Resolução da ficha n.º 1 - Trigonometria - 11.º ano

1) a) $\sin\pi + \cos 3\pi + 2\,tg\, 8\pi + \cos 5\pi =$

$= \sin\pi + \cos\pi + 2\,tg\, 0 + \cos\pi =$

$= 0 - 1 + 2\times 0 - 1 = -2$

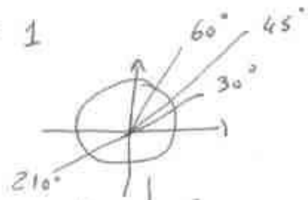
b) $\sin 270° - \cos 90° + tg\, 180° - \sin 90° =$

$= -1 - 0 + 0 - 1 = -2$

c) $2\sin 630° - 3\cos 180° + 5\,tg\, 720° =$

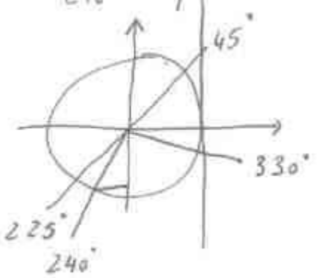
$= 2\sin 270° - 3\cos 180° + 5\,tg\, 0° =$

$= 2\times(-1) - 3\times(-1) + 5\times 0 = -2 + 3 = 1$



d) $\sin 210° + \cos 60° + tg\, 45° =$

$= -\cancel{\frac{1}{2}} + \cancel{\frac{1}{2}} + 1 = 1$



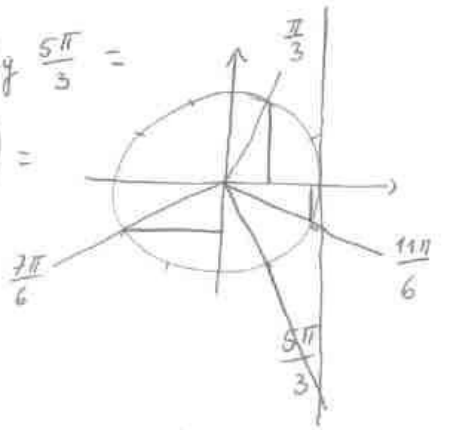
e) $tg\, 225° + \cos 240° + tg\, 330° =$

$= 1 - \frac{1}{2} - \frac{\sqrt{3}}{3} = \frac{1}{2} - \frac{\sqrt{3}}{3} = \frac{3-2\sqrt{3}}{6}$

f) $\dfrac{\cos 1080° - 2\sin 1530°}{\sin(-270°) + tg\, 0°} =$

$= \dfrac{\cos 0° - 2\sin 90°}{\sin 90° + tg\, 0°} = \dfrac{1 - 2\times 1}{1 + 0} = -1$

g) $2\sin\frac{\pi}{3} - 3\cos\frac{7\pi}{6} - \sin\frac{11\pi}{6} + tg\,\frac{5\pi}{3} =$



$= \cancel{2}\times\frac{\sqrt{3}}{\cancel{2}} - 3\times\left(-\frac{\sqrt{3}}{2}\right) - \left(-\frac{1}{2}\right) + \left(-\sqrt{3}\right) =$

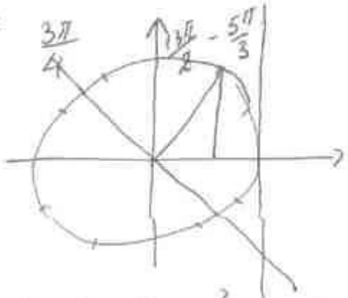
$= \cancel{\sqrt{3}} + \frac{3\sqrt{3}}{2} + \frac{1}{2} - \cancel{\sqrt{3}} =$

$= \frac{1 + 3\sqrt{3}}{2}$

h) $\cos\frac{13\pi}{2} - tg\,\frac{3\pi}{4} - \sin\left(-\frac{5\pi}{3}\right) =$

$= 0 - (-1) - \frac{\sqrt{3}}{2} =$

$= 1 - \frac{\sqrt{3}}{2} = \frac{2-\sqrt{3}}{2}$



2) a) $2 + 3\sin^2 x = 2 + 3(1-\cos^2 x) = 2 + 3 - 3\cos^2 x = 5 - 3\cos^2 x$ //

b) $\left(1+\frac{1}{tg^2 x}\right)(1-\cos^2 x) = \left(1+\frac{1}{\frac{\sin^2 x}{\cos^2 x}}\right)(1-\cos^2 x) =$

$= \left(1+\frac{\cos^2 x}{\sin^2 x}\right)(1-\cos^2 x) = \left(\frac{\sin^2 x+\cos^2 x}{\sin^2 x}\right)(1-\cos^2 x) =$

$= \left(\frac{1}{\sin^2 x}\right)(1-\cos^2 x) = \frac{1-\cos^2 x}{\sin^2 x} = \frac{\sin^2 x}{\sin^2 x} = 1$ //

c) $\frac{\cos x}{1+\sin x} = \frac{\cos x\,(1-\sin x)}{(1+\sin x)(1-\sin x)} = \frac{\cos x\,(1-\sin x)}{1-\sin^2 x} =$

$= \frac{\cancel{\cos x}(1-\sin x)}{\cos^{\cancel{2}} x} = \frac{1-\sin x}{\cos x}$ //

d) $\cos^2 x + \cos^2 x\sin^2 x = 1-\sin^2 x + (1-\sin^2 x)\sin^2 x =$

$= 1-\cancel{\sin^2 x} + \cancel{\sin^2 x} - \sin^4 x = 1-\sin^4 x$ //

e) $(\sin x+\cos x)^2 - (\sin x-\cos x)^2 =$

$= \sin^2 x + 2\sin x\cos x + \cos^2 x - (\sin^2 x - 2\sin x\cos x + \cos^2 x) =$

$= \underbrace{\sin^2 x+\cos^2 x}_{1} + 2\sin x\cos x - (\underbrace{\sin^2 x+\cos^2 x}_{1} - 2\sin x\cos x) =$

$= \cancel{1} + 2\sin x\cos x - \cancel{1} + 2\sin x\cos x = 4\sin x\cos x$ //

f) $\sin x\left(tg\,x + \frac{1}{tg\,x}\right) = \sin x\left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right) =$

$= \frac{\sin^2 x}{\cos x} + \frac{\cancel{\sin x}\cos x}{\cancel{\sin x}} = \frac{\sin^2 x}{\cos x} + \cos x = \frac{\sin^2 x+\cos^2 x}{\cos x} = \frac{1}{\cos x}$ //

g) $\underbrace{(1+tg^2 x)}_{\frac{1}{\cos^2 x}}\underbrace{(-\sin^2 x+1)}_{\cos^2 x} = \frac{1}{\cos^2 x} \times \cos^2 x = 1$ //

③ a) $\cos^2\alpha - \sin^2\alpha - 2\cos^2\alpha = \cos^2\alpha - (1-\cos^2\alpha) - 2\cos^2\alpha =$
$= \cancel{\cos^2\alpha} - 1 + \cancel{\cos^2\alpha} - \cancel{2\cos^2\alpha} = -1$ //

b) $\frac{1}{\cos\alpha\sin\alpha} - \frac{\cos\alpha}{\sin\alpha} = \frac{1}{\cos\alpha\sin\alpha} - \frac{\cos^2\alpha}{\sin\alpha\cos\alpha} =$

$= \frac{1-\cos^2\alpha}{\sin\alpha\cos\alpha} = \frac{\sin^{\cancel{2}}\alpha}{\cancel{\sin}\alpha\cos\alpha} = \frac{\sin\alpha}{\cos\alpha} = tg\,\alpha$ //

c) $\cos^3\alpha + \cos\alpha\sin^2\alpha = \cos\alpha\left(\underbrace{\cos^2\alpha + \sin^2\alpha}_{1}\right) = \cos\alpha$ //

d) $\underbrace{(1+tg^2\alpha)}_{\frac{1}{\cos^2\alpha}}\cos^2\alpha - \sin^2\alpha = \frac{1}{\cos^2\alpha} \times \cos^2\alpha - \sin^2\alpha = 1-\sin^2\alpha = \cos^2\alpha$ //

④ a) Como $\sin^2\alpha + \cos^2\alpha = 1$, então,

$\left(\frac{m+1}{5}\right)^2 + \left(\frac{2m}{5}\right)^2 = 1 \wedge -1 \le \frac{m+1}{5} \le 1 \wedge -1 \le \frac{2m}{5} \le 1$

$\Leftrightarrow \frac{m^2+2m+1}{25} + \frac{4m^2}{25} = 1 \wedge -5 \le m+1 \le 5 \wedge -5 \le 2m \le 5 \Leftrightarrow$

$\Leftrightarrow m^2 + 2m + 1 + 4m^2 = 25 \wedge -6 \le m \le 4 \wedge -\frac{5}{2} \le m \le \frac{5}{2} \Leftrightarrow$

$\Leftrightarrow 5m^2 + 2m - 24 = 0 \wedge \dots \Leftrightarrow$

$\Leftrightarrow (m = 2 \vee m = -2{,}4) \wedge \dots \Leftrightarrow$

$\Leftrightarrow m = 2 \vee m = -2{,}4$

b) $\frac{m+1}{2} = \frac{1}{5} \Leftrightarrow 5m + 5 = 2 \Leftrightarrow 5m = -3 \Leftrightarrow m = -\frac{3}{5}$

c) Como $\frac{\sin^2x + \cos^2x}{\cos^2x} = \frac{1}{\cos^2x}$ então

$tg^2x + 1 = \frac{1}{\cos^2x} \Leftrightarrow$

$\Leftrightarrow (-2m)^2 + 1 = \frac{9}{8} \Leftrightarrow$

$\Leftrightarrow 4m^2 + 1 = \frac{9}{8} \Leftrightarrow$

$\Leftrightarrow 4m^2 = \frac{1}{8} \Leftrightarrow m^2 = \frac{1}{32} \Leftrightarrow$

$\Leftrightarrow m = \pm\sqrt{\frac{1}{32}} = \pm\frac{1}{4\sqrt{2}}$

$\sin^2x + \cos^2x = 1$

$\left(\frac{1}{3}\right)^2 + \cos^2x = 1$

$\frac{1}{9} + \cos^2x = 1$

$\cos^2x = \frac{8}{9}$

d) $\sin x = \frac{m+1}{2} \wedge x \in]0°; 90°[$

$0 < \frac{m+1}{2} < 1 \Leftrightarrow 0 < m+1 < 2 \Leftrightarrow -1 < m < 1$ //

(5) a) $\underbrace{\sin(180°+\alpha)}_{-\sin\alpha} + \underbrace{\cos(180°-\alpha)}_{-\cos\alpha} + \underbrace{tg(360°+\alpha)}_{tg\,\alpha} =$

$= -\sin\alpha - \cos\alpha + tg\,\alpha$ //

b) $\underbrace{\cos(270°-\alpha)}_{-\sin\alpha} - \underbrace{tg(-\alpha)}_{-tg\,\alpha} + \underbrace{\sin(90°+\alpha)}_{\cos\alpha} =$

$= -\sin\alpha + tg\,\alpha + \cos\alpha$ //

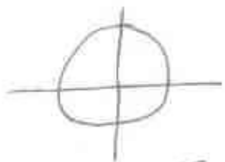
c) $\underbrace{\cos(2\pi+\alpha)}_{\cos\alpha} + \underbrace{\sin(\pi-\alpha)}_{\sin\alpha} - \underbrace{2\sin(2\pi-\alpha)}_{-2\sin\alpha} =$

$= \cos\alpha + \sin\alpha + 2\sin\alpha = \cos\alpha + 3\sin\alpha$ //

d) $\underbrace{\cos(7\pi-\alpha)}_{-\cos\alpha} + \underbrace{\cos(\pi+\alpha)}_{-\cos\alpha} - 2\underbrace{\sin(6\pi+\alpha)}_{\sin\alpha} - \underbrace{\sin(\alpha-\pi)}_{-\sin\alpha} =$

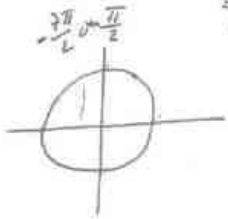
$= -\cos\alpha - \cos\alpha - 2\sin\alpha + \sin\alpha = -2\cos\alpha - \sin\alpha$ //

e) $\underbrace{\cos\left(\frac{3\pi}{2}-\alpha\right)}_{-\sin\alpha} + \underbrace{\cos\left(\frac{7\pi}{2}+\alpha\right)}_{\sin\alpha}$



$= -\cancel{\sin\alpha} + \cancel{\sin\alpha} = 0$ //

f) $\underbrace{\cos\left(\frac{\pi}{2}-\alpha\right)}_{\sin\alpha} + 3\underbrace{\sin\left(\alpha-\frac{7\pi}{2}\right)}_{\cos\alpha} =$



$= \sin\alpha + 3\cos\alpha$ //

⑥ a) $\underbrace{\sin(180°+\alpha)}_{-\sin\alpha} + \underbrace{\cos(180°-\alpha)}_{-\cos\alpha} + \underbrace{tg(360°+\alpha)}_{tg\,\alpha} =$

$= -\sin\alpha - \cos\alpha + tg\,\alpha$

$\boxed{\cos\alpha = \frac{1}{3}}$

$\sin^2\alpha + \cos^2\alpha = 1 \Leftrightarrow$

$\Leftrightarrow \sin^2\alpha + \frac{1}{9} = 1 \Leftrightarrow$

$\Leftrightarrow \sin^2\alpha = \frac{8}{9} \Leftrightarrow$

$\Leftrightarrow \sin\alpha = \pm\frac{\sqrt{8}}{3}$

Como $\alpha \in 1.°Q \Rightarrow \boxed{\sin\alpha = \frac{\sqrt{8}}{3}}$

$tg\,\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{\frac{\sqrt{8}}{\cancel{3}}}{\frac{1}{\cancel{3}}} = \sqrt{8}$

Conclusão:

$-\sin\alpha - \cos\alpha + tg\,\alpha =$

$= -\frac{\sqrt{8}}{3} - \frac{1}{3} + \sqrt{8} =$

$= \frac{-\sqrt{8} - 1 + 3\sqrt{8}}{3} = \frac{2\sqrt{8} - 1}{3}$ //

b) $\underbrace{\cos(270°-\alpha)}_{-\sin\alpha} - \underbrace{tg(-\alpha)}_{-tg\,\alpha} + \underbrace{\sin(90°+\alpha)}_{\cos\alpha} =$

$= -\sin\alpha + tg\,\alpha + \cos\alpha$

$\boxed{\cos\alpha = \frac{1}{3}}$ .... $\sin\alpha = \pm\frac{\sqrt{8}}{3}$

Como $\alpha \in 4.°Q \Rightarrow \boxed{\sin\alpha = -\frac{\sqrt{8}}{3}}$

$tg\,\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{-\frac{\sqrt{8}}{\cancel{3}}}{\frac{1}{\cancel{3}}} = -\sqrt{8}$

Conclusão:

$-\sin\alpha + tg\,\alpha + \cos\alpha =$

$= -\left(-\frac{\sqrt{8}}{3}\right) - \sqrt{8} + \frac{1}{3} =$

$= \frac{\sqrt{8} - 3\sqrt{8} + 1}{3} = \frac{-2\sqrt{8} + 1}{3}$ //

c) $\frac{\overbrace{\cos(\pi-\alpha)}^{-\cos\alpha} - \overbrace{\sin(-\alpha-\pi)}^{\sin\alpha}}{\underbrace{\sin\left(\frac{3\pi}{2}-\alpha\right)}_{-\cos\alpha} - \underbrace{\cos\left(\frac{\pi}{2}-\alpha\right)}_{\sin\alpha}} = \frac{-\cos\alpha - \sin\alpha}{-\cos\alpha - \sin\alpha} = 1$ //

7

a) $\boxed{\sin\alpha = 0,2} \wedge \alpha \in 1.^\circ Q$

$\sin^2\alpha + \cos^2\alpha = 1 \Leftrightarrow$

$\Leftrightarrow (0,2)^2 + \cos^2\alpha = 1 \Leftrightarrow$

$\Leftrightarrow \cos^2\alpha = 0,96 \Leftrightarrow$

$\Leftrightarrow \cos\alpha = \pm\sqrt{0,96}$

Como $\alpha \in 1.^\circ Q \Rightarrow \boxed{\cos\alpha = \sqrt{0,96}} \Leftrightarrow \boxed{\cos\alpha \simeq 0,98}$

$tg\,\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{0,2}{\sqrt{0,96}}$

$\boxed{tg\,\alpha \simeq 0,2}$

b) $\boxed{\cos\beta = 0,8} \wedge \beta \in 4.^\circ Q$

$\sin^2\alpha + \cos^2\alpha = 1 \Leftrightarrow$

$\Leftrightarrow \sin^2\alpha + (0,8)^2 = 1 \Leftrightarrow$

$\Leftrightarrow \sin^2\alpha = 0,36 \Leftrightarrow$

$\Leftrightarrow \sin\alpha = \pm 0,6$

Como $\beta \in 4.^\circ Q \Rightarrow \boxed{\sin\beta = -0,6}$

$tg\,\beta = \frac{-0,6}{0,8} = -0,75$ //

$\boxed{tg\,\beta = -0,75}$

c) $\boxed{tg\,\delta = 3} \wedge \pi < \delta < \frac{3\pi}{2}$

$1 + tg^2\delta = \frac{1}{\cos^2\delta} \Leftrightarrow$

$\Leftrightarrow 1 + 9 = \frac{1}{\cos^2\delta} \Leftrightarrow$

$\Leftrightarrow \cos^2\delta = \frac{1}{10} \Leftrightarrow$

$\Leftrightarrow \cos\delta \simeq \pm 0,32$

Como $\pi < \delta < \frac{3\pi}{2} \Rightarrow \boxed{\cos\delta \simeq -0,32}$

$tg\,\delta = \frac{\sin\delta}{\cos\delta} \Leftrightarrow$

$\Leftrightarrow 3 = \frac{\sin\delta}{-0,32} \Leftrightarrow$

$\Leftrightarrow \boxed{\sin\delta \simeq -0,96}$

d) $tg\,\lambda = -\frac{1}{4} \wedge \lambda \in]0,\pi[$

$1 + tg^2\lambda = \frac{1}{\cos^2\lambda} \Leftrightarrow$

$\Leftrightarrow 1 + \frac{1}{16} = \frac{1}{\cos^2\lambda} \Leftrightarrow$

$\Leftrightarrow \cos^2\lambda = \frac{16}{17} \Leftrightarrow$

$\Leftrightarrow \cos\lambda \simeq \pm 0,97$

Como $\begin{cases} \lambda \in]0,\pi[\\ tg\,\lambda < 0 \end{cases} \Rightarrow \boxed{\cos\lambda \simeq -0,97}$

$tg\,\lambda = \frac{\sin\lambda}{\cos\lambda}$

$\Leftrightarrow -\frac{1}{4} = \frac{\sin\lambda}{-0,97} \Leftrightarrow$

$\Leftrightarrow \boxed{\sin\lambda \simeq 0,24}$

⑧ $\sin(\pi + \alpha) = -\frac{3}{4} \Leftrightarrow -\sin\alpha = -\frac{3}{4} \Leftrightarrow \boxed{\sin\alpha = \frac{3}{4}}$

a)
$\sin^2\alpha + \cos^2\alpha = 1$

$\frac{9}{16} + \cos^2\alpha = 1$

$\cos^2\alpha = \frac{7}{16}$

$\cos\alpha \approx \pm 0{,}66$

Como $\alpha \in 2.^\circ Q \Rightarrow \boxed{\cos\alpha \simeq -0{,}66}$

$tg\,\alpha = \frac{\frac{3}{4}}{-0{,}66}$

$\boxed{tg\,\alpha \simeq -1{,}14}$

b) $\frac{\sin(\pi-\alpha)}{1-\cos\alpha} = \frac{\sin\alpha}{1-\cos\alpha} = \frac{0{,}75}{1+0{,}66} \simeq 0{,}45$

⑨
a) $\sin x = \frac{1}{2} \Leftrightarrow$

$\Leftrightarrow \sin x = \sin\frac{\pi}{6} \Leftrightarrow x = \frac{\pi}{6} + 2k\pi \vee x = \pi - \frac{\pi}{6} + 2k\pi \;;\; k \in \mathbb{Z}$

$\Leftrightarrow x = \frac{\pi}{6} + 2k\pi \vee x = \frac{5\pi}{6} + 2k\pi \;;\; k \in \mathbb{Z}$

b) $2\sin(3x) + \sqrt{3} = 0 \Leftrightarrow \sin(3x) = -\frac{\sqrt{3}}{2} \Leftrightarrow \sin(3x) = \sin\left(-\frac{\pi}{3}\right)$

$\Leftrightarrow 3x = -\frac{\pi}{3} + 2k\pi \vee 3x = \pi + \frac{\pi}{3} + 2k\pi \;;\; k \in \mathbb{Z}$

$\Leftrightarrow x = -\frac{\pi}{9} + \frac{2k\pi}{3} \vee x = \frac{4\pi}{9} + \frac{2k\pi}{3} \;;\; k \in \mathbb{Z}$

c) $-5\sin\left(\frac{x}{2}\right) = 0 \Leftrightarrow \sin\left(\frac{x}{2}\right) = 0 \Leftrightarrow \frac{x}{2} = k\pi \Leftrightarrow x = 2k\pi \;;\; k \in \mathbb{Z}$

d) $\sin\left(2x - \frac{\pi}{3}\right) = \frac{\sqrt{3}}{2} \Leftrightarrow \sin\left(2x - \frac{\pi}{3}\right) = \sin\frac{\pi}{3} \Leftrightarrow$

$\Leftrightarrow 2x - \frac{\pi}{3} = \frac{\pi}{3} + 2k\pi \vee 2x - \cancel{\frac{\pi}{3}} = \pi - \cancel{\frac{\pi}{3}} + 2k\pi \;;\; k \in \mathbb{Z}$

$\Leftrightarrow 2x = \frac{2\pi}{3} + 2k\pi \vee 2x = \pi + 2k\pi \;;\; k \in \mathbb{Z}$

$\Leftrightarrow x = \frac{\pi}{3} + k\pi \vee x = \frac{\pi}{2} + k\pi \;;\; k \in \mathbb{Z}$

e) $\sin^2 x - 1 = 0 \Leftrightarrow \sin^2 x = 1 \Leftrightarrow \sin x = 1 \vee \sin x = -1 \Leftrightarrow$

$\Leftrightarrow x = \frac{\pi}{2} + 2k\pi \vee x = -\frac{\pi}{2} + 2k\pi \Leftrightarrow x = \frac{\pi}{2} + k\pi \;;\; k \in \mathbb{Z}$

f) $\sin^2 x - 3\sin x + 2 = 0$

$\Leftrightarrow \sin x = \dfrac{3 \pm \sqrt{9-8}}{2}$ $\begin{cases} \cancel{\sin x = 2} \\ \sin x = 1 \end{cases}$

$\Leftrightarrow \sin x = 1 \Leftrightarrow x = \frac{\pi}{2} + 2k\pi \;;\; k \in \mathbb{Z}$

g) $\sin x + \sin\frac{\pi}{3} = 0 \Leftrightarrow \sin x = -\sin\frac{\pi}{3} \Leftrightarrow \sin x = \sin\left(-\frac{\pi}{3}\right) \Leftrightarrow$

$\Leftrightarrow x = -\frac{\pi}{3} + 2k\pi \vee x = \pi + \frac{\pi}{3} + 2k\pi \;;\; k \in \mathbb{Z} \Leftrightarrow$

$\Leftrightarrow x = -\frac{\pi}{3} + 2k\pi \vee x = \frac{4\pi}{3} + 2k\pi \;;\; k \in \mathbb{Z}$

h) $\cos x = \cos\frac{\pi}{3} \Leftrightarrow x = \frac{\pi}{3} + 2k\pi \vee x = -\frac{\pi}{3} + 2k\pi \;;\; k \in \mathbb{Z}$

i) $\cos\left(2x + \frac{\pi}{6}\right) = -\frac{\sqrt{3}}{2} \Leftrightarrow \cos\left(2x + \frac{\pi}{6}\right) = \cos\frac{5\pi}{6} \Leftrightarrow$

$\Leftrightarrow 2x + \frac{\pi}{6} = \frac{5\pi}{6} + 2k\pi \vee 2x + \frac{\pi}{6} = -\frac{5\pi}{6} + 2k\pi \;;\; k \in \mathbb{Z}$

$\Leftrightarrow 2x = \frac{2\pi}{3} + 2k\pi \vee 2x = -\pi + 2k\pi \;;\; k \in \mathbb{Z}$

$\Leftrightarrow x = \frac{\pi}{3} + k\pi \vee x = -\frac{\pi}{2} + k\pi \;;\; k \in \mathbb{Z}$

j) $\cos(3x) = -\cos\frac{\pi}{6} \Leftrightarrow \cos(3x) = \cos\frac{5\pi}{6} \Leftrightarrow$

$\Leftrightarrow 3x = \frac{5\pi}{6} + 2k\pi \vee 3x = -\frac{5\pi}{6} + 2k\pi \;;\; k \in \mathbb{Z}$

$\Leftrightarrow x = \frac{5\pi}{18} + \frac{2k\pi}{3} \vee x = -\frac{5\pi}{18} + \frac{2k\pi}{3} \;;\; k \in \mathbb{Z}$

k) $3\cos\left(x + \frac{\pi}{5}\right) = 3 \Leftrightarrow \cos\left(x + \frac{\pi}{5}\right) = 1 \Leftrightarrow x + \frac{\pi}{5} = 2k\pi \;,\; k \in \mathbb{Z}$

$\Leftrightarrow x = -\frac{\pi}{5} + 2k\pi \;;\; k \in \mathbb{Z}$

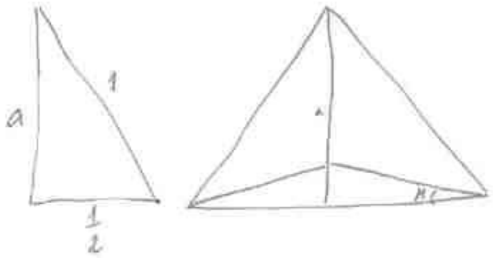
l) $5 - 5\cos^2 x = 0 \Leftrightarrow \cos^2 x = 1 \Leftrightarrow \cos x = 1 \vee \cos x = -1 \Leftrightarrow$

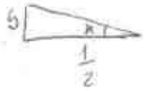
$\Leftrightarrow x = 2k\pi \vee x = \pi + 2k\pi \;;\; k \in \mathbb{Z} \Leftrightarrow x = k\pi \;;\; k \in \mathbb{Z}$

(10)

a) $a^2 + \frac{1}{4} = 1 \Leftrightarrow a^2 = \frac{3}{4}$

$\Leftrightarrow \boxed{a = \frac{\sqrt{3}}{2}}$



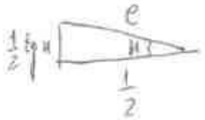


$tg\, x = \frac{b}{\frac{1}{2}}$

$\boxed{b = \frac{1}{2}\, tg\, x}$

$C(x) = \frac{1}{2\cos x} + \frac{1}{2\cos x} + \frac{\sqrt{3}}{2} - \frac{1}{2}\, tg\, x$

$\boxed{C(x) = \frac{1}{\cos x} + \frac{\sqrt{3}}{2} - \frac{1}{2}\, tg\, x}$

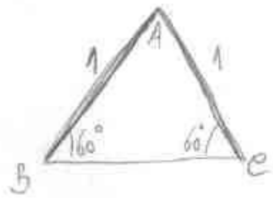


$\cos x = \frac{\frac{1}{2}}{c}$

$\boxed{c = \frac{1}{2\cos x}}$

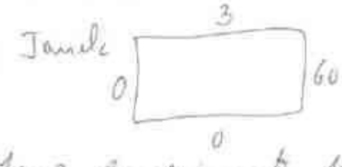
$\boxed{a - b = \frac{\sqrt{3}}{2} - \frac{1}{2}\, tg\, x}$

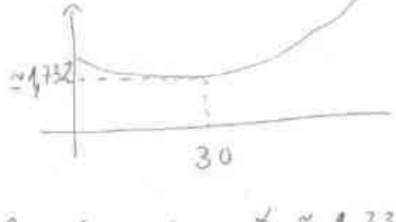
b) $C(60^\circ) = \frac{1}{\cos 60^\circ} + \frac{\sqrt{3}}{2} - \frac{1}{2}\, tg\, 60^\circ = \frac{1}{\frac{1}{2}} + \cancel{\frac{\sqrt{3}}{2}} - \cancel{\frac{1}{2} \times \sqrt{3}} = 2$



Comprimento $= 2$

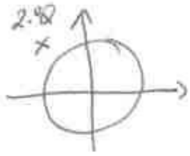
c) $y = \frac{1}{\cos x} + \frac{\sqrt{3}}{2} - \frac{1}{2}\, tg\, x$





Menor comprimento para $x = 30^\circ$. Comprimento $\simeq 1,732$ Km.

(11)

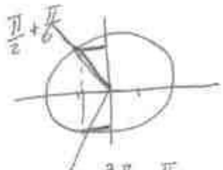


Resposta (D)

(12) $g(x) = \sin(2x)$. Se a função "encolher na horizontal" para metade, o período é π. Resposta (B)

(13) $[0; 2\pi]$ $\quad g(x) = 2\cos x + 1$

$2\cos x + 1 = 0$
$2\cos x = -1$
$\cos x = -\frac{1}{2}$



$\frac{\pi}{2} + \frac{\pi}{6} = \frac{4\pi}{6} = \frac{2\pi}{3}$

$\frac{3\pi}{2} - \frac{\pi}{6} = \frac{8\pi}{6} = \frac{4\pi}{3}$

Resposta (B)

(14) $2\sin x = 1 \Leftrightarrow \sin x = \frac{1}{2} \Leftrightarrow \sin x = \sin\frac{\pi}{6} \Leftrightarrow$
$\Leftrightarrow x = \frac{\pi}{6} + 2k\pi \vee x = \pi - \frac{\pi}{6} + 2k\pi \Leftrightarrow x = \frac{\pi}{6} + 2k\pi \vee x = \frac{5\pi}{6} + 2k\pi;\ k \in \mathbb{Z}$

Resposta (D)

(15) $\sin\left(\frac{\pi}{2} - x\right) = -\frac{5}{13} \Leftrightarrow \boxed{\cos x = -\frac{5}{13}}$. Se $0 < x < \pi$ e $\cos x < 0$, temos $\boxed{x \in 2.^\circ Q}$.

$\sin\left(x - \frac{\pi}{2}\right) + tg(5\pi + x) = \sin\left(-\frac{\pi}{2} + x\right) + tg(5\pi + x) = -\cos x + tg\, x$

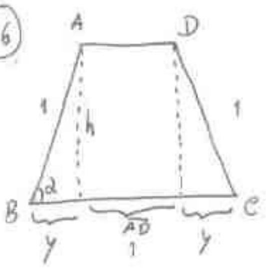
$\frac{1}{\cos^2 x} = 1 + tg^2 x \Leftrightarrow \frac{169}{25} = 1 + tg^2 x \Leftrightarrow tg^2 x = \frac{144}{25} \Leftrightarrow$

$\Leftrightarrow tg\, x = \pm\frac{12}{5} \rightarrow \boxed{tg\, x = -\frac{12}{5}}$

Logo $-\cos x + tg\, x = \frac{5}{13} - \frac{12}{5} = \frac{25 - 156}{65} = -\frac{131}{65}$

(16)

a)



$\sin\alpha = \frac{h}{1} \rightarrow \boxed{h = \sin\alpha}$

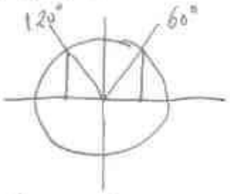
$\cos\alpha = \frac{y}{1} \rightarrow \boxed{y = \cos\alpha}$

$\underline{\underline{\overline{AD}}} = 1 - 2y = \underline{\underline{1 - 2\cos\alpha}}$

$A_{\text{trap}} = \frac{B + b}{2} \times h = \frac{1 + 1 - 2\cos\alpha}{2} \times \sin\alpha = \frac{2 - 2\cos\alpha}{2} \times \sin\alpha =$

$= (1 - \cos\alpha)\sin\alpha = \sin\alpha - \frac{2\sin\alpha\cos\alpha}{2} = \sin\alpha - \frac{1}{2}\sin(2\alpha)$

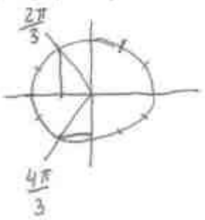
b) $A(60°) = \sin 60° - \frac{1}{2}\sin 120° = \frac{\sqrt{3}}{2} - \frac{1}{2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{4} = \frac{\sqrt{3}}{4}$



(17) $f(x) = 4\cos x - 2 \qquad g(x) = 4\sin(2x) - 2$

a) $f\left(\frac{4\pi}{3}\right) + g\left(\frac{\pi}{3}\right) = 4\cos\left(\frac{4\pi}{3}\right) - 2 + 4\sin\left(\frac{2\pi}{3}\right) - 2 = 4 \times \left(-\frac{1}{2}\right) - 2 + 4 \times \frac{\sqrt{3}}{2} - 2 =$

$= -2 - 2 + 2\sqrt{3} - 2 = 2\sqrt{3} - 6$



b) $g(x) = 0 \Leftrightarrow 4\sin(2x) = 2 \Leftrightarrow \sin(2x) = \frac{1}{2} \Leftrightarrow \sin(2x) = \sin\frac{\pi}{6} \Leftrightarrow$

$\Leftrightarrow 2x = \frac{\pi}{6} + 2k\pi \vee 2x = \pi - \frac{\pi}{6} + 2k\pi \,;\, k \in \mathbb{Z} \Leftrightarrow$

$\Leftrightarrow x = \frac{\pi}{12} + k\pi \vee x = \frac{5\pi}{12} + k\pi \,;\, k \in \mathbb{Z}$

c) $-1 \leq \cos x \leq 1$

$-4 \leq 4\cos x \leq 4$

$-6 \leq 4\cos x - 2 \leq 2$

$D' = [-6; 2]$

d) $f(x) = g(x) \Leftrightarrow \cancel{4}\cos x - \cancel{2} = \cancel{4}\sin(2x) - \cancel{2} \Leftrightarrow$

$\Leftrightarrow \cos x = \sin(2x) \Leftrightarrow \cos x = \cos\left(\frac{\pi}{2} - 2x\right) \Leftrightarrow$

$\Leftrightarrow x = \frac{\pi}{2} - 2x + 2k\pi \vee x = -\frac{\pi}{2} + 2x + 2k\pi \ ; \ k \in \mathbb{Z} \Leftrightarrow$

$\Leftrightarrow 3x = \frac{\pi}{2} + 2k\pi \vee -x = -\frac{\pi}{2} + 2k\pi \ ; \ k \in \mathbb{Z}$

$\Leftrightarrow x = \frac{\pi}{6} + \frac{2k\pi}{3} \vee x = \frac{\pi}{2} - 2k\pi \ ; \ k \in \mathbb{Z}$

$\Leftrightarrow x = \frac{\pi}{6} + \frac{2k\pi}{3} \vee x = \frac{\pi}{2} + 2k\pi \ ; \ k \in \mathbb{Z}$ $\qquad [-\pi; \pi]$

$k = 0 \to x = \frac{\pi}{6}$ ✓ $\vee$ $x = \frac{\pi}{2}$ ✓

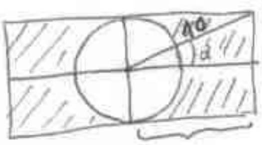
$k = 1 \to x = \frac{5\pi}{6}$ ✓ $\vee$ $\cancel{x = \frac{5\pi}{2}}$ $\qquad C.S. = \left\{ -\frac{\pi}{2} ; \frac{\pi}{6} ; \frac{\pi}{2} ; \frac{5\pi}{6} \right\}$

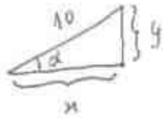
$k = 2 \to \cancel{x = \frac{9\pi}{6}}$ $\vee$ —

$k = -1 \to x = -\frac{\pi}{2}$ ✓ $\vee$ $\cancel{x = -\frac{3\pi}{2}}$

$k = -2 \to \cancel{x = -\frac{7\pi}{6}}$ $\vee$ —

(18)





$\sin\alpha = \frac{y}{10} \to y = 10\sin\alpha$

$\cos\alpha = \frac{x}{10} \to x = 10\cos\alpha$

largura $= 2y = 20\sin\alpha$

comprimento $= 2x = 20\cos\alpha$

$A_{▭} = 20\sin\alpha \times 20\cos\alpha = 400\sin\alpha\cos\alpha$

$A_{\bigcirc} = \pi r^2 = \pi(10\sin\alpha)^2 = 100\pi\sin^2\alpha$

$A_{\text{sombreada}} = 400\sin\alpha\cos\alpha - 100\pi\sin^2\alpha = 100\sin\alpha(4\cos\alpha - \pi\sin\alpha)$