frac{1}{2}}\right)^{-1}=\sqrt[5]{\left(\frac{1}{2}\right)^{-1}}=\sqrt[5]{2}$

36.2. $\left(\sqrt[3]{5}\right)^{-3}=\left[\left(\sqrt[3]{5}\right)^3\right]^{-1}=5^{-1}=\frac{1}{5}$

36.3. $\left(\sqrt[5]{2}\right)^{-1}\times\left(\sqrt[5]{3}\right)^2=\sqrt[5]{2^{-1}}\times\sqrt[5]{3^2}=$

$=\sqrt[5]{\frac{1}{2}\times 3^2}=\sqrt[5]{\frac{9}{2}}$

36.4. $\left(\sqrt[3]{3}\right)^{-3}:\left(\sqrt{3}\right)^{-2}=\sqrt[3]{3^{-3}}:\sqrt{3^{-2}}=$

$=\sqrt[3]{\left(\frac{1}{3}\right)^3}:\sqrt{\left(\frac{1}{3}\right)^2}=$

$=\frac{1}{3}:\frac{1}{3}=1$

37.1. $\sqrt{\sqrt[3]{3}}=\sqrt[6]{3}$

37.2. $\sqrt[5]{\sqrt{32}}=\sqrt[10]{32}=\sqrt[10]{2^5}=\sqrt{2}$

37.3. $\sqrt{5\sqrt{2}}=\sqrt{\sqrt{5^2\times 2}}=\sqrt[4]{50}$

37.4. $\sqrt[4]{2\sqrt[3]{a^4}}=\sqrt[4]{\sqrt[3]{2^3a^4}}=\sqrt[12]{8a^4}$, $a>0$

37.5. $\left(\sqrt{a^5\sqrt{a^3}}\right)^3=\left(\sqrt{\sqrt{\left(a^5\right)^2\times a^3}}\right)^3=\left(\sqrt[4]{a^{13}}\right)^3=$

$=\sqrt[4]{\left(a^{13}\right)^3}=\sqrt[4]{a^{39}}=\sqrt[4]{a^{36}\times a^3}=$

$=a^9\sqrt[4]{a^3}$

37.6. $\sqrt[3]{2\sqrt[3]{2\sqrt{2}}}=\sqrt[3]{\sqrt[3]{2^3\times 2}\times\sqrt{2}}=\sqrt[3]{\sqrt[3]{2^4}\times\sqrt{2}}=$

$=\sqrt[3]{\sqrt[6]{2^8}\times\sqrt[6]{2^3}}=\sqrt[3]{\sqrt[6]{2^8\times 2^3}}=\sqrt[18]{2^{11}}=\sqrt[18]{2048}$

37.7. $\sqrt[3]{2\sqrt{8}}\times\sqrt{3}:\sqrt[3]{2}=$

$=\sqrt[3]{\sqrt{2^2\times 2^3}}\times\sqrt[6]{3^3}:\sqrt[6]{2^2}=$

$=\frac{\sqrt[6]{2^5}\times\sqrt[6]{3^3}}{\sqrt[6]{2^2}}=\sqrt[6]{\frac{2^5\times 3^3}{2^2}}=$

$=\sqrt[6]{2^3\times 3^3}=\sqrt[6]{6^3}=\sqrt{6}$

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38.1. $\frac{1}{\sqrt{3}}=\frac{\sqrt{3}}{\sqrt{3}\times\sqrt{3}}=\frac{\sqrt{3}}{3}$

38.2. $\frac{\sqrt{2}}{\sqrt{6}}=\frac{\sqrt{2}\times\sqrt{6}}{\sqrt{6}\times\sqrt{6}}=\frac{\sqrt{12}}{6}=\frac{\sqrt{4\times 3}}{6}=\frac{2\sqrt{3}}{6}=\frac{\sqrt{3}}{3}$

38.3. $\sqrt{\frac{1}{5}}=\frac{1}{\sqrt{5}}=\frac{\sqrt{5}}{\sqrt{5}\times\sqrt{5}}=\frac{\sqrt{5}}{5}$

38.4. $\sqrt{\frac{5}{3}}=\frac{\sqrt{5}\times\sqrt{3}}{\sqrt{3}\times\sqrt{3}}=\frac{\sqrt{15}}{3}$

38.5. $\frac{1}{2\sqrt{3}}=\frac{1\times\sqrt{3}}{2\sqrt{3}\sqrt{3}}=\frac{\sqrt{3}}{2\times 3}=\frac{\sqrt{3}}{6}$

38.6. $\frac{5}{\sqrt[3]{2}}=\frac{5\times\sqrt[3]{2^2}}{\sqrt[3]{2}\times\sqrt[3]{2^2}}=\frac{5\sqrt[3]{2^2}}{\sqrt[3]{2\times2^2}}=\frac{5\sqrt[3]{2^2}}{\sqrt[3]{2^3}}=\frac{5\sqrt[3]{2^2}}{2}$

38.7. $\sqrt[3]{\frac{1}{5}}=\frac{\sqrt[3]{1}}{\sqrt[3]{5}}=\frac{1\times\sqrt[3]{5^2}}{\sqrt[3]{5}\times\sqrt[3]{5^2}}=\frac{\sqrt[3]{5^2}}{\sqrt[3]{5^3}}=\frac{\sqrt[3]{5^2}}{5}$

38.8. $\sqrt[4]{\frac{1}{2}}=\frac{\sqrt[4]{1}}{\sqrt[4]{2}}=\frac{1\times\sqrt[4]{2^3}}{\sqrt[4]{2}\times\sqrt[4]{2^3}}=\frac{\sqrt[4]{2^3}}{\sqrt[4]{2^4}}=\frac{\sqrt[4]{2^3}}{2}$

38.9. $\frac{3}{2\sqrt[6]{3}}=\frac{3\times\sqrt[6]{3^5}}{2\sqrt[6]{3}\sqrt[6]{3^5}}=\frac{3\sqrt[6]{3^5}}{2\sqrt[6]{3^6}}=\frac{3\sqrt[6]{3^5}}{2\times3}=\frac{\sqrt[6]{3^5}}{2}$

38.10. $\frac{2}{3\sqrt[7]{4}}=\frac{2\times\sqrt[7]{3^3}}{3\sqrt[7]{3^4}\times\sqrt[7]{3^3}}=\frac{2\times\sqrt[7]{3^3}}{3\sqrt[7]{3^7}}=\frac{2\times\sqrt[7]{3^3}}{3\times3}=\frac{2\sqrt[7]{3^3}}{9}$

39.1. $\frac{6}{2-\sqrt{7}}=\frac{6\left(2+\sqrt{7}\right)}{\left(2-\sqrt{7}\right)\left(2+\sqrt{7}\right)}=\frac{6\left(2+\sqrt{7}\right)}{4-7}=$

$=\frac{6\left(2+\sqrt{7}\right)}{-3}=-2\left(2+\sqrt{7}\right)=$

$=-4-2\sqrt{7}$

39.2. $\frac{1}{\sqrt{2}-1}=\frac{\sqrt{2}+1}{\left(\sqrt{2}-1\right)\left(\sqrt{2}+1\right)}=\frac{\sqrt{2}+1}{2-1}=\sqrt{2}+1$

39.3. $\frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}+\sqrt{3}}=\frac{\left(\sqrt{2}-\sqrt{3}\right)\left(\sqrt{2}-\sqrt{3}\right)}{\left(\sqrt{2}+\sqrt{3}\right)\left(\sqrt{2}-\sqrt{3}\right)}=$

$=\frac{2-2\sqrt{2}\sqrt{3}+3}{2-3}=$

$=\frac{5-2\sqrt{6}}{-1}=2\sqrt{6}-5$

39.4. $\frac{30}{3\sqrt{2}-\sqrt{3}}=\frac{30\left(3\sqrt{2}+\sqrt{3}\right)}{\left(3\sqrt{2}-\sqrt{3}\right)\left(3\sqrt{2}+\sqrt{3}\right)}=\frac{30\left(3\sqrt{2}+\sqrt{3}\right)}{18-3}=$

$=\frac{30\left(3\sqrt{2}+\sqrt{3}\right)}{15}=2\left(3\sqrt{2}+\sqrt{3}\right)=$

$=6\sqrt{2}+2\sqrt{3}$

39.5. $\frac{\sqrt{a}}{2\sqrt{a}+\sqrt{b}}=\frac{\sqrt{a}\left(2\sqrt{a}-\sqrt{b}\right)}{\left(2\sqrt{a}+\sqrt{b}\right)\left(2\sqrt{a}-\sqrt{b}\right)}=$

$=\frac{2a-\sqrt{ab}}{4a-b}$, $a>0$ e $b>0$

39.6. $\frac{a}{\sqrt{2+a}+\sqrt{a}}=\frac{a\left(\sqrt{2+a}-\sqrt{a}\right)}{\left(\sqrt{2+a}+\sqrt{a}\right)\left(\sqrt{2+a}-\sqrt{a}\right)}=$

$=\frac{a\left(\sqrt{2+a}-\sqrt{a}\right)}{(2+a)-a}=\frac{a\left(\sqrt{2+a}-\sqrt{a}\right)}{2}$, $a>0$

40.1. $\frac{1}{\sqrt[3]{2}-2}=$

$A^3-B^3=(A-B)\left(A^2+AB+B^2\right)$

$A=\sqrt[3]{2}$ e $B=2=\sqrt[3]{8}$

$=\frac{\left(\sqrt[3]{2}\right)^2+2\sqrt[3]{2}+4}{\left(\sqrt[3]{2}\right)^3-2^3}=$

$=\frac{\sqrt[3]{4}+2\sqrt[3]{2}+4}{2-8}=$

$=-\frac{\sqrt[3]{4}+2\sqrt[3]{2}+4}{6}$

40.2. $\frac{2}{\sqrt[3]{5}+\sqrt[3]{2}}=$

$A^3-B^3=(A-B)\left(A^2+AB+B^2\right)$

$A=\sqrt[3]{5}$ e $B=-\sqrt[3]{2}$

$=\frac{2\left[\left(\sqrt[3]{5}\right)^2+\sqrt[3]{5}\times\left(-\sqrt[3]{2}\right)+\left(-\sqrt[3]{2}\right)^2\right]}{\left(\sqrt[3]{5}\right)^3-\left(-\sqrt[3]{2}\right)^2}=$

$=\frac{2\left(\sqrt[3]{5^2}-\sqrt[3]{10}+\sqrt[3]{4}\right)}{5+2}=$

$=\frac{2}{7}\left(\sqrt[3]{25}-\sqrt[3]{10}+\sqrt[3]{4}\right)$

40.3. $\frac{6}{1+\sqrt[4]{2}}=$

$A^4-B^4=(A-B)\left(A^3+A^2B+AB^2+B^3\right)$

$A=\sqrt[4]{2}$ e $B=-1$

$=\frac{6\left[\left(\sqrt[4]{2}\right)^3+\left(\sqrt[4]{2}\right)^2\times(-1)+\left(\sqrt[4]{2}\right)\times(-1)^2+(-1)^3\right]}{\left(\sqrt{2}\right)^4-(-1)^4}=$

$=\frac{6\left(\sqrt[4]{2^3}-\sqrt[4]{2^2}+\sqrt[4]{2}-1\right)}{2-1}=$

$=6\sqrt[4]{8}+6\sqrt[4]{2}-6\sqrt{2}-6$

40.4. $\frac{1}{\sqrt{2}-\sqrt[4]{3}}=$

$A^4-B^4=(A-B)\left(A^3+A^2B+AB^2+B^3\right)$

$A=\sqrt[4]{4}$ e $B=\sqrt[4]{3}$

$=\frac{1}{\sqrt[4]{4}-\sqrt[4]{3}}=$

$=\frac{\left(\sqrt[4]{4}\right)^3+\left(\sqrt[4]{4}\right)^2\sqrt[4]{3}+\left(\sqrt[4]{4}\right)\left(\sqrt[4]{3}\right)^2+\left(\sqrt[4]{3}\right)^3}{\left(\sqrt[4]{4}\right)^4-\left(\sqrt[4]{3}\right)^4}=$

$=\frac{\sqrt[4]{4^3}+\sqrt[4]{4^2}\sqrt[4]{3}+\sqrt[4]{4}\sqrt[4]{3^2}+\sqrt[4]{3^2}}{4-3}=$

$=\sqrt[4]{64}+\sqrt[4]{48}+\sqrt[4]{36}+\sqrt[4]{27}$

41.1. $\left(3-\sqrt{5}\right)\left(3+\sqrt{5}\right)-\left(2-\sqrt{3}\right)^2=$

$=9-5-\left(4-4\sqrt{3}+3\right)=$

$=4-7+4\sqrt{3}=4\sqrt{3}-3$

41.2. $\left(2-\sqrt{2}\right)^2-\left(\sqrt{6-4\sqrt{2}}\right)^2=$

$=4-4\sqrt{2}+2-\left(6-4\sqrt{2}\right)=$

$=6-4\sqrt{2}-6+4\sqrt{2}=0$

41.3. $\left(3\sqrt{2}-2\sqrt{3}\right)\left(3\sqrt{2}+2\sqrt{3}\right)+\left(1+2\sqrt{2}\right)^2=$

$=\left(3\sqrt{2}\right)^2-\left(2\sqrt{3}\right)^2+1+4\sqrt{2}+8=$

$=18-12+1+4\sqrt{2}+8=15+4\sqrt{2}$

41.4. $\left(1-\frac{1}{2}\sqrt{2}\right)^2-2\left(1-\sqrt{2}\right)\left(1+\frac{\sqrt{2}}{2}\right)^2=$

$=1-\sqrt{2}+\frac{1}{4}\times2-\left(2-2\sqrt{2}\right)\left(1+\sqrt{2}+\frac{2}{4}\right)=$

$=\frac{3}{2}-\sqrt{2}-\left(2-2\sqrt{2}\right)\left(\frac{3}{2}+\sqrt{2}\right)=$

$=\frac{3}{2}-\sqrt{2}-\left(3+2\sqrt{2}-3\sqrt{2}-2\times2\right)=\frac{5}{2}$

41.5. $4\left(-1-\frac{\sqrt{3}}{2}\right)^2-\left(1-\sqrt{3}\right)\left(2+3\sqrt{3}\right)^2=$

$=4\left(1+\sqrt{3}+\frac{3}{4}\right)-\left(1-\sqrt{3}\right)\left(4+12\sqrt{3}+27\right)=$

$=4+4\sqrt{3}+3-\left(1-\sqrt{3}\right)\left(31+12\sqrt{3}\right)=$

$=7+4\sqrt{3}-\left(31+12\sqrt{3}-31\sqrt{3}-36\right)=$

$=7+4\sqrt{3}+5+19\sqrt{3}=$

$=12+23\sqrt{3}$

41.6. $\left(2\sqrt{2-\sqrt{3}}\right)^2\left(-\frac{1}{2}-\frac{\sqrt{3}}{3}\right)^2=$

$=4\left(2-\sqrt{3}\right)\left(\frac{1}{4}+\frac{\sqrt{3}}{3}+\frac{3}{9}\right)=$

$=4\left(2-\sqrt{3}\right)\left(\frac{7}{12}+\frac{\sqrt{3}}{3}\right)=$

$=4\left(\frac{7}{6}+\frac{2\sqrt{3}}{3}-\frac{7\sqrt{3}}{12}-1\right)=$

$=4\left(\frac{1}{6}+\frac{\sqrt{3}}{12}\right)=\frac{2}{3}+\frac{\sqrt{3}}{3}$

41.7. $4\sqrt[6]{\left(x-a\right)^3}:2\sqrt[4]{\left(x-a\right)^2}=$

$=\frac{4\sqrt{x-a}}{2\sqrt{x-a}}=2,\ x-a>0$

41.8. $\sqrt[3]{128}-\sqrt[3]{16}+\sqrt[3]{54}=$

$=\sqrt[3]{64\times2}-\sqrt[3]{8\times2}+\sqrt[3]{27\times2}=$

$=\sqrt[3]{4^3\times2}-\sqrt[3]{2^3\times2}-\sqrt[3]{3^3\times2}=$

$=4\sqrt[3]{2}-2\sqrt[3]{2}+3\sqrt[3]{2}=$

$=5\sqrt[3]{2}$

41.9. $\sqrt[3]{16x^4}+\sqrt[3]{54x^4}-\sqrt[3]{-128x^4}=$

$=\sqrt[3]{8x^3\times2x}+\sqrt[3]{27x^3\times2x}+\sqrt[3]{64x^3\times2x}=$

$=\sqrt[3]{\left(2x\right)^3}\times\sqrt[3]{2x}+\sqrt[3]{\left(3x\right)^3}\times\sqrt[3]{2x}+\sqrt[3]{\left(4x\right)^3}\times\sqrt[3]{2x}=$

$=2x\times\sqrt[3]{2x}+3x\times\sqrt[3]{2x}+4x\times\sqrt[3]{2x}=$

$=\left(2x+3x+4x\right)\sqrt[3]{2x}=$

$=9x\sqrt[3]{2x}$

41.10. $\frac{\sqrt[3]{2\sqrt{3}}\times\sqrt[3]{12}}{\sqrt[6]{27}}=\frac{\sqrt[3]{2\sqrt{3}\times12}}{\sqrt[6]{27}}=$

$=\frac{\sqrt[6]{\left(24\sqrt{3}\right)^2}}{\sqrt[6]{27}}=\sqrt[6]{\frac{24^2\times3}{27}}=$

$=\sqrt[6]{\frac{24^2}{9}}=\sqrt[6]{\left(\frac{24}{3}\right)^2}=\sqrt[3]{8}=2$

41.11. $\frac{2\sqrt[4]{4\sqrt[3]{4^{-1}}}}{\sqrt{2\sqrt[3]{4}}}=\frac{2\sqrt[4]{\sqrt[3]{4^3\times4^{-1}}}}{\sqrt{\sqrt[3]{2^3\times4}}}=$

$=\frac{2\sqrt[4]{\sqrt[3]{4^2}}}{\sqrt[4]{\left(\sqrt[3]{2^5}\right)^2}}=2\times\sqrt[4]{\frac{\sqrt[3]{4^2}}{\sqrt[3]{2^{10}}}}=$

$=2\sqrt[4]{\sqrt[3]{\frac{2^4}{2^{10}}}}=2\sqrt[12]{2^{-6}}=$

$=2\sqrt{2^{-1}}=\sqrt{2^2\times2^{-1}}=\sqrt{2}$

42.1. $4+2\sqrt{3}=\left(a+b\right)^2$

$ab=\sqrt{3}$

$\left(\sqrt{3}\right)^2+1^2=4$

$4+2\sqrt{3}=\left(\sqrt{3}\right)^2+2\sqrt{3}+1^2=$

$=\left(\sqrt{3}+1\right)^2$

$\sqrt{4+2\sqrt{3}}=\sqrt{\left(\sqrt{3}+1\right)^2}=\sqrt{3}+1$

42.2. $36-16\sqrt{2}=\left(a-b\right)^2$

$ab=8\sqrt{2}=2\times4\sqrt{2}$

$8^2+\left(\sqrt{2}\right)^2=66$

$\left(4\sqrt{2}\right)^2+2^2=36$

$36-16\sqrt{2}=\left(4\sqrt{2}\right)^2-16\sqrt{2}+2^2$

$=\left(4\sqrt{2}-2\right)^2$

$\sqrt{36-16\sqrt{2}}=\sqrt{\left(4\sqrt{2}-2\right)^2}=$ $\quad 4\sqrt{2}-2>0$

$=4\sqrt{2}-2$

42.3. $7+2\sqrt{10}=\left(a+b\right)^2$

$ab=\sqrt{10}=\sqrt{5}\sqrt{2}$

$\left(\sqrt{5}\right)^2+\left(\sqrt{2}\right)^2=7$

$7+2\sqrt{10}=\left(\sqrt{5}\right)^2+2\sqrt{10}+\left(\sqrt{2}\right)^2=$

$=\left(\sqrt{5}+\sqrt{2}\right)^2$

$\sqrt{7+2\sqrt{10}}=\sqrt{\left(\sqrt{5}+\sqrt{2}\right)^2}=\sqrt{5}+\sqrt{2}$

43.1. $10-4\sqrt{6}=\left(a-b\right)^2$

$ab=2\sqrt{6}$

$\left(\sqrt{6}\right)^2+2^2=10$

$10-4\sqrt{6}=\left(\sqrt{6}\right)^2-4\sqrt{6}+2^2=$

$=\left(\sqrt{6}-2\right)^2$

$\sqrt{10-4\sqrt{6}}=\sqrt{\left(\sqrt{6}-2\right)^2}=\sqrt{6}-2=$ $\quad \sqrt{6}-2>0$

$=-2+\sqrt{6}=\sqrt{6}-2$

43.2. $29+12\sqrt{5}=\left(a+b\right)^2$

$ab=6\sqrt{5}=3\times2\sqrt{5}=\ldots$

$6^2+\left(\sqrt{5}\right)^2=41\neq29$

$3^2+\left(2\sqrt{5}\right)^2=29$

$29+12\sqrt{5}=3^2+12\sqrt{5}+\left(2\sqrt{5}\right)^2=$

$=\left(3+2\sqrt{5}\right)^2$

$\sqrt{29+12\sqrt{5}}=\sqrt{\left(3+2\sqrt{5}\right)^2}=3+2\sqrt{5}=2\sqrt{5}+3$

43.3. $44+16\sqrt{7}=\left(a+b\right)^2$

$ab=8\sqrt{7}=4\times2\sqrt{7}=\ldots$

$8^2+\left(\sqrt{7}\right)^2=71$

$4^2+\left(2\sqrt{7}\right)^2=44$

$44+16\sqrt{7}=4^2+16\sqrt{7}+\left(2\sqrt{7}\right)^2=$

$=\left(4+2\sqrt{7}\right)^2$

$\sqrt{44+16\sqrt{7}}=\sqrt{\left(4+2\sqrt{7}\right)^2}=4+2\sqrt{7}=2\sqrt{7}+4$

44. $6-2\sqrt{5}=\left(a-b\right)^2$ $\quad ab=\sqrt{5}=1\times\sqrt{5}$

$\left(6-2\sqrt{5}\right)=\left(\sqrt{5}\right)^2-2\sqrt{5}+1^2=$ $\quad \left(\sqrt{5}\right)^2+1^2=6$

$=\left(\sqrt{5}-1\right)^2$

$\sqrt{6-2\sqrt{5}}=\sqrt{\left(\sqrt{5}-1\right)^2}=\sqrt{5}-1$ $\quad \left(\sqrt{5}-1>0\right)$

$\frac{1+\sqrt{5}}{\sqrt{6-2\sqrt{5}}}=\frac{1+\sqrt{5}}{\sqrt{5}-1}=\frac{\left(1+\sqrt{5}\right)\left(\sqrt{5}+1\right)}{\left(\sqrt{5}-1\right)\left(\sqrt{5}+1\right)}=$

$=\frac{1+2\sqrt{5}+5}{5-1}=\frac{6+2\sqrt{5}}{4}=$

$=\frac{3+\sqrt{5}}{2}$

45.1. $x^4 - 10x^2 + 1 = 0$

Fazendo $y = x^2$:

$$y^2 - 10y + 1 = 0 \Leftrightarrow y = \frac{10 \pm \sqrt{100-4}}{2} \Leftrightarrow$$

$$\Leftrightarrow y = \frac{10 \pm \sqrt{96}}{2} \Leftrightarrow y = \frac{10 \pm \sqrt{16 \times 6}}{2} \Leftrightarrow$$

$$\Leftrightarrow y = \frac{10 \pm 4\sqrt{6}}{2} \Leftrightarrow y = 5 \pm 2\sqrt{6}$$

Substituindo y por x^2:

$$x^2 = 5 + 2\sqrt{6} \vee x^2 = 5 - 2\sqrt{6} \Leftrightarrow \qquad \left| 5 - 2\sqrt{6} > 0 \right.$$

$$\Leftrightarrow x = \pm\sqrt{5 + 2\sqrt{6}} \vee x = \sqrt{5 - 2\sqrt{6}}$$

Cálculo auxiliar:

$5 + 2\sqrt{6} = (a+b)^2 \qquad \left| ab = \sqrt{6} = \sqrt{2} \times \sqrt{3} = \ldots \right.$

$5 + 2\sqrt{6} = \left(\sqrt{3}\right)^2 + 2\sqrt{6} + \left(\sqrt{2}\right)^2 = \qquad \left| \left(\sqrt{2}\right)^2 + \left(\sqrt{3}\right)^2 = 5 \right.$

$= \left(\sqrt{3} + \sqrt{2}\right)^2$

$\sqrt{5 + 2\sqrt{6}} = \sqrt{\left(\sqrt{3} + \sqrt{2}\right)^2} = \sqrt{3} + \sqrt{2}$

De igual modo:

$\sqrt{5 - 2\sqrt{6}} = \sqrt{\left(\sqrt{3} - \sqrt{2}\right)^2} = \sqrt{3} - \sqrt{2}$

Então:

$$x = \pm\sqrt{5 + 2\sqrt{6}} \vee x = \pm\sqrt{5 - 2\sqrt{6}} \Leftrightarrow$$

$$\Leftrightarrow x = \sqrt{3} + \sqrt{2} \vee x = -\sqrt{3} - \sqrt{2} \vee x = \sqrt{3} - \sqrt{2} \vee$$

$$\vee x = -\sqrt{3} + \sqrt{2}$$

$$S = \left\{\sqrt{3} + \sqrt{2}\,,\ \sqrt{3} - \sqrt{2}\,,\ -\sqrt{3} + \sqrt{2}\,,\ -\sqrt{3} - \sqrt{2}\right\}$$

45.2. $x^4 - 8\sqrt{3}x^2 - 16 = 0$

Fazendo $y = x^2$:

$$y^2 - 8\sqrt{3}y - 16 = 0 \Leftrightarrow$$

$$\Leftrightarrow y = \frac{8\sqrt{3} \pm \sqrt{192 + 64}}{2} \Leftrightarrow$$

$$\Leftrightarrow y = \frac{8\sqrt{3} \pm 16}{2} \Leftrightarrow$$

$$\Leftrightarrow y = 4\sqrt{3} + 8 \vee y = 4\sqrt{3} - 8$$

Substituindo:

$$x^2 = 4\sqrt{3} + 8 \vee x^2 = 4\sqrt{3} - 8$$

A segunda equação é impossível porque $4\sqrt{3} - 8 < 0$.

$$x^2 = 4\sqrt{3} + 8 \Leftrightarrow x = \pm\sqrt{4\sqrt{3} + 8} \Leftrightarrow$$

$$\Leftrightarrow x = \pm\left(\sqrt{6} + \sqrt{2}\right) \Leftrightarrow$$

$$\Leftrightarrow x = \sqrt{6} + \sqrt{2} \vee x = -\sqrt{6} - \sqrt{2}$$

$$S = \left\{\sqrt{6} + \sqrt{2}\,,\ -\sqrt{6} - \sqrt{2}\right\}$$

Cálculo auxiliar:

$4\sqrt{3} + 8 = (a+b)^2 \qquad \left| ab = 2\sqrt{3} = \sqrt{12} = \sqrt{6} \times \sqrt{2} \ldots \right.$

$4\sqrt{3} + 8 = \left(\sqrt{6}\right)^2 + 2\sqrt{6}\sqrt{2} + \left(\sqrt{2}\right)^2 = \qquad \left| 2^2 + \left(\sqrt{3}\right)^2 = 7 \right.$

$= \left(\sqrt{6} + \sqrt{2}\right)^2 \qquad \left| 1^2 + \left(\sqrt{12}\right)^2 = 13 \right.$

$\sqrt{4\sqrt{3} + 8} = \sqrt{\left(\sqrt{6} + \sqrt{2}\right)^2} = \sqrt{6} + \sqrt{2} \qquad \left| \left(\sqrt{6}\right)^2 + \left(\sqrt{2}\right)^2 = 8 \right.$

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46. a: $\forall a,\ b \in \mathbb{R}^+,\ \sqrt{a^2 + b^2} = a + b$

Proposição falsa

Por exemplo, $\sqrt{3^2 + 4^2} = 5 \neq 3 + 4$.

b: $\forall a,\ b \in \mathbb{R},\ \sqrt{a^2} + \sqrt{b^2} = \sqrt{(a+b)^2}$

Proposição falsa

Por exemplo, $\sqrt{3^2} + \sqrt{(-2)^2} = 3 + 2 = 5$ e $\sqrt{\left[3 + (-2)\right]^2} = 1$

c: $\forall a,\ b \in \mathbb{R},\ \sqrt[3]{a^2} \times \sqrt[3]{b^2} = \sqrt[3]{(a \times b)^2}$

Proposição verdadeira, pois:

$$\sqrt[3]{a^2} \times \sqrt[3]{b^2} = \sqrt[3]{a^2 \times b^2} = \sqrt[3]{(a \times b)^2}$$

d: $\forall a,\ b \in \mathbb{R}^+,\ \sqrt{a^2 \times b^2} = b\sqrt{a}$

Proposição falsa

Por exemplo, $\sqrt{5^2 \times 1^2} = 5$ e $1 \times \sqrt{5} = \sqrt{5}$

e: $\forall a \in \mathbb{R} \setminus \{0\},\ \dfrac{a}{\sqrt[3]{a^2}} = \sqrt[3]{a}$

Proposição verdadeira, pois:

$$\frac{a}{\sqrt[3]{a^2}} = \frac{\sqrt[3]{a^3}}{\sqrt[3]{a^2}} = \sqrt[3]{\frac{a^3}{a^2}} = \sqrt[3]{a}$$

f: Proposição verdadeira

$$\frac{1}{2}\sqrt[3]{6} = \frac{\sqrt[3]{6}}{\sqrt[3]{2^3}} = \sqrt[3]{\frac{6}{8}} = \sqrt[3]{\frac{3}{4}}$$

g: Proposição falsa, pois:

$$\left(\sqrt{\sqrt{2} + 1}\right)^2 = \sqrt{2} + 1$$

$$\left(\sqrt[4]{2} + 1\right)^2 = \left(\sqrt[4]{2}\right) + 2\sqrt[4]{2} + 1 = \sqrt{2} + 1 + 2\sqrt[4]{2}$$

h: Proposição falsa

$$x^8 = 2 \Leftrightarrow x = \sqrt[8]{2} \vee x = -\sqrt[8]{2}$$

i: Proposição falsa, pois:

-1 é solução de $x^4 = 1$ e não é solução de $x^7 = 1$

j: Proposição verdadeira, pois:

$$\left(\sqrt{2}\right)^3 - \sqrt{2} \times \left(\sqrt{2}\right)^2 + \sqrt{2} - \sqrt{2} =$$

$$= \left(\sqrt{2}\right)^3 - \left(\sqrt{2}\right)^3 + 0 = 0$$

k: Proposição falsa, pois:

$$\left(\sqrt[3]{\sqrt[7]{64}}\right)^{14} = \left(\sqrt[21]{2^6}\right)^{14} = \sqrt[21]{\left(2^6\right)^{14}} = \sqrt[3]{\left(2^6\right)^2} =$$

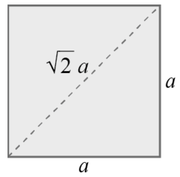
$$= \sqrt[3]{2^{12}} = 2^4 \neq 2^6$$

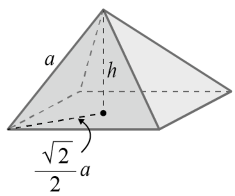
l: Proposição verdadeira, pois:

$$1^3 < 7 < 3^3 \Rightarrow 1 < \sqrt[3]{7} < 3$$

47. O octaedro é formado por duas pirâmides iguais.

É necessário calcular a altura de uma das pirâmides.





Diagonal da base:

$$d^2 = a^2 + a^2$$

$$d^2 = 2a^2 \Leftrightarrow d = \sqrt{2a^2} \Leftrightarrow \qquad (a > 0)$$

$$\Leftrightarrow d = \sqrt{2}a$$

Semidiagonal $= \dfrac{\sqrt{2}a}{2}$

Altura da pirâmide:

$$h^2 + \left(\frac{\sqrt{2}}{2}a\right)^2 = a^2 \Leftrightarrow$$

$$\Leftrightarrow h^2 = a^2 - \frac{1}{2}a^2 \Leftrightarrow$$

$$\Leftrightarrow h^2 = a^2 - \frac{1}{2}a^2 \Leftrightarrow$$

$$\Leftrightarrow h^2 = \frac{1}{2}a^2 \Leftrightarrow h = \sqrt{\frac{1}{2}a^2} \Leftrightarrow$$

$$\Leftrightarrow h = \frac{1}{\sqrt{2}}a \Leftrightarrow h = \frac{\sqrt{2}}{2}a$$

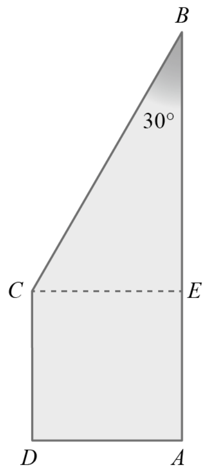
$V_{\text{octaedro}} = 2 \times V_{\text{pirâmide}}$

$$= 2 \times \frac{1}{3} \times A_{\text{base}} \times \text{altura} =$$

$$= \frac{2}{3} \times a^2 \times \frac{\sqrt{2}}{2}a =$$

$$= \frac{\sqrt{2}}{3}a^3$$

48. $\dfrac{\overline{CE}}{\overline{EB}} = \text{tg}\,30°$



$$2\sqrt{2} = \overline{EB} \times \frac{\sqrt{3}}{3} \Leftrightarrow$$

$$\Leftrightarrow 6\sqrt{2} = \overline{EB} \times \sqrt{3} \Leftrightarrow$$

$$\Leftrightarrow \overline{EB} = \frac{6\sqrt{2}}{\sqrt{3}} \Leftrightarrow$$

$$\Leftrightarrow \overline{EB} = \frac{6\sqrt{2}\sqrt{3}}{\sqrt{3}\sqrt{3}} \Leftrightarrow$$

$$\Leftrightarrow \overline{EB} = \frac{6\sqrt{6}}{3} \Leftrightarrow$$

$$\Leftrightarrow \overline{EB} = 2\sqrt{6}$$

$$\overline{BC}^2 = \overline{EB}^2 + \overline{CE}^2$$

$$\overline{BC}^2 = \left(2\sqrt{6}\right)^2 + \left(2\sqrt{2}\right)^2 \Leftrightarrow$$

$$\Leftrightarrow \overline{BC}^2 = 24 + 8 \Leftrightarrow$$

$$\Leftrightarrow \overline{BC} = \sqrt{32} \Leftrightarrow$$

$$\Leftrightarrow \overline{BC} = \sqrt{16 \times 2} \Leftrightarrow \overline{BC} = 4\sqrt{2}$$

Perímetro:

$$\overline{AD} + \overline{DC} + \overline{CB} + \overline{AB} =$$

$$= 2\sqrt{2} + 2\sqrt{2} + 4\sqrt{2} + 2\sqrt{2} + 2\sqrt{6} =$$

$$= 10\sqrt{2} + 2\sqrt{6}$$

Logo, o perímetro do trapézio é $\left(10\sqrt{2} + 2\sqrt{6}\right)$ cm.

49.1. $a^3 = 2\sqrt{50} \Leftrightarrow a = \sqrt[3]{2\sqrt{50}}$

$$\Leftrightarrow a = \sqrt[3]{\sqrt{4 \times 50}} \Leftrightarrow$$

$$\Leftrightarrow a = \sqrt[6]{200}$$

O comprimento da aresta do cubo é $\sqrt[6]{200}$ cm.

49.2. $V_{\text{pirâmide}} = \dfrac{1}{3} \times V_{\text{cubo}} =$

$$= \frac{1}{3} \times 2\sqrt{50} = \frac{10\sqrt{2}}{3}$$

O volume da pirâmide [$ABCDH$] é $\dfrac{10\sqrt{2}}{3}$ cm^3.

49.3. $\overline{HC}^2 = \overline{HG}^2 + \overline{GC}^2$

$$\overline{HC}^2 = \left(\sqrt[6]{200}\right)^2 + \left(\sqrt[6]{200}\right)^2 \Leftrightarrow$$

$$\Leftrightarrow \overline{HC}^2 = 2 \times \left(\sqrt[6]{200}\right)^3 \Leftrightarrow$$

$$\Leftrightarrow \overline{HC} = \sqrt{2\sqrt[6]{200^2}} \Leftrightarrow$$

$$\Leftrightarrow \overline{HC} = \sqrt{2 \times \sqrt[3]{200}} \Leftrightarrow$$

$$\Leftrightarrow \overline{HC} = \sqrt{\sqrt[3]{2^3 \times 200}} \Leftrightarrow$$

$$\Leftrightarrow \overline{HC} = \sqrt[6]{2^3 \times 2 \times 100} \Leftrightarrow$$

$$\Leftrightarrow \overline{HC} = \sqrt[6]{2^4 \times 10^2} \Leftrightarrow$$

$$\Leftrightarrow \overline{HC} = \sqrt[3]{2^2 \times 10} \Leftrightarrow$$

$$\Leftrightarrow \overline{HC} = \sqrt[3]{40} \text{ cm}$$

49.4. $\overline{HB}^2 = \overline{DB}^2 + \overline{HD}^2$

$$\overline{HB}^2 = \left(\sqrt[3]{40}\right)^2 + \left(\sqrt[6]{200}\right)^2 \Leftrightarrow \quad \left|\, \overline{DB} = \overline{HC}\right.$$

$$\Leftrightarrow \overline{HB}^2 = \sqrt[3]{40^2} + \sqrt[6]{200^2} \Leftrightarrow$$

$$\Leftrightarrow \overline{HB}^2 = \sqrt[3]{1600} + \sqrt[3]{200} \Leftrightarrow$$

$$\Leftrightarrow \overline{HB}^2 = \sqrt[3]{8 \times 200} + \sqrt[3]{200} \Leftrightarrow$$

$$\Leftrightarrow \overline{HB}^2 = 2\sqrt[3]{200} + \sqrt[3]{200} \Leftrightarrow$$

$$\Leftrightarrow \overline{HB}^2 = 3\sqrt[3]{200} \Leftrightarrow$$

$$\Leftrightarrow \overline{HB} = \sqrt{3\sqrt[3]{200}} \Leftrightarrow$$

$$\Leftrightarrow \overline{HB} = \sqrt{\sqrt[3]{9 \times 200}} \Leftrightarrow$$

$$\Leftrightarrow \overline{HB} = \sqrt[6]{1800}$$

$$\overline{BH} = \sqrt[6]{1800} \text{ cm}$$

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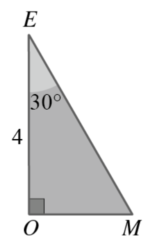
50. $\dfrac{\overline{OE}}{\overline{EM}} = \cos 30°$



$$4 = \overline{EM} \times \frac{\sqrt{3}}{2} \Leftrightarrow$$

$$\Leftrightarrow \overline{EM} = \frac{8}{\sqrt{3}} \Leftrightarrow$$

$$\Leftrightarrow \overline{EM} = \frac{8\sqrt{3}}{3}$$

$$\frac{\overline{OM}}{\overline{OE}} = \text{tg}\,30° \Leftrightarrow \frac{\overline{OM}}{4} = \frac{\sqrt{3}}{3} \Leftrightarrow \overline{OM} = \frac{4\sqrt{3}}{3}$$

$$\overline{AB} = 2\overline{OM} = \frac{8\sqrt{3}}{3}$$

$$A_{\text{base}} = \overline{AB}^2 = \left(\frac{8\sqrt{3}}{3}\right)^2 = \frac{64 \times 3}{9} = \frac{64}{3}$$

$$A_{\text{face}} = \frac{\overline{AM} \times \overline{EM}}{2} = \frac{\frac{8\sqrt{3}}{3} \times \frac{8\sqrt{3}}{3}}{2} = \frac{\frac{64}{3}}{2} = \frac{32}{3}$$

$$A_{\text{total}} = \frac{64}{3} + 4 \times \frac{32}{3} = \frac{64 + 128}{3} = \frac{192}{3} = 64$$

A área total da superfície da pirâmide é 64 cm^2.

51. $\overline{BC} = a$

51.1. $\overline{BG}^2 = a^2 + a^2 \Leftrightarrow \overline{BG} = \sqrt{2a^2} \Leftrightarrow \overline{BG} = \sqrt{2}a \Leftrightarrow$

$$P_{[BGE]} = 3 \times \sqrt{2}a = 3\sqrt{2}a$$

51.2. O triângulo [BGE] é equilátero de lado $\sqrt{2}a$.

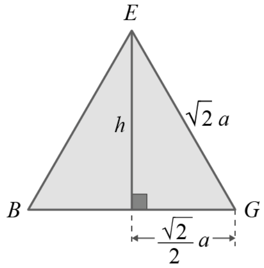
$$h^2+\left(\frac{\sqrt{2}}{2}a\right)^2=\left(\sqrt{2}a\right)^2 \Leftrightarrow$$

$$\Leftrightarrow h^2=2a^2-\frac{1}{2}a^2 \Leftrightarrow$$

$$\Leftrightarrow h^2=\frac{3}{2}a^2 \Leftrightarrow$$

$$\Leftrightarrow h=\sqrt{\frac{3}{2}a^2} \Leftrightarrow$$

$$\Leftrightarrow h=\sqrt{\frac{3}{2}}a \Leftrightarrow h=\frac{\sqrt{3}}{\sqrt{2}}a$$



$$A_{[BGE]}=\frac{\sqrt{2}a\times\frac{\sqrt{3}}{\sqrt{2}}a}{2}=\frac{\sqrt{3}}{2}a^2$$

A área do triângulo [BGE] é $\frac{\sqrt{3}}{2}a^2$.

51.3. $V_{\text{pirâmide}}=\frac{1}{3}A_{[EBF]}\times\overline{FG}$

$$=\frac{1}{3}\times\frac{a\times a}{2}\times a=\frac{a^3}{6}\quad(1)$$

Seja h a altura pedida:

$V_{\text{pirâmide}}=\frac{1}{3}A_{[BGE]}\times h$

$$=\frac{1}{3}\frac{\sqrt{3}}{2}a^2\times h=\frac{\sqrt{3}a^2h}{6}\quad(2)$$

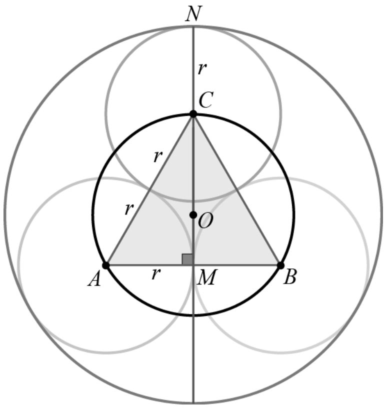
Como (2) = (1), vem:

$$\frac{\sqrt{3}a^2h}{6}=\frac{a^3}{6}\Leftrightarrow\sqrt{3}a^2h=a^3\Leftrightarrow h=\frac{a^3}{\sqrt{3}a^2}\Leftrightarrow$$

$$\Leftrightarrow h=\frac{a}{\sqrt{3}}\Leftrightarrow h=\frac{a\sqrt{3}}{\sqrt{3}\sqrt{3}}\Leftrightarrow h=\frac{\sqrt{3}}{3}a$$

A altura da pirâmide [$EBGF$] é $\frac{\sqrt{3}}{3}a$.

52. Sejam A, B e C os centros das circunferências interiores e O o centro da circunferência exterior.



Seja M o ponto médio de [AB] e r o raio.

$\overline{AC}=2r$ e $\overline{AM}=r$

Como o triângulo [ABC] é equilátero:

$$\overline{AC}^2=\overline{AM}^2+\overline{MC}^2$$

$$(2r)^2=r^2+\overline{MC}^2\Leftrightarrow\overline{MC}^2=4r^2-r^2\Leftrightarrow\overline{MC}=\sqrt{3r^2}\Leftrightarrow$$

$$\Leftrightarrow\overline{MC}=\sqrt{3}r$$

O ponto O também é o centro da circunferência circunscrita ao triângulo [ABC]. Como o triângulo é equilátero, então este ponto também é o ponto de interseção das medianas. Logo:

$$\overline{OC}=\frac{2}{3}\overline{MC}=\frac{2}{3}\sqrt{3}r$$

Seja [ON] o raio da circunferência exterior que passa em C. Então, $\overline{OC}+\overline{CN}$ é igual ao raio da circunferência.

$$\frac{2}{3}\sqrt{3}r+r=6\Leftrightarrow 2\sqrt{3}r+3r=18$$

$$\Leftrightarrow r\left(2\sqrt{3}+3\right)=18\Leftrightarrow r=\frac{18}{2\sqrt{3}+3}\Leftrightarrow$$

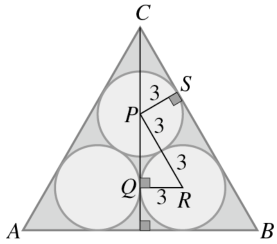
$$\Leftrightarrow r=\frac{18\left(2\sqrt{3}-3\right)}{\left(2\sqrt{3}+3\right)\left(2\sqrt{3}-3\right)}\Leftrightarrow r=\frac{18\left(2\sqrt{3}-3\right)}{12-9}\Leftrightarrow$$

$$\Leftrightarrow r=6\left(2\sqrt{3}-3\right)\Leftrightarrow r=12\sqrt{3}-18$$

O raio de cada uma das circunferências é $\left(18-2\sqrt{3}\right)$ cm.

53. A altura da caixa deve ser igual ao diâmetro dos sabonetes, ou seja, 6 cm.

Para determina as dimensões da base da caixa vamos considerar o seguinte esquema onde se visualiza a planta da caixa com os sabonetes.



$\overline{PR}^2=\overline{QR}^2+\overline{PQ}^2$ Teorema de Pitágoras

$$6^2=3^2+\overline{PQ}^2\Leftrightarrow$$

$$\Leftrightarrow\overline{PQ}=\sqrt{36-9}\Leftrightarrow$$

$$\Leftrightarrow\overline{PQ}=3\sqrt{3}$$

Os triângulos [PSC] e [RQP] são semelhantes por terem dois ângulos iguais

$\left(C\hat{S}P=R\hat{Q}P=90°\ \text{e}\ Q\hat{P}R=P\hat{C}S\right)$

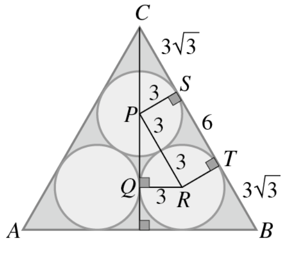
$\frac{\overline{CS}}{\overline{PQ}}=\frac{\overline{QR}}{\overline{PS}}$, ou seja, $\frac{\overline{CS}}{3\sqrt{3}}=\frac{3}{3}\Leftrightarrow\overline{CS}=3\sqrt{3}$

Podemos finalmente, concluir que o lado do triângulo equilátero [ABC] é igual a:

$$\overline{CS}+\overline{ST}+\overline{TB}=3\sqrt{3}+6+3\sqrt{3}=$$

$$=6+6\sqrt{3}$$

A caixa dos sabonetes tem a forma de um prisma triangular regular com 6 cm de altura e $\left(6+6\sqrt{3}\right)$ cm de aresta da base.



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Avaliação 1

1. $\left[-\sqrt[4]{(-2)^2}<-\sqrt[4]{2^3}\Leftrightarrow\left(-\sqrt[4]{(-2)^2}\right)^3>\left(-\sqrt[4]{2^3}\right)^3\right]\Leftrightarrow$

$$\Leftrightarrow\left[\sqrt[4]{4}>\sqrt[4]{8}\Leftrightarrow\left(-\sqrt[4]{4}\right)^3>\left(-\sqrt[4]{8}\right)^3\right]\Leftrightarrow$$

$$\Leftrightarrow\left(\sqrt[4]{4}>\sqrt[4]{8}\Leftrightarrow-\sqrt[4]{4^3}>-\sqrt[4]{8^3}\right)\Leftrightarrow$$

$$\Leftrightarrow\left(\sqrt[4]{4}>\sqrt[4]{8}\Leftrightarrow\sqrt[4]{4^3}<\sqrt[4]{8^3}\right)\Leftrightarrow$$

$$\Leftrightarrow(\text{F}\Leftrightarrow\text{V})\Leftrightarrow\text{F}$$

Resposta: (C)

2. $\sqrt[4]{(-2)^4}=\sqrt[4]{2^4}=2$

Resposta: (A)

3. $x^6=4 \Leftrightarrow x=\pm\sqrt[6]{4} \Leftrightarrow x=\pm\sqrt[6]{2^2} \Leftrightarrow x=\pm\sqrt[3]{2}$

$\Leftrightarrow x=\sqrt[3]{2} \vee x=-\sqrt[3]{2}$

$x^{12}=16 \Leftrightarrow x=\pm\sqrt[12]{2^4} \Leftrightarrow x=\pm\sqrt[3]{2}$

$\Leftrightarrow x=\sqrt[3]{2} \vee x=-\sqrt[3]{2}$

Resposta: (C)

4. $\sqrt{\frac{a}{b}\sqrt[3]{\frac{b}{a}}}=\sqrt{\sqrt[3]{\frac{a^3}{b^3}\times\frac{b}{a}}}=\sqrt[6]{\frac{a^2}{b^2}}=\sqrt[3]{\frac{a}{b}}=\frac{\sqrt[3]{a}}{\sqrt[3]{b}}$

$\frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}}}=\frac{\sqrt[3]{b}}{\sqrt[3]{a}}=\frac{\sqrt[3]{b}\sqrt[3]{a^2}}{\sqrt[3]{a}\sqrt[3]{a^2}}=\frac{\sqrt[3]{ba^2}}{a};\ a,b\in\mathbb{R}^+$

Resposta: (D)

5. $\frac{\left(\sqrt[3]{24}-\sqrt[3]{\sqrt{9}}\right)^4}{\sqrt[3]{3}}=\frac{\left(\sqrt[3]{8\times3}-\sqrt[6]{3^2}\right)^4}{\sqrt[3]{3}}=\frac{\left(\sqrt[3]{8}\sqrt[3]{3}-\sqrt[3]{3}\right)^4}{\sqrt[3]{3}}=$

$=\frac{\left(2\sqrt[3]{3}-\sqrt[3]{3}\right)^4}{\sqrt[3]{3}}=\frac{\left(\sqrt[3]{3}\right)^4}{\sqrt[3]{3}}=\left(\sqrt[3]{3}\right)^3=3$

Resposta: (C)

6. $\left(1-\sqrt{3}\right)^2=1-2\sqrt{3}+3=4-2\sqrt{3}$

- $x^2-2\sqrt{3}=4 \Leftrightarrow x^2=4+2\sqrt{3}$ (não é solução)
- $x^2+4\sqrt{3}=7 \Leftrightarrow x^2=7-4\sqrt{3}$ (não é solução)
- $x^2-2\sqrt{3}x+2=0$

  $4-2\sqrt{3}-2\sqrt{3}\left(1-\sqrt{3}+2\right)=0 \Leftrightarrow$

  $\Leftrightarrow 4-2\sqrt{3}-2\sqrt{3}+6+2=0 \Leftrightarrow$

  $\Leftrightarrow 12-4\sqrt{3}=0$ (falso)
- $x^2+2\sqrt{3}x+2=0$

  $4-2\sqrt{3}+2\sqrt{3}\left(1-\sqrt{3}\right)+2 \Leftrightarrow$

  $\Leftrightarrow 4-2\sqrt{3}+2\sqrt{3}-6+2=0 \Leftrightarrow$

  $\Leftrightarrow 0=0$ (verdadeiro)

Resposta: (D)

7. $r=5 \Leftrightarrow 2r=10$

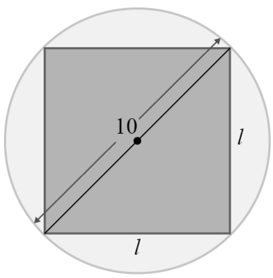
$l^2+l^2=10^2 \Leftrightarrow$

$\Leftrightarrow 2l^2=100 \Leftrightarrow$

$\Leftrightarrow l^2=50 \overset{l>0}{\Leftrightarrow}$

$\Leftrightarrow l=\sqrt{50} \Leftrightarrow$

$\Leftrightarrow l=\sqrt{25\times2} \Leftrightarrow l=5\sqrt{2}$



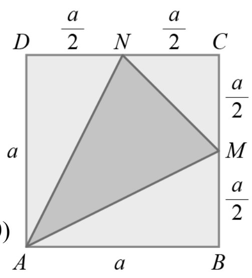
Resposta: (A)

8. Pelo Teorema de Pitágoras:

$\overline{MN}=\sqrt{\left(\frac{a}{2}\right)^2+\left(\frac{a}{2}\right)^2}=$

$=\sqrt{2\times\frac{a^2}{4}}=\sqrt{\frac{a^2}{2}}=$

$=\frac{\sqrt{a^2}}{\sqrt{2}}=\frac{a\sqrt{2}}{\sqrt{2}\sqrt{2}}=\frac{\sqrt{2}}{2}a \quad (a>0)$



$\overline{AM}=\overline{AN}=\sqrt{a^2+\left(\frac{a}{2}\right)^2}=\sqrt{a^2+\frac{a^2}{4}}=\sqrt{\frac{5}{4}a^2}=$

$=\sqrt{\frac{5}{4}}\sqrt{a^2}=\frac{\sqrt{5}}{2}a,\ a>0$

Perímetro $=2\times\frac{\sqrt{5}}{2}a+\frac{\sqrt{2}}{2}a=\frac{2\sqrt{5}+\sqrt{2}}{2}a$

Resposta: (C)

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9.1. $\underset{(1-\sqrt{5})(1+\sqrt{5})}{\sqrt{3}}+\frac{1}{\underset{(1+\sqrt{5})}{1-\sqrt{5}}}+\frac{\sqrt{5}}{\underset{(1-\sqrt{5})}{\sqrt{5}+1}}=$

$=\frac{\sqrt{3}\left(1^2-\left(\sqrt{5}\right)^2\right)+1+\sqrt{5}+\sqrt{5}\left(1-\sqrt{5}\right)}{1^2-\left(\sqrt{5}\right)^2}=$

$=\frac{\sqrt{3}\times(-4)+1+\sqrt{5}+\sqrt{5}-5}{1-5}=$

$=\frac{-4\sqrt{3}+2\sqrt{5}-4}{-4}=$

$=\sqrt{3}-\frac{\sqrt{5}}{2}+1$

9.2. $\frac{\sqrt[3]{24}-\sqrt[3]{81}}{\sqrt{2\sqrt[3]{9}}\times\sqrt{32}}=\frac{\sqrt[3]{8\times3}-\sqrt[3]{27-3}}{\sqrt{\sqrt[3]{8\times9}}\times\sqrt{2^5}}=\frac{2\sqrt[3]{3}-3\sqrt[3]{3}}{\sqrt[6]{2^3\times3^2}\times\sqrt[6]{2^{15}}}=$

$=\frac{-\sqrt[3]{3}}{\sqrt[6]{2^3\times2^{15}\times3^2}}=\frac{-\sqrt[3]{3}}{\sqrt[6]{2^{18}\times3^2}}=\frac{-\sqrt[3]{3}}{\sqrt[6]{2^{18}}\times\sqrt[6]{3^2}}=$

$=\frac{-\sqrt[3]{3}}{2^3\times\sqrt[3]{3}}=-\frac{1}{2^3}=-\frac{1}{8}$

9.3. $\frac{1}{\sqrt[3]{2}-1}=$

$A^3-B^3=(A-B)\left(A^2+AB+B^2\right)$

$A=\sqrt[3]{2}$ e $B=1$

$=\frac{\left(\sqrt[3]{2}\right)^2+\sqrt[3]{2}+1}{\left(\sqrt[3]{2}\right)^3-1^3}=$

$=\sqrt[3]{4}+\sqrt[3]{2}+1$

10. $2x^4-\sqrt{5}x^2-5=0 \Leftrightarrow$ $\quad y=x^2$

$\Leftrightarrow 2y^2-\sqrt{5}y-5=0 \Leftrightarrow$

$\Leftrightarrow y=\frac{\sqrt{5}\pm\sqrt{5+40}}{4} \Leftrightarrow$

$\Leftrightarrow y=\frac{\sqrt{5}\pm\sqrt{9\times5}}{4} \Leftrightarrow$

$\Leftrightarrow y=\frac{\sqrt{5}\pm3\sqrt{5}}{4} \Leftrightarrow$

$\Leftrightarrow y=\sqrt{5} \vee y=-\frac{\sqrt{5}}{2}$

Para $y=x^2$:

$x^2=\sqrt{5} \vee x^2=-\frac{\sqrt{5}}{2}$ (equação impossível)

$\Leftrightarrow x=-\sqrt{\sqrt{5}} \vee x=\sqrt{\sqrt{5}} \Leftrightarrow$

$\Leftrightarrow x=-\sqrt[4]{5} \vee x=\sqrt[4]{5}$

$S=\left\{-\sqrt[4]{5}\ ,\ \sqrt[4]{5}\right\}$

11. $\frac{\sqrt[n]{a^2b^4}}{\sqrt{a}\sqrt[3]{b^2}}=\frac{\sqrt[6]{a^5}}{a} \Leftrightarrow \sqrt[n]{a^2b^4}=\sqrt[6]{a^3}\times\sqrt[6]{b^4}\times\frac{\sqrt[6]{a^5}}{a} \Leftrightarrow$

$\Leftrightarrow \sqrt[n]{a^2b^4}=\frac{\sqrt[6]{a^8b^4}}{a} \Leftrightarrow \sqrt[n]{a^2b^4}=\sqrt[6]{\frac{a^8b^4}{a^6}} \Leftrightarrow$

$\Leftrightarrow \sqrt[n]{a^2b^4}=\sqrt[6]{a^2b^4}$

Logo, $n=6$

12.1. $(x-2)^2 + x^2 = \left(\sqrt{5}\right)^2 \Leftrightarrow$

$\Leftrightarrow x^2 - 4x + 4 + x^2 - 5 = 0 \Leftrightarrow$

$\Leftrightarrow 2x^2 - 4x - 1 = 0 \Leftrightarrow$

$\Leftrightarrow x = \dfrac{4 \pm \sqrt{16+8}}{4} \Leftrightarrow x = \dfrac{4 \pm \sqrt{24}}{4} \Leftrightarrow$

$\Leftrightarrow x = \dfrac{4 \pm \sqrt{16+8}}{4} \Leftrightarrow x = \dfrac{4 \pm \sqrt{24}}{4} \Leftrightarrow$

$\Leftrightarrow x = \dfrac{2 \pm \sqrt{6}}{2}$

Como $x > 0$, temos $x = \dfrac{2+\sqrt{6}}{2}$.

12.2. $A_{\text{quadrado}} = (x-2)^2 = \left(\dfrac{2+\sqrt{6}}{2} - 2\right)^2 = \left(\dfrac{2+\sqrt{6}-4}{2}\right)^2 =$

$= \left(\dfrac{\sqrt{6}-2}{2}\right)^2 = \dfrac{6-4\sqrt{6}+4}{4} = \dfrac{10-4\sqrt{6}}{4} = \dfrac{5}{2} - \sqrt{6}$

A área do quadrado é $\left(\dfrac{5}{2} - \sqrt{6}\right)$ cm².

13. Seja a a aresta do cubo.

$\overline{EB}^2 = a^2 + a^2$

$\overline{EC}^2 = \overline{EB}^2 + \overline{BC}^2$

$\overline{EC}^2 = a^2 + a^2 + a^2 \Leftrightarrow \overline{EC} = \sqrt{3a^2} \Leftrightarrow$

$\Leftrightarrow \overline{EC} = \sqrt{3}a \quad \left(\overline{EC} > 0 \text{ e } a > 0\right)$

Como $\overline{EC} = \sqrt{12}$, temos:

$\sqrt{3}a = \sqrt{12} \Leftrightarrow a = \dfrac{\sqrt{12}}{\sqrt{3}} \Leftrightarrow a = \sqrt{\dfrac{12}{3}} \Leftrightarrow a = 2$

$V_{\text{cubo}} = 2^3 = 8$

$V_{\text{pirâmide}} = \dfrac{1}{3} \times 2^2 \times 2 = \dfrac{8}{3}$, logo $V = 8 - \dfrac{8}{3} = \dfrac{16}{3}$

O volume da parte do cubo não ocupada pela pirâmide é $\dfrac{16}{3}$ cm³.

14. $2x^6 - \sqrt{3}x^3 - 3 = 0$

Para $x = \sqrt[6]{3}$:

$2\left(\sqrt[6]{3}\right)^6 - \sqrt{3} \times \left(\sqrt[6]{3}\right)^3 - 3 = 0 \Leftrightarrow$

$\Leftrightarrow 2 \times 3 - \sqrt{3} \times \sqrt[6]{3^3} - 3 = 0 \Leftrightarrow$

$\Leftrightarrow 6 - \sqrt[3]{3} \times \sqrt{3} - 3 = 0 \Leftrightarrow$

$\Leftrightarrow 6 - 3 - 3 = 0 \Leftrightarrow 0 = 0$

Para $x = -\sqrt[6]{\dfrac{3}{4}}$:

$2 \times \left(-\sqrt[6]{\dfrac{3}{4}}\right)^6 - \sqrt{3} \times \left(-\sqrt[6]{\dfrac{3}{4}}\right)^3 - 3 = 0 \Leftrightarrow$

$\Leftrightarrow 2 \times \dfrac{3}{4} + \sqrt{3} \times \sqrt[6]{\left(\dfrac{3}{4}\right)^3} - 3 = 0 \Leftrightarrow$

$\Leftrightarrow \dfrac{3}{2} + \sqrt{3} \times \sqrt{\dfrac{3}{4}} - 3 = 0 \Leftrightarrow$

$\Leftrightarrow \dfrac{3}{2} + \sqrt{3} \times \dfrac{\sqrt{3}}{2} - 3 = 0 \Leftrightarrow$

$\Leftrightarrow \dfrac{3}{2} + \dfrac{3}{2} - 3 = 0 \Leftrightarrow 0 = 0$

15. $7 - 4\sqrt{3} = (a-b)^2$ $\quad\quad ab = 2\sqrt{3}$

$7 - 4\sqrt{3} = 2^2 - 4\sqrt{3} + \left(\sqrt{3}\right)^2 =$ $\quad\quad 2^2 + \left(\sqrt{3}\right)^2 = 7$

$= \left(2 - \sqrt{3}\right)^2$

$\sqrt{7 - 4\sqrt{3}} = \sqrt{\left(2-\sqrt{3}\right)^2} = 2 - \sqrt{3} \quad \left(2 - \sqrt{3} > 0\right)$

De igual modo:

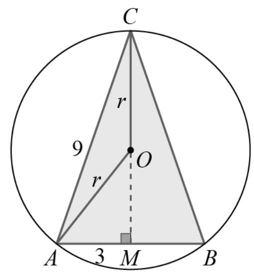
$\sqrt{7 + 4\sqrt{3}} = \sqrt{\left(2+\sqrt{3}\right)^2} = 2 + \sqrt{3}$

$\sqrt{7 - 4\sqrt{3}} + \sqrt{7 + 4\sqrt{3}} = 2 - \sqrt{3} + 2 + \sqrt{3} = 4$

16. $\overline{AB} = 6$ cm

$\overline{AC} = \overline{BC} = 9$ cm

Seja M o ponto médio de [AB].



$\overline{AC}^2 = \overline{AM}^2 + \overline{MC}^2$

$9^2 = 3^2 + \overline{MC}^2 \Leftrightarrow$

$\Leftrightarrow \overline{MC} = \sqrt{81 - 27} \Leftrightarrow$

$\Leftrightarrow \overline{MC} = \sqrt{54} \Leftrightarrow \quad \left(\overline{MC} > 0\right)$

$\Leftrightarrow \overline{MC} = 3\sqrt{6}$

$\overline{MO} = \overline{MC} - r \Leftrightarrow$

$\Leftrightarrow \overline{MO} = 3\sqrt{6} - r$

$\overline{AM}^2 + \overline{MO}^2 = r^2$

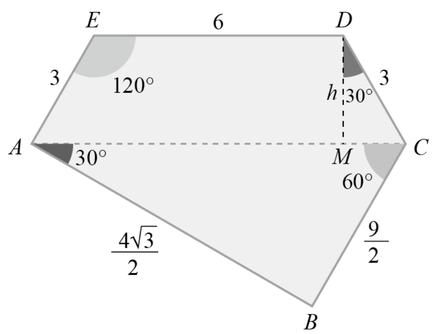
$9 + \left(3\sqrt{6} - r\right)^2 = r^2 \Leftrightarrow 9 + 54 - 6\sqrt{6}r + r^2 = r^2 \Leftrightarrow$

$\Leftrightarrow 6\sqrt{6}r = 63 \Leftrightarrow r = \dfrac{63}{6\sqrt{6}} \Leftrightarrow r = \dfrac{21}{2\sqrt{6}} \Leftrightarrow$

$\Leftrightarrow r = \dfrac{21\sqrt{6}}{2 \times \sqrt{6} \times \sqrt{6}} \Leftrightarrow r = \dfrac{21\sqrt{6}}{12} \Leftrightarrow r = \dfrac{7}{4}\sqrt{6}$

O raio da circunferência circunscrita ao triângulo é $\dfrac{7\sqrt{6}}{4}$ cm.

17. Como $\overline{AE} = \overline{DC} = 3$ e $ED \parallel AC$, então [$ACDE$] é um trapézio isósceles.



Logo, $E\hat{D}C = A\hat{E}D = 120°$

Seja $M \in [AD]$ tal que $DM \perp AC$ e $M\hat{D}C = 30°$

Seja h a altura do trapézio.

$\dfrac{\overline{DM}}{\overline{DC}} = \cos 30° \Leftrightarrow h = 3 \times \dfrac{\sqrt{3}}{2} \Leftrightarrow h = \dfrac{3\sqrt{3}}{3}$

$\dfrac{\overline{MC}}{\overline{DC}} = \sin 30° \Leftrightarrow x = 3 \times \dfrac{1}{2} \Leftrightarrow x = \dfrac{3}{2}$

$\overline{AC} = \overline{ED} + 2x = 6 + 2 \times \dfrac{3}{2} = 9$

Se $B\hat{A}C = 30°$ e $A\hat{C}B = 60°$, então $C\hat{B}A = 90°$, ou seja, o triângulo [ABC] é retângulo em B.

$$\frac{\overline{BC}}{\overline{AC}} = \sin 30° \Leftrightarrow \frac{\overline{BC}}{9} = \frac{1}{2} \Leftrightarrow \overline{BC} = \frac{9}{2}$$

$$\frac{\overline{AB}}{\overline{AC}} = \cos 30° \Leftrightarrow \overline{AB} = 9 \times \frac{\sqrt{3}}{2} \Leftrightarrow \overline{AB} = \frac{9\sqrt{3}}{2}$$

$$A = A_{\text{trapézio}} + A_{\text{triângulo}} =$$

$$= \frac{6+9}{2} \times \frac{3\sqrt{3}}{2} + \frac{\frac{9}{2} \times \frac{9\sqrt{3}}{2}}{2} =$$

$$= \frac{45\sqrt{3}}{4} + \frac{81\sqrt{3}}{8} = \frac{90\sqrt{3} + 81\sqrt{3}}{8} = \frac{171\sqrt{3}}{8}$$

A área do pentágono $[ABCDE]$ é $\frac{171\sqrt{3}}{8}$ cm^2.

18.1. Sendo $2r$ é a diagonal espacial do cubo:

$$d^2 = a^2 + a^2 \Leftrightarrow$$

$$\Leftrightarrow (2r)^2 = d^2 + a^2 \Leftrightarrow$$

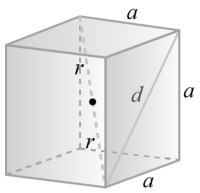
$$\Leftrightarrow (2r)^2 = a^2 + a^2 + a^2 \Leftrightarrow$$

$$\Leftrightarrow 4r^2 = 3a^2 \Leftrightarrow$$

$$\Leftrightarrow r^2 = \frac{3}{4}a^2 \Leftrightarrow$$

$$\Leftrightarrow r = \sqrt{\frac{3}{4}a^2} \Leftrightarrow$$

$$\Leftrightarrow r = \frac{\sqrt{3}}{2}a \text{ , } a > 0$$



18.2. $V = \frac{4}{3}\pi^3$

Como $r = \frac{\sqrt{3}}{2}a$:

$$V = \frac{4}{3}\pi \times \left(\frac{\sqrt{3}}{2}a\right)^3 \Leftrightarrow V = \frac{4}{3}\pi \times \left(\frac{\sqrt{3}}{2}\right)^2 \times \frac{\sqrt{3}}{2} \times a^3 \Leftrightarrow$$

$$\Leftrightarrow V = \frac{4}{3}\pi \times \frac{3}{4} \times \frac{\sqrt{3}}{2}a^3 \Leftrightarrow$$

$$\Leftrightarrow V = \pi \times \frac{\sqrt{3}}{2}a^3 \Leftrightarrow$$

$$\Leftrightarrow 2V = \pi\sqrt{3}a^3 \Leftrightarrow$$

$$\Leftrightarrow a^3 = \frac{2V}{\pi\sqrt{3}} \Leftrightarrow$$

$$\Leftrightarrow a = \sqrt[3]{\frac{2V \times \sqrt{3}}{\pi\sqrt{3}\sqrt{3}}} \Leftrightarrow$$

$$\Leftrightarrow a = \sqrt[3]{\frac{2\sqrt{3}V}{3\pi}}$$

2.2. Potências de expoente racional

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Atividade inicial 2

1.1. Se $x = 8^{\frac{1}{3}}$, temos $x^3 = \left(8^{\frac{1}{3}}\right)^3 = 8^{\frac{1}{3}\times 3} = 8^1 = 8$.

Se $x^3 = 8$, então $x = \sqrt[3]{8}$. Logo, $8^{\frac{1}{3}} = \sqrt[3]{8}$.

1.2. Se $x = 2^{\frac{1}{4}}$, vem $x^4 = \left(2^{\frac{1}{4}}\right)^4 = 2^{\frac{1}{4}\times 4} = 2^1 = 2$.

Portanto, se $x^4 = 2$, então $x = \sqrt[4]{2}$. Logo, $2^{\frac{1}{4}} = \sqrt[4]{2}$.

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1.1. $25^{\frac{1}{2}} = \sqrt{25} = 5$ **1.2.** $\left(\frac{1}{9}\right)^{\frac{1}{2}} = \sqrt{\frac{1}{9}} = \frac{1}{3}$

1.3. $27^{\frac{1}{3}} = \sqrt[3]{27} = \sqrt[3]{3^3} = 3$ **1.4.** $\left(\frac{4}{9}\right)^{0,5} = \left(\frac{4}{9}\right)^{\frac{1}{2}} = \sqrt{\frac{4}{9}} = \frac{2}{3}$

1.5. $0,01^{\frac{1}{2}} = \sqrt{0,01} = 0,1$

2.1. $7^{\frac{1}{5}} = \sqrt[5]{7}$ **2.2.** $0,01^{\frac{1}{3}} = \sqrt[3]{0,01}$ **2.3.** $3^{\frac{1}{20}} = \sqrt[20]{3}$

2.4. $a^{\frac{1}{6}} = \sqrt[6]{a}$, $a > 0$ **2.5.** $\left(\frac{a}{8}\right)^{\frac{1}{5}} = \sqrt[5]{\frac{a}{8}}$, $a \geq 0$

3.1. $\sqrt{2} = 2^{\frac{1}{2}}$ **3.2.** $\sqrt[6]{5} = 5^{\frac{1}{6}}$ **3.3.** $\sqrt[3]{\frac{1}{3}} = \left(\frac{1}{3}\right)^{\frac{1}{3}}$

3.4. $\sqrt[4]{a} = a^{\frac{1}{4}}$, $a \geq 0$ **3.5.** $\sqrt[5]{\frac{2}{3}} = \left(\frac{2}{3}\right)^{\frac{1}{5}}$

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4.1. $27^{\frac{2}{3}} = \sqrt[3]{27^2} = \sqrt[3]{\left(3^3\right)^2} = \sqrt[3]{\left(3^2\right)^3} = 3^2 = 9$

4.2. $64^{\frac{5}{6}} = \sqrt[6]{64^5} = \sqrt[6]{\left(2^6\right)^5} = \sqrt[6]{\left(2^5\right)^6} = 2^5 = 32$

4.3. $8^{\frac{5}{3}} = \sqrt[3]{8^5} = \sqrt[3]{\left(2^3\right)^5} = \sqrt[3]{\left(2^5\right)^3} = 2^5 = 32$

4.4. $\left(\frac{1}{16}\right)^{\frac{3}{2}} = \sqrt{\left(\frac{1}{16}\right)^3} = \sqrt{\left[\left(\frac{1}{2}\right)^4\right]^3} = \sqrt{\left(\frac{1}{2}\right)^{12}} = \sqrt{\left[\left(\frac{1}{2}\right)^6\right]^2} =$

$= \left(\frac{1}{2}\right)^6 = \frac{1}{64}$

4.5. $\left(8a^3\right)^{\frac{4}{3}} = \sqrt[3]{\left(2^3a^3\right)^4} = \sqrt[3]{\left[(2a)^3\right]^4} = \sqrt[3]{\left[(2a)^4\right]^3} =$

$= (2a)^4 = 16a^4$, $a \geq 0$

4.6. $1000^{\frac{2}{3}} = \sqrt[3]{\left(10^3\right)^2} = \sqrt[3]{\left(10^2\right)^3} = 10^2 = 100$

4.7. $64^{1,5} = \left(2^6\right)^{\frac{3}{2}} = \sqrt{\left(10^2\right)^3} = \sqrt{\left(2^9\right)^2} = 2^9 = 512$

4.8. $32^{0,4} = 32^{\frac{4}{10}} = 32^{\frac{2}{5}} = \sqrt[5]{\left(2^5\right)^2} = \sqrt[5]{\left(2^2\right)^5} = 4$

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5.1. $\sqrt[8]{2^5} = 2^{\frac{5}{8}}$ **5.2.** $\sqrt[10]{2^4} = 2^{\frac{4}{10}} = 2^{\frac{2}{5}}$

5.3. $\sqrt[3]{\left(\frac{1}{3}\right)^5} = \left(\frac{1}{3}\right)^{\frac{5}{3}}$ **5.4.** $\sqrt[12]{a^9} = a^{\frac{9}{12}} = a^{\frac{3}{4}}$, $a > 0$

5.5. $\sqrt[6]{\left(\frac{a}{5}\right)^3} = \left(\frac{a}{5}\right)^{\frac{3}{6}} = \left(\frac{a}{5}\right)^{\frac{1}{2}}$, $a > 0$

6.1. $\sqrt{\frac{1}{3}} = \frac{1}{\sqrt{3}} = \frac{1}{3^{\frac{1}{2}}} = 3^{-\frac{1}{2}}$

6.2. $\sqrt[6]{\frac{1}{81}} = \frac{1}{\sqrt[6]{81}} = \frac{1}{\sqrt[6]{3^4}} = \frac{1}{\sqrt[3]{3^2}} = \frac{1}{9^{\frac{1}{3}}} = 9^{-\frac{1}{3}}$

6.3. $\sqrt[6]{\frac{1}{243}} = \frac{1}{\sqrt[6]{3^5}} = \frac{1}{3^{\frac{5}{6}}} = 3^{-\frac{5}{6}}$

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7.1. $2^{\frac{2}{3}} : 2^{\frac{1}{4}} = \sqrt[3]{2^2} : \sqrt[4]{2} = \sqrt[12]{2^8} : \sqrt[12]{2^3} =$

$= \sqrt[12]{2^8 : 2^3} = \sqrt[12]{2^{8-3}} = \sqrt[12]{2^5} = 2^{\frac{5}{12}}$

7.2. $a^{\frac{m}{n}} : a^{\frac{p}{q}} = \sqrt[n]{a^m} : \sqrt[q]{a^p} = \sqrt[nq]{a^{mq}} : \sqrt[nq]{a^{np}} =$

$= \sqrt[nq]{a^{mq} - a^{np}} = \sqrt[nq]{a^{mq-np}} = a^{\frac{mq-np}{nq}} =$

$= a^{\frac{mq}{nq} - \frac{np}{nq}} = a^{\frac{m}{n} - \frac{p}{q}}$, $a > 0$

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8.1. $8^{\frac{2}{3}} \times 4^{\frac{2}{3}} = \sqrt[3]{8^2} \times \sqrt[3]{4^2} = \sqrt[3]{8^2 \times 4^2} =$

$= \sqrt[3]{(8\times 4)^2} = (8\times 4)^{\frac{2}{3}} = 32^{\frac{2}{3}}$

8.2. $a^{\frac{m}{n}} \times b^{\frac{m}{n}} = \sqrt[n]{a^m} \times \sqrt[n]{b^m} = \sqrt[n]{a^m \times b^m} =$

$= \sqrt[n]{(a\times b)^m} = (a\times b)^{\frac{m}{n}}$, $a > 0$ e $b > 0$

9.1. $\left(a^{\frac{3}{2}}\right)^{\frac{2}{5}} = \left(\sqrt{a^3}\right)^{\frac{2}{5}} = \sqrt[5]{\left(\sqrt{a^3}\right)^2} = \sqrt[5]{\sqrt{\left(a^3\right)^2}} =$

$= \sqrt[10]{a^6} = a^{\frac{6}{10}} = a^{\frac{3}{5}}$

9.2. $\left(2^{0,5}\right)^{\frac{3}{4}} = \left(2^{\frac{1}{2}}\right)^{\frac{3}{4}} = \left(\sqrt{2}\right)^{\frac{3}{4}} = \sqrt[4]{\left(\sqrt{2}\right)^3} =$

$= \sqrt[4]{\sqrt{2^3}} = \sqrt[8]{2^3} = 2^{\frac{3}{8}}$

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10.1. $\dfrac{2^{\frac{1}{2}} \times 2^{-3}}{\left(2^{-5}\right)^{\frac{1}{2}}} = \dfrac{2^{\frac{1}{2}-3}}{2^{-\frac{5}{2}}} = \dfrac{2^{-\frac{5}{2}}}{2^{-\frac{5}{2}}} = 1$

10.2. $\dfrac{5^{\frac{2}{3}} \times 4^{-\frac{3}{2}}}{5^{-\frac{1}{3}} \times 4^{\frac{1}{2}}} - 5\times 2^{-4} = \dfrac{5^{\frac{2}{3}}}{5^{-\frac{1}{3}}} \times \dfrac{4^{-\frac{3}{2}}}{4^{\frac{1}{2}}} - 5\times 2^{-4} =$

$= 5^{\frac{2}{3}+\frac{1}{3}} \times 4^{-\frac{3}{2}-\frac{1}{2}} - 5\times 2^{-4} =$

$= 5^1 \times 4^{-2} - 5\times 2^{-4} =$

$= 5\times \frac{1}{4^2} - 5\times \frac{1}{2^4} =$

$= \frac{5}{16} - \frac{5}{16} =$

$= 0$

10.3. $\dfrac{100^{\frac{1}{2}}+0{,}1^0}{11^{-3}\times\left(3\times 2^2-1\right)^4}=\dfrac{\sqrt{100}+1}{11^{-3}\times 11^4}=\dfrac{10+1}{11^{-3+4}}=\dfrac{11}{11}=1$

11. $\overline{AB}^2+\overline{AE}^2=\overline{BE}^2$ $\quad\overline{AB}=\overline{AE}$

$2\overline{AB}^2=\left(\sqrt[4]{2}\right)^2\Leftrightarrow\overline{AB}^2=\dfrac{\sqrt[4]{2^2}}{2}\Leftrightarrow\overline{AB}^2=\dfrac{\sqrt{2}}{2}$

$\overline{AB}^2$ é a área do quadrado $[ABDE]$.

Área de $[ABCE]=\dfrac{\sqrt{2}}{2}+\dfrac{1}{2}\dfrac{\sqrt{2}}{2}=$

$=\dfrac{3}{2}\dfrac{\sqrt{2}}{2}=3\dfrac{\sqrt{2}}{4}=3\sqrt{\dfrac{2}{16}}=$

$=3\sqrt{\dfrac{1}{8}}=3\sqrt{\dfrac{1}{2^3}}=3\times\left(2^{-3}\right)^{\frac{1}{2}}=$

$=3\times 2^{-\frac{3}{2}}$

A área do quadrilátero é $\left(3\times 2^{-\frac{3}{2}}\right)$ cm^2.

12. $\overline{AC}^2=\overline{AB}^2+\overline{BC}^2$

$\left(2\sqrt[4]{32}\right)^2=2\overline{AB}^2\Leftrightarrow$

$\Leftrightarrow\overline{AB}^2=\dfrac{4\sqrt[4]{32^2}}{2}\Leftrightarrow$

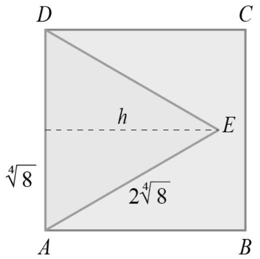
$\Leftrightarrow\overline{AB}^2=2\sqrt{32}\Leftrightarrow$

$\Leftrightarrow\overline{AB}=\sqrt{2\sqrt{32}}\Leftrightarrow$

$\Leftrightarrow\overline{AB}=\sqrt{\sqrt{4\times 32}}\Leftrightarrow$

$\Leftrightarrow\overline{AB}=\sqrt[4]{2^2\times 2^5}\Leftrightarrow\overline{AB}=\sqrt[4]{2^4\times 2^3}\Leftrightarrow$

$\Leftrightarrow\overline{AB}=2\sqrt[4]{8}$



Altura do triângulo, h:

$h^2+\left(\sqrt[4]{8}\right)^2=\left(2\sqrt[4]{8}\right)^2\Leftrightarrow h^2=4\sqrt[4]{8^2}-\sqrt[4]{8^2}\Leftrightarrow$

$\Leftrightarrow h^2=3\sqrt{8}\Leftrightarrow h=\sqrt{\sqrt{9\times 8}}\Leftrightarrow h=\sqrt[4]{72}$

$A_{[AED]}=\dfrac{2\sqrt[4]{8}\times\sqrt[4]{72}}{2}=$

$=\sqrt[4]{8\times 72}=\sqrt[4]{2^3\times 2\times 36}=$

$=\sqrt[4]{2^4\times 6^2}=\sqrt{2^2\times 6}=$

$=2\sqrt{6}=2\times 6^{\frac{1}{2}}$

A área do triângulo é $\left(2\times 6^{\frac{1}{2}}\right)$ cm^2.

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13.1. Altura do prisma: $h=\dfrac{\sqrt{3}}{9}$ m

$A_{\text{base}}=1{,}92$ m^2

Aresta da base: a

$a=\sqrt{1{,}92}=\sqrt{\dfrac{192}{100}}=$

192	2
96	2
48	2
24	2
12	2
6	2
3	3
1	

$=\dfrac{\sqrt{192}}{\sqrt{100}}=\dfrac{\sqrt{2^6\times 3}}{10}=$

$=\dfrac{2^3\times\sqrt{3}}{10}=\dfrac{8\sqrt{3}}{10}$

$=\dfrac{4\sqrt{3}}{5}$

$A_{\text{lateral}}=4\times a\times h=4\times\dfrac{\sqrt{3}}{9}\times\dfrac{4\sqrt{3}}{5}=\dfrac{16\times 3}{45}=\dfrac{16}{15}$

A área da superfície lateral do prisma é $\dfrac{16}{15}$ m^2.

13.2. Seja r o raio da esfera.

$2r=a\Leftrightarrow 2r=\dfrac{4\sqrt{3}}{5}\Leftrightarrow r=\dfrac{2\sqrt{3}}{5}$

$\dfrac{h}{r}=\dfrac{\frac{\sqrt{3}}{9}}{\frac{2\sqrt{3}}{5}}=\dfrac{5\sqrt{3}}{9\times 2\sqrt{3}}=\dfrac{5}{18}$

14. $a=\overline{AF}=\sqrt{24}$

$b=\overline{CD}=9$

$C=\left(\left(\sqrt{24}\right)^{\frac{2}{3}}+9^{\frac{2}{3}}\right)^{\frac{3}{2}}=\left(\sqrt[3]{\left(\sqrt{24}\right)^2}+\sqrt[3]{9^2}\right)^{\frac{3}{2}}=$

$=\left(\sqrt[3]{24}+\sqrt[3]{\left(3^2\right)^2}\right)^{\frac{3}{2}}=\left(\sqrt[3]{2^3\times 3}+\sqrt[3]{3^3\times 3}\right)^{\frac{3}{2}}=$

$=\left(2\sqrt[3]{3}+3\sqrt[3]{3}\right)^{\frac{3}{2}}=\left(5\sqrt[3]{3}\right)^{\frac{3}{2}}=\left(5^3\times\left(\sqrt[3]{3}\right)^3\right)^{\frac{1}{2}}=$

$=\left(5^3\times 3\right)^{\frac{1}{2}}=\left(5\times 5^2\times 3\right)^{\frac{1}{2}}=\left(5^2\right)^{\frac{1}{2}}\times 15^{\frac{1}{2}}=$

$=5\times 15^{\frac{1}{2}}$

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Atividades complementares

15.1. $8^{\frac{1}{3}}=\left(2^3\right)^{\frac{1}{3}}=2^1=2$

15.2. $\left(\dfrac{1}{25}\right)^{0,5}=\left[\left(\dfrac{1}{5}\right)^2\right]^{0,5}=\left(\dfrac{1}{5}\right)^1=\dfrac{1}{5}$

15.3. $\left(\dfrac{64}{27}\right)^{\frac{1}{3}}=\left(\dfrac{4^3}{3^3}\right)^{\frac{1}{3}}=\left(\dfrac{4}{3}\right)^{3\times\frac{1}{3}}=\dfrac{4}{3}$

15.4. $10\,000^{0,25}=\left(10^4\right)^{0,25}=10^1=10$

15.5. $1{,}44^{\frac{1}{2}}=\sqrt{1{,}44}=1{,}2$

15.6. $\left(\dfrac{1}{32}\right)^{0,2}=\left(\dfrac{1}{2^5}\right)^{0,2}=\left[\left(\dfrac{1}{2}\right)^5\right]^{0,2}=\left(\dfrac{1}{2}\right)^{5\times 0,2}=\left(\dfrac{1}{2}\right)^1=\dfrac{1}{2}$

15.7. $64^{\frac{1}{6}}=\left(2^6\right)^{\frac{1}{6}}=2$

15.8. $\left(\dfrac{1}{81}\right)^{0,25}=\left(\dfrac{1}{3^4}\right)^{0,25}=\left[\left(\dfrac{1}{3}\right)^4\right]^{0,25}=\dfrac{1^{4\times 0,25}}{3}=\dfrac{1}{3}$

16.1. $5^{\frac{1}{2}}=\sqrt{5}$

16.2. $\left(\dfrac{1}{2}\right)^{\frac{3}{2}}=\sqrt{\left(\dfrac{1}{2}\right)^3}=\sqrt{\dfrac{1}{8}}$

16.3. $x^{-\frac{1}{2}}=\dfrac{1}{x^{\frac{1}{2}}}=\left(\dfrac{1}{x}\right)^{\frac{1}{2}}=\sqrt{\dfrac{1}{x}},\ x\in\mathbb{Q}^+$

16.4. $2\times\left(\dfrac{3}{2}\right)^{\frac{1}{2}}=2\sqrt{\dfrac{3}{2}}=\sqrt{4\times\dfrac{3}{2}}=\sqrt{6}$

16.5. $\frac{2^{\frac{1}{2}}}{4}=\frac{\sqrt{2}}{4}=\sqrt{\frac{2}{4^2}}=\sqrt{\frac{2}{16}}=\sqrt{\frac{1}{8}}$

16.6. $\frac{a^{\frac{5}{2}}}{2a^2}=\frac{a^{\frac{5}{2}-2}}{2}=\frac{a^{\frac{1}{2}}}{2}=\frac{\sqrt{a}}{2}=\sqrt{\frac{a}{4}}$, $a\in\mathbb{Q}^+$

17.1. $\sqrt[3]{7}=7^{\frac{1}{3}}$ **17.2.** $\sqrt[4]{20}=20^{\frac{1}{4}}$

17.3. $\sqrt[5]{\frac{1}{7}}=\left(\frac{1}{7}\right)^{\frac{1}{5}}$ **17.4.** $\sqrt[8]{\frac{2}{a}}=\left(\frac{2}{a}\right)^{\frac{1}{8}}$, $a>0$

17.5. $\sqrt{0{,}1}=0{,}1^{0{,}5}$ **17.6.** $\sqrt[20]{ab}=(ab)^{\frac{1}{20}}$, $ab>0$

18.1. $1000^{\frac{2}{3}}=\left(10^3\right)^{\frac{2}{3}}=10^2=100$

18.2. $32^{0{,}4}=\left(2^5\right)^{0{,}4}=2^2=4$

18.3. $8^{\frac{4}{3}}=\left(2^3\right)^{\frac{4}{3}}=2^{3\times\frac{4}{3}}=2^4=16$

18.4. $64^{\frac{2}{3}}=\left(2^6\right)^{\frac{2}{3}}=2^{6\times\frac{2}{3}}=2^4=16$

18.5. $9^{1{,}5}=\left(3^2\right)^{1{,}5}=3^{2\times1{,}5}=3^3=27$

18.6. $16^{\frac{5}{4}}=\left(2^4\right)^{\frac{5}{4}}=2^5=32$

19.1. $2^{\frac{3}{4}}=\sqrt[4]{2^3}=\sqrt[4]{8}$

19.2. $4^{\frac{2}{3}}=\sqrt[3]{4^2}=\sqrt[3]{16}$

20.1. $\sqrt[3]{2^2}=2^{\frac{2}{3}}$ **20.2.** $\sqrt[4]{3^5}=3^{\frac{5}{4}}$

20.3. $\sqrt[5]{a^{20}}=a^{\frac{20}{5}}=a^4$

20.4. $\sqrt[3]{2\sqrt{2}}=\sqrt[3]{\sqrt{8}}=\sqrt[6]{8}=\sqrt[6]{2^3}=\sqrt{2}=2^{\frac{1}{2}}$

20.5. $\sqrt[5]{\frac{1}{2}\sqrt[3]{5}}=\sqrt[5]{\sqrt[3]{\left(\frac{1}{2}\right)^3\times5}}=\sqrt[15]{\frac{1}{8}\times5}=\sqrt[15]{\frac{5}{8}}=\left(\frac{5}{18}\right)^{\frac{1}{15}}$

20.6. $\sqrt{\sqrt{\sqrt{3}}}=\sqrt[2\times2\times2]{3}=\sqrt[8]{3}=3^{\frac{1}{8}}$

21.1. $\sqrt{5}=5^{\frac{1}{2}}$ **21.2.** $\sqrt{5^3}=5^{\frac{3}{2}}$

21.3. $\sqrt[3]{625}=\sqrt[3]{25\times25}=\sqrt[3]{5^4}=5^{\frac{4}{3}}$

21.4. $\sqrt{\sqrt{5^3}}=\sqrt[4]{5^3}=5^{\frac{3}{4}}$ **21.5.** $\sqrt[3]{\sqrt[4]{5^5}}=\sqrt[12]{5^5}=5^{\frac{5}{12}}$

21.6. $\sqrt[3]{\sqrt{\sqrt{5}}}=\sqrt[3\times2\times2]{5}=5^{\frac{1}{12}}$

22.1. $\sqrt[3]{\frac{1}{25}}=\left(\frac{1}{5^2}\right)^{\frac{1}{3}}=\left(5^{-2}\right)^{\frac{1}{3}}=5^{-\frac{2}{3}}$

22.2. $\frac{1}{\sqrt{5}}=\frac{1}{5^{\frac{1}{2}}}=\left(\frac{1}{5}\right)^{\frac{1}{2}}=5^{-\frac{1}{2}}$

22.3. $\sqrt[3]{\frac{1}{10^5}}=\left(10^{-5}\right)^{\frac{1}{3}}=10^{-\frac{5}{3}}$

22.4. $\frac{1}{\sqrt[4]{5^3}}=\frac{1}{5^{\frac{3}{4}}}=5^{-\frac{3}{4}}$

23.1. $5^{\frac{2}{3}}\times5^{\frac{1}{2}}=5^{\frac{2}{3}+\frac{1}{2}}=5^{\frac{7}{6}}$

23.2. $10^{\frac{3}{5}}:10^{-\frac{1}{2}}=10^{\frac{3}{5}-\left(-\frac{1}{2}\right)}=10^{\frac{11}{10}}$

23.3. $2^{\frac{1}{3}}\times2^{\frac{3}{4}}:2^{\frac{13}{12}}=2^{\frac{1}{3}+\frac{3}{4}}:2^{\frac{13}{12}}=2^{\frac{13}{12}}:2^{\frac{13}{12}}=1$

23.4. $\frac{\left(\frac{1}{2}\right)^{\frac{1}{3}}\times\left(\frac{1}{2}\right)^{\frac{1}{2}}}{4^{\frac{5}{6}}:2^{\frac{5}{6}}}=\frac{\left(\frac{1}{2}\right)^{\frac{1}{3}+\frac{1}{2}}}{(4:2)^{\frac{5}{6}}}=\frac{\left(\frac{1}{2}\right)^{\frac{5}{6}}}{2^{\frac{5}{6}}}=\left(\frac{1}{2}:2\right)^{\frac{5}{6}}=\left(\frac{1}{4}\right)^{\frac{5}{6}}$

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24.1. $9^{\frac{1}{2}}\times3^{\frac{1}{2}}=27^{\frac{1}{2}}=\sqrt{3^3}=3\sqrt{3}$

24.2. $\left(20^{\frac{1}{5}}:4^{\frac{1}{5}}\right)^2=\left[(20:4)^{\frac{1}{5}}\right]^2=\left(5^{\frac{1}{5}}\right)^2=\sqrt[5]{5^2}=\sqrt[5]{25}$

24.3. $6^{-\frac{2}{3}}\times6^{-\frac{4}{3}}:3^{-2}=6^{-\frac{2}{3}-\frac{4}{3}}:3^{-2}=6^{-2}:3^{-2}=(6:3)^{-2}=$

$=2^{-2}=\frac{1}{4}$

24.4. $\left(9^{\frac{1}{3}}\right)^{-\frac{1}{2}}:3^{-\frac{1}{6}}:3^{-\frac{7}{6}}=$

$=9^{-\frac{1}{6}}:3^{-\frac{1}{6}}:3^{-\frac{7}{6}}=$

$=(9:3)^{-\frac{1}{6}}:3^{-\frac{7}{6}}=$

$=3^{-\frac{1}{6}}:3^{-\frac{7}{6}}=3^{-\frac{1}{6}+\frac{7}{6}}=$

$=3^{\frac{6}{6}}=3$

24.5. $\frac{3^{\frac{1}{3}}\times3^{-2}}{\left(3^{-5}\right)^{\frac{1}{3}}}=\frac{3^{\frac{1}{3}-2}}{3^{-\frac{5}{3}}}=\frac{3^{\frac{1}{3}-\frac{6}{3}}}{3^{-\frac{5}{3}}}=\frac{3^{-\frac{5}{3}}}{3^{-\frac{5}{3}}}=1$

24.6. $\frac{27^{-\frac{3}{2}}\times27^{-3}:27^{-2}}{\sqrt{27^{-1}}}=\frac{27^{-\frac{3}{2}-3}:27^{-2}}{27^{-\frac{1}{2}}}=$

$=\frac{27^{-\frac{9}{2}+2}}{27^{-\frac{1}{2}}}=\frac{27^{-\frac{5}{2}}}{27^{-\frac{1}{2}}}=27^{-\frac{5}{2}+\frac{1}{2}}=$

$=27^{-\frac{4}{2}}=27^{-2}=\frac{1}{27^2}=$

$=\frac{1}{729}$

24.7. $\frac{1000^{\frac{1}{3}}+0{,}01^0}{11^{-\frac{1}{3}}\times\left[3^2\times2-\left(2^3-1\right)\right]^{\frac{1}{3}}}=$

$=\frac{\left(10^3\right)^{\frac{1}{3}}+1}{11^{-\frac{1}{3}}\times(18-7)^{\frac{1}{3}}}=\frac{10^1+1}{11^{-\frac{1}{3}}\times11^{\frac{1}{3}}}=$

$=\frac{11}{-\frac{1}{3}+\frac{1}{3}}=\frac{11}{11^0}=$

$=11$

24.8. $\frac{\sqrt{a^3}:\sqrt[3]{a^2}}{a^{\frac{2}{3}}:\sqrt{a}}=\frac{a^{\frac{3}{2}}:a^{\frac{2}{3}}}{a^{\frac{2}{3}}:a^{\frac{1}{2}}}=$

$=\frac{a^{\frac{3}{2}-\frac{2}{3}}}{a^{\frac{2}{3}-\frac{1}{2}}}=a^{\frac{3}{2}-\frac{2}{3}-\frac{2}{3}+\frac{1}{2}}=$

$=a^{\frac{4}{2}-\frac{4}{3}}=a^{2-\frac{4}{3}}=a^{\frac{2}{3}}=$

$=\sqrt[3]{a^2}$, $a>0$

25.1. $\left(8^{\frac{1}{2}}\right)^3 = \left(2^3\right)^{\frac{3}{2}} = 2^{\frac{9}{2}} = 2^{\frac{8}{2}+\frac{1}{2}} = 2^4 \times 2^{\frac{1}{2}} = 16\sqrt{2}$

25.2. $\left((-2)^4\right)^{\frac{1}{3}} = \left(2^4\right)^{\frac{1}{3}} = 2^{\frac{4}{3}} = 2^{1+\frac{1}{3}} = 2 \times 2^{\frac{1}{3}} = 2\sqrt[3]{2}$

25.3. $\left[\left(\frac{2}{3}\right)^{\frac{1}{4}}\right]^{-\frac{1}{2}} = \left(\frac{2}{3}\right)^{-\frac{1}{8}} = \left(\frac{3}{2}\right)^{\frac{1}{8}} = \frac{\sqrt[3]{12}}{2} =$

$= \frac{\sqrt[3]{2}}{\sqrt[3]{2}} = \frac{\sqrt[3]{2}\sqrt[3]{2^2}}{\sqrt[3]{2} \times \sqrt[3]{2^2}} = \frac{\sqrt[3]{3 \times 2^2}}{2} = \frac{\sqrt[3]{12}}{2}$

25.4. $\left[\left(\frac{8}{3}\right)^{\frac{1}{2}}\right]^{-\frac{1}{3}} = \left(\frac{8}{3}\right)^{-\frac{1}{6}} = \left(\frac{3}{8}\right)^{\frac{1}{6}} = \sqrt[6]{\frac{3}{8}} = \frac{\sqrt[6]{3}}{\sqrt[6]{8}} =$

$= \frac{\sqrt[6]{3}}{\sqrt[6]{2^3}} = \frac{\sqrt[6]{3}}{\sqrt{2}} = \frac{\sqrt[6]{3}\sqrt{2}}{\sqrt{2}\sqrt{2}} = \frac{\sqrt[6]{3}\sqrt[6]{2^3}}{2} =$

$= \frac{\sqrt[6]{3 \times 2^3}}{2} = \frac{\sqrt[6]{24}}{2}$

25.5. $\frac{\left(a^{\frac{3}{2}}\right)^{\frac{1}{3}}}{\left(a^5\right)^{\frac{1}{6}}} = \frac{a^{\frac{1}{2}}}{a^{\frac{5}{6}}} = a^{\frac{1}{2}-\frac{5}{6}} = a^{-\frac{1}{3}} =$

$= \frac{1}{a^{\frac{1}{3}}} = \frac{1}{\sqrt[3]{a}} = \frac{\sqrt[3]{a^2}}{\sqrt[3]{a} \times \sqrt[3]{a^2}} = \frac{\sqrt[3]{a^2}}{a}$, $a > 0$

25.6. $\frac{\left(a^{\frac{1}{6}}\right)^4}{a^{-\frac{1}{2}}} = \frac{a^{\frac{4}{6}}}{a^{-\frac{1}{2}}} = a^{\frac{4}{6}+\frac{3}{6}} = a^{\frac{7}{6}} = a^{1+\frac{1}{6}} = a \times a^{\frac{1}{2}} = a\sqrt[6]{a}$, $a > 0$

26.1. $\frac{\left(3^{\frac{1}{2}} + 2^{\frac{1}{2}}\right)^2 - 5}{\sqrt{3\sqrt{2}}} = \frac{\left(3^{\frac{1}{2}}\right)^2 + 2 \times 3^{\frac{1}{2}} \times 2^{\frac{1}{2}} + \left(2^{\frac{1}{2}}\right)^2 - 5}{\sqrt{\sqrt{3^2 \times 2}}} =$

$= \frac{3^1 + 2 \times 6^{\frac{1}{2}} + 2 - 5}{\sqrt[6]{18}} =$

$= \frac{2\sqrt{6}}{\sqrt[4]{18}} = \frac{2\sqrt[4]{6^2}}{\sqrt[4]{18}} =$

$= 2\sqrt[4]{\frac{36}{18}} = 2\sqrt[4]{2}$

26.2. $\frac{\left(3^2 \times 5\right)^{\frac{1}{2}} - \left(5 \times 5^{\frac{1}{2}}\right)^{\frac{1}{3}}}{\left(2^2 \times 5\right)^{\frac{1}{2}} : 4^{\frac{1}{4}}} = \frac{\left(3^2\right)^{\frac{1}{2}} \times 5^{\frac{1}{2}} - \left(5^1 \times 5^{\frac{1}{2}}\right)^{\frac{1}{3}}}{\left(2^2\right)^{\frac{1}{2}} \times 5^{\frac{1}{2}} : \left(2^2\right)^{\frac{1}{4}}} =$

$= \frac{3\sqrt{5} - \left(5^{\frac{3}{2}}\right)^{\frac{1}{3}}}{2\sqrt{5} : 2^{\frac{1}{2}}} = \frac{3\sqrt{5} - 5^{\frac{1}{2}}}{2\sqrt{5} : \sqrt{2}} = \frac{3\sqrt{5} - \sqrt{5}}{2\sqrt{5} : \sqrt{2}} = \frac{2\sqrt{5}}{2\frac{\sqrt{5}}{\sqrt{2}}} =$

$= \frac{\sqrt{5} \times \sqrt{2}}{\sqrt{5}} = \sqrt{2}$

26.3. $\left(\sqrt{\frac{2}{2^{\frac{1}{3}}}}\right)^{\frac{1}{4}} = \left(\sqrt{2^{1-\frac{1}{3}}}\right)^{\frac{1}{4}} = \left(\sqrt{2^{\frac{2}{3}}}\right)^{\frac{1}{4}} = \left(\left(2^{\frac{2}{3}}\right)^{\frac{1}{2}}\right)^{\frac{1}{4}} =$

$= 2^{\frac{1}{12}} = \sqrt[12]{2}$

26.4. $\left[\sqrt{2\sqrt{2\sqrt{2}}} : \left(\sqrt[8]{2}\right)^3\right]^{\frac{1}{2}} =$

$= \left[\sqrt{2\sqrt{\sqrt{8}}} : \left(2^{\frac{3}{8}}\right)\right]^{\frac{1}{2}} =$

$= \left(\sqrt{2\sqrt[4]{8}} : 2^{\frac{3}{8}}\right)^{\frac{1}{2}} =$

$= \left(\sqrt{\sqrt{2^4 \times 2^3}} : 2^{\frac{3}{8}}\right)^{\frac{1}{2}} =$

$= \left(\sqrt{2^7} : 2^{\frac{3}{8}}\right)^{\frac{1}{2}} =$

$= \left(2^{\frac{7}{8}} : 2^{\frac{3}{8}}\right)^{\frac{1}{2}} = \left(2^{\frac{7}{8}-\frac{3}{8}}\right)^{\frac{1}{2}} = \left(2^{\frac{4}{8}}\right)^{\frac{1}{2}} =$

$= 2^{\frac{1}{4}} = \sqrt[4]{2}$

26.5. $\left(\sqrt{\frac{1}{a}} \times a^{\frac{2}{3}}\right) : \sqrt[6]{\sqrt{a}} =$

$= \left[\left(\frac{1}{a}\right)^{\frac{1}{2}} \times a^{\frac{2}{3}}\right] : \sqrt[12]{a} =$

$= \left[\left(a^{-1}\right)^{\frac{1}{2}} \times a^{\frac{2}{3}}\right] : a^{\frac{1}{12}} =$

$= \left(a^{-\frac{1}{2}} \times a^{\frac{2}{3}}\right) : a^{\frac{1}{12}} =$

$= a^{-\frac{1}{2}+\frac{2}{3}} : a^{\frac{1}{12}} =$

$= a^{\frac{1}{6}} : a^{\frac{1}{12}} = a^{\frac{1}{6}-\frac{1}{12}} =$

$= a^{\frac{1}{12}} = \sqrt[12]{a}$, $a > 0$

27. Seja $\overline{EF} = a$.

27.1. $a^2 + a^2 = d^2 \Leftrightarrow$

$\Leftrightarrow 2a^2 = d^2 \Leftrightarrow$

$\Leftrightarrow a^2 = \frac{d^2}{2} \Leftrightarrow$

$\Leftrightarrow a = \sqrt{\frac{d^2}{2}} \Leftrightarrow$

$\Leftrightarrow a = d\sqrt{\frac{1}{2}} \Leftrightarrow a = d \times \left(\frac{1}{2}\right)^{\frac{1}{2}} \Leftrightarrow$

$\Leftrightarrow a = d \times 2^{-\frac{1}{2}}$, $a > 0$ e $d > 0$

A medida da aresta da base do prisma é $d \times 2^{-\frac{1}{2}}$.

27.2. $\overline{AE} = d$

$V_{\text{prisma}} = A_{\text{base}} \times \text{altura} = a^2 \times d = \left(d \times 2^{-\frac{1}{2}}\right)^2 \times d =$

$= d^2 \times \left(2^{-\frac{1}{2}}\right)^2 \times d = d^3 \times 2^{-1} = \frac{d^3}{2}$

$V_{\text{prisma}} = 27$

$\frac{d^3}{2} = 27 \Leftrightarrow d^3 = 54$

Sabemos que $2a^2 = d^2$.

$d=\sqrt{2a^2}=\sqrt{2}a$ e $d^3=\left(\sqrt{2}a\right)^3$

$d^3=\left(\sqrt{2}\right)^3a^3 \Leftrightarrow d^3=2\sqrt{2}a^3$

$d^3=54 \Leftrightarrow 2\sqrt{2}a^3=54 \Leftrightarrow$

$\Leftrightarrow a^3=\frac{54}{2\sqrt{2}} \Leftrightarrow a^3=\frac{27\sqrt{2}}{\sqrt{2}\times\sqrt{2}} \Leftrightarrow$

$\Leftrightarrow a^3=\frac{27\sqrt{2}}{2} \Leftrightarrow a=\sqrt[3]{\frac{27\sqrt{2}}{2}} \Leftrightarrow$

$\Leftrightarrow a=\sqrt[3]{3^3\times\frac{\sqrt{2}}{2}} \Leftrightarrow a=3\sqrt[3]{\sqrt{\frac{2}{4}}} \Leftrightarrow$

$\Leftrightarrow a=3\sqrt[6]{\frac{1}{2}} \Leftrightarrow a=3\times\left(\frac{1}{2}\right)^{\frac{1}{6}} \Leftrightarrow$

$\Leftrightarrow a=3\times 2^{-\frac{1}{6}}$

A medida da aresta da base do prisma é $3\times 2^{-\frac{1}{6}}$.

28. $a=2^{2,5}=2^{2+\frac{1}{2}}=2^2\times 2^{\frac{1}{2}}=4\sqrt{2}$

$b=8^{0,5}=\left(2^3\right)^{\frac{1}{2}}=\sqrt{2^2\times 2}=2\sqrt{2}$

$c=3\times 2^{\frac{1}{2}}=3\sqrt{2}$

$s=\frac{a+b+c}{2}=\frac{4\sqrt{2}+2\sqrt{2}+3\sqrt{2}}{2}=\frac{9}{2}\sqrt{2}$

$A=\sqrt{\frac{9}{2}\sqrt{2}\left(\frac{9}{2}\sqrt{2}-4\sqrt{2}\right)\left(\frac{9}{2}\sqrt{2}-2\sqrt{2}\right)\left(\frac{9}{2}\sqrt{2}-3\sqrt{2}\right)}=$

$=\sqrt{\frac{9}{2}\sqrt{2}\times\frac{1}{2}\sqrt{2}\times\frac{5}{2}\sqrt{2}\times\frac{3}{2}\sqrt{2}}=$

$=\sqrt{\frac{9}{2}\times\frac{1}{2}\times\frac{5}{2}\times\frac{3}{2}\times\left(\sqrt{2}\right)^4}=$

$=\sqrt{\frac{9}{4}}\times\sqrt{\frac{15}{4}}\times\sqrt{2^2}=$

$=\frac{3}{2}\times\frac{\sqrt{15}}{2}\times 2=\frac{3}{2}\sqrt{15}$

A área do triângulo [*ABC*] é $\frac{3}{2}\sqrt{15}$ cm².

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29. *a*: $\left(2{,}3^{\frac{1}{2}}\right)^2=2{,}3^1=2{,}3$ (Proposição verdadeira)

b: $\left(\frac{9}{49}\right)^{-0,5}=\left(\frac{49}{9}\right)^{\frac{1}{2}}=\sqrt{\frac{49}{9}}=\frac{7}{3}$ (Proposição falsa)

c: $\left(\frac{1}{2}\right)^{\frac{1}{6}}\times\left(\frac{1}{2}\right)^{\frac{1}{3}}=\left(\frac{1}{2}\right)^{\frac{1}{6}+\frac{1}{3}}=\left(\frac{1}{2}\right)^{\frac{1}{2}}=\sqrt{\frac{1}{2}}$

(Proposição verdadeira)

d: $\left(\frac{1}{2}\right)^{\frac{1}{3}}\times\left(\frac{1}{2}\right)^{\frac{1}{6}}:\sqrt{\frac{1}{2}}=\left(\frac{1}{2}\right)^{\frac{1}{2}}:\left(\frac{1}{2}\right)^{\frac{1}{2}}=1$ (Proposição falsa)

e: $\frac{50^{\frac{1}{2}}}{2^{\frac{1}{2}}}\times 4^{\frac{1}{2}}=\left(\frac{50}{2}\right)^{\frac{1}{2}}\times\sqrt{4}=\sqrt{25}\times 2=5\times 2=10$

(Proposição verdadeira)

f: $\left(\sqrt{\frac{1}{4}}\right)^{\frac{1}{3}}=\left(\frac{1}{2}\right)^{\frac{1}{3}}=2^{-\frac{1}{3}}$ (Proposição verdadeira)

g: $1000^{-\frac{2}{3}}=\left(10^3\right)^{-\frac{2}{3}}=10^{-2}=\frac{1}{100}$ (Proposição verdadeira)

h: $\sqrt[3]{\sqrt{a^3}}=\sqrt[6]{a^3}=\sqrt{a}=a^{\frac{1}{2}}$ (Proposição verdadeira)

i: $\sqrt{a\sqrt[3]{\frac{1}{a}}}=\sqrt{\sqrt[3]{a^3\times\frac{1}{a}}}=\sqrt[6]{a^2}=\sqrt[3]{a}=a^{\frac{1}{3}}$

(Proposição verdadeira)

30. $\forall a,\ b\in\mathbb{R},\ \left(a^4b^8\right)^{\frac{1}{4}}=ab^2$

Por exemplo, se $a=-2$ e $b=1$:

$\left(a^4b^8\right)^{\frac{1}{4}}=\left((-2)^4\times 1^8\right)^{\frac{1}{4}}=\left(2^4\right)^{\frac{1}{4}}=2$ e $ab^2=-2\times 1^2=-2$

31.1. $\frac{23\times 3^{\frac{1}{4}}+2\times 3^{\frac{1}{4}}}{5\sqrt{3}}=\frac{(23+2)\times 3^{\frac{1}{4}}}{5\times 3^{\frac{1}{2}}}=\frac{25}{5}\times 3^{\frac{1}{4}-\frac{1}{2}}=$

$=5\times 3^{-\frac{1}{4}}=5\times\left(\sqrt[4]{3}\right)^{-1}=5\times\frac{1}{\sqrt[4]{3}}=5\times\frac{\sqrt[4]{3^3}}{\sqrt[4]{3}\sqrt[4]{3^3}}=\frac{5}{3}\sqrt[4]{27}$

31.2. $\frac{16^{-\frac{3}{4}}\times 8^{\frac{5}{3}}}{2^{-\frac{1}{2}}}:\sqrt[3]{2}=\frac{\left(2^4\right)^{-\frac{3}{4}}\times\left(2^3\right)^{\frac{5}{3}}}{2^{-\frac{1}{2}}}:2^{\frac{1}{3}}=$

$=\frac{2^{-3}\times 2^5}{2^{-\frac{1}{2}}}:2^{\frac{1}{3}}=\frac{2^2}{2^{-\frac{1}{2}}}:2^{\frac{1}{3}}=$

$=2^{2+\frac{1}{2}}:2^{\frac{1}{3}}=2^{2+\frac{1}{2}-\frac{1}{3}}=2^{2+\frac{1}{6}}=$

$=2^2\times 2^{\frac{1}{6}}=4\sqrt[6]{2}$

31.3. $\frac{\sqrt{18}-\sqrt{50}+\sqrt{12}-\sqrt{75}}{2^{\frac{3}{2}}+3^{\frac{3}{2}}}=$

$=\frac{\sqrt{9\times 2}-\sqrt{25\times 2}+\sqrt{4\times 3}-\sqrt{25\times 3}}{\sqrt{2^3}+\sqrt{3^3}}=$

$=\frac{3\sqrt{2}-5\sqrt{2}+2\sqrt{3}-5\sqrt{3}}{\sqrt{2^2\times 2}+\sqrt{3^2\times 3}}=\frac{2\sqrt{2}-3\sqrt{2}}{2\sqrt{2}+3\sqrt{2}}=$

$=\frac{-\left(2\sqrt{2}+3\sqrt{2}\right)}{2\sqrt{2}+3\sqrt{2}}=-1$

32. Seja *a* a aresta do cubo.

$A_b=a^2$; $V=a^3 \Leftrightarrow a=\sqrt[3]{V}$

Então, $A_b=a^2=\left(\sqrt[3]{V}\right)^2=\sqrt[3]{V^2}=V^{\frac{2}{3}}$.

$A_b=V^{\frac{2}{3}}$

33. Seja *a* a aresta do cubo.

$a=2r \Leftrightarrow r=\frac{a}{2}$

33.1. $V=a^3 \Leftrightarrow V=(2r)^3 \Leftrightarrow V=8r^3 \Leftrightarrow r^3=\frac{V}{8} \Leftrightarrow$

$\Leftrightarrow r=\sqrt[3]{\frac{V}{8}} \Leftrightarrow r=\left(\frac{V}{8}\right)^{\frac{1}{3}}$

33.2. $V_{\text{esfera}}=\frac{4}{3}\pi r^3=\frac{4}{3}\pi\left(\frac{a}{2}\right)^3=\frac{4}{3}\pi\times\frac{a^3}{8}=\frac{4}{3}\times\frac{1}{8}\times\pi\times V=\frac{\pi}{6}V$

34. Sejam $r=4$ cm e $h=2$ cm.

$A=\pi\times 4\times\left(4^2+2^2\right)^{\frac{1}{2}}=4\pi(20)^{\frac{1}{2}}=4\pi\sqrt{20}=4\pi\sqrt{4\times 5}=$

$=4\pi\times 2\sqrt{5}=8\pi\sqrt{5}$

A área da superfície do cone é $8\pi\sqrt{5}$ cm².

Avaliação 2

1. • $5^{\frac{1}{3}} \times 5^{\frac{1}{3}} = \left(5^{\frac{1}{3}}\right)^2 = 5^{\frac{2}{3}} = \sqrt[3]{5^2}$

• $5^{\frac{1}{6}} : 5^{\frac{1}{3}} = 5^{\frac{1}{6}-\frac{1}{3}} = 5^{-\frac{3}{10}}$

• $\frac{2\sqrt[6]{5}}{5^{\frac{1}{2}}} = \frac{2\sqrt[6]{5}}{\sqrt{5}} = \frac{2\sqrt[6]{5}}{\sqrt[6]{5^3}} = 2\sqrt[6]{\frac{5}{5^3}} = 2\sqrt[6]{\frac{1}{5^2}} = 2\sqrt[3]{\frac{1}{5}}$

• $10^{\frac{1}{3}} : \left(2^{\frac{1}{2}}\right)^{\frac{2}{3}} = 10^{\frac{1}{3}} : 2^{\frac{1}{3}} = (10:2)^{\frac{1}{3}} = 5^{\frac{1}{3}} = \sqrt[3]{5}$

Resposta: (D)

2. $A \times B = \left(4+8^{\frac{1}{2}}\right)^{\frac{1}{3}} \times \left(4-8^{\frac{1}{2}}\right)^{\frac{1}{3}} =$

$= \left[\left(4+8^{\frac{1}{2}}\right)\left(4-8^{\frac{1}{2}}\right)\right]^{\frac{1}{3}} =$

$= \left[4^2 - \left(8^{\frac{1}{2}}\right)^2\right]^{\frac{1}{3}} = (16-8)^{\frac{1}{3}} =$

$= 8^{\frac{1}{3}} = \left(2^3\right)^{\frac{1}{3}} = 2$

Resposta: (B)

3. $\left(2\sqrt{2}\right)^{\frac{1}{2}} : 2^{\frac{1}{2}} = \left(2\sqrt{2}:2\right)^{\frac{1}{2}} = \left(\sqrt{2}\right)^{\frac{1}{2}} = \left(2^{\frac{1}{2}}\right)^{\frac{1}{2}} = 2^{\frac{1}{4}}$

Resposta: (C)

4. $\left[(5xy)^{\frac{1}{2}}\right]^{\frac{2}{3}} \times \left(25x^2y^2\right)^{\frac{1}{3}} \times \sqrt{x^2} =$

$= (5xy)^{\frac{1}{3}}\left(25x^2y^2\right)^{\frac{1}{3}} \times x =$

$= \left(5xy \times 25x^2y^2\right)^{\frac{1}{3}} \times x =$

$= \left(5^3x^3y^3\right)^{\frac{1}{3}} x =$

$= 5xy \times x = 5x^2y$

Resposta: (A)

5. $\sqrt{2^{2n}+2^{2n}} = \sqrt{2 \times 2^{2n}} = \sqrt{2^{2n+1}} = \left(2^{2n+1}\right)^{\frac{1}{2}} =$

$= 2^{\frac{1}{2}(2n+1)} = 2^{n+\frac{1}{2}}$

Resposta: (B)

6. $2 \times 4^n = 2 \times \left(2^2\right)^n = 2^1 \times 2^{2n} = 2^{2n+1}$

Resposta: (C)

7. Se $a \in \mathbb{N}$, então $a > 0$ e $-a < 0$.

• $(-a)^3 < 0$ e $(-2a)^{-5} < 0$. Logo, $(-a)^3 \times (-2a)^{-5} > 0$.

• $\left(\frac{1}{2}\right)^{-a} > 0$ e $(-a)^{\frac{1}{3}} < 0$. Logo, $\left(\frac{1}{2}\right)^{-a} \times (-a)^{\frac{1}{3}} < 0$.

Resposta: (B)

8. $1^{\frac{1}{2}} = 1$. Logo, $\exists x \in \mathbb{R}^+ : x^{\frac{1}{2}} = x$

Resposta: (A)

9. $x^3 = 3^{\frac{1}{2}} \Leftrightarrow x = \sqrt[3]{3^{\frac{1}{2}}} \Leftrightarrow x = \sqrt[3]{\sqrt{3}} \Leftrightarrow x = \sqrt[6]{3}$

Resposta: (C)

10. • $a^{\frac{1}{n}} \times a^{\frac{1}{n}} = \left(a^{\frac{1}{n}}\right)^2 = a^{\frac{2}{n}}$

• $a^{\frac{1}{2n}} \times a^{\frac{1}{2n}} = \left(a^{\frac{1}{2n}}\right)^2 = a^{\frac{2}{2n}} = a^{\frac{1}{n}}$

Resposta: (B)

11.1. $\frac{2^{\frac{1}{2}} \times 3^{\frac{1}{2}} : 6^{\frac{2}{3}}}{\left(6^{\frac{1}{2}} \times 6^{\frac{1}{3}}\right)^{\frac{3}{5}}} = \frac{(2\times 3)^{\frac{1}{2}} : 6^{\frac{2}{3}}}{\left(6^{\frac{1}{2}+\frac{1}{3}}\right)^{\frac{3}{5}}} = \frac{6^{\frac{1}{2}} : 6^{\frac{2}{3}}}{\left(6^{\frac{5}{6}}\right)^{\frac{3}{5}}} =$

$= \frac{6^{\frac{1}{2}-\frac{2}{3}}}{6^{\frac{1}{2}}} = 6^{\frac{1}{2}-\frac{2}{3}-\frac{1}{2}} = 6^{-\frac{2}{3}} =$

$= \frac{1}{6^{\frac{2}{3}}} = \frac{1}{\sqrt[3]{6^2}} = \frac{\sqrt[3]{6}}{\sqrt[3]{6^2}\sqrt[3]{6}} = \frac{\sqrt[3]{6}}{6}$

11.2. $\frac{2^{\frac{1}{2}} \times 3}{\sqrt{2} \times 3^{-2}} \times \left(\frac{1}{2}\right)^2 = \frac{\sqrt{2} \times 3}{\sqrt{2} \times 3^{-2}} \times \frac{1}{4} =$

$= \frac{3}{3^{-2}} \times \frac{1}{4} = 3 \times 3^2 \times \frac{1}{4} = \frac{27}{4}$

11.3. $\left(\sqrt[2n]{(a-1)^2}\right)^2 : (a-1)^{\frac{1}{n}} =$

$= \left(\sqrt[n]{a-1}\right)^2 : (a-1)^{\frac{1}{n}} = \sqrt[n]{(a-1)^2} : (a-1)^{\frac{1}{n}} =$

$= (a-1)^{\frac{2}{n}-\frac{1}{n}} = (a-1)^{\frac{1}{n}} = \sqrt[n]{a-1},\ \ a-1>0$ e $n \in \mathbb{N}$

12. $\overline{EH} = \sqrt[4]{32}$

$\overline{DE}^2 + \overline{DH}^2 = \overline{EH}^2$

Como $\overline{DE} = \overline{DH}$:

$2\overline{DE}^2 = \left(\sqrt[4]{32}\right)^2 \Leftrightarrow \overline{DE}^2 = \frac{\sqrt[4]{32^2}}{2} \Leftrightarrow \overline{DE}^2 = \frac{\sqrt{32}}{3} \Leftrightarrow$

$\Leftrightarrow \overline{DE}^2 = \sqrt{\frac{32}{4}} \Leftrightarrow \overline{DE}^2 = \sqrt{8} \Leftrightarrow$

$\Leftrightarrow \overline{DE} = \sqrt{\sqrt{8}} \Leftrightarrow \overline{DE} = \sqrt[4]{8}$

$A_{\text{colorida}} = \left(2\overline{DE}\right)^2 - \left(\overline{EH}\right)^2 = \left(2\sqrt[4]{8}\right)^2 - \left(\sqrt[4]{32}\right)^2 =$

$= 4\sqrt[4]{8^2} - \sqrt[4]{32^2} = 4\sqrt{8} - \sqrt{32} =$

$= 4\sqrt{4\times 2} - \sqrt{16 \times 2} = 8\sqrt{2} - 4\sqrt{2} =$

$= 4\sqrt{2}$

ou

$A_{\text{colorida}} = 4 \times \frac{\overline{DE} \times \overline{DE}}{2} = 2 \times \sqrt[4]{8} \times \sqrt[4]{8} =$

$= 2 \times \sqrt[4]{8^2} = 2\sqrt{8} = 4\sqrt{2}$

13. $\overline{AG} = 3^{\frac{1}{4}}$

Seja a aresta do cubo.

$\overline{AG} = \pm\sqrt{a^2+a^2+a^2}$

$\sqrt{3a^2} = 3^{\frac{1}{4}} \Leftrightarrow \sqrt{3}a = 3^{\frac{1}{4}} \Leftrightarrow a = \frac{3^{\frac{1}{4}}}{\sqrt{3}} \Leftrightarrow$

$\Leftrightarrow d = \frac{3^{\frac{1}{4}}}{3^{\frac{1}{2}}} \Leftrightarrow a = 3^{\frac{1}{4}-\frac{1}{2}} \Leftrightarrow a = 3^{-\frac{1}{4}}$

$$V = a^3 = \left(3^{-\frac{1}{4}}\right)^3 = 3^{-\frac{3}{4}} = \frac{1}{3^{\frac{3}{4}}} = \frac{1}{\sqrt[4]{3^3}} = \frac{1 \times \sqrt[4]{3}}{\sqrt[4]{3^3}\sqrt[4]{3}} = \frac{\sqrt[4]{3}}{3}$$

A medida do volume do cubo é $\frac{\sqrt[4]{3}}{3}$.

14. $\overline{AD} = a$

14.1. $\overline{HD}^2 + \overline{AD}^2 = \overline{HA}^2 \Leftrightarrow$

$\Leftrightarrow \overline{HD}^2 + a^2 = \left(\sqrt{5}a\right)^2 \Leftrightarrow$

$\Leftrightarrow \overline{HD}^2 = 5a^2 - a^2 \Leftrightarrow$

$\Leftrightarrow \overline{HD} = \sqrt{4a^2} \Leftrightarrow \quad \left| \left(\overline{HD} > 0 \text{ e } a > 0\right)\right.$

$\Leftrightarrow \overline{HD} = 2a$

Altura do prisma: $2a$

$V = A_{base} \times \text{altura}$

$= a^2 \times 2a = 2a^3$

A medida do volume do prisma é $2a^3$.

14.2. Considerando $[ACD]$ para base, temos:

$$V_{\text{pirâmide}} = \frac{1}{3} \times \frac{\overline{AD} \times \overline{DC}}{2} \times \overline{HD} =$$

$$= \frac{1}{3} \times \frac{a \times a}{2} \times 2a =$$

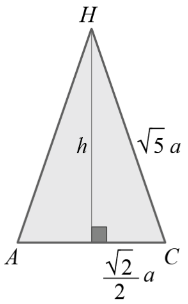
$$= \frac{1}{3}a^3$$

A medida do volume da pirâmide é $\frac{1}{3}a^3$.

14.3. $\overline{AC} = \sqrt{a^2 + a^2} = \sqrt{2a^2} = \sqrt{2}a$

$\overline{AH} = \overline{HC} = \sqrt{5}a$

Seja h a altura do triângulo.



$$h^2 + \left(\frac{\sqrt{2}}{2}a\right)^2 = \left(\sqrt{5}a\right)^2 \Leftrightarrow$$

$$\Leftrightarrow h^2 = 5a^2 - \frac{1}{2}a^2 \Leftrightarrow$$

$$\Leftrightarrow h^2 = \sqrt{\frac{9}{2}a^2} \Leftrightarrow h = \frac{3}{\sqrt{2}}a$$

$$A_{[ACH]} = \frac{\overline{AC} \times h}{2} = \frac{\sqrt{2}a \times \frac{3}{\sqrt{2}}a}{2} = \frac{3}{2}a^2$$

A medida da área do triângulo $[ACH]$ é $\frac{3}{2}a^2$.

14.4. Se a base da pirâmide é $[ACH]$ e a altura é x, então o seu volume é:

$$V = \frac{1}{3} \times A_{[ACH]} \times x$$

$$= \frac{1}{3} \times \frac{3}{2}a^2 \times x$$

$$= \frac{1}{2}a^2 x$$

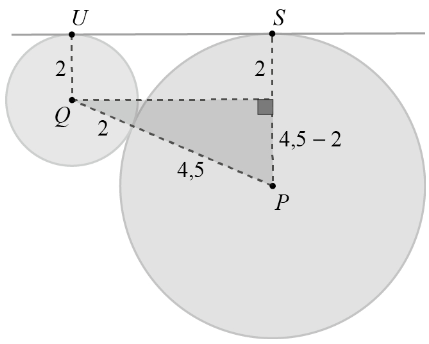
Sabemos que $V = \frac{1}{3}a^3$.

$$\frac{1}{2}a^2 x = \frac{1}{3}a^3 \Leftrightarrow 3a^2 x = 2a^3 \Leftrightarrow x = \frac{2a^3}{3a^2} \Leftrightarrow$$

$$\Leftrightarrow x = \frac{2}{3}a$$

A altura da pirâmide relativa à base $[ACH]$ é $\frac{2}{3}a$

15.

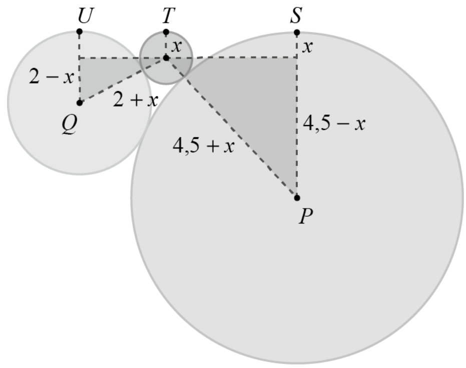


$$\overline{SU}^2 + (4{,}5 - 2)^2 = (4{,}5 + 2)^2 \Leftrightarrow$$

$$\Leftrightarrow \overline{SU}^2 = 6.5^2 - 2{,}5^2 \Leftrightarrow$$

$$\Leftrightarrow SU = \sqrt{42{,}25 - 6{,}25} \Leftrightarrow$$

$$\Leftrightarrow \overline{SU} = 6$$



Aplicando o Teorema de Pitágoras:

$$\overline{ST}^2 = (4{,}5 + x)^2 - (4{,}5 - x)^2 =$$

$$= (4{,}5)^2 + 9x + x^2 - (4{,}5)^2 + 9x - x^2 =$$

$$= 18x$$

$$\overline{ST} = \sqrt{18x}$$

$$\overline{TU}^2 = (2 + x)^2 - (2 - x)^2 =$$

$$= 4 + 4x + x^2 - 4 + 4x - x^2 =$$

$$= 8x$$

$$\overline{TU} = \sqrt{8x}$$

Sabemos que $\overline{SU} = 6$ e $\overline{SU} = \overline{ST} + \overline{TU}$

Logo:

$$\overline{ST} + \overline{TU} = 6$$

$$\sqrt{18x} + \sqrt{8x} = 6 \Leftrightarrow$$

$$\Leftrightarrow \sqrt{9 \times 2x} + \sqrt{4 \times 2x} = 6 \Leftrightarrow$$

$$\Leftrightarrow 3 \times \sqrt{2x} + 2\sqrt{2x} = 6$$

$$\Leftrightarrow 5\sqrt{2x} = 6 \Leftrightarrow \sqrt{2x} = \frac{6}{5}$$

Como $x > 0$:

$$2x = \frac{36}{25} \Leftrightarrow x = \frac{18}{25} \Leftrightarrow x = 0{,}72$$

O raio da circunferência de centro R é igual a 0,72 cm.

2.3. Operações com polinómios

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Atividade inicial 3

1.1. Monómios constantes: $C(x)=-1$ e $D(x)=0$

1.2. $D(x)=0$

1.3. Monómios na forma canónica: $A(x)=\frac{1}{2}x^2$, $C(x)=-1$ e $D(x)=0$

1.4. $B(x)=\frac{1}{6}x\times(-3)x=-\frac{3}{6}x^2=-\frac{1}{2}x^2=E(x)$

$B(x)$ e $E(x)$ são monómios iguais.

1.5. Os monómios $A(x)=\frac{1}{2}x^2$ e $E(x)=-\frac{1}{2}x^2$ são semelhantes mas não iguais.

2.

Monómio	Forma canónica	Parte numérica ou coeficiente	Parte literal	Grau
$A(x)$	$\frac{1}{2}x^2$	$\frac{1}{2}$	x^2	2
$B(x)$	$-\frac{1}{2}x^2$	$-\frac{1}{2}$	x^2	2
$C(x)$	– 1	– 1	Não tem	0
$D(x)$	0	0	Não tem	Indeterminado
$E(x)$	$-\frac{1}{2}x^2$	$-\frac{1}{2}$	x^2	2

3. $A(x)+B(x)=\frac{1}{2}x^2-\frac{1}{2}x^2=0$. $A(x)$ + $B(x)$ é o monómio nulo.

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1.1. $2x^2+3+\frac{x^2}{2}-\frac{1}{2}x+x^4=x^4+\left(2+\frac{1}{2}\right)x^2-\frac{1}{2}x+3=$

$=x^4+\frac{5}{2}x^2-\frac{1}{2}x+3$

Grau: 4, termos nulos: $0x^3$; termo independente: 3

1.2. $\frac{2x^2-2}{2}+x^3-x^2=\frac{2x^2}{2}-\frac{2}{2}+x^3-x^2=x^3-1$

Grau: 3, termos nulos: $0x^2$ e $0x$; termo independente: – 1

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2. $A(x)=\frac{1}{2}x^2+\frac{2}{3}x-1$; $B(x)=3x^3-\frac{1}{2}x^2+\frac{1}{3}x+3$; $C(x)=6x^2-12$

2.1. $A(x)+B(x)=\left(\frac{1}{2}x^2+\frac{2}{3}x-1\right)+\left(3x^3-\frac{1}{2}x^2+\frac{1}{3}x+3\right)=$

$=3x^3+\frac{1}{2}x^2-\frac{1}{2}x^2+\frac{2}{3}x+\frac{1}{3}x-1+3=3x^3+x+2$

2.2. $A(x)-B(x)=\left(\frac{1}{2}x^2+\frac{2}{3}x-1\right)-\left(3x^3-\frac{1}{2}x^2+\frac{1}{3}x+3\right)=$

$=\frac{1}{2}x^2+\frac{2}{3}x-1-3x^3+\frac{1}{2}x^2-\frac{1}{3}x-3=-3x^3+x^2+\frac{1}{3}x-4$

2.3. $A(x)\times C(x)=\left(\frac{1}{2}x^2+\frac{2}{3}x-1\right)\times\left(6x^2-12\right)=$

$=3x^4-6x^2+4x^3-8x-6x^2+12=$

$=3x^4+4x^3-12x^2-8x+12$

3.1. $A(x)$ é de grau 5 e $B(x)$ é de grau 3.

3.2. O grau de $A(x)\times B(x)$ é 8 = 5 + 3.

3.3. $A(x)\times B(x)=\left(x^5-x^2\right)\left(x^3-2x+1\right)=$

$=x^8-2x^6+x^5-x^5+2x^3-x^2=$

$=x^8-2x^6+2x^3-x^2$

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4.1. a)

$$\begin{array}{r|l} 5x^3 & x^2 \\ \cline{2-2} 0 & 5x \end{array}$$

$Q(x)=5x$; $R(x)=0$

b)

$$\begin{array}{rrrr|l} 2x^3 & +\,0x^2 & +\,3x & +\,0 & x \\ \cline{5-5} -2x^3 & & & & 2x^2+3 \\ \cline{1-4} & & 3x & & \\ & & -\,3x & & \\ \cline{3-3} & & 0 & & \end{array}$$

$Q(x)=2x^2+3$; $R(x)=0$

c)

$$\begin{array}{rr|l} 2x & -\,3 & x-5 \\ \cline{3-3} -\,2x & +\,10 & 2 \\ \cline{1-2} & 7 & \end{array}$$

$Q(x)=2$; $R(x)=7$

d)

$$\begin{array}{rr|l} x^2 & +\,2x & -x+1 \\ \cline{3-3} -\,x^2 & +\,x & -x-3 \\ \cline{1-2} & 3x & \\ & -\,3x+3 & \\ \cline{2-2} & 3 & \end{array}$$

$Q(x)=-x-3$; $R(x)=3$

e)

$$\begin{array}{rrrr|l} 3x^3 & -\,2x^2 & +\,x & -\,1 & x^2+x+1 \\ \cline{5-5} -\,3x^3 & -\,3x^2 & -\,3x & & 3x-5 \\ \cline{1-4} & -\,5x^2 & -\,2x & -\,1 & \\ & +\,5x^2 & +\,5x & +\,5 & \\ \cline{2-4} & & 3x & +\,4 & \end{array}$$

$Q(x)=3x-5$; $R(x)=3x+4$

f)

$$\begin{array}{rrrrr|l} 2x^4 & -\,0x^3 & -\,3x^2 & +\,0x & +\,5 & 3x^2-2x+1 \\ \cline{6-6} -2x^4 & +\,\frac{4}{3}x^3 & -\,\frac{2}{3}x^2 & & & \frac{2}{3}x^2+\frac{4}{9}x-\frac{25}{27} \\ \cline{1-5} & \frac{4}{3}x^3 & -\,\frac{11}{3}x^2 & +\,0x & +\,5 & \\ & -\frac{4}{3}x^3 & +\,\frac{8}{9}x^2 & -\,\frac{4}{9}x & +\,5 & \\ \cline{2-5} & & -\,\frac{25}{9}x^2 & -\,\frac{4}{9}x & +\,5 & \\ & & +\,\frac{25}{9}x^2 & -\,\frac{50}{27}x & +\,\frac{25}{27} & \\ \cline{3-5} & & & -\,\frac{62}{27}x & +\,\frac{160}{27} & \end{array}$$

$Q(x)=\frac{2}{3}x^2+\frac{4}{9}x-\frac{25}{27}$; $R(x)=-\frac{62}{27}x+\frac{160}{27}$

4.2.

$$\begin{array}{l|l} A(x) & x^2+1 \\ \hline 1 & x-1 \end{array}$$

$A(x)=(x^2+1)(x-1)+1 \Leftrightarrow A(x)=x^3-x^2+x-1+1 \Leftrightarrow$

$\Leftrightarrow A(x)=x^3-x^2+x$

4.3. Seja $H(x)$ a expressão para a altura do triângulo.

$\dfrac{(5x+3)\times H(x)}{2}=10x^2+6x \Leftrightarrow$

$\Leftrightarrow (5x+3)\times H(x)=20x^2+12x \Leftrightarrow$

$\Leftrightarrow H(x)=(20x^2+12x):(5x+3)$

$$\begin{array}{rrl|l} 20x^2 & +\ 12x & & 5x+3 \\ \cline{4-4} -20x^2 & -\ 12x & & 4x \\ \cline{1-2} & 0 & & \end{array}$$

$H(x)=4x$

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5. $A(x)=12x^3+\dfrac{2}{3}x-1$; $B(x)=x+\dfrac{1}{3}$

5.1. a)

$$\begin{array}{rrrr|l} 12x^3 & +\ 0x^2 & +\ \frac{2}{3}x & -\ 1 & x+\frac{1}{3} \\ \cline{5-5} -12x^3 & -\ 4x^2 & & & 12x^2-4x+2 \\ \cline{1-4} & -\ 4x^2 & +\ \frac{2}{3}x & -\ 1 & \\ & +\ 4x^2 & +\ \frac{4}{3}x & & \\ \cline{2-4} & & 2x & -\ 1 & \\ & & -\ 2x & -\ \frac{2}{3} & \\ \cline{3-4} & & & -\ \frac{5}{3} & \end{array}$$

$Q(x)=12x^2-4x+2$; $R(x)=-\dfrac{5}{3}$

b)

$$\begin{array}{r|rrrr} & 12 & 0 & \frac{2}{3} & -1 \\ -\frac{1}{3} & & -4 & \frac{4}{3} & -\frac{2}{3} \\ \hline & 12 & -4 & 2 & \boxed{-\frac{5}{3}} \end{array}$$

$Q(x)=12x^2-4x+2$; $R(x)=-\dfrac{5}{3}$

5.2. $A\left(-\dfrac{1}{3}\right)=12\times\left(-\dfrac{1}{3}\right)^3+\dfrac{2}{3}\times\left(-\dfrac{1}{3}\right)-1=$

$=-\dfrac{12}{27}-\dfrac{2}{9}-1=\dfrac{4}{9}-\dfrac{2}{9}-\dfrac{9}{9}=-\dfrac{15}{9}=-\dfrac{5}{3}$

6.1. $A(x)=x^3-2x^2-3x-5$

$B(x)=x-2$

$$\begin{array}{r|rrrr} & 1 & -2 & -3 & -5 \\ 2 & & 2 & 0 & -6 \\ \hline & 1 & 0 & -3 & \boxed{-11} \end{array}$$

$A(2)=2^3-2\times2^2-3\times2-5=-11$

$Q(x)=x^2-3$; $R(x)=-11$; $A(2)=-11$

6.2. $A(x)=7x^3+2x^2-5x+10$; $B(x)=x+2$

$$\begin{array}{r|rrrr} & 7 & 2 & -5 & 10 \\ -2 & & -14 & 24 & -38 \\ \hline & 7 & -12 & 19 & \boxed{-28} \end{array}$$

$A(-2)=7\times(-2)^3+2\times(-2)^2-5\times(-2)+10=-28$

$Q(x)=7x^2-12x+19$; $R(x)=-28$; $A(-2)=-28$

6.3. $A(x)=6x^3+2x-3$; $B(x)=x-5$

$$\begin{array}{r|rrrr} & 6 & 0 & 2 & -3 \\ 5 & & 30 & 150 & 760 \\ \hline & 6 & 30 & 152 & \boxed{757} \end{array}$$

$A(5)=6\times5^3+2\times5-3=757$; $Q(x)=6x^2+30x+152$;

$R(x)=757$; $A(5)=757$

6.4. $A(x)=x^5+1$; $B(x)=x+3$

$$\begin{array}{r|rrrrrr} & 1 & 0 & 0 & 0 & 0 & 1 \\ -3 & & -3 & 9 & -27 & 81 & -243 \\ \hline & 1 & -3 & 9 & -27 & 81 & \boxed{-242} \end{array}$$

$A(-3)=(-3)^5+1=-242$

$Q(x)=x^4-3x^3+9x^2-27x+81$; $R(x)=-242$;

$A(-3)=-242$

6.5. $A(x)=x^4-x^3-1$; $B(x)=x-\dfrac{1}{3}$

$$\begin{array}{r|rrrrr} & 1 & -1 & 0 & 0 & -1 \\ \frac{1}{3} & & \frac{1}{3} & -\frac{2}{9} & -\frac{2}{27} & -\frac{2}{81} \\ \hline & 1 & -\frac{2}{3} & -\frac{2}{9} & -\frac{2}{27} & \boxed{-\frac{83}{81}} \end{array}$$

$A\left(\dfrac{1}{3}\right)=\left(\dfrac{1}{3}\right)^4-\left(\dfrac{1}{3}\right)^3-1=$

$=\dfrac{1}{81}-\dfrac{1}{27}-1=\dfrac{1}{81}-\dfrac{3}{81}-\dfrac{81}{81}=-\dfrac{83}{81}$

$Q(x)=x^3-\dfrac{2}{3}x^2-\dfrac{2}{9}x-\dfrac{2}{27}$; $R(x)=-\dfrac{83}{81}$; $A\left(\dfrac{1}{3}\right)=-\dfrac{83}{81}$

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7.1. $3x^2-4x+1$; $3x-2=3\left(x-\dfrac{2}{3}\right)$

$$\begin{array}{r|rrr} & 3 & -4 & -1 \\ \frac{2}{3} & & 2 & -\frac{4}{3} \\ \hline & 3 & -2 & \boxed{-\frac{1}{3}} \end{array}$$

$3Q(x)=3x-2 \Leftrightarrow Q(x)=x-\dfrac{2}{3}$

$Q(x)=x-\dfrac{2}{3}$; $R(x)=-\dfrac{1}{3}$

7.2. x^4-1 ; $\dfrac{1}{2}x+1=\dfrac{1}{2}(x+2)$

$$\begin{array}{r|rrrrr} & 1 & 0 & 0 & 0 & -1 \\ -2 & & -2 & 4 & -8 & 16 \\ \hline & 1 & -2 & 4 & -8 & \boxed{15} \end{array}$$

$\dfrac{1}{2}Q(x)=x^3-2x^2+4x-8 \Leftrightarrow$

$\Leftrightarrow Q(x)=2x^3-4x^2+8x-16$

$Q(x)=2x^3-4x^2+8x-16$; $R(x)=15$

7.3. $x^4 - 3x^2 - 2x$; $2x + 1 = 2\left(x + \frac{1}{2}\right)$

	1	0	-3	-2	0
$-\frac{1}{2}$		$-\frac{1}{2}$	$\frac{1}{4}$	$\frac{11}{8}$	$\frac{5}{16}$
	1	$-\frac{1}{2}$	$-\frac{11}{4}$	$-\frac{5}{8}$	$\frac{5}{16}$

$$2Q(x) = x^3 - \frac{1}{2}x^2 - \frac{11}{4}x - \frac{5}{8} \Leftrightarrow$$

$$\Leftrightarrow Q(x) = \frac{1}{2}x^3 - \frac{1}{4}x^2 - \frac{11}{8}x - \frac{5}{16}$$

$$Q(x) = \frac{1}{2}x^3 - \frac{1}{4}x^2 - \frac{11}{8}x - \frac{5}{16} \ ; \ R(x) = \frac{5}{16}$$

8. $-2x^2 - 18x + 16 = mx^2 + (n-3)x + 4p \Leftrightarrow$

$$\Leftrightarrow \begin{cases} -2 = m \\ -18 = n - 3 \\ 16 = 4p \end{cases} \Leftrightarrow \begin{cases} m = -2 \\ n = -15 \\ p = 4 \end{cases}$$

Logo, $m = -2$, $n = -15$ e $p = 4$.

9. $x^2 - 8x + 7 = (x-k)^2 - b \Leftrightarrow$

$$\Leftrightarrow x^2 - 8x + 7 = x^2 - 2kx + k^2 - b$$

$$\Leftrightarrow \begin{cases} -8 = -2k \\ 7 = k^2 - b \end{cases} \Leftrightarrow \begin{cases} k = 4 \\ 7 = 4^2 - b \end{cases} \Leftrightarrow \begin{cases} k = 4 \\ b = 9 \end{cases}$$

Logo, $k = 4$ e $b = 9$.

10. $x^3 - 2 = (x+1)(ax^2 + bx + c) + d$

$$\Leftrightarrow x^3 - 2 = ax^3 + bx^2 + cx + ax^2 + bx + c + d$$

$$\Leftrightarrow x^3 - 2 = ax^3 + (a+b)x^2 + (b+c)x + (c+d)$$

$$\Leftrightarrow \begin{cases} a = 1 \\ a + b = 0 \\ b + c = + \\ c + d = -2 \end{cases} \Leftrightarrow \begin{cases} a = 1 \\ 1 + b = 0 \\ b + c = 0 \\ c + d = -2 \end{cases} \Leftrightarrow \begin{cases} a = 1 \\ b = -1 \\ -1 + c = 0 \\ c + d = -2 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} a = 1 \\ b = -1 \\ c = 1 \\ 1 + d = -2 \end{cases} \Leftrightarrow \begin{cases} a = 1 \\ b = -1 \\ c = 1 \\ d = -3 \end{cases}$$

Logo, $a = 1$, $b = -1$, $c = 1$ e $d = -3$.

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11. $P(x) = x^3 - 2x^2 + 3x + 5$

$$P(-3) = (-3)^3 - 2\times(-3)^2 + 3\times(-3) + 5$$

$$= -27 - 18 - 9 + 5 = -49$$

O resto da divisão de $P(x)$ por $x + 3$ é $P(-3) = -49$.

12. $P(x) = x^2 - 3x + a$

$$P(5) = 5^2 - 3\times 5 + a = 25 - 15 + a = 10 + a$$

$$P(5) = 0 \Leftrightarrow 10 + a = 0 \Leftrightarrow a = -10$$

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13. $P(x) = 4x^3 - 13x - 6$

a) $P(2) = 4\times 2^3 - 13\times 2 - 6 = 32 - 26 - 6 = 0$

$P(2) = 0$. Logo, $P(x)$ é divisível por $x - 2$.

b) $2x + 3 = 2\left(x + \frac{3}{2}\right)$

$$P\left(-\frac{3}{2}\right) = 4\times\left(-\frac{3}{2}\right)^3 - 13\times\left(-\frac{3}{2}\right) - 6 =$$

$$= -4\times\frac{27}{8} + \frac{39}{2} - 6 = -\frac{27}{2} + \frac{39}{2} - 6$$

$$= -\frac{12}{2} + 6 = 0$$

$P\left(-\frac{3}{2}\right) = 0$. Logo, $P(x)$ é divisível por $(2x + 3)$.

14. $P(x) = x^3 + 8x^2 - 7$

14.1. $P(-1) = (-1)^3 + 8\times(-1)^2 - 7 = -1 + 8 - 7 = 0$

$P(-1) = 0$. Logo, $P(x)$ é divisível por $x + 1$.

14.2.

	1	8	0	-7
-1		-1	-7	7
	1	7	-7	0

$$P(x) = (x+1)(x^2 + 7x - 7)$$

15. $P(x) = x^4 - x^3 + x - 1$

$$P(1) = 1^4 - 1^3 + 1 - 1 = 0$$

$P(1) = 0$. Logo, $P(x)$ é divisível por $x - 1$.

	1	-1	1	-1
1		1	0	1
	1	0	1	0

$$P(x) = (x-1)(x^3 + 1)$$

Logo, $Q(x) = x^3 + 1$.

16. $P(x) = 2x^3 - 5x^2 + x + a$

$$P(2) = 2\times 2^3 - 5\times 2^2 + 2 + a =$$

$$= 16 - 20 + 2 + a = -2 + a$$

$$P(2) = 0 \Leftrightarrow -2 + a = 0 \Leftrightarrow a = 2$$

17. $P(x) = ax^3 + (1-3a)x^2 + 2ax + a - 6$

$$P(2) = a\times 2^3 + (1-3a)\times 2^2 + 2a\times 2 + a - 6$$

$$= 8a + 4 - 12a + 4a + a - 6$$

$$= a - 2$$

$$P(2) = 0 \Leftrightarrow a - 2 = 0 \Leftrightarrow a = 2$$

$$P(x) = 2x^3 - 5x^2 + 4x - 4$$

	2	-5	4	-4
2		4	-2	4
	2	-1	2	0

$$P(x) = (x-2)(2x^2 - x + 2)$$

Logo, $Q(x) = 2x^2 - x + 2$.

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Atividades complementares

18. $3x^5 - x^{10} + 7 - x^2 = -x^{10} + 3x^5 - x^2 + 7$

18.1. O termo independente é 7.

18.2. O coeficiente do termo de grau 2 é -1.

18.3. O polinómio tem grau 10.

19. $P(x) = \frac{(x-2)^2}{3} - \frac{x-1}{2} - \frac{5}{3}$

19.1. $P(x)=\dfrac{x^2-4x+4}{3}-\dfrac{x}{2}+\dfrac{1}{2}-\dfrac{5}{3}$

$=\dfrac{x^2}{3}-\dfrac{4}{3}x+\dfrac{4}{3}-\dfrac{x}{2}+\dfrac{1}{2}-\dfrac{5}{3}$

$=\dfrac{1}{3}x^2-\dfrac{11}{6}x+\dfrac{1}{6}$

19.2. a) O polinómio tem grau 2.

b) O termo independente é $\dfrac{1}{6}$.

20.1. Um polinómio completo de grau 4 tem cinco termos.

20.2. Um polinómio completo de grau $n + 1$ tem $n + 2$ termos.

20.3. Um polinómio completo de grau $n - 2$ tem $n - 2 + 1 = n - 1$ termos.

21. Um polinómio de grau n tem pelo menos um termo (o termo de grau n).

22. $A(x)=5x^4-3x-3$; $B(x)=3x^4-6x^2-3x+3$

22.1. $A(x)+B(x)=5x^4-3x-3+\left(3x^4-6x^2-3x+3\right)=$

$=8x^4-6x^2-6x$

22.2. $A(x)-B(x)=5x^4-3x-3-\left(3x^4-6x^2-3x+3\right)=$

$=5x^4-3x-3-3x^4+6x^2+3x-3=$

$=2x^4+6x^2-6$

22.3. $2A(x)-\dfrac{10}{3}B(x)=2\left(5x^4-3x-3\right)-\dfrac{10}{3}\left(3x^4-6x^2-3x+3\right)=$

$=10x^4-6x-6-10x^4+20x^2+10x-10=$

$=20x^2+4x-16$

23. $C(x)\times D(x)=\left(x^2+9x+3\right)\left(x^3-3x+1\right)=$

$=x^5-3x^3+x^2+9x^4-27x^2+9x+3x^3-9x+3=$

$=x^5+9x^4-26x^2+3$

24.1. $\left(x^2+2-4x\right)(3x-2)=$

$=3x^3-2x^2+6x-4-12x^2+8x=$

$=3x^3-14x^2+14x-4$

24.2. $(x-3)(x+2)-(2x+2)^2=$

$=x^2+2x-3x-6-\left(4x^2+8x+4\right)=$

$=x^2-x-6-4x^2-8x-4=$

$=-3x^2-9x-10$

24.3. $\left[4x^2-3x\left(\dfrac{2}{3}x+1\right)\right]\times(4x-1)=$

$=\left(4x^2-2x^2-3x\right)\times(4x-1)=$

$=\left(2x^2-3x\right)(4x-1)=$

$=8x^3-2x^2-12x^2+3x=8x^3-14x^2+3x$

24.4. $(2x+1)(x-1)-(x+4)(x-4)=$

$=2x^2-2x+x-1-\left(x^2-16\right)=$

$=x^2-x+15$

25.1. $A(x)+B(x)$ é de grau 5.

25.2. $A(x)-B(x)$ é de grau 5.

25.3. $A(x)\times B(x)$ é de grau 5 + 3 = 8.

25.4. $\left[A(x)\right]^2$ é de grau 5 × 2 = 10.

26. $\left(-x^2-x+1\right)\times A(x)=x^5-29x+12$

2 + grau ($A(x)$) = 5, logo grau ($A(x)$) = 3.

27.1. $A(x)=-x^3+3x^2+2$; $B(x)=-x^2-2x+1$

$$\begin{array}{rrrr|l}
-x^3 & +\,3x^2 & +\,0x & +\,2 & -x^2-2x+1 \\ \cline{5-5}
+x^3 & +\,2x^2 & -\,x & & x-5 \\ \cline{1-4}
 & 5x^2 & -\,x & +\,2 & \\
 & -\,5x^2 & -\,10x & +\,5 & \\ \cline{2-4}
 & & -\,11x & +\,7 &
\end{array}$$

$Q(x)=x-5$; $R(x)=-11x+7$

27.2. $A(x)=x^3+1$; $B(x)=x+1$

$$\begin{array}{rrrr|l}
x^3 & +\,0x^2 & +\,0x & +\,1 & x+1 \\ \cline{5-5}
-x^3 & -\,x^2 & & & x^2-x+1 \\ \cline{1-4}
 & -\,x^2 & +\,0x & +\,1 & \\
 & +\,x^2 & +\,x & & \\ \cline{2-4}
 & & x & +\,1 & \\
 & & -\,x & -\,1 & \\ \cline{3-4}
 & & & 0 &
\end{array}$$

$Q(x)=x^2-x+1$; $R(x)=0$

27.3. $A(x)=3x+2$; $B(x)=x+1$

$$\begin{array}{rr|l}
3x & +\,2 & x+1 \\ \cline{3-3}
-\,3x & -\,3 & 3 \\ \cline{1-2}
 & -\,1 &
\end{array}$$

$Q(x)=3$; $R(x)=-1$

27.4. $A(x)=\dfrac{1}{2}x^3+2x^2-22x+1$; $B(x)=\dfrac{1}{3}x+3$

$$\begin{array}{rrrr|l}
\frac{1}{2}x^3 & +\,2x^2 & -\,22x & +\,1 & \frac{1}{3}x+3 \\ \cline{5-5}
-\frac{1}{2}x^3 & -\,\frac{9}{2}x^2 & & & \frac{2}{3}x^2-\frac{15}{2}x+\frac{3}{2} \\ \cline{1-4}
 & -\frac{5}{2}x^2 & -\,22x & +\,1 & \\
 & \frac{5}{2}x^2 & +\,\frac{45}{2}x & & \\ \cline{2-4}
 & & \frac{1}{2}x & +\,1 & \\
 & & -\,\frac{1}{2}x & -\,\frac{9}{2} & \\ \cline{3-4}
 & & & -\,\frac{7}{2} &
\end{array}$$

$Q(x)=\dfrac{3}{2}x^2-\dfrac{15}{2}x+\dfrac{3}{2}$; $R(x)=-\dfrac{7}{2}$

28.1. $\dfrac{x^2+5x+3}{x+2}=x+3-\dfrac{3}{x+2}$

$$\begin{array}{rrr|l}
x^2 & +\,5x & +\,3 & x+2 \\ \cline{4-4}
-\,x^2 & -\,2x & & x+3 \\ \cline{1-3}
 & 3x & +\,3 & \\
 & -\,3x & -\,6 & \\ \cline{2-3}
 & & -\,3 &
\end{array}$$

28.2. $\dfrac{5x^2+3x-1}{x+3}=5x-12+\dfrac{35}{x+3}$

$$\begin{array}{rrr|l}
5x^2 & +\,3x & -\,1 & x+2 \\ \cline{4-4}
-\,5x^2 & -\,15x & & x+3 \\ \cline{1-3}
 & -\,12x & -\,1 & \\
 & +\,12x & +\,36 & \\ \cline{2-3}
 & & 35 &
\end{array}$$

28.3. $\dfrac{x^3-x+3}{x^2+1}=x-\dfrac{2x-3}{x^2+1}$

$$\begin{array}{rrr|l}
x^3 & -\,x & +\,3 & x+1 \\ \cline{4-4}
-\,x^3 & -\,x & & 3 \\ \cline{1-3}
 & -\,2x & +\,3 &
\end{array}$$

29.1. $A(x)=3x^2-5x+4$; $B(x)=x-2$

$$\begin{array}{c|ccc}
 & 3 & -5 & 4 \\
2 & & 6 & 2 \\ \hline
 & 3 & 1 & \boxed{6}
\end{array}$$

$Q(x)=3x+1$; $R(x)=6$

29.2. $A(x)=-x^4-x^3+x^2+5$; $B(x)=x+2$

$$\begin{array}{c|ccccc}
 & -1 & -1 & 1 & 0 & 5 \\
-2 & & 2 & -2 & 2 & -4 \\ \hline
 & -1 & 1 & -1 & 2 & \boxed{1}
\end{array}$$

$Q(x)=-x^3+x^2-x+2$; $R(x)=1$

29.3. $A(x)=x^4-x^3+1$; $B(x)=x+2$

$$\begin{array}{c|ccccc}
 & 1 & -1 & 0 & 0 & 1 \\
-2 & & -2 & 6 & -12 & 24 \\ \hline
 & 1 & -3 & 6 & -12 & \boxed{25}
\end{array}$$

$Q(x)=x^3-3x^2+6x-12$; $R(x)=25$

29.4. $A(x)=8x^3-2x-1$; $B(x)=x+\dfrac{1}{2}$

$$\begin{array}{c|cccc}
 & 8 & 0 & -2 & -1 \\
-\frac{1}{2} & & -4 & 2 & 0 \\ \hline
 & 8 & -4 & 0 & \boxed{-1}
\end{array}$$

$Q(x)=8x^2-4x$; $R(x)=-1$

29.5. $A(x)=x^5+1$; $B(x)=x+1$

$$\begin{array}{c|cccccc}
 & 1 & 0 & 0 & 0 & 0 & 1 \\
-1 & & -1 & 1 & -1 & 1 & -1 \\ \hline
 & 1 & -1 & 1 & -1 & 1 & \boxed{0}
\end{array}$$

$Q(x)=x^4-x^3+x^2-x+1$; $R(x)=0$

30.1. $A(x)=2x^3-4x+1$; $B(x)=2x-4=2(x-2)$

$$\begin{array}{c|cccc}
 & 2 & 0 & -4 & 1 \\
2 & & 4 & 8 & 8 \\ \hline
 & 2 & 4 & 4 & \boxed{9}
\end{array}$$

$2Q(x)=2x^2+4x+4$; $Q(x)=x^2+2x+x$; $R(x)=9$

30.2. $A(x)=4x^3-3$; $B(x)=2x-1=2\left(x-\dfrac{1}{2}\right)$

$$\begin{array}{c|cccc}
 & 4 & 0 & 0 & -3 \\
\frac{1}{2} & & 2 & 1 & \frac{1}{2} \\ \hline
 & 4 & 2 & 1 & \boxed{-\frac{5}{2}}
\end{array}$$

$2Q(x)=4x^2+2x+1$; $Q(x)=2x^2+x+\dfrac{1}{2}$; $R(x)=-\dfrac{5}{2}$

30.3. $A(x)=8x^2-5x+3$; $B(x)=\dfrac{1}{2}x+1=\dfrac{1}{2}(x+2)$

$$\begin{array}{c|ccc}
 & 8 & -5 & 3 \\
-2 & & -16 & 42 \\ \hline
 & 8 & -21 & \boxed{45}
\end{array}$$

$\dfrac{1}{2}Q(x)=8x-21$; $Q(x)=16x-42$; $R(x)=45$

31.

$$\begin{array}{rrrr|l}
3x^2 & - & x & +\ 1 & 2x-3 \\ \cline{5-5}
-3x^2 & + & \frac{9}{2}x & & \frac{3}{2}x+\frac{7}{4} \\ \cline{1-4}
 & & \frac{7}{2}x & +\ 1 & \\
 & - & \frac{7}{2}x & +\ \frac{21}{4} & \\ \cline{2-4}
 & & & \frac{25}{4} &
\end{array}$$

$$\begin{array}{rrrr|l}
3x^2 & - & x & +\ 1 & x-\frac{3}{2} \\ \cline{5-5}
-3x^2 & + & \frac{9}{2}x & & 3x+\frac{7}{2} \\ \cline{1-4}
 & & \frac{7}{2}x & +\ 1 & \\
 & - & \frac{7}{2}x & +\ \frac{21}{4} & \\ \cline{2-4}
 & & & \frac{25}{4} &
\end{array}$$

$R(x)=\dfrac{25}{4}$, nos dois casos.

32.1. $A(x)=x^2-3x+1$; $B(x)=x+1$

$a\times A(x)=x+b\times(B(x))^2$

$\Leftrightarrow a(x^2-3x+1)=x+b(x+1)^2 \Leftrightarrow$

$\Leftrightarrow ax^2-3ax+a=x+b(x^2+2x+1) \Leftrightarrow$

$\Leftrightarrow ax^2-3ax+a=bx^2+(1+2b)x+b \Leftrightarrow$

$$\Leftrightarrow \begin{cases} a=b \\ -3a=1+2b \\ a=b \end{cases} \Leftrightarrow \begin{cases} a=b \\ -3b=1+2b \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} a=b \\ 5b=-1 \end{cases} \Leftrightarrow \begin{cases} a=-\dfrac{1}{5} \\ b=-\dfrac{1}{5} \end{cases}$$

32.2. $A(x)-1=B(x)\times(ax+b)+c$

$x^2-3x=(x+1)(ax+b)+c \Leftrightarrow$

$\Leftrightarrow x^2-3x=ax^2+bx+ax+b+c \Leftrightarrow$

$\Leftrightarrow x^2-3x=ax^2+(b+a)x+b+c \Leftrightarrow$

$$\Leftrightarrow \begin{cases} a=1 \\ b+a=-3 \\ b+c=0 \end{cases} \Leftrightarrow \begin{cases} a=1 \\ b+1=-3 \\ c=-b \end{cases} \Leftrightarrow \begin{cases} a=1 \\ b=-4 \\ c=4 \end{cases}$$

Logo, $a = 1$, $b = -4$ e $c = 4$.

33. $A(x)=2x^3-4x^2+3x+1$

33.1. $B(x)=x-2$

$A(2)=2\times 2^3-4\times 2^2+3\times 2+1=$

$=2\times 8-4\times 4+6+1=7$

$R=A(2)=7$

33.2. $C(x)=x+3$

$A(-3)=2\times(-3)^3-4\times(-3)^2+3\times(-3)+1=$

$=-54-36-9+1=-98$

$R=A(-3)=-98$

33.3. $D(x)=x=x-0$; $D(0)=1$; $R=D(0)=1$

33.4. $E(x)=2x-1=2\left(x-\dfrac{1}{2}\right)$

$A\left(\dfrac{1}{2}\right)=2\times\left(\dfrac{1}{2}\right)^3-4\times\left(\dfrac{1}{2}\right)^2+3\times\dfrac{1}{2}+1=$

$=\dfrac{2}{8}-1+\dfrac{3}{2}+1=\dfrac{7}{4}$

$R=A\left(\dfrac{1}{2}\right)=\dfrac{7}{4}$

34. Seja $P(x)=a_0x^n+a_1x^{n-1}+...+a_{n-1}x+a_n$ um polinómio qualquer e $B(x)=x=x-0$.

O resto da divisão de $P(x)$ por $B(x)$ é igual a $P(0)$ e

$P(0)=a_0\times 0^n+a_1\times a^{n-1}+...+a_{n-1}\times a+a_n=a_n$

Logo, o resto da divisão de $P(x)$ por x é o termo independente a_n.

35. $P(x)=x^4-x^2+ax+2$; $B(x)=x-1$

$P(1)=1^4-1^2+a+2=a+2$

$P(1)=2 \Leftrightarrow a+2=2 \Leftrightarrow a=0$

36. $A(x)=x^3-3x+2$

36.1. $B(x)=x-1$; $A(1)=1-3+2=0$

Como $A(1)=0$, $A(x)$ é divisível por $x-1$.

36.2. $B(x)=x+3$

$A(-3)=(-3)^3-3\times(-3)+2=-27+9+2=-16$

Como $A(-3)\neq 0$, $A(x)$ não é divisível por $x+3$.

37. $A(x)=x^3-3x^2+5x+9$

37.1. $A(-1)=(-1)^3-3\times(-1)^2+5\times(-1)+9=-1-3-5+9=0$

Como $A(-1)=0$, $A(x)$ é divisível por $x+1$.

37.2.

	1	− 3	5	9
− 1		− 1	4	− 9
	1	− 4	9	0

$Q(x)=x^2-4x+9$ e tem grau 2.

38. $A(x)=x^3-6x^2+11x-6$

38.1. $A(1)=1^3-6\times1^2+11\times1-6=$

$=1-6+5=0$

38.2.

	1	− 6	11	− 6
1		1	− 5	6
	1	− 5	6	0

$Q(x)=x^2-5x+6$

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39. x^2+4 é um polinómio do 2.º grau. Se for o quadrado de um polinómio, terá de ser de um polinómio do 1.º grau.

$x^2+4=(ax+b)^2 \Leftrightarrow$

$\Leftrightarrow x^2+4=a^2x^2+2abx+b^2$

$$\Leftrightarrow \begin{cases} a^2=1 \\ 2ab=0 \\ b^2=4 \end{cases} \Leftrightarrow \begin{cases} a=1 \vee a=-1 \\ a=0 \vee b=0 \\ b=2 \vee b=-2 \end{cases}$$

O sistema é impossível. Logo, x^2+4 não é o quadrado de um polinómio.

40. Seja $P_n(x)=(x+4)^n+(x+3)^{2n}-1$

$P_n(-3)=(-3+4)^n+(-3+3)^{2n}-1=1^n+0^n-1=0$

$P_n(-4)=(-4+4)^n+(-4+3)^{2n}-1=0^n+(-1)^{2n}-1=$

$=\left[(-1)^2\right]^n-1=1^n-1=$

$=1-1=0$

Como $\forall n\in\mathbb{N},\ P_n(-3)=0$ e $P_n(-4)=0$, então $P_n(x)$ é divisível por $x+3$ e por $x+4$. Logo, $\forall n\in\mathbb{N},\ P_n(x)$ é divisível por $(x+3)(x+4)$.

41. Seja $P_n(x)=x^{2n+1}-nx-1+n$.

$P_n(1)=1^{2n+1}-n-1+n=1-1=0$

Como $\forall n\in\mathbb{N},\ P_n(1)=0$, podemos concluir que $\forall n\in\mathbb{N},\ P_n(x)$ é divisível por $x+1$.

42. $P(x)=x^{200}-x$

$P(1)=1^{200}-1=1-1=0$

O resto da divisão de $x^{200}-x$ por $x-1$ é 0.

Resposta: (D)

43. $P(x)=x^n-1$; $P(1)=1^n-1=0$

$\forall n\in\mathbb{N},\ P(1)=0$. Logo, $\forall n\in\mathbb{N},\ P(x)$ é divisível por $x-1$.

44. $P(x)=x^n+a^n$

$P(-a)=(-a)^n+a^n=(-1)^n\times a^n+a^n=$

$=-a^n+a^n=0$

pois, se n é ímpar, então $(-1)^n=-1$.

Se n é ímpar, $P(-a)=0$. Logo, se n é ímpar $P(x)$ é divisível por $x+a$.

45. $P(x)=x^3-2x+1$

45.1. $P(-2)=(-2)^3-2\times(-2)+1=-8+4+1=-3$

Como $P(-2)=-3$, o resto da divisão de $P(x)$ por $x+2$ é -3.

45.2.

	1	0	− 2	1
− 2		− 2	4	− 4
	1	− 2	2	− 3

$P(x)=(x+2)\times(x^2-2x+2)-3$

$\Leftrightarrow P(x)+3=(x+2)(x^2-2x+2)$

$Q(x)=x^2-2x+2$

46. Grau de $A(x)$: $n+2$

Grau de $B(x)$: $n-1$

Grau de $Q(x)=(n+2)-(n-1)=n+2-n+1=3$

Grau de $R(x)$ é menor que o de $B(x)$.

O grau de $Q(x)$ é 3 e o de $R(x)$ é menor que $n-1$ ou $R(x)$ é o polinómio nulo.

47. Como $x(x-1)$ tem grau 2, $R(x)$, resto da divisão de $P(x)$ por $x(x-1)$, tem grau não superior a 1.

$R(x)=ax+b$

Então, $P(x)=x(x-1)Q(x)+ax+b$.

Sabemos que $P(0)=9$ e $P(1)=7$, pelo que:

$$\begin{cases} P(0)=9 \\ P(1)=7 \end{cases} \Leftrightarrow \begin{cases} 0\times(0-1)\times Q(0)+a\times0+b=9 \\ 1\times(1-1)\times Q(1)+a+b=7 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} 0+b=9 \\ 0+a+b=7 \end{cases} \Leftrightarrow \begin{cases} b=9 \\ a+9=7 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} b=9 \\ a=-2 \end{cases}$$

Logo, $R(x)=-2x+9$.

48. $P(a+3)=2a^2-3a+1$

48.1. a) $a+3=-1 \Leftrightarrow a=-4$

$P(-1)=2\times(-4)^2-3\times(-4)+1=$

$=32+12+1=45$

b) $a+3=2 \Leftrightarrow a=2-3 \Leftrightarrow a=-1$

$P(2)=2\times(-1)^2-3\times(-1)+1=2+3+1=6$

48.2. $a+3=x \Leftrightarrow a=x-3$

$P(x)=2(x-3)^2-3(x-3)+1=$

$=2(x^2-6x+9)-3x+9+1=$

$=2x^2-12x+18-3x+9+1=$

$=2x^2-15x+28$

$P(x)=0 \Leftrightarrow 2x^2-15x+28=0 \Leftrightarrow$

$\Leftrightarrow x=\dfrac{15\pm\sqrt{15^2-4\times2\times28}}{4} \Leftrightarrow$

$\Leftrightarrow x=\dfrac{15\pm1}{4} \Leftrightarrow$

$\Leftrightarrow x=\dfrac{7}{2} \vee x=4$

Os zeros de $P(x)$ são $\dfrac{7}{2}$ e 4.

49. $P(x)=x^3-px+1$; $2x-4=2(x-2)$

$P(2)=0 \Leftrightarrow 2^3-2p+1=0 \Leftrightarrow 2p=9 \Leftrightarrow p=\dfrac{9}{2}$

50. $P(x)=(p-1)x^3-2x^2-3qx+1$

Se $p=3q+2$, então $p-1=3q+2-1=3q+1$ e $P(x)=(3q+1)x^3-2x^2-3qx+1$.

$P(1)=(3q+1)\times1^3-2\times1^2-3q\times1+1=$

$=\cancel{3q}+1-2-\cancel{3q}+1=0$

Portanto, se $p=3q+2$, então o polinómio $P(x)$ é divisível por $x-1$.

51. $A(x)=(m-2)x^3-2x+m$

$2x-1=2\left(x-\dfrac{1}{2}\right)$

$A\left(\dfrac{1}{2}\right)=0 \Leftrightarrow (m-2)\times\left(\dfrac{1}{2}\right)^3-2\times\dfrac{1}{2}+m=0 \Leftrightarrow$

$\Leftrightarrow \dfrac{1}{8}m-2\times\dfrac{1}{8}-1+m=0 \Leftrightarrow \dfrac{1}{8}m+m=\dfrac{5}{4} \Leftrightarrow$

$\Leftrightarrow m+8m=10 \Leftrightarrow 9m=10 \Leftrightarrow m=\dfrac{10}{9}$

52. $P(x)=ax^2+bx+c$

$$\begin{cases} P(0)=0 \\ P(3)=0 \\ P(-3)=1 \end{cases} \Leftrightarrow \begin{cases} c=0 \\ 9a+3b+c=0 \\ 9a-3b+c=1 \end{cases} \Leftrightarrow \begin{cases} c=0 \\ 9a=-3b \\ -3b-3b=1 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} c=0 \\ 9a=-3\times\left(-\dfrac{1}{6}\right) \\ b=-\dfrac{1}{6} \end{cases} \Leftrightarrow \begin{cases} a=0 \\ a=\dfrac{1}{18} \\ b=-\dfrac{1}{6} \end{cases}$$

$P(x)=\dfrac{1}{18}x^2-\dfrac{1}{6}x$

53.1. $P(x)=2x^{2n}-(n+1)x-(n+3)$

$P(-1)=2\times(-1)^{2n}-(n+1)\times(-1)-n-3=$

$=2\times\left((-1)^2\right)^n+n+1-n-3=$

$=2\times1^n-2=2-2=0$

Como $\forall n\in\mathbb{N},\ P(-1)=0$, então $\forall n\in\mathbb{N},\ P(x)$ é divisível por $x+1$.

53.2. $B(x)=(x+1)^n+3x^{n+1}+2$

$A(x)=x=x-0$

$B(0)=(0+1)^n+3\times0^{n+1}+2=$

$=1+0+2=3$

Como $\forall n\in\mathbb{N},\ B(0)=3$, então o resto da divisão de $B(x)$ por x é 3, $\forall n\in\mathbb{N}$.

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Avaliação 3

1. A expressão $2x^{-1}+3x=\dfrac{2}{x}+3x$ não representa um polinómio.

Resposta: (B)

2. $P(x)=(2x-1)^2-(2x-3)(2x+3)=$

$=4x^2-4x+1-(4x^2-9)$

$=4x+10$

O polinómio $P(x)$ tem grau 1.

Resposta: (B)

3. $P(x)=(x-20)(x-15)(x-n)$

$P(10)=400 \Leftrightarrow (10-20)(10-15)(10-n)=400 \Leftrightarrow$

$\Leftrightarrow -10\times(-5)\times(10-n)=400 \Leftrightarrow$

$\Leftrightarrow 50\times(10-n)=400 \Leftrightarrow$

$\Leftrightarrow 10-n=\dfrac{400}{50} \Leftrightarrow$

$\Leftrightarrow 10-n=8 \Leftrightarrow$

$\Leftrightarrow n=2$

$P(x)=(x-20)(x-15)(x-2)$

$P(x)=0 \Leftrightarrow x=20 \vee x=15 \vee x=2$

$V=20\times15\times2=600$

Resposta: (A)

4. Da definição de divisão inteira, temos:

$P(x)=Q(x)\times(x^2-x-2)+(2x-1)$

O resto da divisão de $P(x)$ por $x+1$ é $P(-1)$.

$P(-1)=Q(-1)\times\left((-1)^2-(-1)-2\right)+\left(2\times(-1)-1\right)=$

$=Q(-1)\times(1+1-2)+(-2-1)=$

$=Q(-1)\times0+(-3)=$

$=-3$

Resposta: (A)

5. • Se $A(x)$ tem grau 2 e $B(x)$ tem grau 3, então $A(x)+B(x)$ tem grau 3.

• Se $\left[A(x)+B(x)\right]$ tem grau 3 e $C(x)$ tem grau 4, então $\left[A(x)+B(x)\right]\times C(x)$ tem grau 3 + 4 = 7.

Resposta: (B)

6. $P(x) = x^{1001}$

$A(x) = 2x + 2 = 2(x+1)$

O resto da divisão de $P(x)$ por $2(x+1)$ é $P(-1)$.

$P(-1) = (-1)^{1001} = -1$

Resposta: (A)

7. $A(x) = \left[B(x)\right]^2 \Leftrightarrow$

$\Leftrightarrow ax^2 + bx + 1 = (cx+1)^2 \Leftrightarrow$

$\Leftrightarrow ax^2 + bx + 1 = c^2x^2 + 2cx + 1 \Leftrightarrow$

$\Leftrightarrow \begin{cases} a = c^2 \\ b = 2c \end{cases}$

Se $b = 2c$, então $b^2 = (2c)^2 = 4c^2$. Como $c^2 = a$, $b^2 = 4a$.

Resposta: (D)

8. $P(a-1) = a^3 - 3a + 5$

$a - 1 = 1 \Leftrightarrow a = 2$

$P(2-1) = 2^3 - 3 \times 2 + 5$

$P(1) = 8 - 6 + 5 = 7$

Resposta: (D)

9. $3x - 2 \Leftrightarrow 3\left(x - \frac{2}{3}\right)$

	3	16	0	− 8
$\frac{2}{3}$		2	12	8
	3	18	12	0

$3Q(x) = 3x^2 + 18x + 12 \Leftrightarrow Q(x) = x^2 + 6x + 4$

$Q(0) = 4$

Resposta: (C)

10. $A(x) = (x+a)(x+3a)$; $B(x) = x - a$

$A(x) = x^2 + 3ax + ax + 3a^2$

$= x^2 + 4ax + 3a^2$

	1	$4a$	$3a^2$
a		a	$5a^2$
	1	$5a$	$8a^2$

$Q(x) = x + 5a$ e $R(x) = 8a^2$

Resposta: (C)

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11. $P(x) = (m^2 - 1)x^4 + (m+1)x^3 + x^2 + 3$

$\begin{cases} m^2 - 2 = 0 \\ m + 1 = 0 \end{cases} \Leftrightarrow \begin{cases} m^2 = 1 \\ m = -1 \end{cases} \Leftrightarrow \begin{cases} m = -1 \vee m = 1 \\ m = -1 \end{cases} \Leftrightarrow m = -1$

12. $P(x) = x^3 - 2x^2 + 3x - 5$

$P\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^3 - 2 \times \left(\frac{1}{2}\right)^2 + 3 \times \frac{1}{2} - 5 =$

$= \frac{1}{8} - \frac{1}{2} + \frac{3}{2} - 5 =$

$= \frac{1}{8} + 1 - 5$

$= \frac{1}{8} - 4 = -\frac{31}{8}$

O resto da divisão de $P(x)$ por $B(x)$ é $-\frac{31}{8}$.

13. $x^2 - 1 = (x-1)(x+1)$

$P(x) = x^3 - mx^2 + nx + 1$

$\begin{cases} P(-1) = 0 \\ P(1) = 0 \end{cases} \Leftrightarrow \begin{cases} -1 - m - n + 1 = 0 \\ 1 - m + n + 1 = 0 \end{cases} \Leftrightarrow \begin{cases} m + n = 0 \\ m - n = 2 \end{cases} \Leftrightarrow$

$\Leftrightarrow \begin{cases} m = -n \\ -n - n = 2 \end{cases} \Leftrightarrow \begin{cases} m = -n \\ -2n = 2 \end{cases} \Leftrightarrow \begin{cases} m = 1 \\ n = -1 \end{cases}$

$P(x)$ é divisível por $x^2 - 1$ se $m = 1$ e $n = -1$.

14.

	1	a	$2a$	$-2a$	8
α					− 8
				− 4	0

14.1. O polinómio-divisor é $x - \alpha$.

$\alpha \times (-4) = -8 \Leftrightarrow \alpha = 2$

O polinómio-divisor é $x - 2$.

14.2.

	1	a	$2a$	$-2a$	8
2		2	$2a + 4$	$8a + 8$	− 8
	1	$a + 2$	$4a + 4$	$6a + 8$	0

$60 + 8 = -4 \Leftrightarrow 60 = -12 \Leftrightarrow a = -2$

14.3. $a = -2$; $2a = -4$; $-2a = 4$

$P(x) = x^4 - 2x^3 - 4x^2 + 4x + 8$

15. $(x-1)^2 = x^2 - 2x + 1$

$$
\begin{array}{rrrrl|l}
ax^3 & +\ bx^2 & +\ 0x & -\ 1 & & x^2 - 2x + 1 \\
-\ ax^3 & -\ 2ax^2 & -\ ax & & & ax + 2a + b \\
\hline
 & (2a+b)x^2 & -\ ax & -\ 1 & & \\
 & -\ (2a+b)x^2 & +\ (4a+2b)x & -\ 2a - b & & \\
\hline
 & & (3a+2b)x & -\ 2a - b - 1 & &
\end{array}
$$

$(3a + 2b)x - 2a - b - 1 = 0x + 0 \Leftrightarrow$

$\begin{cases} 3a + 2b = 0 \\ -2a - b - 1 = 0 \end{cases} \Leftrightarrow \begin{cases} 3a + 2(-2a - 1) = 0 \\ b = -2a - 1 \end{cases} \Leftrightarrow$

$\Leftrightarrow \begin{cases} 3a - 4a - 2 = 0 \\ b = -2a - 1 \end{cases} \Leftrightarrow \begin{cases} a = -2 \\ b = 3 \end{cases}$

$a = -2$ e $b = 3$

16. $P(x) = x^2 + bx + 1$

$P(2) = P(-2) \Leftrightarrow$

$\Leftrightarrow 4 + 2b + 1 = 4 - 2b + 1 \Leftrightarrow$

$\Leftrightarrow 2b + 2b = 0 \Leftrightarrow$

$\Leftrightarrow 4b = 0 \Leftrightarrow$

$\Leftrightarrow b = 0$

17. $A(x) = x^3 + \frac{a}{2}x + b$

$B(x) = -x^4 + bx + a$

$\begin{cases} A(-2) = 0 \\ B(-2) = 0 \end{cases} \Leftrightarrow \begin{cases} -8 - a + b = 0 \\ -16 - 2b + a = 0 \end{cases} \Leftrightarrow$

$\Leftrightarrow \begin{cases} a = b - 8 \\ -2b + b - 8 = 16 \end{cases} \Leftrightarrow$

$\Leftrightarrow \begin{cases} a = b - 8 \\ -b = 24 \end{cases} \Leftrightarrow$

$\Leftrightarrow \begin{cases} a = -32 \\ b = -24 \end{cases}$

$a = -32$ e $b = -24$

18. O resto da divisão inteira de $P(x)$ por $x^2 - 1$ tem grau não superior a 1.

$R(x) = ax + b$

$P(x) = Q(x) \times (x^2 - 1) + (ax + b)$

$\begin{cases} P(1) = 3 \\ P(-1) = 1 \end{cases} \Leftrightarrow \begin{cases} Q(1) \times (1^2 - 1) + (a + b) = 3 \\ Q(-1) \times ((-1)^2 - 1) + (-a + b) = 1 \end{cases} \Leftrightarrow$

$\Leftrightarrow \begin{cases} 0 \times Q(1) + a + b = 3 \\ 0 \times Q(-1) - a + b = 1 \end{cases} \Leftrightarrow \begin{cases} 0 + a + b = 3 \\ 0 + b = 1 + a \end{cases} \Leftrightarrow$

$\Leftrightarrow \begin{cases} a + 1 + a = 3 \\ b = 1 + a \end{cases} \Leftrightarrow \begin{cases} 2a = 2 \\ b = 1 + a \end{cases} \Leftrightarrow \begin{cases} a = 1 \\ b = 2 \end{cases}$

$R(x) = x + 2$

19. $P(x) = (x - 2)(x - 3)(ax + b)$

O termo x^3 é ax^3. Logo, $a = 2$

$P(x) = (x - 2)(x - 3)(2x + b)$

$P(1) = -2 \Leftrightarrow (1 - 2)(1 - 3)(2 + b) = -2 \Leftrightarrow$

$\Leftrightarrow -1 \times (-2) \times (2 + b) = -2 \Leftrightarrow$

$\Leftrightarrow 2 + b = -1 \Leftrightarrow$

$\Leftrightarrow b = -3$

$P(x) = (x - 2)(x - 3)(2x - 3)$

O termo independente é $P(0)$.

$P(0) = -2 \times (-3) \times (-3) = -18$

20. $A(x) = B(x) + x^2 + 1$

$A(1) = 0$; $B(2) = 0$

- $A(1) = B(1) + 1^2 + 1 \Leftrightarrow$ $\quad | A(1) = 0$

  $\Leftrightarrow 0 = B(1) + 2 \Leftrightarrow$

  $\Leftrightarrow B(1) = -2$

- $A(2) = B(2) + 2^2 + 1 \Leftrightarrow$ $\quad | B(2) = 0$

  $\Leftrightarrow A(2) = 0 + 5 \Leftrightarrow$

  $\Leftrightarrow A(2) = 5$

$A(2) - B(1) = 5 + 2 = 7$

21. $P(x) = ax^2 + bx + c$

$P(0) = 1 \Leftrightarrow c = 1$

$P(x) = ax^2 + bx + 1$

$P(1) + P(2) = (a + b + 1) + (4a + 2b + 1) =$

$= 5a + 3b + 2$

$P(2) - 4P(1) = (4a + 2b + 1) - 4(a + b + 1) =$

$= 4a + 2b + 1 - 4a - 4b - 4$

$= -2b - 3$

$\begin{cases} P(1) + P(2) = 11 \\ P(2) - 4P(1) = 1 \end{cases} \Leftrightarrow$

$\Leftrightarrow \begin{cases} 5a + 3b + 2 = 11 \\ -2b - 3 = 1 \end{cases} \Leftrightarrow \begin{cases} 5a - 6 + 2 = 11 \\ b = -2 \end{cases} \Leftrightarrow$

$\Leftrightarrow \begin{cases} 5a = 15 \\ b = -2 \end{cases} \Leftrightarrow \begin{cases} a = 3 \\ b = -2 \end{cases}$

$P(x) = 3x^2 - 2x + 1$

$P(3) = 3 \times 9 - 2 \times 3 + 1 = 27 - 6 + 1 = 22$

22. $P(x) = (a + 4)x^3 + 3ax^2 + (a + 2)x - 4a$

$P(-1) = 0 \Leftrightarrow$

$\Leftrightarrow (a + 4) \times (-1) + 3a \times 1 + (a + 2) \times (-1) - 4a = 0 \Leftrightarrow$

$\Leftrightarrow -a - 4 + 3a - a - 2 - 4a = 0 \Leftrightarrow$

$\Leftrightarrow -3a = 6 \Leftrightarrow$

$\Leftrightarrow a = -2$

Assim:

$a + 4 = -2 + 4 = 2$

$3a = 3 \times (-2) = -6$

$a + 2 = -2 + 2 = 0$

$-4a = -4 \times (-2) = 8$

Logo, $P(x) = 2x^3 - 6x^2 + 8$.

	2	−6	0	8
−1		−2	8	−8
	2	−8	8	0

$Q(x) = 2x^2 - 8x + 8$

2.4. Fatorização de polinómios. Resolução de equações e inequações de grau superior ao segundo

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Atividade inicial 4

1.1. $P(x) = 2x^3 + 3x^2 + 4x - 7$

$B(x) = x + 2$

$R = P(-2) = 2\times(-2)^3 + 3\times(-2)^2 + 4\times(-2) - 7$

$= -16 + 12 - 8 - 7 = -19$

1.2. $P(x) = 3x^4 - 21x^2 - 3x - 5$

$B(x) = x - 3$

$R = P(3) = 3\times 3^4 \times 21\times 3^2 - 3\times 3 - 5 =$

$= 243 - 189 - 9 - 5 = 40$

1.3. $P(x) = -2x^3 + 11x^2 + 7x - 6$

$B(x) = x + 6$

$R = P(-6) = -2\times(-6)^3 + 11\times(-6)^2 + 7\times(-6) - 6 =$

$= 432 + 396 - 42 - 6 = 780$

2.1. $A(x) = x^2 + 4$

$A(x)$ não tem zeros porque a equação $x^2 + 4 = 0$ é impossível em $\mathbb{R}$.

2.2. $B(x) = (x-3)^5$

$(x-3)^5 = 0 \Leftrightarrow x = 3$

$B(x)$ tem um zero.

2.3. $C(x) = x^2 + 2x + 1$

$x^2 + 2x + 1 = 0 \Leftrightarrow (x+1)^2 = 0 \Leftrightarrow x = -1$

$C(x)$ tem um zero.

2.4. $D(x) = (x^2+3)(x-1)$

$(x^2+3)(x-1) = 0 \Leftrightarrow x^2 + 3 = 0 \vee x - 1 = 0 \Leftrightarrow x = 1$

$D(x)$ tem um zero.

3.1. $A(x) = x^2 - x = x(x-1)$

3.2. $B(x) = \frac{1}{4} - x^2 = \left(\frac{1}{2} - x\right)\left(\frac{1}{2} + x\right)$

3.3. $C(x) = 7x^2 - 14x = 7x(x-2)$

3.4. $D(x) = 1 - (x-3)^2 = [1-(x-3)][1+(x-3)] =$

$= (4-x)(x-2)$

3.5. $E(x) = 12x^3 - 3x^2 = 3x^2(4x-1)$

3.6. $F(x) = (x-1)^2 - 2(x-1) =$

$= (x-1)[(x-1)-2] = (x-1)(x-3)$

3.7. $G(x) = 16 - 4x^2 = 4^2 - (2x)^2 = (4-2x)(4+2x)$

3.8. $H(x) = (2x-3)^2 - (2x-3)^3 = (2x-3)^2(1-(2x-3)) =$

$= (2x-3)^2(4-2x)$

3.9. $I(x) = \frac{1}{4}x^2 - 1 = \left(\frac{1}{2}x\right)^2 - 1^2 = \left(\frac{1}{2}x - 1\right)\left(\frac{1}{2}x + 1\right)$

3.10. $J(x) = (1-3x)^2 - (2-5x)^2 =$

$= [(1-3x)-(2-5x)][(1-3x)+(2-5x)] =$

$= (2x-1)(3-8x)$

3.11. $K(x) = \left(\frac{1}{2} - x\right)^2 - 25 = \left(\frac{1}{2} - x - 5\right)\left(\frac{1}{2} - x + 5\right) =$

$= \left(-x - \frac{9}{2}\right)\left(-x + \frac{11}{2}\right) = \left(x + \frac{9}{2}\right)\left(x - \frac{11}{2}\right)$

3.12. $L(x) = \left(\frac{1}{2} - x\right)^2 - (1-x)^2 =$

$= \left[\left(\frac{1}{2} - x\right) - (1-x)\right]\left[\left(\frac{1}{2} - x\right) + (1-x)\right] =$

$= -\frac{1}{2}\left(\frac{3}{2} - 2x\right) = x - \frac{3}{4}$

4.1. $D_6 = \{-6\ ,\ -3\ ,\ -2\ ,\ -1\ ,\ 1\ ,\ 2\ ,\ 3\ ,\ 6\}$

4.2. $D_{27} = \{-27\ ,\ -9\ ,\ -3\ ,\ -1\ ,\ 1\ ,\ 3\ ,\ 9\ ,\ 27\}$

4.3. $D_8 = \{-8\ ,\ -4\ ,\ -2\ ,\ -1\ ,\ 1\ ,\ 2\ ,\ 4\ ,\ 8\}$

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1.1. $2x^2 - x - 1 = 0 \Leftrightarrow$

$\Leftrightarrow x = \frac{1 \pm \sqrt{1+8}}{4} \Leftrightarrow x = \frac{1 \pm 3}{4} \Leftrightarrow$

$\Leftrightarrow x = -\frac{1}{2} \vee x = 1$

$2x^2 - x - 1 = 2\left(x + \frac{1}{2}\right)(x-1)$

1.2. $3x^2 - 7x + 2 = 0 \Leftrightarrow x = \frac{7 \pm \sqrt{49-24}}{6} \Leftrightarrow$

$\Leftrightarrow x = \frac{7 \pm 5}{6} \Leftrightarrow x = \frac{1}{3} \vee x = 2$

$3x^2 - 7x + 2 = 3\left(x - \frac{1}{3}\right)(x-2)$

1.3. $5x^2 + 10x + 5 = 0 \Leftrightarrow$

$\Leftrightarrow x = \frac{-10 \pm \sqrt{100-100}}{10} \Leftrightarrow$

$\Leftrightarrow x = -1$ (– 1 é uma raiz dupla)

$5x^2 + 10x + 5 = 5(x+1)^2$

1.4. $7x^2 - 2x + 6 = 0 \Leftrightarrow x = \frac{2 \pm \sqrt{4-168}}{2}$

O polinómio não é fatorizável porque não tem raízes reais.

1.5. $x^2 - 4x - 4 = 0 \Leftrightarrow x = \frac{4 \pm \sqrt{16+16}}{2} \Leftrightarrow$

$\Leftrightarrow x = \frac{4 \pm \sqrt{16\times 2}}{2} \Leftrightarrow x = \frac{4 \pm \sqrt{16}\times\sqrt{2}}{2} \Leftrightarrow$

$\Leftrightarrow x = \frac{4 \pm 4\sqrt{2}}{2} \Leftrightarrow x = 2 \pm 2\sqrt{2}$

$x^2 - 4x - 4 = \left(x - 2 + 2\sqrt{2}\right)\left(x - 2 - 2\sqrt{2}\right)$

1.6. $x^2 - \sqrt{2}x - 4 = 0 \Leftrightarrow x = \frac{\sqrt{2} \pm \sqrt{2+16}}{2} \Leftrightarrow$

$\Leftrightarrow x = \frac{\sqrt{2} \pm \sqrt{9\times 2}}{2} \Leftrightarrow x = \frac{\sqrt{2} \pm 3\sqrt{2}}{2} \Leftrightarrow$

$\Leftrightarrow x = \frac{\sqrt{2} + 3\sqrt{2}}{2} \vee x = \frac{\sqrt{2} - 3\sqrt{2}}{2} \Leftrightarrow$

$\Leftrightarrow x = 2\sqrt{2} \vee x = -\sqrt{2}$

$x^2 - \sqrt{2}x - 4 = \left(x - 2\sqrt{2}\right)\left(x + \sqrt{2}\right)$

2.1. $9-4\sqrt{2}=\left(a-b\right)^2=a^2-2ab+b^2$

$-2ab=-4\sqrt{2}\Leftrightarrow ab=2\sqrt{2}$

Se $a=2$ e $b=\sqrt{2}$, $a^2+b^2=4+2=6\neq 9$.

Se $a=2\sqrt{2}$ e $b=1$, $a^2+b^2=8+1=9$.

$9-4\sqrt{2}=\left(2\sqrt{2}-1\right)^2$

$\sqrt{9-4\sqrt{2}}=\sqrt{\left(2\sqrt{2}-1\right)^2}=2\sqrt{2}-1 \qquad \left(2\sqrt{2}-1>0\right)$

2.2. $x^2+2\sqrt{2}x+x+2\sqrt{2}=0\Leftrightarrow$

$\Leftrightarrow x^2+\left(2\sqrt{2}+1\right)x+2\sqrt{2}=0\Leftrightarrow$

$\Leftrightarrow x=\dfrac{-2\sqrt{2}-1\pm\sqrt{\left(2\sqrt{2}+1\right)^2-8\sqrt{2}}}{2}\Leftrightarrow$

$\Leftrightarrow x=\dfrac{-2\sqrt{2}-1\pm\sqrt{8+4\sqrt{2}+1-8\sqrt{2}}}{2}\Leftrightarrow$

$\Leftrightarrow x=\dfrac{-2\sqrt{2}-1\pm\sqrt{9-4\sqrt{2}}}{2}\Leftrightarrow$

$\Leftrightarrow x=\dfrac{-2\sqrt{2}-1\pm\left(2\sqrt{2}-1\right)}{2}\Leftrightarrow$

$\Leftrightarrow x=\dfrac{\cancel{-2\sqrt{2}}-1+\cancel{2\sqrt{2}}-1}{2}\vee x=\dfrac{-2\sqrt{2}-1-2\sqrt{2}+1}{2}\Leftrightarrow$

$\Leftrightarrow x=-1\vee x=\dfrac{-4\sqrt{2}}{2}\Leftrightarrow$

$\Leftrightarrow x=-1\vee x=-2\sqrt{2}$

$x^2+2\sqrt{2}x+x+2\sqrt{2}=\left(x+1\right)\left(x+2\sqrt{2}\right)$

3. Por exemplo, $P\left(x\right)=\left(x-2\right)\left(x+3\right)$ e $P\left(x\right)=2\left(x-2\right)\left(x+3\right)$

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4.1. $A\left(x\right)=3x^3+7x^2-22x-8$

	3	7	−22	−8
$-\frac{1}{3}$		−1	−2	8
	3	6	−24	0

$3x^2+6x-24=0\Leftrightarrow$

$\Leftrightarrow x=\dfrac{-6\pm\sqrt{36+288}}{6}\Leftrightarrow x=\dfrac{-6\pm\sqrt{324}}{6}\Leftrightarrow$

$\Leftrightarrow x=\dfrac{-6\pm 18}{6}\Leftrightarrow x=x=-4\vee x=2$

$A\left(x\right)=3\left(x-2\right)\left(x+\dfrac{1}{3}\right)\left(x+4\right)$

4.2. $B\left(x\right)=2x^3-x^2-3x$

$=x\left(2x^2-x-3\right)$

$2x^2-x-3=0\Leftrightarrow x=\dfrac{1\pm\sqrt{1+24}}{4}\Leftrightarrow$

$\Leftrightarrow x=\dfrac{1\pm 5}{4}\Leftrightarrow x=-1\vee x=\dfrac{3}{2}$

$B\left(x\right)=2x\left(x+1\right)\left(x-\dfrac{3}{2}\right)$

4.3. $C\left(x\right)=x^4-x^3-x^2+x=$

$=x\left(x^3-x^2-x+1\right)$

	1	−1	−1	1
1		1	0	−1
	1	0	−1	0

$C\left(x\right)=x\left(x-1\right)\left(x^2-1\right)=$

$=x\left(x-1\right)\left(x-1\right)\left(x+1\right)=$

$=x\left(x-1\right)^2\left(x+1\right)$

4.4. $D\left(x\right)=x^4+3x^3+11x^2+27x+18$

	1	3	11	27	18
−1		−1	−2	−9	−18
	1	2	9	18	0
−2		−2	0	−18	
	1	0	9	0	

$D\left(x\right)=\left(x+1\right)\left(x+2\right)\left(x^2+9\right)$

x^2+9 não pode ser fatorizado porque não tem raízes reais.

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5.1. $P\left(x\right)=2x^3+9x^2+7x-6$

Divisores inteiros de – 6: –1, 1 , –2, 2, –3, 3, –6, 6.

$P\left(-1\right)=-6$; $P\left(1\right)=12$; $P\left(-2\right)=0$

– 2 é uma raiz de $P(x)$

	2	9	7	−6
−2		−4	−10	6
	2	5	−3	0

$2x^2+5x-3=0\Leftrightarrow x=\dfrac{-5\pm\sqrt{25+24}}{4}\Leftrightarrow$

$\Leftrightarrow x=\dfrac{-5\pm 7}{4}\Leftrightarrow x=-3\vee x=\dfrac{1}{2}$

$P\left(x\right)=2\left(x+2\right)\left(x+3\right)\left(x-\dfrac{1}{2}\right)$

5.2. $P\left(x\right)=2x^3+x^2-11x-10$

Divisores inteiros de –10: –1, 1, –2, 2, –5, 5, –10, 10

$P\left(-1\right)=0$

– 1 é uma raiz de $P(x)$

	2	1	−11	−10
−1		−2	1	10
	2	−1	−10	0

$2x^2-x-10=0\Leftrightarrow x=\dfrac{1\pm\sqrt{1+80}}{4}\Leftrightarrow$

$\Leftrightarrow x=\dfrac{1\pm 9}{4}\Leftrightarrow x=-2\vee x=\dfrac{5}{2}$

$P\left(x\right)=2\left(x+1\right)\left(x+2\right)\left(x-\dfrac{5}{2}\right)$

5.3. $P\left(x\right)=4x^3-20x^2-x+5$

Divisores inteiros de 5: 1, –1, 5, –5

$P\left(1\right)=-12$; $P\left(-1\right)=-18$; $P\left(5\right)=0$

5 é uma raiz de $P(x)$

	4	−20	−1	5
5		20	0	−5
	4	0	−1	0

$4x^2-1=0\Leftrightarrow x^2=\dfrac{1}{4}\Leftrightarrow x=-\dfrac{1}{2}\vee x=\dfrac{1}{2}$

$P\left(x\right)=4\left(x-5\right)\left(x+\dfrac{1}{2}\right)\left(x-\dfrac{1}{2}\right)$

5.4. $P\left(x\right)=9x^3-9x^2-x+1$

Divisores inteiros de 1: –1, 1

$P(-1) = -16$; $P(1) = 0$

1 é raiz de $P(x)$

	9	−9	−1	1
1		9	0	−1
	9	0	−1	0

$9x^2 - 1 = 0 \Leftrightarrow x^2 = \frac{1}{9} \Leftrightarrow x = -\frac{1}{3} \vee x = \frac{1}{3}$

$P(x) = 9(x-1)\left(x+\frac{1}{3}\right)\left(x-\frac{1}{3}\right)$

5.5. $P(x) = (x+1)^3 - 4x - 4$

$= (x+1)^3 - 4(x+1)$

$= (x+1)\left[(x+1)^2 - 4\right]$

$= (x+1)(x+1-2)(x+1+2)$

$= (x+1)(x-1)(x+3)$

5.6. $P(x) = (x^3-1)^2 - (3x+1)^2 =$

$= \left[(x^3-1)-(3x+1)\right]\left[(x^3-1)+(3x+1)\right]$

$= (x^3 - 1 - 3x - 1)(x^3 - 1 + 3x + 1)$

$= (x^3 - 3x - 2)(x^3 + 3x)$

$= (x+1)^2(x+2)x(x^2+3)$

$= x(x-2)(x+1)^2(x^2+3)$

Cálculos auxiliares:

Seja $A(x) = x^3 - 3x - 2$.

Divisores de – 2: –1, 1, –2, 2

$A(-1) = 0$

–1 é zero de $A(x)$

	1	0	−3	−2
−1		−1	1	2
	1	−1	−2	0

$x^2 - x - 2 = 0 \Leftrightarrow x = \frac{1 \pm \sqrt{1+8}}{2} \Leftrightarrow$

$\Leftrightarrow x = -1 \vee x = 2$

$A(x) = (x+1)(x+1)(x-2) =$

$= (x+1)^2(x+2)$

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6.1. $A(x) = 2x^3 + x + 3$

Divisores de 3: –1, 1, –3, 3

$A(-1) = 0$

–1 é raiz de $A(x)$

	2	0	1	3
−1		−2	2	−3
	2	−2	3	0

$2x^3 + x + 3 = 0 \Leftrightarrow (x+1)(2x^2 - 2x + 3) = 0 \Leftrightarrow$

$\Leftrightarrow x + 1 = 0 \vee 2x^2 - 2x + 3 = 0 \Leftrightarrow$

$\Leftrightarrow x = -1 \vee x = \frac{2 \pm \sqrt{4-24}}{4} \Leftrightarrow$

$\Leftrightarrow x = -1 \vee x \in \varnothing \Leftrightarrow$

$\Leftrightarrow x = -1$

$S = \{-1\}$

6.2. $A(x) = 3x^3 - 12x^2 - x + 4$

Divisores de – 4: –1, 1, –2, 2, –4, 4

$A(-1) = -10$; $A(1) = -6$; $A(-2) = -66$; $A(2) = -22$

$A(-4) = -376$; $A(4) = 0$

	3	−12	−1	4
4		12	0	−4
	3	0	−1	0

$3x^3 - 12x^2 - x + 4 = 0 \Leftrightarrow$

$\Leftrightarrow (x-4)(3x^2-1) = 0 \Leftrightarrow$

$\Leftrightarrow x - 4 = 0 \vee 3x^2 - 1 = 0 \Leftrightarrow$

$\Leftrightarrow x = 4 \vee x^2 = \frac{1}{3} \Leftrightarrow$

$\Leftrightarrow x = 4 \vee x = -\sqrt{\frac{1}{3}} \vee x = \sqrt{\frac{1}{3}} \Leftrightarrow$

$\Leftrightarrow x = 4 \vee x = -\frac{1}{\sqrt{3}} \vee x = \frac{1}{\sqrt{3}} \Leftrightarrow$

$\Leftrightarrow x = 4 \vee x = -\frac{\sqrt{3}}{3} \vee x = \frac{\sqrt{3}}{3}$

$S = \left\{-\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}, 4\right\}$

6.3. $4x^4 + 4x^3 + x^2 = 0 \Leftrightarrow$

$\Leftrightarrow x^2(4x^2 + 4x + 1) = 0 \Leftrightarrow$

$\Leftrightarrow x^2 = 0 \vee 4x^2 + 4x + 1 = 0 \Leftrightarrow$

$\Leftrightarrow x = 0 \vee x = \frac{-4 \pm \sqrt{16-16}}{8} \Leftrightarrow$

$\Leftrightarrow x = 0 \vee x = -\frac{1}{2}$

$S = \left\{-\frac{1}{2}, 0\right\}$

6.4. $4x^4 - 3x^2 = x \Leftrightarrow 4x^4 - 3x^2 - x = 0 \Leftrightarrow$

$\Leftrightarrow x(4x^3 - 3x - 1) = 0$

Fatorização de $A(x) = 4x^3 - 3x - 1$

Divisores de 1: –1, 1

$A(-1) = -2$; $A(1) = 0$

	4	0	−3	−1
1		4	4	1
	4	4	1	0

$x(4x^3 - 3x - 1) = 0 \Leftrightarrow$

$\Leftrightarrow x(x-1)(4x^2 + 4x + 1) = 0 \Leftrightarrow$

$\Leftrightarrow x = 0 \vee x - 1 = 0 \vee (2x+1)^2 = 0 \Leftrightarrow$

$\Leftrightarrow x = 0 \vee x = 1 \vee 2x + 1 = 0 \Leftrightarrow$

$\Leftrightarrow x = 0 \vee x = 1 \vee x = -\frac{1}{2}$

$S = \left\{-\frac{1}{2}, 0, 1\right\}$

7. $P(x) = 12x^3 - 8x^2 - x + 1$

7.1. Divisores de 1: –1 e 1

$P(-1) = -18$ e $P(1) = 4$

Os divisores inteiros do termo independente não são raízes de $P(x)$. Logo, $P(x)$ não tem raízes inteiras.

7.2.

	12	−8	−1	1
$-\frac{1}{3}$		−4	4	−1
	12	−12	3	0

$12x^3-8x^2-x+1=0 \Leftrightarrow$

$\Leftrightarrow \left(x+\frac{1}{3}\right)\left(12x^2-12x+3\right)=0 \Leftrightarrow$

$\Leftrightarrow x+\frac{1}{3}=0 \vee 3\left(4x^2-4x+1\right)=0 \Leftrightarrow$

$\Leftrightarrow x=-\frac{1}{3} \vee (2x-1)^2=0 \Leftrightarrow$

$\Leftrightarrow x=-\frac{1}{3} \vee 2x-1=0 \Leftrightarrow$

$\Leftrightarrow x=-\frac{1}{3} \vee x=\frac{1}{2}$

$S=\left\{-\frac{1}{3}\ ,\ \frac{1}{2}\right\}$

8. $P(x)=16x^4-8x^2+1$

	16	0	–8	0	1
$\frac{1}{2}$		8	4	– 2	–1
	16	8	– 4	–2	0
$\frac{1}{2}$		8	8	2	
	16	16	4	0	

$16x^2+16x+4=0 \Leftrightarrow 4x^2+4x+1=0 \Leftrightarrow$

$\Leftrightarrow x=\frac{-4\pm\sqrt{16-16}}{8} \Leftrightarrow x=-\frac{1}{2}$

$S=\left\{-\frac{1}{2}\ ,\ \frac{1}{2}\right\}$

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9.1. $4x^4-25x^2+6=0$

Considerando $x^2=y$:

$4y^2-25y+6=0 \Leftrightarrow$

$\Leftrightarrow y^2=\frac{25\pm\sqrt{25^2-96}}{8} \Leftrightarrow y=\frac{25\pm\sqrt{529}}{8} \Leftrightarrow$

$\Leftrightarrow y=\frac{25\pm 23}{8} \Leftrightarrow$

$\Leftrightarrow y=6 \vee y=\frac{1}{4} \qquad \left| y=x^2\right.$

$\Leftrightarrow x^2=6 \vee x^2=\frac{1}{4}$

$\Leftrightarrow x=-\sqrt{6} \vee x=\sqrt{6} \vee x=-\frac{1}{2} \vee x=\frac{1}{2}$

$S=\left\{-\sqrt{6}\ ,\ -\frac{1}{2}\ ,\ \frac{1}{2}\ ,\ \sqrt{6}\right\}$

9.2. $3x^4-2x^2-8=0$

Considerando $x^2=y$:

$3y^2-2y-8=0 \Leftrightarrow$

$\Leftrightarrow y=\frac{2\pm\sqrt{4+96}}{6} \Leftrightarrow y=\frac{2\pm 10}{6} \Leftrightarrow$

$\Leftrightarrow y=2 \vee y=-\frac{4}{3} \Leftrightarrow \qquad \left| y=x^2\right.$

$\Leftrightarrow x^2=2 \vee x^2=-\frac{4}{3} \Leftrightarrow$

$\Leftrightarrow x=-\sqrt{2} \vee x=\sqrt{2}$

$S=\left\{-\sqrt{2}\ ,\ \sqrt{2}\right\}$

10. $x^6+7x^3-8=0 \Leftrightarrow \left(x^3\right)^2+7x^3-8=0$

Considerando $y=x^3$:

$y^2+7y-8=0 \Leftrightarrow y=\frac{-7\pm\sqrt{49+32}}{2} \Leftrightarrow$

$\Leftrightarrow y=\frac{-7\pm 9}{2} \Leftrightarrow$

$\Leftrightarrow y=1 \vee y=-8 \Leftrightarrow \left| y=x^3\right.$

Para $x^3=1 \vee x^3=-8 \Leftrightarrow$

$\Leftrightarrow x=\sqrt[3]{1} \vee x=\sqrt[3]{-8} \Leftrightarrow$

$\Leftrightarrow x=1 \vee x=-2$

$S=\{-2\ ,\ 1\}$

11.1. $P(x)=x^4-3x^2-4$

Considerando $y=x^2$:

$x^4-3x^2-4=0 \Leftrightarrow y^2-3y-4=0 \Leftrightarrow$

$\Leftrightarrow y=\frac{3\pm\sqrt{9+16}}{2} \Leftrightarrow y=\frac{3\pm 5}{2} \Leftrightarrow$

$\Leftrightarrow y=-1 \vee y=4$

$y^2-3y-4=(y+1)(y-4)$

Como $y=x^2$, vem:

$x^4=3x^2-4=\left(x^2+1\right)\left(x^2-4\right)=$

$=\left(x^2+1\right)(x-2)(x+2)=$

$=(x-2)(x+2)\left(x^2+1\right)$

11.2. $P(x)=x^4-6x^2+8$

Se $y=x^2$:

$x^4-6x^2+8=0 \Leftrightarrow$

$\Leftrightarrow y^2-6y+8=0 \Leftrightarrow$

$\Leftrightarrow y=\frac{6\pm\sqrt{36-32}}{2} \Leftrightarrow$

$\Leftrightarrow y=\frac{6\pm 2}{2} \Leftrightarrow y=2 \vee y=4$

$y^2-6y+8=(y-2)(y-4)$

Como $y=x^2$:

$x^4-6x^2+8=\left(x^2-2\right)\left(x^2-4\right)=$

$=\left(x-\sqrt{2}\right)\left(x+\sqrt{2}\right)(x-2)(x+2)$

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12. $(x-3)(x+1)\le 0$

$x-3=0 \Leftrightarrow x=3\ ;\ x+2=0 \Leftrightarrow x=2$

x	$-\infty$	– 2		3	$+\infty$
$x-3$	–	–	–	0	+
$(x+2)$	–	0	+	+	+
$(x-3)(x+2)$	+	0	–	0	+

$(x-3)(x+2)\le 0 \Leftrightarrow x\in[-2\ ,\ 3]$

$S=[-2\ ,\ 3]$

13.1. $-x(x+3)\le 0 \Leftrightarrow$

$\Leftrightarrow (-x\le 0 \wedge x+3\ge 0)\vee(-x\ge 0 \wedge x+3\le 0) \Leftrightarrow$

$\Leftrightarrow (x\ge 0 \wedge x\ge -3)\vee(x\le 0 \wedge x\le -3) \Leftrightarrow$

$\Leftrightarrow x\ge 0 \vee x\le -3 \Leftrightarrow x\in\left]-\infty\ ,\ -3\right]\cup\left[0\ ,\ +\infty\right[$

$S=\left]-\infty\ ,\ -3\right]\cup\left[0\ ,\ +\infty\right[$

x	$-\infty$	-3		0	$+\infty$
$-x$	+	+	+	0	–
$x+3$	–	0	+	+	+
$-x(x+3)$	–	0	+	0	–

$-x(x+3) \leq 0 \Leftrightarrow x \in]-\infty\,,\ -3] \cup [0\,,\ +\infty[$

$S =]-\infty\,,\ -3] \cup [0\,,\ +\infty[$

13.2. $(1-x)(x-2) > 0 \Leftrightarrow$

$\Leftrightarrow (1-x > 0 \wedge x-2 > 0) \vee (1-x < 0 \wedge x-2 < 0) \Leftrightarrow$

$\Leftrightarrow (x < 1 \wedge x > 2) \vee (x > 1 \wedge x < 2) \Leftrightarrow$

$\Leftrightarrow x \in \varnothing \vee x \in]1\,,\ 2[\ \Leftrightarrow\ x \in]1\,,\ 2[$

$S =]1\,,\ 2[$

x		1		2	
$1-x$	+	0	–	–	–
$x-2$	–	–	–	0	+
$(1-x)(x-2)$	–	0	+	0	–

$(1-x)(x-2) > 0 \Leftrightarrow x \in]1\,,\ 2[$

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14.1. $-x^2 - 1 \leq 0 \Leftrightarrow x^2 + 1 \geq 0$

$\Leftrightarrow x \in \mathbb{R}$ (Condição universal)

$S = \mathbb{R}$

14.2. $-(x-1)^2 \geq 0 \Leftrightarrow (x-1)^2 \leq 0 \Leftrightarrow x = 1$

$S = \{1\}$

14.3. $x^2 - 5x + 6 \geq 0$

$x^2 - 5x + 6 = 0 \Leftrightarrow x = \dfrac{5 \pm \sqrt{25-24}}{2} \ \Leftrightarrow\ x = 2 \vee x = 3$

$x^2 - 5x + 6 \geq 0 \Leftrightarrow (x-2)(x-3) \geq 0$

x	$-\infty$	2		3	$+\infty$
$(x-2)$	–	0	+	+	+
$(x-3)$	–	–	–	0	+
$(x-2)(x-3)$	+	0	–	0	+

$(x-2)(x-3) \geq 0 \Leftrightarrow x \in]-\infty\,,\ 2] \cup [3\,,\ +\infty[$

$S =]-\infty\,,\ 2] \cup [3\,,\ +\infty[$

14.4. $(x-1)^2 < (x-1)(x+1) \Leftrightarrow$

$\Leftrightarrow (x-1)^2 - (x-1)(x+1) < 0 \Leftrightarrow$

$\Leftrightarrow (x-1)\left[(x-1)-(x+1)\right] < 0 \Leftrightarrow$

$\Leftrightarrow (x-1)(-2) < 0 \Leftrightarrow$

$\Leftrightarrow x - 1 > 0 \Leftrightarrow x > 1$

$S =]1\,,\ +\infty[$

15.1. $2x^3 - 3x^2 \geq 0 \Leftrightarrow$

$\Leftrightarrow x^2(2x-3) \geq 0 \Leftrightarrow x \in \{0\} \cup \left[\dfrac{3}{2}\,,\ +\infty\right[$

x	$-\infty$	0		$\frac{3}{2}$	$+\infty$
x^2	+	0	+	+	+
$2x-3$	–	–	–	0	+
$x^2(2x-3)$	–	0	–	0	+

$S = \{0\} \cup \left[\dfrac{3}{2}\,,\ +\infty\right[$

15.2. $x^3 - x < 0 \Leftrightarrow$

$\Leftrightarrow x(x^2-1) < 0 \Leftrightarrow$

$\Leftrightarrow x(x-1)(x+1) < 0 \Leftrightarrow$

$\Leftrightarrow x \in]-\infty\,,\ -1[\cup]0\,,\ 1[$

x	$-\infty$	-1		0		1	$+\infty$
x	–	–	–	0	+	+	+
$x+1$	–	0	+	+	+	+	+
$x-1$	–	–	–	–	–	0	+
$x(x-1)(x+1)$	–	0	+	0	–	0	+

$S =]-\infty\,,\ -1[\cup]0\,,\ 1[$

15.3. $P(x) = x^3 - x^2 - 4x + 4 < 0$

Divisores de 4: –1, 1, –2, 2, –4 e 4

$P(-1) = 6\ ;\ P(1) = 0$

1 é raiz de $P(x)$

$$\begin{array}{c|cccc} & 1 & -1 & -4 & 4 \\ 1 & & 1 & 0 & -4 \\ \hline & 1 & 0 & -4 & \boxed{0} \end{array}$$

$x^3 - x^2 + 4x + 4 < 0 \Leftrightarrow (x-1)(x^2-4) < 0 \Leftrightarrow$

$\Leftrightarrow (x-1)(x-2)(x+2) < 0 \Leftrightarrow$

$\Leftrightarrow x \in]-\infty\,,\ -2[\cup]1\,,\ 2[$

x	$-\infty$	-2		1		2	$+\infty$
$x-1$	–	–	–	0	+	+	+
$x-2$	–	–	–	–	–	0	+
$x+2$	–	0	+	+	+	+	+
$(x-1)(x-1)(x+1)$	–	0	+	0	–	0	+

$S =]-\infty\,,\ -2[\cup]1\,,\ 2[$

15.4. $x^3 - 3x^2 > 6 - 2x \Leftrightarrow x^3 - 3x^2 + 2x - 6 > 0$

Seja $P(x) = x^3 - 3x^2 + 2x - 6$

Divisores de – 6: –1, 1, –2, 2, –3, 3, –6 e 6

$P(-1) = -12\ ;\ P(1) = -6\ ;\ P(-2) = -30$

$P(2) = -6\ ;\ P(-3) = -66\ ;\ P(3) = 0$

3 é uma raiz de $P(x)$.

$$\begin{array}{c|cccc} & 1 & -3 & 2 & -6 \\ 3 & & 3 & 0 & 6 \\ \hline & 1 & 0 & 2 & \boxed{0} \end{array}$$

$x^3 - 3x^2 + 2x - 6 > 0 \Leftrightarrow (x-3)(x^2+2) > 0$

Como $x^2 + 2 > 0,\ \forall x \in \mathbb{R}$, então:

$(x-3)(x^2+2) > 0 \Leftrightarrow x - 3 > 0 \Leftrightarrow x > 3$

$S =]3\,,\ +\infty[$

15.5. $x^4 \geq x^2 \Leftrightarrow x^4 - x^2 \geq 0 \Leftrightarrow$

$\Leftrightarrow x^2(x^2-1) \geq 0 \Leftrightarrow$

$\Leftrightarrow x^2(x-1)(x+1) \geq 0 \Leftrightarrow$

$\Leftrightarrow x \in]-\infty\,,\ -1] \cup \{0\} \cup [1\,,\ +\infty[$

x	$-\infty$	-1		0		1	$+\infty$
x^2	+	+	+	0	+	+	+
$x-1$	–	–	–	–	–	0	+
$x+1$	–	0	+	+	+	+	+
$x^2(x-1)(x+1)$	+	0	–	0	–	0	+

$S =]-\infty\,,\ -1[\cup \{0\} \cup]1\,,\ +\infty[$

15.6. $x^3 + 9x > 6x^2 \Leftrightarrow$

$\Leftrightarrow x^3 - 6x^2 + 9x > 0 \Leftrightarrow$

$\Leftrightarrow x\left(x^2 - 6x + 9\right) > 0 \Leftrightarrow$

$\Leftrightarrow x\left(x - 3\right)^2 > 0 \Leftrightarrow$

$\Leftrightarrow x \in \left]0\ ,\ 3\right[\cup \left]3\ ,\ +\infty\right[$

x	$-\infty$	0		3	$+\infty$
x	–	0	+	+	+
$(x-3)^2$	+	+	+	0	+
$x(x-3)^2$	–	0	+	0	+

$S = \left]0\ ,\ 3\right[\cup \left]3\ ,\ +\infty\right[$

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Atividades complementares

16.1. $A(x) = 0 \Leftrightarrow 3x^2 + 7x + 2 = 0 \Leftrightarrow$

$\Leftrightarrow x = \frac{-7 \pm \sqrt{49 - 24}}{6} \Leftrightarrow x = \frac{-7 \pm 5}{6} \Leftrightarrow$

$\Leftrightarrow x = -2 \vee x = -\frac{1}{3}$

$A(x) = 3\left(x + \frac{1}{3}\right)(x + 2)$

16.2. $B(x) = 0 \Leftrightarrow 2x^2 + 9x + 4 = 0 \Leftrightarrow$

$\Leftrightarrow x = \frac{-9 \pm \sqrt{81 - 32}}{4} \Leftrightarrow x = \frac{-9 \pm 7}{4} \Leftrightarrow$

$\Leftrightarrow x = -4 \vee x = -\frac{1}{2}$

$B(x) = 2(x + 4)\left(x + \frac{1}{2}\right)$

16.3. $C(x) = 0 \Leftrightarrow 6x^2 + 19x + 10 = 0 \Leftrightarrow$

$\Leftrightarrow x = \frac{-19 \pm \sqrt{19^2 - 240}}{12} \Leftrightarrow x = \frac{-19 \pm 11}{12} \Leftrightarrow$

$\Leftrightarrow x = -\frac{5}{2} \vee x = -\frac{2}{3}$

$C(x) = 6\left(x + \frac{5}{2}\right)\left(x + \frac{2}{3}\right)$

16.4. $D(x) = 0 \Leftrightarrow 12x^2 + 19x + 4 = 0 \Leftrightarrow$

$\Leftrightarrow x = \frac{-19 \pm \sqrt{19^2 - 192}}{24} \Leftrightarrow x = \frac{-19 \pm 13}{24} \Leftrightarrow$

$\Leftrightarrow x = -\frac{4}{3} \vee x = -\frac{1}{4}$

$D(x) = 12\left(x + \frac{4}{3}\right)\left(x + \frac{1}{4}\right)$

16.5. $E(x) = 0 \Leftrightarrow 8x^2 - 3x - 5 = 0 \Leftrightarrow$

$\Leftrightarrow x = \frac{3 \pm \sqrt{9 + 169}}{16} \Leftrightarrow x = \frac{3 \pm 13}{16} \Leftrightarrow$

$\Leftrightarrow x = 1 \vee x = -\frac{5}{8}$

$E(x) = 8(x - 1)\left(x + \frac{5}{8}\right)$

16.6. $F(x) = 9x^2 + 6x + 1 =$

$= (3x)^2 + 2 \times 3x + 1^2 =$

$= (3x + 1)^2 = (3x + 1)(3x + 1)$

17.1. $P(x) = x^3 - 8\ ;\ P(2) = 0$

	1	0	0	– 8
2		2	4	8
	1	2	4	0

$x^2 + 2x + 4 = 0 \Leftrightarrow$

$\Leftrightarrow x = \frac{-2 \pm \sqrt{4 - 16}}{2} \Leftrightarrow$

$\Leftrightarrow x \in \varnothing$

$P(x) = (x - 2)\left(x^2 + 2x + 4\right)$

17.2. $P(x) = x^3 + 5x^2 - x - 5\ ;\ P(-5) = 0$

	1	5	– 1	– 5
– 5		– 5	0	5
	1	0	– 1	0

$P(x) = (x + 5)\left(x^2 - 1\right) = (x + 5)(x - 1)(x + 1)$

17.3. $P(x) = x^3 + 19x^2 - 42x =$

$= x\left(x^2 + 19x - 42\right) =$

$x^2 + 19x - 42 = 0 \Leftrightarrow x = \frac{-19 \pm \sqrt{19^2 + 168}}{2} \Leftrightarrow$

$\Leftrightarrow x = \frac{-19 \pm 23}{2} \Leftrightarrow x = -21 \vee x = 2$

$P(x) = x(x - 2)(x + 21)$

17.4. $P(x) = x^4 + x^3 - 3x^2 - x + 2$

	1	1	– 3	– 1	2
1		1	2	– 1	– 2
	1	2	– 1	– 2	0
1		1	3	2	
	1	3	2	0	

$x^2 + 3x + 2 = 0 \Leftrightarrow$

$\Leftrightarrow x = \frac{-3 \pm \sqrt{9 - 8}}{2} \Leftrightarrow x = -2 \vee x = -1$

$P(x) = (x - 1)^2 (x + 1)(x + 2)$

18. Por exemplo:

18.1. $P(x) = (x - 1)^3$

18.2. $P(x) = x^2 (x - 1)^2$

19.1. $P(x) = x^3 - 2x^2 - 5x + 6$

Divisores de 6: –1, 1, –2, 2, –3, 3, –6 e 6

$P(-1) = 8\ ;\ P(1) = 0\ ;\ P(-2) = 0$

$P(2) = -4\ ;\ P(-3) = -24\ ;\ P(3) = 0$

$P(x)$ é do 3.º grau e tem as raízes 1, –2 e 3.
Logo, $P(x) = (x - 1)(x + 2)(x - 3)$.

19.2. $P(x) = x^3 + 3x^2 - 6x - 8$

Divisores de –8: –1, 1, –2, 2, –4, 4, –8 e 8

$P(-1) = 0$

–1 é uma raiz de $P(x)$.

	1	3	– 6	– 8
– 1		– 1	– 2	8
	1	2	– 8	0

$x^2 + 2x - 8 = 0 \Leftrightarrow x = \frac{-2 \pm \sqrt{4 + 32}}{2} \Leftrightarrow$

$\Leftrightarrow x = -4 \vee x = 2$

$P(x) = (x + 1)(x + 4)(x - 2)$

19.3. $P(x) = 2x^3 + x^2 - 4x - 3$

Divisores de –3: –1, 1, –3 e 3

$P(-1)=0$

-1 é uma raiz de $P(x)$

$$\begin{array}{c|cccc} & 2 & 1 & -4 & -3 \\ -1 & & -2 & 1 & 3 \\ \hline & 2 & -1 & -3 & \boxed{0} \end{array}$$

$2x^2-x-3=0 \Leftrightarrow$

$\Leftrightarrow x=\dfrac{1\pm\sqrt{1+24}}{4} \Leftrightarrow$

$\Leftrightarrow x=\dfrac{1\pm 5}{4} \Leftrightarrow x=\dfrac{3}{2} \vee x=-1$

$P(x)=(x+1)\left(x-\dfrac{3}{2}\right)(x+1) \Leftrightarrow$

$=(x+1)^2\left(x-\dfrac{3}{2}\right)$

20.1. $P(x)=x^3-x^2-2x+2$

Divisores de 2: –1, 1, 2 e –2

$P(-1)=2 \;;\; P(1)=0$

1 é uma raiz de $P(x)$.

$$\begin{array}{c|cccc} & 1 & -1 & -2 & 2 \\ 1 & & 1 & 0 & -2 \\ \hline & 1 & 0 & -2 & \boxed{0} \end{array}$$

$P(x)=(x-1)(x^2-2)=(x-1)\left(x^2-\left(\sqrt{2}\right)^2\right)$

$P(x)=(x-1)\left(x-\sqrt{2}\right)\left(x+\sqrt{2}\right)$

20.2. $P(x)=x^3-3x^2+x+1$

Divisores de 1: 1 e –1

$P(1)=0$

$$\begin{array}{c|cccc} & 1 & -3 & 1 & 1 \\ 1 & & 1 & -2 & -1 \\ \hline & 1 & -2 & -1 & \boxed{0} \end{array}$$

$x^2-2x-1=0 \Leftrightarrow$

$\Leftrightarrow x=\dfrac{2\pm\sqrt{4+4}}{2} \Leftrightarrow x=\dfrac{2\pm 2\sqrt{2}}{2} \Leftrightarrow$

$\Leftrightarrow x=1-\sqrt{2} \vee x=1+\sqrt{2}$

$P(x)=(x-1)\left(x-1+\sqrt{2}\right)\left(x-1-\sqrt{2}\right)$

21.1. $P(x)=x^4-3x^3+x^2+4$

Divisores de 4: –1, 1, –2, 2, –4 e 4

$P(-1)=9 \;;\; P(1)=3 \;;\; P(-2)=48 \;;\; P(2)=0$

Verificação se 2 é uma raiz dupla:

$$\begin{array}{c|ccccc} & 1 & -3 & 1 & 0 & 4 \\ 2 & & 2 & -2 & -2 & -4 \\ \hline & 1 & -1 & -1 & -2 & \boxed{0} \\ 2 & & 2 & 2 & 2 & \\ \hline & 1 & 1 & 1 & \boxed{0} & \end{array}$$

$x^2+x+1=0 \Leftrightarrow x=\dfrac{-1\pm\sqrt{1-4}}{2} \Leftrightarrow x\in\varnothing$

$x^4-3x^3+x^2+4=0 \Leftrightarrow (x-2)^2\left(x^2+x+1\right)^2=0 \Leftrightarrow$

$\Leftrightarrow x=2$

$S=\{2\}$

21.2. $P(x)=2x^3+11x^2-7x-6$

Divisores de –6: –1, 1, –2, 2, –3, 3, –6 e 6

$P(1)=0$

$$\begin{array}{c|cccc} & 2 & 11 & -7 & -6 \\ 1 & & 2 & 13 & 6 \\ \hline & 2 & 13 & 6 & \boxed{0} \end{array}$$

$2x^2+13x+6=0 \Leftrightarrow$

$\Leftrightarrow x=\dfrac{-13\pm\sqrt{169-48}}{4} \Leftrightarrow x=\dfrac{-13\pm 11}{4} \Leftrightarrow$

$\Leftrightarrow x=-6 \vee x=-\dfrac{1}{2}$

$2x^3+11x^2-7x-6=0 \Leftrightarrow$

$\Leftrightarrow x=1 \vee x=-6 \vee x=-\dfrac{1}{2}$

$S=\left\{-6\;,\;-\dfrac{1}{2}\;,\;1\right\}$

22.1. $x^3+3x^2+x=-3 \Leftrightarrow x^3+3x^2+x+3=0$

$P(x)=x^3+3x^2+x+3$

Divisores de 3: –1, 1, –3 e 3

$P(-1)=4 \;;\; P(1)=8 \;;\; P(-3)=0$

$$\begin{array}{c|cccc} & 1 & 3 & 1 & 3 \\ -3 & & -3 & 0 & -3 \\ \hline & 1 & 0 & 1 & \boxed{0} \end{array}$$

$x^3+3x^2+x+3=0 \Leftrightarrow (x+3)\left(x^2+1\right)=0 \Leftrightarrow$

$\Leftrightarrow x+3=0 \vee x^2+1=0 \Leftrightarrow$

$\Leftrightarrow x=-3 \vee x\in\varnothing \Leftrightarrow x=-3$

$S=\{-3\}$

22.2. $x^3-x^2=x-1 \Leftrightarrow x^3-x^2-x+1=0$

$P(x)=x^3-x^2-x+1$

Divisores de 1: –1 e 1

$P(-1)=0$

$$\begin{array}{c|cccc} & 1 & -1 & -1 & 1 \\ -1 & & -1 & 2 & -1 \\ \hline & 1 & -2 & 1 & \boxed{0} \end{array}$$

$x^3-x^2-x+1=0 \Leftrightarrow (x-1)\left(x^2-2x+1\right)=0 \Leftrightarrow$

$\Leftrightarrow x-1=0 \vee (x-1)^2=0 \Leftrightarrow$

$\Leftrightarrow x=1 \vee x=-1$

$S=\{-1\;,\;1\}$

23.1. $7x^4-30x^2+8=0 \Leftrightarrow 7y^2-30y+8=0 \Leftrightarrow$ $\quad \left| x^2=y \right.$

$\Leftrightarrow y=\dfrac{30\pm\sqrt{900-224}}{14} \Leftrightarrow y=\dfrac{30\pm 26}{14} \Leftrightarrow$

$\Leftrightarrow y=\dfrac{2}{7} \vee y=4 \Leftrightarrow x^2=\dfrac{2}{7} \vee x^2=4$ $\quad \left| y=x^2 \right.$

$\Leftrightarrow x=-\sqrt{\dfrac{2}{7}} \vee x=\sqrt{\dfrac{2}{7}} \vee x=-2 \vee x=2 \Leftrightarrow$

$\Leftrightarrow x=-\dfrac{\sqrt{2}\sqrt{7}}{7} \vee x=\dfrac{\sqrt{2}\sqrt{7}}{7} \vee x=-2 \vee x=2 \Leftrightarrow$

$\Leftrightarrow x=-\dfrac{\sqrt{14}}{7} \vee x=\dfrac{\sqrt{14}}{7} \vee x=-2 \vee x=2$

$S=\left\{-2\;,\;-\dfrac{\sqrt{14}}{7}\;,\;\dfrac{\sqrt{14}}{7}\;,\;2\right\}$

23.2. $12x^4-20x^2=8 \Leftrightarrow 12x^4-20x^2-8=0 \Leftrightarrow$ $\quad \left| x^2=y \right.$

$\Leftrightarrow 12y^2-20y-8=0 \Leftrightarrow 3y^2-5y-2=0 \Leftrightarrow$

$\Leftrightarrow y=\dfrac{5\pm\sqrt{25+24}}{6} \Leftrightarrow y=2 \vee y=-\dfrac{1}{3} \Leftrightarrow$ $\quad \left| y=x^2 \right.$

$\Leftrightarrow x^2=2 \vee x^2=-\dfrac{1}{3} \Leftrightarrow x=-\sqrt{2} \vee x=\sqrt{2} \vee x\in\varnothing \Leftrightarrow$

$\Leftrightarrow x=-\sqrt{2} \vee x=\sqrt{2}$

$S=\left\{-\sqrt{2}\;,\;\sqrt{2}\right\}$

24.1. $P(x)=2x^4+13x^2-15$

$2x^4+13x^2-15=0 \Leftrightarrow 2y^2+13y-15=0 \Leftrightarrow$ $\quad | \; x^2=y$

$\Leftrightarrow 2y^2+13y-15=0 \Leftrightarrow$

$\Leftrightarrow y=\dfrac{-13\pm\sqrt{169+120}}{4} \Leftrightarrow$

$\Leftrightarrow y=\dfrac{-13\pm 17}{4} \Leftrightarrow y=-\dfrac{15}{2} \vee y=1$

$2y^2+13y-15=2(y+15)(y-2)$ $\quad | \; y=x^2$

$2x^4+13x^2-15=2\left(x^2+\dfrac{15}{2}\right)\left(x^2-1\right)=$

$=\left(2x^2+15\right)(x-1)(x+1)$

$P(x)=\left(2x^2+15\right)(x-1)(x+1)$

24.2. $Q(x)=x^4+x^2-2 \Leftrightarrow x^4+x^2-2=0 \Leftrightarrow$ $\quad | \; x^2=y$

$\Leftrightarrow y^2+y-2=0 \Leftrightarrow y=\dfrac{-1\pm\sqrt{1+9}}{2} \Leftrightarrow$

$\Leftrightarrow y=1 \vee y=-2$

$y^2+y-2=(y-1)(y+2)$ $\quad | \; y=x^2$

$x^4+x^2-2=\left(x^2-1\right)\left(x^2+2\right)=(x-1)(x+1)\left(x^2+2\right)$

25.1. $(x-5)(-x+2)\le 0 \Leftrightarrow x\in \left]-\infty\,,\,2\right]\cup\left[5\,,\,+\infty\right[$

x	$-\infty$	2		5	$+\infty$
$x-5$	$-$	$-$	$-$	0	$+$
$-x+2$	$+$	0	$-$	$-$	$-$
$(x-5)(-x+2)$	$-$	0	$+$	0	$-$

$S=\left]-\infty\,,\,2\right]\cup\left[5\,,\,+\infty\right[$

25.2. $-\left(x-\sqrt{2}\right)(-x+3)<0 \Leftrightarrow$

$\Leftrightarrow \left(x-\sqrt{2}\right)(-x+3)>0 \Leftrightarrow x\in\left]\sqrt{2}\,,\,3\right[$

x	$-\infty$	$\sqrt{2}$		3	$+\infty$
$x-\sqrt{2}$	$-$	0	$+$	$+$	$+$
$-x+3$	$+$	$+$	$+$	0	$-$
$\left(x-\sqrt{2}\right)(-x+3)$	$-$	0	$+$	0	$-$

$S=\left]\sqrt{2}\,,\,3\right[$

26.1. $2x^2-x-1=0 \Leftrightarrow$

$\Leftrightarrow x=\dfrac{1\pm\sqrt{1+8}}{4} \Leftrightarrow x=\dfrac{1\pm 3}{4}$

$\Leftrightarrow x=-\dfrac{1}{2} \vee x=1$

$2x^2-x-1\ge 0 \Leftrightarrow 2\left(x+\dfrac{1}{2}\right)(x-1)\ge 0$

$\Leftrightarrow \left]-\infty\,,\,-\dfrac{1}{2}\right]\cup\left[1\,,\,+\infty\right[$

x	$-\infty$	$-\dfrac{1}{2}$		1	$+\infty$
$x+\dfrac{1}{2}$	$-$	0	$+$	$+$	$+$
$x-1$	$-$	$-$	$-$	0	$+$
$2x^2-x-1$	$+$	0	$-$	0	$+$

$S=\left]-\infty\,,\,-\dfrac{1}{2}\right]\cup\left[1\,,\,+\infty\right[$

26.2. $(x-2)(x-3)\le x^2-4 \Leftrightarrow$

$\Leftrightarrow (x-2)(x-3)-(x-2)(x+2)\le 0 \Leftrightarrow$

$\Leftrightarrow (x-2)\left[(x-3)-(x+2)\right]\le 0 \Leftrightarrow$

$\Leftrightarrow (x-2)(-5)\le 0 \Leftrightarrow$

$\Leftrightarrow 5(x-2)\ge 0 \Leftrightarrow$

$\Leftrightarrow x\ge 2$

$S=\left[2\,,\,+\infty\right[$

27.1. $(2x-3)^2\le 0 \Leftrightarrow 2x-3=0 \Leftrightarrow$

$\Leftrightarrow x=\dfrac{3}{2}$

$S=\left\{\dfrac{3}{2}\right\}$

27.2. $\left(x-\sqrt{2}\right)^3>0 \Leftrightarrow x-\sqrt{2}>0 \Leftrightarrow x>\sqrt{2}$

$S=\left]\sqrt{2}\,,\,+\infty\right[$

27.3. $(x-1)^4<0 \Leftrightarrow x\in\varnothing$

$S=\varnothing$

28.1. $P(x)=x^3-2x^2+x=$

$=x\left(x^2-2x+1\right)=$

$=x(x-1)^2$

$x^3-2x^2+x\le 0 \Leftrightarrow x(x-1)^2\le 0$

x	$-\infty$	0		1	$+\infty$
x	$-$	0	$+$	$+$	$+$
$(x-1)^2$	$+$	$+$	$+$	0	$+$
x^3-2x^2+x	$-$	0	$+$	0	$+$

$S=\left]-\infty\,,\,0\right]\cup\{1\}$

28.2. $2x^3+9x^2+7x-6\le 0$

	2	9	7	-6
-2		-4	-10	6
	2	5	-3	0

$2x^2+5x-3=0 \Leftrightarrow$

$\Leftrightarrow x=\dfrac{-5\pm\sqrt{25+24}}{4} \Leftrightarrow x=\dfrac{-5\pm 7}{4} \Leftrightarrow$

$\Leftrightarrow x=-3 \vee x=\dfrac{1}{2}$

$2x^3+9x^2+7x-6\le 0 \Leftrightarrow$

$\Leftrightarrow 2(x+2)(x+3)\left(x-\dfrac{1}{2}\right)\le 0 \Leftrightarrow$

$\Leftrightarrow x\in\left]-\infty\,,\,-3\right]\cup\left[-2\,,\,\dfrac{1}{2}\right]$

x	$-\infty$	-3		-2		$\dfrac{1}{2}$	$+\infty$
$x+2$	$-$	$-$	$-$	0	$+$	$+$	$+$
$x+3$	$-$	0	$+$	$+$	$+$	$+$	$+$
$x-\dfrac{1}{2}$	$-$	$-$	$-$	$-$	$-$	0	$+$
$2x^3+9x^2+7x-6$	$-$	0	$+$	0	$-$	0	$+$

$S=\left]-\infty\,,\,-3\right]\cup\left[-2\,,\,\dfrac{1}{2}\right]$

28.3. $x^3 - x^2 + 9x - 9 > 0 \Leftrightarrow$

$\Leftrightarrow x^2(x-1) + 9(x-1) > 0 \Leftrightarrow$

$\Leftrightarrow (x-1)(x^2+9) > 0 \Leftrightarrow$

$\Leftrightarrow x - 1 > 0 \Leftrightarrow \qquad (x^2 + 9 > 0,\ \forall x \in \mathbb{R})$

$\Leftrightarrow x > 1$

$S =]1\ ,\ +\infty[$

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29.1. $P(x)$ tem três zeros distintos: 3, –2 e 1

29.2. –2 tem multiplicidade 2 porque 2 é o maior número natural para o qual $P(x)$ é divisível por $(x+2)^2$

30. Por exemplo, $(x^2+2)(x-2)^2$.

31.1. $x^3 - x^2 > 0 \Leftrightarrow x^2(x-1) > 0$

$\Leftrightarrow x \in]1\ ,\ +\infty[$

x	$-\infty$	0		1	$+\infty$
x^2	+	0	+	+	+
$x - 1$	–	–	–	0	+
$x^2(x-1)$	–	0	–	0	+

31.2. $1 - x^2 > 0 \Leftrightarrow (1-x)(1+x) > 0$

$\Leftrightarrow x \in]-1\ ,\ 1[$

x	$-\infty$	-1		1	$+\infty$
$1 - x$	+	+	+	0	–
$1 + x$	–	0	+	+	+
$1 - x^2$	–	0	+	0	–

32.1. Uma potência de expoente par é sempre não negativa.

32.2. Uma potência de expoente ímpar tem o sinal da base.

32.3. $(x-3)^6 = 0 \Leftrightarrow x = 3$

Se $x \neq 3$, $(x-3)^6 > 0$. Logo, $(x-3)^6 \leq 0 \Leftrightarrow x = 3$

33.1. $0^4 + 0^2 = 0$ e, para $x \neq 0$, $x^4 + x^2 > 0$ porque a soma de dois números positivos é um número positivo.

33.2. $x^{2n} + x^{2n+2} = (x^n)^2 + (x^{n+1})^2$

A soma de dois quadrados é nula se e somente se as bases forem nulas.

34.1. $x^2 - 7x + 6 = 0 \Leftrightarrow x = \dfrac{7 \pm \sqrt{49-24}}{2} \Leftrightarrow$

$\Leftrightarrow x = \dfrac{7 \pm 5}{2} \Leftrightarrow x = 6 \vee x = 1$

$P(x) = (x-1)(x-6)$

34.2. $6x^2 - 5x + 1 = 0 \Leftrightarrow x = \dfrac{5 \pm \sqrt{25-24}}{12} \Leftrightarrow$

$\Leftrightarrow x = \dfrac{4}{12} \vee x = \dfrac{6}{12} \Leftrightarrow x = \dfrac{1}{3} \vee x = \dfrac{1}{2}$

$P(x) = 6\left(x - \dfrac{1}{3}\right)\left(x - \dfrac{1}{2}\right)$

34.3. $P(x) = 4x^3 - 8x^2 - x + 2$

	4	– 8	– 1	2
$\frac{1}{2}$		2	– 3	– 2
	4	– 6	– 4	0

$4x^2 - 6x - 4 = 0 \Leftrightarrow x = \dfrac{6 \pm \sqrt{36+64}}{8} \Leftrightarrow$

$\Leftrightarrow x = \dfrac{6 \pm 10}{8} \Leftrightarrow x = 2 \vee x = -\dfrac{1}{2}$

$P(x) = 4\left(x - \dfrac{1}{2}\right)\left(x + \dfrac{1}{2}\right)(x-2)$

34.4. $P(x) = x^5 - 5x^3 + 4x = x(x^4 - 5x^2 + 4)$

$x^4 - 5x^2 + 4 = 0 \Leftrightarrow \qquad | x^2 = y$

$\Leftrightarrow y^2 - 5y + 4 = 0 \Leftrightarrow$

$\Leftrightarrow y = \dfrac{5 \pm \sqrt{25-16}}{2} \Leftrightarrow$

$\Leftrightarrow y = 1 \vee y = 4$

$y^5 - 5y + 4 = (y-1)(y-4) \qquad | y = x^2$

$x^4 - 5x^2 + 4 = (x^2+1)(x^2-4)$

$P(x) = x(x^2-1)(x^2-4) = x(x-1)(x+1)(x-2)(x+2)$

34.5. $P(x) = 36x^4 - 13x^2 + 1$

$36x^4 - 13x^2 + 1 = 0 \qquad | x^2 = y$

$36y^2 - 13y + 1 = 0 \Leftrightarrow$

$\Leftrightarrow y = \dfrac{13 \pm \sqrt{13^2 - 144}}{72} \Leftrightarrow$

$\Leftrightarrow y = \dfrac{1}{9} \vee y = \dfrac{1}{4}$

$36y^2 - 13y + 1 = 36\left(y - \dfrac{1}{9}\right)\left(y - \dfrac{1}{4}\right) \qquad | y = x^2$

$36x^4 - 13x^2 + 1 = 36\left(x^2 - \dfrac{1}{9}\right)\left(x^2 - \dfrac{1}{4}\right)$

$P(x) = 36\left(x - \dfrac{1}{3}\right)\left(x + \dfrac{1}{3}\right)\left(x - \dfrac{1}{2}\right)\left(x + \dfrac{1}{2}\right)$

34.6. $P(x) = -4x^3 + 3x + 1$

Divisores de 1: –1 e 1

$P(-1) = 2$; $P(1) = 0$

	– 4	0	3	1
1		– 4	– 4	– 1
	– 4	– 4	– 1	0

$-4x^2 - 4x - 1 = 0 \Leftrightarrow x = \dfrac{4 \pm \sqrt{16-16}}{-8} \Leftrightarrow x = -\dfrac{1}{2}$

$P(x) = -4(x-1)\left(x + \dfrac{1}{2}\right)^2$

35.1. $P(x) = x^3 + 5x^2 + 8x + 4$

	1	5	8	4
– 2		– 2	– 6	– 4
	1	3	2	0
–2		– 2	– 2	
	1	1	0	

$P(x) = (x+2)^2(x+1)$

–2 tem grau de multiplicidade 2.

35.2. $P(x) = 2x^5 + 8x^4 - 32x^2 - 32x$

	2	8	0	– 32	–32	0
– 2		– 4	– 8	16	22	0
	2	4	– 8	– 16	0	0
–2		– 4	0	16	0	
	2	0	– 8	0	0	
– 2		– 4	8	0		
	2	– 4	0	0		

$P(x) = (x+2)^3(2x^2 - 4x)$

$P(x) = 2x(x+2)^3(x-2)$

–2 tem grau de multiplicidade 3.

36. $P(x) = x^3 + 2x^2 - 13x + 10$

36.1.

	1	2	−13	10
1		1	3	−10
	1	3	−10	0

$x^2 + 3x - 10 = 0 \Leftrightarrow x = \dfrac{-3 \pm \sqrt{9+40}}{2}$

$\Leftrightarrow x = -5 \vee x = 2$

$S = \{-5\ ,\ 1\ ,\ 2\}$

36.2. $P(x) = (x+5)(x-1)(x-2)$

37.1. $(1-x)(5x-4) = 0 \Leftrightarrow$

$\Leftrightarrow 1 - x = 0 \vee 5x - 4 = 0 \Leftrightarrow$

$\Leftrightarrow x = 1 \vee x = \dfrac{4}{5}$

$S = \left\{1\ ,\ \dfrac{4}{5}\right\}$

37.2. $(x^2 - x)(7 - 3x) = 0 \Leftrightarrow$

$\Leftrightarrow x^2 - x = 0 \vee 7 - 3x = 0 \Leftrightarrow$

$\Leftrightarrow x(x-1) = 0 \vee 3x = 7 \Leftrightarrow$

$\Leftrightarrow x = 0 \vee x = 1 \vee x = \dfrac{7}{3}$

$S = \left\{0\ ,\ 1\ ,\ \dfrac{7}{3}\right\}$

37.3. $(x^2 - 2x + 1)(25 - x^2) = 0 \Leftrightarrow$

$\Leftrightarrow x^2 - 2x + 1 = 0 \vee 25 - x^2 = 0 \Leftrightarrow$

$\Leftrightarrow (x-1)^2 = 0 \vee x^2 = 25 \Leftrightarrow$

$\Leftrightarrow x = 1 \vee x = -5 \vee x = 5$

$S = \{-5\ ,\ 1\ ,\ 5\}$

38.1. $x^2 - 9x + 8 = 0 \Leftrightarrow x = \dfrac{9 \pm \sqrt{81-32}}{2} \Leftrightarrow x = 1 \vee x = 8$

$x^2 - 9x + 8 = (x-1)(x-8)$

$P(x) = x^4 - 6x^3 + 7x^2 + mx + n$

$P(x)$ é divisível por $x - 1$ e por $x - 8$

$\begin{cases} P(1) = 0 \\ P(8) = 0 \end{cases} \Leftrightarrow \begin{cases} 1 - 6 + 7 + m + n = 0 \\ 8^4 - 6 \times 8^3 + 7 \times 8^2 + 8m + n \end{cases} \Leftrightarrow$

$\Leftrightarrow \begin{cases} m + n + 2 = 0 \\ 8m + n + 1472 = 0 \end{cases}$

Logo, $m + n = -2$.

38.2. $\begin{cases} n = -m - 2 \\ 8m - m - 2 + 1472 = 0 \end{cases} \Leftrightarrow \begin{cases} n = -m - 2 \\ 7m = -1470 \end{cases} \Leftrightarrow \begin{cases} n = 208 \\ m = -210 \end{cases}$

$P(x) = x^4 - 6x^3 + 7x^2 - 210x + 208$

	1	−6	7	−210	208
1		1	−5	2	−208
	1	−5	2	−208	0
8		8	24	208	
	1	3	26	0	

$x^2 + 3x + 26 = 0 \Leftrightarrow x = \dfrac{-3 \pm \sqrt{9-104}}{2} \Leftrightarrow$

$\Leftrightarrow x \in \varnothing$

$P(x) = (x-1)(x-8)(x^2 + 3x + 26)$

$P(x) = 0 \Leftrightarrow x = 1 \vee x = 8$

$S = \{1\ ,\ 8\}$

39. $P(x) = x^4 - 5x^3 - 7x^2 + 41x - 30$

$x^2 + 2x - 3 = 0 \Leftrightarrow x = \dfrac{-2 \pm \sqrt{4+12}}{2} \Leftrightarrow$

$\Leftrightarrow x = -3 \vee x = 1$

$P(-3) = P(1) = 0$

	1	−5	−7	41	−30
1		1	−4	−11	30
	1	−4	−11	30	0
−3		−3	21	−30	
	1	−7	10	0	

$x^2 - 7x + 10 = 0 \Leftrightarrow x = \dfrac{7 \pm \sqrt{49-40}}{2} \Leftrightarrow$

$\Leftrightarrow x = 2 \vee x = 5$

$P(x) = 0 \Leftrightarrow x = -3 \vee x = 1 \vee x = 2 \vee x = 5$

$S = \{-3\ ,\ 1\ ,\ 2\ ,\ 5\}$

40.1. $(x-1)^2 - (2x-3)^2 = 0 \Leftrightarrow$

$\Leftrightarrow (x - 1 - 2x + 3)(x - 1 + 2x - 3) = 0 \Leftrightarrow$

$\Leftrightarrow -x + 2 = 0 \vee 3x - 4 = 0 \Leftrightarrow$

$\Leftrightarrow x = 2 \vee x = \dfrac{4}{3}$

$S = \left\{\dfrac{4}{3}\ ,\ 2\right\}$

40.2. $(x^3 + x)(8x^2 - 2x - 1) = 0 \Leftrightarrow$

$\Leftrightarrow x^3 + x = 0 \vee 8x^2 - 2x - 1 = 0 \Leftrightarrow$

$\Leftrightarrow x(x^2 + 1) = 0 \vee x = \dfrac{2 \pm \sqrt{4+32}}{16} \Leftrightarrow$

$\Leftrightarrow x = 0 \vee x^2 + 1 = 0 \vee x = \dfrac{2 \pm 6}{16} \Leftrightarrow$

$\Leftrightarrow x = 0 \vee x \in \varnothing \vee x = -\dfrac{1}{4} \vee x = \dfrac{1}{2} \Leftrightarrow$

$\Leftrightarrow x = -\dfrac{1}{4} \vee x = 0 \vee x = \dfrac{1}{2}$

$S = \left\{-\dfrac{1}{4}\ ,\ 0\ ,\ \dfrac{1}{2}\right\}$

40.3. $P(x) = x^4 - 3x^3 - 2x^2 + 12x - 8$

	1	−3	−2	12	−8
2		2	−2	−8	8
	1	−1	−4	4	0
2		2	2	−4	
	1	1	−2	0	

$x^2 + x - 2 = 0 \Leftrightarrow x = \dfrac{-1 \pm \sqrt{1+8}}{2} \Leftrightarrow$

$\Leftrightarrow x = -2 \vee x = 1$

$P(x) = 0 \Leftrightarrow (x-2)^2(x+2)(x-1) = 0 \Leftrightarrow$

$\Leftrightarrow x = 2 \vee x = -2 \vee x = 1$

$S = \{-2\ ,\ 1\ ,\ 2\}$

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41. $P(x) = -x^3 - 3x^2 + 25x + 75$

$P(-3) = 0$

$$\begin{array}{r|rrrr} & -1 & -3 & 25 & 75 \\ -3 & & 3 & 0 & -75 \\ \hline & -1 & 0 & 25 & \boxed{0} \end{array}$$

$P(x)=0 \Leftrightarrow (x+3)(-x^2+25)=0 \Leftrightarrow$

$\Leftrightarrow x+3=0 \vee -x^2+25=0 \Leftrightarrow$

$\Leftrightarrow x=-3 \vee x=-5 \vee x=5$

$S=\{-5\ ,\ -3\ ,\ 5\}$

42.1. $(x+2)(x+3)>0 \Leftrightarrow$

$\Leftrightarrow (x+2>0 \wedge x+3>0) \vee (x+2<0 \wedge x+3<0) \Leftrightarrow$

$\Leftrightarrow (x>-2 \wedge x>-3) \vee (x<-2 \wedge x<-3) \Leftrightarrow$

$\Leftrightarrow x>-2 \vee x<-3 \Leftrightarrow$

$\Leftrightarrow x \in]-\infty\ ,\ -3[\cup]-2\ ,\ +\infty[$

$S=]-\infty\ ,\ -3[\cup]-2\ ,\ +\infty[$

42.2. $(2-x)(x-\sqrt{2})<0 \Leftrightarrow$

$\Leftrightarrow (2-x<0 \wedge x-\sqrt{2}>0) \vee (2-x>0 \wedge x-\sqrt{2}<0) \Leftrightarrow$

$\Leftrightarrow (x>2 \wedge x>\sqrt{2}) \vee (x<2 \wedge x<\sqrt{2}) \Leftrightarrow$

$\Leftrightarrow x>2 \vee x<\sqrt{2} \Leftrightarrow$

$\Leftrightarrow x \in]-\infty\ ,\ \sqrt{2}[\cup]2\ ,\ +\infty[$

$S=]-\infty\ ,\ \sqrt{2}[\cup]2\ ,\ +\infty[$

42.3. $2x^2+7x<0 \Leftrightarrow x(2x+7)<0 \Leftrightarrow$

$\Leftrightarrow (x<0 \wedge 2x+7>0) \vee (x>0 \wedge 2x+7<0) \Leftrightarrow$

$\Leftrightarrow \left(x<0 \wedge x>-\frac{7}{2}\right) \vee \left(x>0 \wedge x<-\frac{7}{2}\right) \Leftrightarrow$

$\Leftrightarrow -\frac{7}{2}<x<0 \vee x \in \varnothing \Leftrightarrow$

$\Leftrightarrow x \in \left]-\frac{7}{2}\ ,\ 0\right[$

$S=\left]-\frac{7}{2}\ ,\ 0\right[$

42.4. $(x-3)(2-x)(x+\sqrt{2})>0 \Leftrightarrow$

$\Leftrightarrow x \in]-\infty\ ,\ -\sqrt{2}[\cup]2\ ,\ 3[$

x	$-\infty$	$-\sqrt{2}$		2		3	$+\infty$
$x-3$	$-$	$-$	$-$	$-$	$-$	0	$+$
$2-x$	$+$	$+$	$+$	0	$-$	$-$	$-$
$x+\sqrt{2}$	$-$	0	$+$	$+$	$+$	$+$	$+$
P	$+$	0	$-$	0	$+$	0	$-$

$S=]-\infty\ ,\ -\sqrt{2}[\cup]2\ ,\ 3[$

43.1. $x^2-5x+4=0 \Leftrightarrow x=\frac{5\pm\sqrt{25-16}}{2} \Leftrightarrow x=1 \vee x=4$

$x^2-5x+4>0 \Leftrightarrow (x-1)(x-4)>0 \Leftrightarrow$

$\Leftrightarrow (x-1>0 \wedge x-4>0) \vee (x-1<0 \wedge x-4<0) \Leftrightarrow$

$\Leftrightarrow (x>1 \wedge x>4) \vee (x<1 \wedge x<4) \Leftrightarrow$

$\Leftrightarrow x>4 \vee x<1 \Leftrightarrow$

$\Leftrightarrow x \in]-\infty\ ,\ 1[\cup]4\ ,\ +\infty[$

$S=]-\infty\ ,\ 1[\cup]4\ ,\ +\infty[$

43.2. $x^2+x-12=0 \Leftrightarrow x=\frac{-1\pm\sqrt{1+48}}{2} \Leftrightarrow x=-4 \vee x=3$

$x^2+x-12 \leq 0 \Leftrightarrow$

$\Leftrightarrow (x+4)(x-3) \leq 0 \Leftrightarrow$

$\Leftrightarrow (x+4 \leq 0 \wedge x-3 \geq 0) \vee (x+4 \geq 0 \wedge x-3 \leq 0) \Leftrightarrow$

$\Leftrightarrow (x \leq -4 \wedge x \geq 3) \vee (x \geq -4 \wedge x \leq 3) \Leftrightarrow$

$\Leftrightarrow x \in \varnothing \vee -4 \leq x \leq 3 \Leftrightarrow$

$\Leftrightarrow x \in [-4\ ,\ 3]$

$S=[-4\ ,\ 3]$

43.3. $x^2-6x+8=0 \Leftrightarrow x=\frac{6\pm\sqrt{36-32}}{2} \Leftrightarrow x=2 \vee x=4$

$-x^2+6x-8 \geq 0 \Leftrightarrow$

$\Leftrightarrow x^2-6x+8 \leq 0 \Leftrightarrow$

$\Leftrightarrow (x-2)(x-4) \leq 0 \Leftrightarrow$

$\Leftrightarrow (x-2 \leq 0 \wedge x-4 \geq 0) \vee (x-2 \geq 0 \wedge x-4 \leq 0) \Leftrightarrow$

$\Leftrightarrow (x \leq 2 \wedge x \geq 4) \vee (x \geq 2 \wedge x \leq 4) \Leftrightarrow$

$\Leftrightarrow x \in \varnothing \vee 2 \leq x \leq 4 \Leftrightarrow$

$\Leftrightarrow x \in [2\ ,\ 4]$

$S=[2\ ,\ 4]$

44.1. $(x-3)^4>0 \Leftrightarrow x-3 \neq 0 \Leftrightarrow x \neq 3 \Leftrightarrow$

$\Leftrightarrow x \in \mathbb{R} \setminus \{3\}$

$S=\mathbb{R} \setminus \{3\}$

44.2. $(x-3)^2(x-1)<0$

x	$-\infty$	1		3	$+\infty$
$(x-3)^2$	$+$	$+$	$+$	0	$+$
$x-1$	$-$	0	$+$	$+$	$+$
P	$-$	0	$+$	0	$+$

$(x-3)^2+(x-1)<0 \Leftrightarrow x \in]-\infty\ ,\ 1[$

$S=]-\infty\ ,\ 1[$

44.3. $x^3-6x \geq 0 \Leftrightarrow$

$\Leftrightarrow x(x^2-6) \geq 0 \Leftrightarrow$

$\Leftrightarrow x(x-\sqrt{6})(x+\sqrt{6}) \geq 0$

x	$-\infty$	$-\sqrt{6}$		0		$\sqrt{6}$	$+\infty$
x	$-$	$-$	$-$	0	$+$	$+$	$+$
$x-\sqrt{6}$	$-$	0	$+$	$+$	$+$	$+$	$+$
$x+\sqrt{6}$	$-$	$-$	$-$	$-$	$-$	0	$+$
P	$-$	0	$+$	0	$-$	0	$+$

$x^2-6x \geq 0 \Leftrightarrow x \in [-\sqrt{6}\ ,\ 0] \cup [\sqrt{6}\ ,\ +\infty[$

$S=[-\sqrt{6}\ ,\ 0[\cup [\sqrt{6}\ ,\ +\infty[$

44.4. $x^5+2x^4+x^3<0 \Leftrightarrow$

$\Leftrightarrow x^3(x^2+2x+1)<0 \Leftrightarrow$

$\Leftrightarrow x^3(x+1)^2<0 \Leftrightarrow$

$\Leftrightarrow x<0 \wedge x \neq -1 \Leftrightarrow$

$\Leftrightarrow x \in]-\infty\ ,\ -1[\cup]-1\ ,\ 0[$

$S=]-\infty\ ,\ -1[\cup]-1\ ,\ 0[$

45. $P(x) = ax^2 + bx + c$

$$\begin{cases} P(0) = -20 \\ P(1) + P(2) = -18 \\ P(1) - 3P(2) = 6 \end{cases} \Leftrightarrow \begin{cases} c = -20 \\ a + b + c + 4a + 2b + c = -18 \\ a + b + c - 3(4a + 2b + c) = 6 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} c = -20 \\ 5a + 3b = -18 + 40 \\ a + b - 12a - 6b + c - 3c = 6 \end{cases} \Leftrightarrow \begin{cases} c = -20 \\ 5a + 3b = 22 \\ -11a - 5b = 6 - 40 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} c = -20 \\ 3b = 22 - 5a \\ 11a + 5b = 34 \end{cases} \Leftrightarrow \begin{cases} c = -20 \\ b = \dfrac{22 - 5a}{3} \\ 11a + 5 \times \dfrac{22 - 5a}{3} = 34 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} c = -20 \\ b = \dfrac{22 - 5a}{3} \\ 33a + 110 - 25a = 102 \end{cases} \Leftrightarrow \begin{cases} c = -20 \\ b = \dfrac{22 - 5a}{3} \\ 8a = -8 \end{cases} \Leftrightarrow \begin{cases} c = -20 \\ b = 9 \\ a = -1 \end{cases}$$

$P(x) = -x^2 + 9x - 20$

$P(x) = 0 \Leftrightarrow -x^2 + 9x - 20 = 0 \Leftrightarrow$

$\Leftrightarrow x = \dfrac{-9 \pm \sqrt{81 - 80}}{-2} \Leftrightarrow$

$\Leftrightarrow x = 4 \vee x = 5$

$P(x) < 0 \Leftrightarrow -(x - 4)(x - 5) < 0 \Leftrightarrow$

$\Leftrightarrow (x - 4)(x - 5) > 0 \Leftrightarrow$

$\Leftrightarrow (x - 4 > 0 \wedge x - 5 > 0) \vee (x - 4 < 0 \wedge x - 5 < 0) \Leftrightarrow$

$\Leftrightarrow (x > 4 \wedge x > 5) \vee (x < 4 \wedge x < 5) \Leftrightarrow$

$\Leftrightarrow x > 5 \vee x < 4 \Leftrightarrow x \in]-\infty, 4[\cup]5, +\infty[$

$S =]-\infty, 4[\cup]5, +\infty[$

46.1. $Q(x) = x^2 - 9x - 10$

$x^2 - 9x - 10 = 0 \Leftrightarrow x = \dfrac{9 \pm \sqrt{81 + 40}}{2} \Leftrightarrow$

$\Leftrightarrow x = \dfrac{9 \pm 11}{2} \Leftrightarrow x = -1 \vee x = 10$

$Q(x) = (x + 1)(x - 10)$

$P(x) = x^4 - 13x^3 + 30x^2 + 4x - 40$

	1	−13	30	4	−40
−1		−1	14	−44	40
	1	−14	44	−40	0
10		10	−40	40	
	1	−4	4	0	

$S(x) = x^2 - 4x + 4$

46.2. $x^2 - 4x + 4 = 0 \Leftrightarrow x = \dfrac{4 \pm \sqrt{16 - 16}}{2}$

$\Leftrightarrow x = 2$

$x^2 - 4x + 4 = (x - 2)^2$

$P(x) = (x + 1)(x - 10)(x - 2)^2$

x	$-\infty$	-1		2		10	$+\infty$
$x + 1$	−	0	+	+	+	+	+
$x - 10$	−	−	−	−	−	0	+
$(x + 2)^2$	+	+	+	0	+	+	+
$P(x)$	+	0	−	0	−	0	+

$P(x) < 0 \Leftrightarrow$

$\Leftrightarrow x \in]-1, 2[\cup]2, 10[$

$S =]-1, 2[\cup]2, 10[$

47. $P(x) = x^3 - 6x^2 + 3x + 10$

Divisores de 10: –1, 1, –2, 2, –5, 5, –10 e 10

$P(-1) = 0$; $P(1) = 8$; $P(-2) = -28$; $P(2) = 0$

$P(-5) = -280$; $P(5) = 0$

$P(x) = (x + 1)(x - 2)(x - 5)$

47.1. $P(x) = 0 \Leftrightarrow x = -1 \vee x = 2 \vee x = 5$

$S = \{-1, 2, 5\}$

47.2. $P(x) \geq 0 \Leftrightarrow (x + 1)(x - 2)(x - 5) \geq 0$

x	$-\infty$	-1		2		5	$+\infty$
$x + 1$	−	0	+	+	+	+	+
$x - 2$	−	−	−	0	+	+	+
$x - 5$	−	−	−	−	−	0	+
$P(x)$	−	0	+	0	−	0	+

$P(x) > 0 \Leftrightarrow$

$\Leftrightarrow x \in [-1, 2] \cup [5, +\infty[$

$S = [-1, 2] \cup [5, +\infty[$

48. $P(x) = x^5 - 2x^4 + x^3 - 2x^2 - 2x + 4$

48.1.

	1	−2	1	−2	−2	4
2		2	0	2	0	−4
	1	0	1	0	−2	0

$Q(x) = x^4 + x^2 - 2$

$Q(2) = 2^4 + 2^2 - 2 = 18$

$P(x) = (x - 2)Q(x)$ e $Q(2) \neq 0$

$Q(x) = x^4 + x^2 - 2$

48.2. $Q(x) = x^4 + x^2 - 2 =$ $\quad | \; y = x^2$

$= y^2 + y - 2$

$y^2 + y - 2 = 0 \Leftrightarrow$

$\Leftrightarrow y = \dfrac{-1 \pm \sqrt{1 + 8}}{2} \Leftrightarrow$

$\Leftrightarrow y = -2 \vee y = 1$

$y^2 + y - 2 = (y + 2)(y - 1)$ $\quad | \; y = x^2$

$x^4 + x - 2 =$

$= (x^2 + 2)(x^2 - 1) =$

$= (x^2 + 2)(x - 1)(x + 1)$

$P(x) = (x - 2)(x - 1)(x + 1)(x^2 + 2)$

48.3. $x^2 + x > 0, \ \forall x \in \mathbb{R}$

x	$-\infty$	-1		1		2	$+\infty$
$x - 2$	−	−	−	−	−	0	+
$x - 1$	−	−	−	0	+	+	+
$x + 1$	−	0	+	+	+	+	+
$P(x)$	−	0	+	0	−	0	+

$P(x) \geq 0 \Leftrightarrow x \in [-1, 1] \cup [2, +\infty[$

$S = [-1, 1] \cup [2, +\infty[$

49. $P(x) = 4x^5 + 8x^4 + x^3 - 5x^2 - x + 1$

49.1.

	4	8	1	−5	−1	1
−1		−4	−4	3	2	−1
	4	4	−3	−2	1	0
−1		−4	0	3	−1	
	4	0	−3	1	0	
−1		−4	4	−1		
	4	−4	1	0		
−1		4	0			
	4	0	$1 \neq 0$			

-1 tem grau de multiplicidade 3.

49.2. $P(x) = (x+1)^3 (4x^2 - 4x + 1)$

$= (x+1)^3 (2x-1)^2$

$2x - 1 = 0 \Leftrightarrow x = \frac{1}{2}$

x	$-\infty$	-1		$\frac{1}{2}$	$+\infty$
$(x-1)^3$	$-$	0	$+$	$+$	$+$
$(2x-1)^2$	$+$	$+$	$+$	0	$+$
P	$-$	0	$+$	0	$+$

$P(x) \leq 0 \Leftrightarrow x \in \left]-\infty\ ,\ -1\right] \cup \left\{\frac{1}{2}\right\}$

$S = \left]-\infty\ ,\ 1\right] \cup \left\{\frac{1}{2}\right\}$

50. $P(x) = 2x^3 + 7x^2 + ax - b$

50.1.

	2	7	a	$-b$
−2		−4	−6	$-2a + 12$
	2	3	$a - 6$	$-2a - b + 12$
−2		−4	2	
	2	−1	$a - 4$	

$$\begin{cases} a - 4 = 0 \\ -2a - b + 12 = 0 \end{cases} \Leftrightarrow \begin{cases} a = 4 \\ -8 - b + 12 = 0 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} a = 4 \\ b = 4 \end{cases}$$

$a = 4$ e $b = 4$

50.2. $P(x) = (x+2)^2 (2x-1)$

51. $ax^3 - 4ax^2 + (3a+1)x - a + 1 = 0$

51.1. $a \times 3^3 - 4a \times 3^2 + (3a+1) \times 3 - a + 1 = 0$

$\Leftrightarrow 27a - 36a + 9a + 3 - a + 1 = 0$

$\Leftrightarrow -a + 4 = 0 \Leftrightarrow a = 4$

51.2. $4x^3 - 16x^2 + 13x - 3 = 0$

	4	−16	13	−3
3		12	−12	3
	4	−4	1	0

$4x^2 - 4x + 1 = 0 \Leftrightarrow (2x-1)^2 = 0 \Leftrightarrow x = \frac{1}{2}$

$S = \left\{\frac{1}{2}\ ,\ 3\right\}$

52. $A(x) = x^2 + ax + 1$

$B(x) = x^2 + bx + 1$

$C(x) = x^4 + 1$

52.1. $A(x) \times B(x) = C(x)$

$(x^2 + ax + 1) \times (x^2 + bx + 1) = x^4 + 1 \Leftrightarrow$

$\Leftrightarrow x^4 + bx^3 + x^2 + ax^3 + abx^2 + ax + x^2 + bx + 1 = x^4 + 1 \Leftrightarrow$

$\Leftrightarrow x^4 + (a+b)x^3 + (ab+2)x^2 + (a+b)x + 1 = x^4 + 1 \Leftrightarrow$

$$\Leftrightarrow \begin{cases} a + b = 0 \\ ab + 2 = 0 \end{cases} \Leftrightarrow \begin{cases} b = -a \\ -a \times a = -2 \end{cases} \Leftrightarrow \begin{cases} b = -a \\ a^2 = a \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} b = -a \\ a = \sqrt{2} \vee a = -\sqrt{2} \end{cases}$$

$a = \sqrt{2}$ e $b = -\sqrt{2}$ ou $a = -\sqrt{2}$ e $b = \sqrt{2}$

52.2. $C(x) = \left(x^2 - \sqrt{2}x + 1\right)\left(x^2 + \sqrt{2}x + 1\right)$

Como $\left(\pm\sqrt{2}\right)^2 - 4 = -2 < 0$, os polinómios $x^2 - \sqrt{2}x + 1$ e $x^2 + \sqrt{2}x + 1$ ao tem raízes.

53. $P(x) = x^{n+2} - 2x^{n+1} + x^n - x^2 + 2x - 1,\ n \in \mathbb{N}$

53.1. $P(2) = 2^{n+2} - 2(2)^{n+1} + 2^n - 2^2 + 2(2) - 1 =$

$= 2^{n+2} - 2^1 \times 2^{n+1} + 2^n - \cancel{4} + \cancel{4} - 1 =$

$= 2^{n+2} - 2^{n+1+1} + 2^n - 1 =$

$= \cancel{2^{n+2}} - \cancel{2^{n+2}} + 2^n - 1 =$

$= 2^n - 1$

53.2. $P(x) = x^n \times x^2 - 2x^n \times x + x^n - (x^2 - 2x + 1) =$

$= x^n (x^2 - 2x + 1) - (x^2 - 2x + 1) =$

$= (x^2 - 2x + 1)(x^n - 1) =$

$= (x-1)^2 (x^n - 1)$

53.3. $P(1) = (1-1)^2 (1^n - 1) = 0 \times 0 = 0$

Vimos que 1 é raiz de $x - 1$ e de $x^n - 1$.
Vamos dividir $x^n - 1$ por $x - 1$.

$$x^n - 1 = x^n \underbrace{+ 0x^{n-1} + 0x^{n-2} + \ldots + 0x}_{n - 1 \text{ zeros}} - 1$$

$n - 1$ zeros

	1	0	0	0	…	0	−1
1		1	1	1	…	1	1
	1	1	1	1	…	1	0

$$x^n - 1 = (x-1)\overbrace{\left(x^{n-1} + x^{n-2} + \ldots + x + 1\right)}^{n \text{ termos}}$$

Como $P(x) = (x-1)^2 (x^n - 1)$ e substituindo $x^n - 1$:

$P(x) = (x-1)^2 \times (x-1)\left(x^{n-1} + x^{n-2} + \ldots + x + 1\right)$

$P(x) = (x-1)^3 \times Q(x)$ com $Q(x) = \underbrace{x^{n-1} + x^{n+2} + \ldots + x + 1}_{x \text{ parcelas}}$

$Q(1) = 1 + 1 + \ldots + 1 + 1 = n \times 1 = n$

$P(x) = (x-1)^3 \times Q(x)$ com $Q(1) \neq 0$

A raiz 1 tem multiplicidade 3.

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Avaliação 4

1. $(x+1)^2 > 0 \Leftrightarrow x + 1 \neq 0 \Leftrightarrow x \neq -1 \Leftrightarrow$

$\Leftrightarrow x \in \mathbb{R} \setminus \{-1\}$

$\left(x^2-1\right)\leq 0 \Leftrightarrow (x-1)(x+1)\leq 0 \Leftrightarrow$

$\Leftrightarrow (x-1\leq 0 \wedge x+1\geq 0)\vee(x-1\geq 0\wedge x+1\leq 0) \Leftrightarrow$

$\Leftrightarrow (x\leq 1\wedge x\geq -1)\vee(x\geq 1\wedge x\leq -1) \Leftrightarrow$

$\Leftrightarrow -1\leq x\leq 1 \vee x\in\varnothing \Leftrightarrow x\in[-1\ ,\ 1]$

$\left(x^2+4\right)(x+1)^2\leq 0\Leftrightarrow$

$\Leftrightarrow (x+1)^2\leq 0 \Leftrightarrow \qquad \left(\forall x\in\mathbb{R},\ x^2+4>0\right)$

$\Leftrightarrow x+1=0\Leftrightarrow x=-1$

$S=\{-1\}$

Resposta: (C)

2. $P(x)=a(x-1)(x-2)(x-3)(x-4)(x-5)$

$P(6)=1\Leftrightarrow a\times 5\times 4\times 3\times 2\times 1=1\Leftrightarrow$

$\Leftrightarrow a=\frac{1}{120}$

$P(x)=\frac{1}{120}(x-1)(x-2)(x-3)(x-4)(x-5)$

$P(0)=\frac{1}{120}(0-1)(0-2)(0-3)(0-4)(0-5)=$

$=\frac{1}{120}\times(-1)\times(-2)\times(-3)\times(-4)\times(-5)=$

$=\frac{1}{120}\times(-120)=-1$

Resposta: (C)

3. $A(x)=x^3-2x-4\ ;\ A(2)=0$

	1	0	−2	−4
2		2	4	4
	1	2	2	0

$x^2+2x+2=0\Leftrightarrow x=\frac{-2\pm\sqrt{4-8}}{2}\Leftrightarrow$

$\Leftrightarrow x\in\varnothing$

$S=\{2\}$

Resposta: (D)

4. Seja $P(x)=2x^3-x^2-2x+1$

Divisores de 1: – 1 e 1

Verifiquemos se – 1 ou 1 são raízes de $P(x)$

	2	−1	−2	1
−1		−2	3	−1
	2	−3	1	0
1		2	−1	
	2	−1	0	

$2x^3-x^2-2x+1=0\Leftrightarrow$

$\Leftrightarrow (x+1)(x-1)(2x-1)=0\Leftrightarrow$

$\Leftrightarrow x=-1\vee x=1\vee x=\frac{1}{2}$

Resposta: (D)

5. $P(x)=(x-1)(x-2)^2(x-3)^3\times Q(x)$; 1 + 2 + 3 = 6

$P(x)$ tem grau maior ou igual a 6.

Resposta: (A)

6. $P(x)=x^2+ax-4\ ;\ Q(x)=(x+b)(x-b)=x^2-b^2$

$P(x)=Q(x)\Leftrightarrow\begin{cases}a=0\\-b^2=-4\end{cases}\Leftrightarrow\begin{cases}a=0\\b=2\vee b=-2\end{cases}$

$\Leftrightarrow a=0\wedge(b=2\vee b=-2)$

Resposta: (A)

7. $P(x)=x^4+x^2+k$

$P(a)=a^4+a^2+k$

$P(-a)=(-a)^4+(-a)^2+k=a^4+a^2+k=P(a)$

$P(x)$ é divisível por $x-a\Rightarrow$

$\Rightarrow P(a)=0\Rightarrow P(-a)=0$

$\Rightarrow P(x)$ é divisível por $x+a$.

Resposta: (B)

8. Sabemos que $P(x)=(x-3)^2\times Q(x)$ como $Q(3)\neq 0$

Se $P(x)=(x-3)^1\times Q(x)$, então $Q(x)=(x-3)Q_1(x)$

pelo que $Q(3)=0$

Resposta: (B)

9. $(x-2)^2(x-1)^3\leq 0$

x	$-\infty$	1		2	$+\infty$
$(x-2)^2$	+	+	+	0	+
$(x-1)^3$	−	0	+	+	+
P	−	0	+	0	+

$S=]-\infty\ ,\ 1]\cup\{2\}$

Resposta: (C)

10. $P(x)=2x^4-5x^3-2x^2-4x+k$

$(2x-1)=2\left(x-\frac{1}{2}\right)$

$P\left(\frac{1}{2}\right)=0\Leftrightarrow$

$\Leftrightarrow 2\times\left(\frac{1}{2}\right)^4-5\times\left(\frac{1}{2}\right)^3-2\times\left(\frac{1}{2}\right)^2-4\times\frac{1}{2}+k=0\Leftrightarrow$

$\Leftrightarrow\frac{1}{8}-\frac{5}{8}-\frac{1}{2}-2+k=0\Leftrightarrow k=3$

Resposta: (A)

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11. $P(x)=2x^3-4x^2+mx+4$

11.1. $2x-1=2\left(x-\frac{1}{2}\right)$

$P\left(\frac{1}{2}\right)=3\Leftrightarrow$

$\Leftrightarrow 2\times\left(\frac{1}{2}\right)^3-4\times\left(\frac{1}{2}\right)^2+m\times\frac{1}{2}+4=3\Leftrightarrow$

$\Leftrightarrow\frac{2}{8}-\cancel{1}+\frac{m}{2}+\cancel{1}=0\Leftrightarrow$

$\Leftrightarrow\frac{1}{4}+\frac{2m}{4}=0\Leftrightarrow 2m=-1\Leftrightarrow$

$\Leftrightarrow m=-\frac{1}{2}$

11.2. $P(x)=2x^3-4x^2-2x+4$

	2	−4	−2	4
2		4	0	−4
	2	0	−2	0

$P(x)=(x-2)\left(x^2-1\right)=2(x-2)\left(x^2-1\right)=$

$=2(x-2)(x-1)(x+1)$

11.3. $2x^3-4x^2-2x+4\leq 0 \Leftrightarrow$

$\Leftrightarrow 2(x-2)(x-1)(x+1)\leq 0 \Leftrightarrow x\in\left]-\infty\ ,\ -1\right]\cup\left[1\ ,\ 2\right]$

x	$-\infty$	-1		1		2	$+\infty$
$x-2$	$-$	$-$	$-$	$-$	$-$	0	$+$
$x-1$	$-$	$-$	$-$	0	$+$	$+$	$+$
$x+1$	$-$	0	$+$	$+$	$+$	$+$	$+$
$P(x)$	$-$	0	$+$	0	$-$	0	$+$

$S=\left]-\infty\ ,\ -1\right]\cup\left[1\ ,\ 2\right]$

12. $2x^4+7x^3-2x^2-13x+6=0$

$$\begin{array}{r|rrrrr}
 & 2 & 7 & -2 & -13 & 6 \\
\frac{1}{2} & & 1 & 4 & 1 & -6 \\
\hline
 & 2 & 8 & 2 & -12 & \boxed{0} \\
-2 & & -4 & -8 & 12 & \\
\hline
 & 2 & 4 & -6 & \boxed{0} &
\end{array}$$

$2x^2+4x-6=0 \Leftrightarrow x^2+2x-3=0 \Leftrightarrow$

$\Leftrightarrow x=\dfrac{-2\pm\sqrt{4+12}}{2} \Leftrightarrow x=-3 \vee x=1 \Leftrightarrow$

$S=\left\{-3\ ,\ -2\ ,\ \dfrac{1}{2}\ ,\ 1\right\}$

13. $x^4+x^2-6\leq 0$

$x^4+x^2-6=0 \Leftrightarrow$ $\quad\quad |\ x^2=y$

$\Leftrightarrow y^2+y-6=0 \Leftrightarrow$

$\Leftrightarrow y=\dfrac{-1\pm\sqrt{1+24}}{2} \Leftrightarrow y=-3 \vee y=2$

$y^2+y-6=(y+3)(y-2)$ $\quad\quad |\ y=x^2$

$x^4+x^2-6=(x^2+3)(x^2-2)$

$x^4+x^2-6\leq 0 \Leftrightarrow$

$\Leftrightarrow (x^2+3)(x^2-2)\leq 0 \Leftrightarrow$ $\quad\quad |\ \forall x\in\mathbb{R},\ x^2+3>0$

$\Leftrightarrow (x-\sqrt{2})(x+\sqrt{2})\leq 0 \Leftrightarrow$

$\Leftrightarrow x\in\left[-\sqrt{2}\ ,\ \sqrt{2}\right]$

x	$-\infty$	$-\sqrt{2}$		$\sqrt{2}$	$+\infty$
$x-\sqrt{2}$	$-$	$-$	$-$	0	$+$
$x+\sqrt{2}$	$-$	0	$+$	$+$	$+$
P	$+$	0	$-$	0	$+$

$S=\left[-\sqrt{2}\ ,\ \sqrt{2}\right]$

14. $P(x)=(x+2)^2(ax+b)$

$\begin{cases}P(-1)=4\\P(0)=4\end{cases} \Leftrightarrow \begin{cases}-a+b=4\\4b=4\end{cases} \Leftrightarrow \begin{cases}a=1-4\\b=1\end{cases} \Leftrightarrow \begin{cases}a=-3\\b=1\end{cases}$

$P(x)=(x+2)^2(-3x+1)$

15. $P(y-1)=y^2-5y+6$

$x=y-1 \Leftrightarrow y=x+1$

$P(x)=(x+1)^2-5(x+1)+6$

$P(x)=x^2+2x+1-5x-5+6$

$P(x)=x^2-3x+2$

$P(x)=0 \Leftrightarrow x^2-3x+2=0 \Leftrightarrow$

$\Leftrightarrow x=\dfrac{3\pm\sqrt{9-8}}{2} \Leftrightarrow x=1 \vee x=2$

$S=\{1\ ,\ 2\}$

16. $P(x)=-2x^3-7x^2-4x+k$

16.1.

$$\begin{array}{r|rrrr}
 & -2 & -7 & -4 & k \\
-2 & & 4 & 6 & -4 \\
\hline
 & -2 & -3 & 2 & \boxed{k-4} \\
-2 & & 4 & -2 & \\
\hline
 & -2 & 1 & \boxed{0} &
\end{array}$$

$k-4=0 \Leftrightarrow k=4$

16.2. $P(x)=(x+2)^2(-2x+1)$

$P(x)-2x=4 \Leftrightarrow (x+2)^2(-2x+1)-2x-4=0 \Leftrightarrow$

$\Leftrightarrow (x+2)^2(-2x+1)-2(x+2)=0 \Leftrightarrow$

$\Leftrightarrow (x+2)\big((x+2)(-2x+1)-2\big)=0 \Leftrightarrow$

$\Leftrightarrow (x+2)(-2x^2+x-4x+2-2)=0 \Leftrightarrow$

$\Leftrightarrow (x+2)(-2x^2-3x)=0 \Leftrightarrow$

$\Leftrightarrow x(x+2)(-2x-3)=0 \Leftrightarrow$

$\Leftrightarrow x=0 \vee x=-2 \vee x=-\dfrac{3}{2}$

$\{x\in\mathbb{R}:\ P(x)-2x=4\}=\left\{-2\ ,\ -\dfrac{3}{2}\ ,\ 0\right\}$

17. $P(x)=ax^2+bx+c$

$\begin{cases}P(-3)=0\\P(0)+3=0\\P(0)+P(1)=5\end{cases} \Leftrightarrow \begin{cases}9a-3b+c=0\\c+3=0\\c+a+b+c=5\end{cases} \Leftrightarrow \begin{cases}c=-3\\a+b=11\\9a-3b=3\end{cases} \Leftrightarrow$

$\Leftrightarrow \begin{cases}c=-3\\b=11-a\\3a-b=1\end{cases} \Leftrightarrow \begin{cases}c=-3\\b=11-a\\3a-(11-a)=1\end{cases} \Leftrightarrow \begin{cases}c=-3\\b=11-a\\4a=12\end{cases} \Leftrightarrow \begin{cases}c=-3\\b=8\\a=3\end{cases}$

$P(x)=3x^2+8x-3$

18. $P(x)=x^3-bx^2+bx-4\ ;\ P(b)=0$

$b^3-b\times b^2+bb-4=0 \Leftrightarrow b^2=4 \Leftrightarrow b=-2 \vee b=2$

19. $P(x)=x^4-ax^2+(a-1)x,\ a\in\mathbb{R}$

19.1. $P(3)=0 \Leftrightarrow 81-9a+3(a-1)=0 \Leftrightarrow$

$\Leftrightarrow 81-9a+3a-3=0 \Leftrightarrow$

$\Leftrightarrow -6a+78=0 \Leftrightarrow a=13$

19.2. $P(x)=x^4-13x^2+12x=$

$=x(x^3-13x+13)$

$$\begin{array}{r|rrrr}
 & 1 & 0 & -13 & 12 \\
3 & & 3 & 9 & -12 \\
\hline
 & 1 & 3 & -4 & \boxed{0}
\end{array}$$

$x^2+3x-4=0 \Leftrightarrow$

$\Leftrightarrow x=\dfrac{-3\pm\sqrt{9+16}}{2} \Leftrightarrow$

$\Leftrightarrow x=-4 \vee x=1$

$P(x)=x(x-3)(x-1)(x+4)$

20. $P(x)=2x^4+ax^3+bx^2+c;\ a,b\in\mathbb{R}$

$\begin{cases}P(-1)=0\\P(0)=1\\P(1)=4\end{cases} \Leftrightarrow \begin{cases}2-a+b+c=0\\c=1\\2+a+b+c=4\end{cases} \Leftrightarrow \begin{cases}-a+b=-3\\c=1\\a+b=1\end{cases} \Leftrightarrow$

$\Leftrightarrow \begin{cases}c=1\\a=b+3\\b+3+b=1\end{cases} \Leftrightarrow \begin{cases}c=1\\a=2\\b=-1\end{cases}$

$P(x)=2x^4+2x^3-x^2+1$

$$\begin{array}{r|rrrrr}
 & 2 & 2 & -1 & 0 & 1 \\
-1 & & -2 & 0 & 1 & -1 \\
\hline
 & 2 & 0 & -1 & 1 & \boxed{0} \\
-1 & & -2 & 2 & -1 & \\
\hline
 & 2 & -2 & 1 & \boxed{0} &
\end{array}$$

$P(x)=(x+1)^2(2x^2-2x+1)$

$Q(x)=2x^2-2x+1$

$Q(-1)=2+2+2=5$

$P(x)=(x+1)^2\times Q(x)$ com $Q(-1)=5\neq 0$

21. $P(x)=x^3-ax^2-bx+b$

21.1. $x^2-4=(x-2)(x+2)$

$$\begin{cases}P(-2)=0\\P(2)=0\end{cases}\Leftrightarrow\begin{cases}-8-4a+2b+b=0\\8-4a-2b+b=0\end{cases}\Leftrightarrow$$

$$\Leftrightarrow\begin{cases}-4a+3b=8\\-4a-b=-8\end{cases}\Leftrightarrow\begin{cases}-4a-12a+24=8\\b=-4a+8\end{cases}\Leftrightarrow$$

$$\Leftrightarrow\begin{cases}-16a=-16\\b=-4a+8\end{cases}\Leftrightarrow\begin{cases}a=1\\b=4\end{cases}$$

$a=1$ e $b=4$

21.2. x^2+4 não tem raízes.

$$\begin{array}{rrrr|l}
x^3 & -\ ax^2 & -\ bx & +\ b & x^2+4 \\
\cline{5-5}
-x^3 & & -\ 4x & & x-a \\
\cline{1-4}
 & -\ ax^2 & +\ (-b-4)x & b & \\
 & ax^2 & & +\ 4a & \\
\cline{2-4}
 & & (-b-4)x & +\ 4a+b &
\end{array}$$

$$(-b-4)x+4a+b=0\Leftrightarrow\begin{cases}-b-4=0\\4a+b=0\end{cases}\Leftrightarrow\begin{cases}b=-4\\a=1\end{cases}$$

$a=1$ e $b=-4$

21.3. -4 é uma raiz dupla.

$$\begin{array}{r|llll}
 & 1 & -a & -b & b \\
-4 & & -4 & 16+4a & -64-16a+4b \\
\hline
 & 1 & -4-a & 16+4a-b & \boxed{-64-16a+5b} \\
-4 & & -4 & 32+4a & \\
\hline
 & 1 & -8-a & \boxed{48+8a-b} &
\end{array}$$

$$\begin{cases}-64-16a+5b=0\\48+8a-b=0\end{cases}\Leftrightarrow\begin{cases}-16a+5(8a+48)=64\\b=8a+48\end{cases}\Leftrightarrow$$

$$\Leftrightarrow\begin{cases}-16a+40a+240=64\\b=8a+48\end{cases}\Leftrightarrow\begin{cases}24a=-176\\b=8a+48\end{cases}\Leftrightarrow$$

$$\Leftrightarrow\begin{cases}a=\dfrac{-176}{24}\\b=8a+48\end{cases}\Leftrightarrow\begin{cases}a=-\dfrac{22}{3}\\b=-8\times\dfrac{22}{3}+48\end{cases}\Leftrightarrow\begin{cases}a=-\dfrac{22}{3}\\b=-\dfrac{32}{3}\end{cases}$$

$a=-\dfrac{22}{3}$ e $b=-\dfrac{32}{3}$

22. $P(x)=x^3+ax+b$

22.1.

$$\begin{array}{r|llll}
 & 1 & 0 & a & b \\
3 & & 3 & 9 & 3a+27 \\
\hline
 & 1 & 3 & a+9 & \boxed{3a+b+27} \\
3 & & 3 & 18 & \\
\hline
 & 1 & 6 & \boxed{a+27} &
\end{array}$$

$$\begin{cases}a+27=0\\3a+b+27=0\end{cases}\Leftrightarrow\begin{cases}a=-27\\-3\times 27+b+27=0\end{cases}\Leftrightarrow\begin{cases}a=-27\\b=54\end{cases}$$

$a=-27$ e $b=54$

22.2. $P(x)=x^3-27x+54=$

$=(x-3)^2(x+6)$

$(x-3)^2(x+6)>0\Leftrightarrow$ $\quad\Big|\ \forall x\in\mathbb{R},\ (x-3)^2\geq 0$

$\Leftrightarrow x\neq 3\wedge x+6>0\Leftrightarrow$

$\Leftrightarrow x\neq 3\wedge x>-6\Leftrightarrow$

$\Leftrightarrow x\in\,]-6\ ,\ 3[\,\cup\,]3\ ,\ +\infty[$

$S=\,]-6\ ,\ 3[\,\cup\,]3\ ,\ +\infty[$

23. $A(x)=ax^3-8x^2+(2a+1)x-2$

$3x-2=3\left(x-\dfrac{2}{3}\right)$

$P\left(\dfrac{2}{3}\right)=0\Leftrightarrow$

$$\Leftrightarrow a\times\left(\frac{2}{3}\right)^3-8\times\left(\frac{2}{3}\right)^2+(2a+1)\times\frac{2}{3}-2=0\Leftrightarrow$$

$$\Leftrightarrow a\times\frac{8}{27}-8\times\frac{4}{9}+\frac{4}{3}a+\frac{2}{3}-2=0\Leftrightarrow$$

$$\Leftrightarrow\frac{8a}{27}+\frac{36a}{27}=\frac{32}{9}-\frac{6}{9}+\frac{18}{9}\Leftrightarrow$$

$$\Leftrightarrow\frac{449}{27}=\underset{(3)}{\frac{44}{9}}\Leftrightarrow 44a=44\times 3\Leftrightarrow a=3$$

$A(x)=3x^3-8x^2+7x-2$

$$\begin{array}{r|rrrr}
 & 3 & -8 & 7 & -2 \\
\frac{2}{3} & & 2 & -4 & 2 \\
\hline
 & 3 & -6 & 3 & \boxed{0}
\end{array}$$

$3x^2-6x+3=0\Leftrightarrow$

$\Leftrightarrow x^2-2x+1=0\Leftrightarrow$

$\Leftrightarrow(x-1)^2$

$A(x)=3\left(x-\dfrac{2}{3}\right)(x-1)^2$

$A(x)>2x-2\Leftrightarrow$

$$\Leftrightarrow 3\left(x-\frac{2}{3}\right)(x-1)^2-2(x-1)>0\Leftrightarrow$$

$$\Leftrightarrow(x-1)\big[(3x-2)(x-1)-2\big]>0\Leftrightarrow$$

$$\Leftrightarrow(x-1)\big(3x^2-3x-2x+2-2\big)>0\Leftrightarrow$$

$$\Leftrightarrow(x-1)\big(3x^2-5x\big)>0\Leftrightarrow$$

$$\Leftrightarrow x(x-1)(3x-5)>0$$

x	$-\infty$	0		1		$\frac{5}{3}$	$+\infty$
x	−	0	+	+	+	+	+
$x-1$	−	−	−	0	+	+	+
$3x-5$	−	−	−		−	0	+
P	−	0	+	0	−	0	+

$S=\,]0\ ,\ 1[\,\cup\left]\dfrac{5}{3}\ ,\ +\infty\right[$

24. $P(x)=x^n-1$

24.1. $P(1)=1^n-1=0$

Como $P(1)=0$, $P(x)$ é divisível por $x-1$.

Logo, existe um polinómio $Q(x)$ tal que $P(x)=(x-1)\times Q(x)$.

24.2. Vamos dividir $x^n - 1$ por $x - 1$.

$$x^n - 1 = x^n + \underbrace{0x^{n-1} + 0x^{n-2} + ... + 0x}_{n-1 \text{ zeros}} - 1$$

$n - 1$ zeros

	1	0	0	0	…	0	– 1
1		1	1	1	…	1	1
	1	1	1	1	…	1	0

n termos

$$P(x) = x^n - 1 = (x-1)\left(\overbrace{x^{n-1} + x^{n-2} + ... + x + 1}^{n \text{ termos}}\right)$$

$$P(x) = (x-1)Q(x) \text{ com } Q(x) = x^{n-1} + x^{n-2} + ... + x + 1$$

Logo, $Q(1) = 1^{n-1} + 1^{n-2} + ... + 1 + 1 = n \times 1 = n$.

Avaliação global

1. $\sqrt{2\sqrt[3]{16}} = \sqrt{\sqrt[3]{2^3 \times 2^4}} = \sqrt[6]{2^7} = \sqrt[6]{2^6 \times 2} = 2\sqrt[6]{2}$

Resposta: (D)

2. $\dfrac{a}{\sqrt[n]{a^{n-3}}} = \dfrac{a\sqrt[n]{a^3}}{\sqrt[n]{a^{n-3}}\sqrt[n]{a^3}} = \dfrac{a\sqrt[n]{a^3}}{\sqrt[n]{a^n}} = \dfrac{a\sqrt[n]{a^3}}{a} = \sqrt[n]{a^3}$, $a > 0$

Resposta: (B)

3. $\dfrac{\sqrt{6}}{\sqrt{3} - \sqrt{6}} = \dfrac{\sqrt{6}\left(\sqrt{3} + \sqrt{6}\right)}{\left(\sqrt{3} - \sqrt{6}\right)\left(\sqrt{3} + \sqrt{6}\right)} = \dfrac{\sqrt{18} + 6}{3 - 6} = \dfrac{\sqrt{9 \times 2} + 6}{-3} =$

$= \dfrac{3\sqrt{2} + 6}{-3} = -\sqrt{2} - 2$

Resposta: (D)

4. $9 + 4\sqrt{2} = (a+b)^2 = a^2 + 2ab + b^2$

$2ab = 4\sqrt{2}$; $ab = 2\sqrt{2}$

Se $a = 2$ e $b = \sqrt{2}$, $a^2 + b^2 = 6$.

Se $a = 1$ e $b = 2\sqrt{2}$, $a^2 + b^2 = 9$.

$9 + 4\sqrt{2} = \left(1 + 2\sqrt{2}\right)^2$

$\sqrt{9 + 4\sqrt{2}} = \sqrt{\left(1 + 2\sqrt{2}\right)^2} = 1 + 2\sqrt{2}$

Resposta: (B)

5. $\left[1 + \left(3^{\frac{1}{2}} + 3^{\frac{3}{2}}\right)^2\right]^{\frac{1}{2}} = \sqrt{1 + \left(\sqrt{3} + \sqrt{3^3}\right)^2} = \sqrt{1 + \left(\sqrt{3} + 3\sqrt{3}\right)^2} =$

$= \sqrt{1 + \left(4\sqrt{3}\right)^2} = \sqrt{1 + 16 \times 3} = \sqrt{49} = 7$

Resposta: (B)

6. $\underset{\text{grau } 2}{A(x)} \times \underset{\text{grau } 3}{B(x)} = \underset{\text{grau } 5}{C(x)}$

Resposta: (B)

7. $A(x) = (x-2) \times (3x-1) + R$

$= 2(x-2) \times \dfrac{1}{2}(3x-1) + R$

$= (2x-4) \times \left(\dfrac{3}{2}x - \dfrac{1}{2}\right) + R$

Resposta: (B)

8. $d = 4$ cm = diâmetro da esfera

= diâmetro espacial do cubo

8.1. Seja a o comprimento da aresta do cubo.

$a^2 + a^2 + a^2 = d^2 \Leftrightarrow 3a^2 = d^2 \Leftrightarrow$

$\Leftrightarrow a^2 = \dfrac{1}{3}d^2 \Leftrightarrow a = \sqrt{\dfrac{1}{3}d^2} \Leftrightarrow a = \sqrt{\dfrac{1}{3}}d \Leftrightarrow$

$\Leftrightarrow a = \dfrac{1}{\sqrt{3}}d \Leftrightarrow a = \dfrac{\sqrt{3}}{3}d$; $a > 0$ e $d > 0$

$a = \left(\dfrac{\sqrt{3}}{3} \times 4\right)$ cm = $\dfrac{4\sqrt{3}}{3}$ cm

O comprimento da aresta é $\dfrac{4\sqrt{3}}{3}$ cm.

8.2. $V_{\text{cubo}} = a^3 = \left(\dfrac{4\sqrt{3}}{3}\right)^3 = \dfrac{4^3 \times \left(\sqrt{3}\right)^3}{3^3} = \dfrac{64 \times 3\sqrt{3}}{3^3} = \dfrac{64\sqrt{3}}{9}$

O volume do cubo é $\dfrac{64\sqrt{3}}{9}$ cm^3.

8.3. Raio da esfera: $r = \dfrac{4}{2}$ cm = 2 cm

$V_{\text{esfera}} = \dfrac{4}{3}\pi r^3 = \dfrac{4}{3}\pi \times 2^3 = \dfrac{32\pi}{3}$

$\dfrac{V_{\text{esfera}}}{V_{\text{cubo}}} = \dfrac{\frac{32\pi}{3}}{\frac{64\sqrt{3}}{9}} = \dfrac{32\pi}{3} \times \dfrac{9}{64\sqrt{3}} = \dfrac{3\pi}{2\sqrt{3}} =$

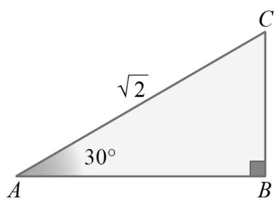
$= \dfrac{3\pi\sqrt{3}}{2\sqrt{3} \times \sqrt{3}} = \dfrac{3\sqrt{3}\pi}{2 \times 3} = \dfrac{\sqrt{3}}{2}\pi$

9. $\dfrac{\overline{AB}}{\sqrt{2}} = \cos(30°)$

$\overline{AB} = \sqrt{2} \times \dfrac{\sqrt{3}}{2} = \dfrac{\sqrt{6}}{2}$

$\dfrac{\overline{BC}}{\sqrt{2}} = \sin(30)°$

$\overline{BC} = \sqrt{2} \times \dfrac{1}{2} = \dfrac{\sqrt{2}}{2}$



$A_{[ABC]} = \dfrac{\overline{AB} \times \overline{BC}}{2} = \dfrac{\frac{\sqrt{6}}{2} \times \frac{\sqrt{2}}{2}}{2} = \dfrac{\sqrt{12}}{4} \times \dfrac{1}{2} = \dfrac{\sqrt{4 \times 3}}{8} =$

$= \dfrac{2\sqrt{3}}{8} = \dfrac{\sqrt{3}}{4}$

A área do triângulo é $\dfrac{\sqrt{3}}{4}$ cm^2.

10. $\dfrac{\left(\frac{1}{2}\right)^{\frac{1}{3}} : \left(\frac{1}{2}\right)^{\frac{2}{3}} \times \sqrt[3]{12}}{\sqrt[3]{3}} = \dfrac{\left(\frac{1}{2}\right)^{\frac{1}{3} - \frac{2}{3}} \times \sqrt[3]{12}}{\sqrt[3]{3}} =$

$= \dfrac{\left(\frac{1}{2}\right)^{-\frac{1}{3}} \times \sqrt[3]{12}}{\sqrt[3]{3}} = \dfrac{2^{\frac{1}{3}} \times \sqrt[3]{12}}{\sqrt[3]{3}} = \dfrac{\sqrt[3]{2} \times \sqrt[3]{12}}{\sqrt[3]{3}} =$

$= \sqrt[3]{\dfrac{2 \times 12}{3}} = \sqrt[3]{8} = 2$

11. $P(x) = (x-2)(x+3)^2(x-4)^3(x-1)^4$

1 + 2 + 3 + 4 = 10. $P(x)$ tem grau 10

12. $A(x)=3x^5-x^4+2x^3+4x-3$

$B(x)=x^3-2x+1$

$$\begin{array}{rrrrrrr|l}
3x^5 & -x^4 & +2x^3 & +0x^2 & +4x & -3 & & x^3-2x+1 \\
-3x^5 & & +6x^3 & -3x^2 & & & & 3x^2-x+8 \\
\hline
 & -x^4 & +8x^3 & -3x^2 & +4x & -3 & & \\
 & +x^4 & & -2x^2 & +x & & & \\
\hline
 & & 8x^3 & -5x^2 & +5x & -3 & & \\
 & & -8x^3 & & +16x & -8 & & \\
\hline
 & & & -5x^2 & +21x & -11 & &
\end{array}$$

$Q(x)=3x^2-x+8$ e $R(x)=-5x^2+21x-11$

13.

$$\begin{array}{c|c}
P(x) & x^2+x+1 \\
\hline
-x+13 & x^2-x
\end{array}$$

$P(x)=(x^2-x)(x^2+x+1)-x+13=$

$=x^4+x^3+x^2-x^3-x^2-x-x+13=$

$=x^4-2x+13$

14. $P(x)=x^3+x^2-3ax-4a$

$B(x)=x^2-x-4$

$$\begin{array}{rrrr|l}
x^3 & +x^2 & -3ax & -4a & x^2-x-4 \\
-x^3 & +x^2 & +4x & & x+2 \\
\hline
 & 2x^2 & +(4-3a)x & -4a & \\
 & -2x^2 & +2x & +8 & \\
\hline
 & & (6-3a)x & +8-4a &
\end{array}$$

$(6-3a)x+8-4a=0 \Leftrightarrow$

$\Leftrightarrow \begin{cases} 6-3a=0 \\ 8-4a=0 \end{cases} \Leftrightarrow \begin{cases} 3a=6 \\ 4a=8 \end{cases} \Leftrightarrow a=2$

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15. $P(x)=(x-2)(x-4)(x-5)\times Q(x)+(x+1)$

$\begin{cases} P(2)=A \\ P(4)=B \\ P(5)=C \end{cases} \Leftrightarrow \begin{cases} 0\times Q(x)+3=A \\ 0\times Q(x)+4=B \\ 0\times Q(x)+5=C \end{cases} \Leftrightarrow \begin{cases} A=3 \\ B=4 \\ C=5 \end{cases}$

$A\times B\times C=3\times 4\times 5=90$

16. $P(x)=x^{100}+x+1$

$P(x)=Q(x)\times(x^2-1)+R(x)$

$R(x)=ax+b$ porque o grau do divisor é 2.

$\begin{cases} P(1)=Q(1)\times 0+R(1) \\ P(-1)=Q(-1)\times 0+R(-1) \end{cases} \Leftrightarrow$

$\Leftrightarrow \begin{cases} 1^{100}+1+1=a\times 1+b \\ (-1)^{100}-1+1=a\times(-1)+b \end{cases} \Leftrightarrow \begin{cases} a+b=3 \\ -a+b=1 \end{cases} \Leftrightarrow$

$\Leftrightarrow \begin{cases} b-1+b=3 \\ a=b-1 \end{cases} \Leftrightarrow \begin{cases} 2b=4 \\ a=b-1 \end{cases} \Leftrightarrow \begin{cases} b=2 \\ a=1 \end{cases}$

$R(x)=x+2$

17. $[P(x)]^3=x^2\times P(x) \Leftrightarrow \dfrac{[P(x)]^3}{P(x)}=x^2 \Leftrightarrow$

$\Leftrightarrow [P(x)]^2=x^2 \Leftrightarrow P(x)=x \vee P(x)=-x$

Logo, $P(x)$ tem grau 1.

18. $P(x)+x\times P(2-x)=x^2+3$

18.1. $P(0)+0\times P(2)=3 \Leftrightarrow P(0)=3$

$P(1)+1\times P(2-1)=1^2+3 \Leftrightarrow P(1)+P(1)=4 \Leftrightarrow$

$\Leftrightarrow 2P(1)=4 \Leftrightarrow P(1)=2$

$P(2)+2\times P(2-2)=4+3 \Leftrightarrow$

$\Leftrightarrow P(2)+2\times P(0)=7 \Leftrightarrow$

$\Leftrightarrow P(2)+2\times 3=7 \Leftrightarrow P(2)=1$

18.2. $P(x)=ax+b$

$\begin{cases} P(0)=3 \\ P(1)=2 \\ P(2)=1 \end{cases} \Leftrightarrow \begin{cases} b=3 \\ a+b=2 \\ 2a+b=1 \end{cases} \Leftrightarrow \begin{cases} b=3 \\ a+3=2 \\ 2a+3=1 \end{cases} \Leftrightarrow$

$\Leftrightarrow \begin{cases} b=3 \\ a=-1 \\ 2a=-2 \end{cases} \Leftrightarrow \begin{cases} b=3 \\ a=-1 \end{cases}$

$P(x)=-x+3$

19. $P(x)=x^3+ax^2+bx$

$\begin{cases} P(2)=2 \\ P(1)=4 \end{cases} \Leftrightarrow \begin{cases} 8+4a+2b=2 \\ 1+a+b=4 \end{cases} \Leftrightarrow \begin{cases} 4a+2b=-6 \\ a+b=3 \end{cases} \Leftrightarrow$

$\Leftrightarrow \begin{cases} 2a+3-a=-3 \\ b=3-a \end{cases} \Leftrightarrow \begin{cases} a=-6 \\ b=9 \end{cases}$

$a=-6$ e $b=9$

20. $V(x)=A(x)\times H(x)$

$V(x)=x^3+7x^2+14x+8$

$H(x)=x+1$

$A(x)$ é o quociente de $V(x)$ por $H(x)$

	1	7	14	8
−1		−1	−6	−8
	1	6	8	0

$A(x)=x^2+6x+8$

21. $P(x)=(x-1)\times Q(x)+3$

$Q(x)=(x-2)\times Q_1(x)+2$

Logo:

$P(x)=(x-1)\times[(x-2)\times Q_1(x)+2]+3=$

$=(x-1)\times(x-2)Q_1(x)+2(x-1)+3=$

$P(x)=(x-1)\times(x-2)Q_1(x)+2x+1$

$R(x)=2x+1$

22. $(x^2-4)(x-3)\le 0 \Leftrightarrow$

$\Leftrightarrow (x-2)(x+2)(x-3)\le 0 \Leftrightarrow$

$\Leftrightarrow x\in\,]-\infty\,,\,-2]\cup[2\,,\,3]$

x	$-\infty$	-2		2		3	$+\infty$
$x-2$	−	−	−	0	+	+	+
$x+2$	−	0	+	+	+	+	+
$x-3$	−	−	−	−	−	0	+
P	−	0	+	0	−	0	+

$S=\,]-\infty\,,\,-2]\cup[2\,,\,3]$

23. $P(x)=x^3-6x^2+11x-6$

23.1. $P(3)=3^3-6\times 3^2+11\times 3-6=$

$=27-54+33-6=0$

Se $P(3)=0$, então $P(x)$ é divisível por $x-3$.

23.2.

	1	−6	11	−6
3		3	−9	6
	1	−3	2	$\boxed{0}$

$x^2-3x+2=0 \Leftrightarrow x=\frac{3\pm\sqrt{9-8}}{2} \Leftrightarrow$

$\Leftrightarrow x=1 \vee x=2$

$P(x)=(x-1)(x-2)(x-3)$

$P(x)>0 \Leftrightarrow (x-1)(x-2)(x-3)>0 \Leftrightarrow$

$\Leftrightarrow x\in \left]1\ ,\ 2\right[\cup\left]3\ ,\ +\infty\right[$

x	$-\infty$	1		2		3	$+\infty$
$x-1$	−	0	+	+	+	+	+
$x-2$	−	−	−	0	+	+	+
$x-3$	−	−	−	−	−	0	+
$P(x)$	−	0	+	0	−	0	+

$S=\left]1\ ,\ 2\right[\cup\left]3\ ,\ +\infty\right[$

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24. $P(x)=8x^5+4x^4-10x^3-x^2+4x-1$

24.1.

	8	4	−10	−1	4	−1
$\frac{1}{2}$		4	4	−3	−2	1
	8	8	−6	−4	2	$\boxed{0}$
$\frac{1}{2}$		4	6	0	−2	
	8	12	0	−4	$\boxed{0}$	
$\frac{1}{2}$		4	8	4		
	8	16	8	$\boxed{0}$		
$\frac{1}{2}$		4	10			
	8	20	$\boxed{18\neq 0}$			

$P(x)=\left(x-\frac{1}{2}\right)^3\left(8x^2-16x+8\right)$ e $\frac{1}{2}$ não é raiz de $8x^2-16x+8$.

Logo, $\frac{1}{2}$ é uma raiz de grau de multiplicidade 3.

24.2. $8x^2+16x+8=0 \Leftrightarrow$

$\Leftrightarrow 8\left(x^2+2x+1\right)=8(x+1)^2$

$P(x)=8\left(x-\frac{1}{2}\right)^3(x+1)^2$

$P(x)\leq 0 \Leftrightarrow x\in\left]-\infty\ ,\ \frac{1}{2}\right]$

x	$-\infty$	−1		$\frac{1}{2}$	$+\infty$
$\left(x-\frac{1}{2}\right)^3$	−	−	−	0	+
$(x+1)^2$	+	0	+	+	+
$P(x)$	−	0	−	0	+

$S=\left]-\infty\ ,\ \frac{1}{2}\right]$

25. $P(-1)=-1$; $P(2)=-7$

$P(x)=(x+1)(x-2)\times Q(x)+R(x)$

$R(x)=ax+b$, dado que $(x+1)(x+2)$ é do 2.º grau

$P(x)=(x+1)(x-2)\times Q(x)+ax+b$

$\begin{cases}P(-1)=-1\\P(2)=-7\end{cases} \Leftrightarrow \begin{cases}0\times Q(-1)+a\times(-1)+b=-1\\0\times Q(2)+a\times 2+b=-7\end{cases} \Leftrightarrow$

$\Leftrightarrow \begin{cases}-a+b=-1\\P(2)=-7\end{cases} \Leftrightarrow \begin{cases}b=a-1\\2a+a-1=-7\end{cases} \Leftrightarrow \begin{cases}b=a-4\\3a=-6\end{cases} \Leftrightarrow$

$\Leftrightarrow \begin{cases}b=-3\\a=-2\end{cases}$

$R(x)=-2x-3$

26. $P(x)=3x^3+mx^2+nx-2$

x^2+x+2 não tem raízes reais.

$$\begin{array}{rrrr|l}
3x^3 & +\ mx^2 & +\ nx & -\ 2 & \ x^2+x+2 \\ \cline{5-5}
-\ 3x^3 & -\ 3x^2 & -\ 6x & & \ 3x+m-3 \\ \hline
 & (m-3)\,x^2 & +\ (n-6)x & -\ 2 & \\
 & -\ (m-3)\,x^2 & +\ (-\,m+3)\,x & -\ 2m+6 & \\ \hline
 & & (n-m-3)x & -\ 2m+4 &
\end{array}$$

$(n-m-3)x-2m+4=0 \Leftrightarrow \begin{cases}n-m-3=0\\-2m+4=0\end{cases} \Leftrightarrow$

$\Leftrightarrow \begin{cases}n-2-3=0\\m=2\end{cases} \Leftrightarrow \begin{cases}n=5\\m=2\end{cases}$

$m=2$ e $n=5$

Outro processo:

$3x^3+mx^2+nx-2=\left(x^2+x+2\right)(ax+b) \Leftrightarrow$

$\Leftrightarrow 3x^3+mx^2+nx-2=ax^3+bx^2+ax^2+bx+2ax+2b \Leftrightarrow$

$\Leftrightarrow 3x^3+mx^2+nx-2=ax^3+(a+b)x^2+(2a+b)x+2b \Leftrightarrow$

$\Leftrightarrow \begin{cases}a=3\\a+b=m\\2a+b=n\\2b=-2\end{cases} \Leftrightarrow \begin{cases}a=3\\3-1=m\\6-1=n\\b=-1\end{cases} \Leftrightarrow \begin{cases}a=3\\m=2\\n=5\\b=-1\end{cases}$

$m=2$ e $n=5$

27. $\left(B(x)=0 \Leftrightarrow x=1\right)\wedge\left(\forall x\in\mathbb{R},\ B(x)<0 \Leftrightarrow x\in\left]1\ ,\ +\infty\right[\right) \Rightarrow$

$\Rightarrow \left(\forall x\in\mathbb{R},\ B(x)>0 \Leftrightarrow x\in\left]-\infty\ ,\ 1\right[\right)$

27.1.

x	$-\infty$	1		2	$+\infty$
$2-x$	+	+	+	0	−
$B(x)$	+	0	−	−	−
$(2-x)\,B(x)$	+	0	−	0	+

$(2-x)B(x)\leq 0 \Leftrightarrow x\in[1\ ,\ 2]$

$S=[1\ ,\ 2]$

27.2. $x^2-3x+2=0 \Leftrightarrow x=\frac{3\pm\sqrt{9-8}}{2} \Leftrightarrow x=1 \vee x=2$

$\left(x^2-3x+2\right)B(x)>0 \Leftrightarrow (x-1)(x-2)B(x)>0$

x	$-\infty$	1		2	$+\infty$
$x-1$	−	0	+	+	+
$x-2$	−	−	−	0	+
$B(x)$	+	0	−	−	−
P	+	0	+	0	−

$(x-1)(x-2)B(x)>0 \Leftrightarrow \left]-\infty\,,\ 1\right[\cup\left]1\,,\ 2\right[$

$S=\left]-\infty\,,\ 1\right[\cup\left]1\,,\ 2\right[$

28. Pretende-se provar que:

$P(x)$ é divisível por $x^2-a^2 \Rightarrow$

$\Rightarrow P(x)$ é divisível por $x-a$

$P(x)$ é divisível por $x^2-a^2 \Rightarrow$

$\Rightarrow P(x)=\left(x^2-a^2\right)Q(x)$

$\Rightarrow P(x)=(x-a)(x+a)Q(x)$

$\Rightarrow P(x)=(x-a)\times Q_1(x)$ sendo $Q_1(x)=(x+a)Q(x)$

$\Rightarrow P(x)$ é divisível por $x-a$

Portanto, $b(x)$ é condição suficiente para que se verifique $a(x)$.

29. $P(x)=(x-n)^2\,Q(x)$ com $Q(n)\neq 0$

29.1. $x^3-mx-16=(x-n)^2(ax+b)$, dado que $(x-n)^2$ é um polinómio do 2.º grau.

$x^3-mx-16=\left(x^2-2nx+n^2\right)(ax+b)\Leftrightarrow$

$\Leftrightarrow x^3-mx-16=ax^3+bx^2-2anx^2-2bnx+an^2x+bn^2\Leftrightarrow$

$\Leftrightarrow x^3-mx-16=ax^3(b-2an)x^2+\left(an^2-2bn\right)x+bn^2\Leftrightarrow$

$$\Leftrightarrow\begin{cases}a=1\\ b-2an=0\\ an^2-2bn=-m\\ bn^2=-16\end{cases}\Leftrightarrow\begin{cases}a=1\\ b=2n\\ n^2-2\times(2n)n=-m\\ 2n\times n^2=-16\end{cases}\Leftrightarrow$$

$$\Leftrightarrow\begin{cases}a=1\\ b=2n\\ n^2-4n^2=-m\\ n^3=-8\end{cases}\Leftrightarrow\begin{cases}a=1\\ b=2\times(-2)\\ -3n^2=-m\\ n=-2\end{cases}\Leftrightarrow\begin{cases}a=1\\ b=-4\\ m=12\\ n=-2\end{cases}$$

$m = 12$ e $n = -2$

29.2. $x^3-mx-16=(x-n)^2(ax+b)$

$=(x+2)^2(x-4)\qquad (a=1\,,\ b=-4\,,\ m=12 \text{ e } n=-2)$

$P(x)=(x+2)^2(x-4)$

30. $P(x)=8x^3+4x^2-2x-1$

30.1. $P(x)=8(x-a)^2(x+a)\Leftrightarrow$

$\Leftrightarrow 8x^3+4x^2-2x-1=8\left(x^2-2ax+a^2\right)(x+a)\Leftrightarrow$

$8x^3+4x^2-2x-1=8\left(x^3+ax^2-2ax^2-2a^2x+a^2x+a^3\right)$

$\Leftrightarrow 8x^3+4x^2-2x-1=8x^3-8ax^2-8a^2x+8a^3\Leftrightarrow$

$$\Leftrightarrow\begin{cases}-8a=4\\ -8a^2=-2\\ 8a^3=-1\end{cases}\Leftrightarrow\begin{cases}a=-\frac{1}{2}\\ a^2=\frac{1}{4}\\ a^3=-\frac{1}{8}\end{cases}\Leftrightarrow a=-\frac{1}{2}$$

$P(x)=8\left(x+\frac{1}{2}\right)^2\left(x-\frac{1}{2}\right)$

30.2. $P(x)<0\Leftrightarrow 8\left(x+\frac{1}{2}\right)^2\left(x-\frac{1}{2}\right)<0\Leftrightarrow$

$\Leftrightarrow x-\frac{1}{2}<0\wedge x+\frac{1}{2}\neq 0\Leftrightarrow$

$\Leftrightarrow x<\frac{1}{2}\wedge x\neq-\frac{1}{2}$

$S=\left]-\infty\,,\ -\frac{1}{2}\right[\cup\left]-\frac{1}{2}\,,\ \frac{1}{2}\right[$

Ou

x	$-\infty$	$-\frac{1}{2}$		$\frac{1}{2}$	$+\infty$
$\left(x+\frac{1}{2}\right)^2$	+	0	+	+	+
$x-\frac{1}{2}$	−	−	−	0	+
$P(x)$	−	0	−	0	+

$S=\left]-\infty\,,\ -\frac{1}{2}\right[\cup\left]-\frac{1}{2}\,,\ \frac{1}{2}\right[$

31. $P(x)=x^{5n}+x^{4n}-x^n-1,\ n\in\mathbb{N}$

31.1. $P(x)=x^{4n}\left(x^n+1\right)-\left(x^n+1\right)=$

$=\left(x^n+1\right)\left(x^{4n}-1\right)=$

$=\left(x^{4n}-1\right)\left(x^n+1\right)$

31.2. $P(x)=\left(x^{4n}-1\right)\left(x^n+1\right)=$

$=\left[\left(x^{2n}\right)^2-1\right]\left(x^n+1\right)=$

$=\left(x^{2n}-1\right)\left(x^{2n}+1\right)\left(x^n+1\right)=$

$=\left(x^n-1\right)\left(x^n+1\right)\left(x^{2n}+1\right)\left(x^n+1\right)=$

$=\left(x^n-1\right)\left(x^n+1\right)^2\left(x^{2n}+1\right)$

Vamos dividir x^n-1 por $x-1$, usando a Regra de Ruffini.

$x^n-1=x^n+\underbrace{0x^{n-1}+0x^{n-2}+\ldots+0x}_{n-1\text{ zeros}}-1$

$n-1$ zeros

	1	0	0	0	…	0	−1
1		1	1	1	…	1	1
	1	1	1	1	…	1	$\boxed{0}$

$x^n-1=(x-1)\overbrace{\left(x^{n-1}+x^{n-2}+\ldots+x+1\right)}^{n\text{ termos}}$

$P(x)=(x-1)\left(x^{n-1}+x^{n-2}+\ldots+x+1\right)\left(x^n+1\right)^2\left(x^{2n}+1\right)$

$P(x)=(x-1)\times Q(x)$ sendo

$Q(x)=\left(x^{n-1}+x^{n-2}+\ldots+x+1\right)\left(x^n+1\right)^2\left(x^{2n}+1\right)$

$Q(1)=\underbrace{(1+1+\ldots+1+1)}_{n\text{ vezes}}(1+1)^2(1+1)=$

$=x\times1\times4\times2=$

$=8n\neq 0$

Como $P(x)=(x-1)Q(x)$ e $Q(1)\neq 0$, 1 é uma raiz simples de $P(x)$.