33

$tg(\pi-\alpha) = \frac{\overline{CA}}{3}$

$-tg\alpha = \frac{\overline{CA}}{3}$

$\boxed{\overline{CA} = -3tg\alpha}$

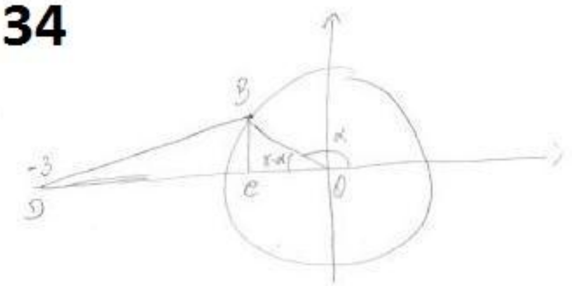
$\cos(\pi-\alpha) = \frac{3}{\overline{AO}}$

$-\cos\alpha = \frac{3}{\overline{AO}}$

$\boxed{\overline{AO} = -\frac{3}{\cos\alpha}}$

Perímetro$_{\triangle}$ $= 2\overline{AO} + 2\overline{CA} = 2\left(-\frac{3}{\cos\alpha} - 3tg\alpha\right) = -6tg\alpha - \frac{6}{\cos\alpha}$ cqd.

34



$\sin(\pi-\alpha) = \frac{\overline{BC}}{1}$

$\boxed{\sin\alpha = \overline{BC}}$

$\cos(\pi-\alpha) = \frac{\overline{OC}}{1}$

$\boxed{-\cos\alpha = \overline{OC}}$

$\overline{DC} = \overline{DO} - \overline{OC} =$

$= 3 - (-\cos\alpha)$

$= 3 + \cos\alpha$

$A = \frac{\overline{DC} \times \overline{BC}}{2} =$

$= \frac{(3+\cos\alpha) \times \sin\alpha}{2}$

$= \frac{1}{2}(3+\cos\alpha)\sin\alpha$ C

35

$\sin\alpha = \frac{\overline{QR}}{\overline{PR}} \Leftrightarrow \sin\alpha = \frac{\overline{QR}}{4} \Leftrightarrow \boxed{\overline{QR} = 4\sin\alpha}$

$\cos\alpha = \frac{\overline{PQ}}{4} \Leftrightarrow \boxed{\overline{PQ} = 4\cos\alpha}$

$A(\alpha) = \cancel{2} \times \frac{\overline{PQ} \times \overline{QR}}{\cancel{2}} =$

$= 4\sin\alpha \times 4\cos\alpha =$

$= 16\sin\alpha\cos\alpha$ c.q.d.

Agora,

$A(\theta) = 16\sin\theta\cos\theta$

$tg\,\theta = 2\sqrt{2}$

$1 + tg^2\theta = \frac{1}{\cos^2\theta}$

$1 + (2\sqrt{2})^2 = \frac{1}{\cos^2\theta}$

$1 + 2^2 \times (\sqrt{2})^2 = \frac{1}{\cos^2\theta}$

$1 + 8 = \frac{1}{\cos^2\theta}$

$9 = \frac{1}{\cos^2\theta}$

$\cos^2\theta = \frac{1}{9}$

$\cos\theta = \pm\frac{1}{3}$

Como $\theta \in \left]0, \frac{\pi}{2}\right[$, $\boxed{\cos\theta = \frac{1}{3}}$

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta + \left(\frac{1}{3}\right)^2 = 1$

$\sin^2\theta = 1 - \frac{1}{9}$

$\sin^2\theta = \frac{8}{9}$

$\sin\theta = \pm\frac{2\sqrt{2}}{3}$

Como $\theta \in \left]0, \frac{\pi}{2}\right[$, $\boxed{\sin\theta = \frac{2\sqrt{2}}{3}}$

Assim,

$A(\theta) = 16\sin\theta\cos\theta =$

$= 16 \times \frac{2\sqrt{2}}{3} \times \frac{1}{3} =$

$= \frac{32\sqrt{2}}{9}$

36

$\overline{AB} = \operatorname{sen}\alpha$

$\overline{OB} = \cos\alpha$

$\overline{OC} = 1$

$\overline{BC} = 1 - \cos\alpha$

$\overline{DC} = tg\,\alpha$

A  $= \frac{\overline{OC} \times \overline{CD}}{2} = \frac{1 \times tg\,\alpha}{2} = \frac{tg\,\alpha}{2}$

A  $= \frac{\overline{OB} \times \overline{AB}}{2} = \frac{\cos\alpha \times \operatorname{sen}\alpha}{2} =$

$= \frac{\overbrace{2\cos\alpha\operatorname{sen}\alpha}^{\operatorname{sen}(2\alpha)}}{4} = \frac{\operatorname{sen}(2\alpha)}{4}$

Logo $A_{sombreada} = \frac{tg\,\alpha}{2} - \frac{\operatorname{sen}(2\alpha)}{4}$ (B)

37

$$d(t) = 1 + \frac{1}{2}\,\text{sen}\left(\pi t + \frac{\pi}{6}\right)$$

$$d(0) = 1 + \frac{1}{2}\,\text{sen}\left(\frac{\pi}{6}\right) = 1 + \frac{1}{2} \times \frac{1}{2} = 1 + \frac{1}{4} = \frac{5}{4}$$

$d(t) = d(0) \Leftrightarrow$

$\Leftrightarrow 1 + \frac{1}{2}\,\text{sen}\left(\pi t + \frac{\pi}{6}\right) = \frac{5}{4} \Leftrightarrow$

$\Leftrightarrow \frac{1}{2}\,\text{sen}\left(\pi t + \frac{\pi}{6}\right) = \frac{1}{4}$

$\Leftrightarrow \text{sen}\left(\pi t + \frac{\pi}{6}\right) = \frac{\frac{1}{4}}{\frac{1}{2}}$

$\Leftrightarrow \text{sen}\left(\pi t + \frac{\pi}{6}\right) = \frac{1}{2}$

$\Leftrightarrow \text{sen}\left(\pi t + \frac{\pi}{6}\right) = \text{sen}\left(\frac{\pi}{6}\right)$

$\Leftrightarrow \pi t + \cancel{\frac{\pi}{6}} = \cancel{\frac{\pi}{6}} + 2k\pi \lor \pi t + \frac{\pi}{6} = \pi - \frac{\pi}{6} + 2k\pi \,;\, k \in \mathbb{Z}$

$\Leftrightarrow \cancel{\pi} t = 2k\cancel{\pi} \lor \underset{(3)}{\pi t} = \frac{2\pi}{3} + \underset{(3)}{2k\pi} \,;\, k \in \mathbb{Z}$

$\Leftrightarrow t = 2k \lor 3\cancel{\pi} t = 2\cancel{\pi} + 6k\cancel{\pi} \,;\, k \in \mathbb{Z}$

$\Leftrightarrow t = 2k \lor 3t = 2 + 6k \,;\, k \in \mathbb{Z}$

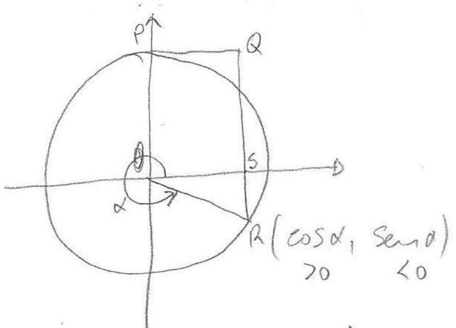
$\Leftrightarrow t = 2k \lor t = \frac{2+6k}{3} \,;\, k \in \mathbb{Z}$

$k = 0 \hookrightarrow t = 0 \lor t = \frac{2}{3}$ $\qquad t \in \,]0, 3]$

$k = 1 \hookrightarrow t = 2 \lor t = \frac{8}{3}$ $\qquad$ R: $t = \frac{2}{3} \lor t = 2 \lor t = \frac{8}{3}$

$k = 2 \hookrightarrow \cancel{t = 4} \lor \cancel{t = \frac{14}{3}}$

38



$\overline{OS} = \cos\alpha$

$\overline{SR} = -\text{sen}\,\alpha$

$\overline{OP} = 1$

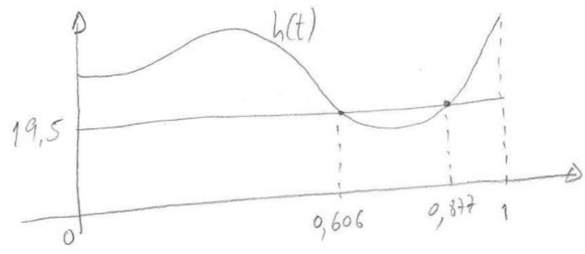
$$\begin{cases} \overline{QR} = 1 + \overline{SR} = 1 - \text{sen}\,\alpha = B \\ \overline{PO} = 1 = b \\ \overline{OS} = \cos\alpha = h \end{cases}$$

$$A = \frac{B+b}{2} \times h = \frac{1 - \text{sen}\,\alpha + 1}{2} \times \cos\alpha = \frac{2 - \text{sen}\,\alpha}{2} \times \cos\alpha =$$

$$= \frac{2\cos\alpha - \text{sen}\,\alpha\cos\alpha}{2} = \frac{\cancel{2}\cos\alpha}{\cancel{2}} - \frac{\text{sen}\,\alpha\cos\alpha}{2} =$$

$$= \cos\alpha - \frac{\text{sen}\,\alpha\cos\alpha}{2} \quad (D)$$

39



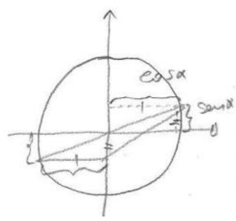
$b - a = 0{,}877 - 0{,}606 \simeq 0{,}27$

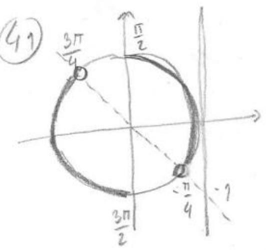
Durante 0,27 minutos, a distância do ponto P do tabuleiro a um ponto fixo do vale foi inferior a 19,5m

40 $\overline{OR} = \operatorname{sen}\alpha$, logo h = altura do triângulo $= 2\operatorname{sen}\alpha$
$\overline{QR} = \cos\alpha$, logo b = base " " $= \cos\alpha$

$$A = \frac{b \times h}{2} = \frac{\cancel{2}\operatorname{sen}\alpha \times \cos\alpha}{\cancel{2}} = \operatorname{sen}\alpha\cos\alpha =$$

$$= \frac{2\operatorname{sen}\alpha\cos\alpha}{2} = \frac{\operatorname{sen}(2\alpha)}{2}$$ (D)



41) 

$\left]\frac{3\pi}{4}, \frac{3\pi}{2}\right[$ (B)

42) $\cancel{3}+\frac{sen(x-1)}{1-x}=\cancel{3} \Leftrightarrow \frac{sen(x-1)}{1-x}=0 \Leftrightarrow \underset{1-x\neq 0}{sen(x-1)=0} \Leftrightarrow x-1=k\pi, k\in\mathbb{Z}$

$\Leftrightarrow x=1+k\pi, k\in\mathbb{Z}$

$K=0 \rightarrow x=1 \notin]4,5[$ $\qquad S=\{1+\pi\}$

$K=1 \rightarrow x=1+\pi \in]4,5[$

$K=2 \rightarrow x=2+\pi \notin]4,5[$

43) $\left(2x\,sen\alpha+\frac{\cos\alpha}{x}\right)^2 = 4x^2 sen^2\alpha + \frac{4\cancel{x}\,sen\alpha\cos\alpha}{\cancel{x}} + \frac{\cos^2\alpha}{x^2}, \quad x\neq 0$

Se o termo independente de x é igual a 1, então

$4\,sen\alpha\cos\alpha = 1 \Leftrightarrow 2\times 2\,sen\alpha\cos\alpha = 1 \Leftrightarrow 2\,sen(2\alpha)=1 \Leftrightarrow sen(2\alpha)=\frac{1}{2} \Leftrightarrow$

$\Leftrightarrow 2\alpha=\frac{\pi}{6}+2k\pi \vee 2\alpha=\pi-\frac{\pi}{6}+2k\pi, k\in\mathbb{Z} \wedge \alpha\in]\pi,2\pi[$

$\Leftrightarrow \alpha=\frac{\pi}{12}+k\pi \vee \alpha=\frac{5\pi}{12}+k\pi, k\in\mathbb{Z}$ Como $\alpha\in]\pi,2\pi[$, então os valores de α são $\frac{13\pi}{12}$ e $\frac{17\pi}{12}$

$K=0 \rightarrow \alpha=\frac{\pi}{12} \vee \alpha=\frac{5\pi}{12}$

$K=1 \rightarrow \alpha=\frac{13\pi}{12} \vee \alpha=\frac{17\pi}{12}$

$K=2 \rightarrow \alpha=\frac{25\pi}{12} \vee \alpha=\frac{29\pi}{12}$

44) base $=\overline{AB}=-\cos(2\alpha)$

altura $=\overline{CD}-\overline{OB}=tg\alpha-sen(2\alpha)$

$A=\frac{-\cos(2\alpha)\times(tg\alpha-sen(2\alpha))}{2}=\frac{-\cos(2\alpha)\left(\frac{sen\alpha}{\cos\alpha}-2\,sen\alpha\cos\alpha\right)}{2}=$

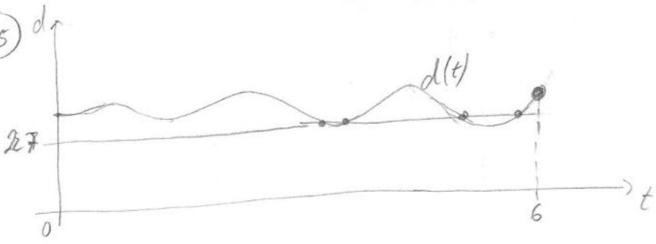
$=\frac{-\cos(2\alpha)\left(\frac{sen\alpha-2\,sen\alpha\cos^2\alpha}{\cos\alpha}\right)}{2}=\frac{-\cos(2\alpha)\left(\frac{sen\alpha(1-2\cos^2\alpha)}{\cos\alpha}\right)}{2}=$

$=\frac{-\cos(2\alpha)\,tg\alpha\,(1-2\cos^2\alpha)}{2}=\frac{-\cos(2\alpha)\,tg\alpha\,(sen^2\alpha+\cos^2\alpha-2\cos^2\alpha)}{2}$

$=\frac{-\cos(2\alpha)\,tg\alpha\,(sen^2\alpha-\cos^2\alpha)}{2}=\frac{\cos(2\alpha)\,tg\alpha\,(\cos^2\alpha-sen^2\alpha)}{2}=$

$=\frac{\cos(2\alpha)\,tg\alpha\,\cos(2\alpha)}{2}=\frac{tg\alpha\cos^2(2\alpha)}{2}$ c.q.d.

(45)



Como se pode observar, o n.º de soluções da equação $d(t)=27$ é 4. Significa que a criança esteve à mesma distância de 27 dm do muro, por 4 vezes, nos primeiros 6 segundos.

(46)

$0 \leq \arccos x \leq \pi$

$0 \leq \arccos(-2x) \leq \pi$

$0+1 \leq 1+\arccos(-2x) \leq 1+\pi$

$D' = [1, 1+\pi]$ R: (A)

$-1 \leq -2x \leq 1$

$1 \geq 2x \geq -1$

$\frac{1}{2} \geq x \geq -\frac{1}{2}$

$D = \left[-\frac{1}{2}, \frac{1}{2}\right]$

ou

$\arccos(x): [-1,1] \to [0,\pi]$

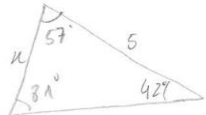
$\arccos(2x): \left[-\frac{1}{2},\frac{1}{2}\right] \to [0,\pi]$

$\arccos(-2x): \left[-\frac{1}{2},\frac{1}{2}\right] \to [0,\pi]$

$1+\arccos(-2x): \underbrace{\left[-\frac{1}{2},\frac{1}{2}\right]}_{D} \to \underbrace{[1, 1+\pi]}_{D'}$

(47) $\arcsin(1) + \arccos\left(-\frac{1}{2}\right) = \frac{\pi}{2} + \frac{2\pi}{3} = \frac{7\pi}{6}$ (A)

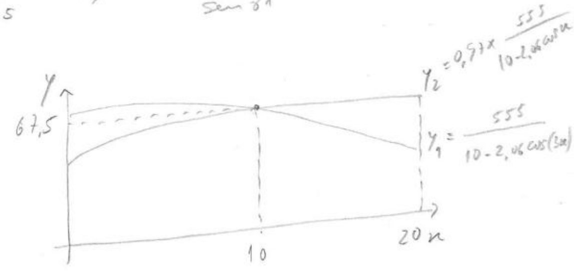
(48)



$\frac{\operatorname{sen} 42°}{x} = \frac{\operatorname{sen} 81°}{5} \Leftrightarrow x = \frac{5 \operatorname{sen} 42°}{\operatorname{sen} 81°} \Leftrightarrow x \approx 3{,}39$ (C)

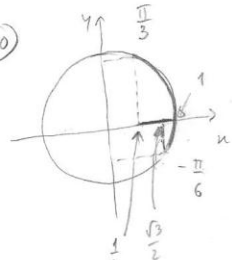
(49) $d(3\alpha) = 0{,}97\, d(\alpha) \Leftrightarrow$

$\underbrace{\frac{555}{10-2{,}06\cos(3\alpha)}}_{y_1} = \underbrace{0{,}97 \times \frac{555}{10-2{,}06\cos\alpha}}_{y_2}$



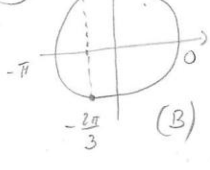
R: $\alpha \approx 10°$

(50)



$D' = \left[\frac{1}{2}, 1\right]$ (C)

(51)

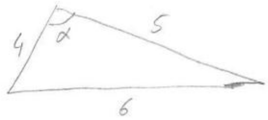


(B)

$2\cos x + 1 = 0$

$\cos x = -\frac{1}{2}$

(52)



$6^2 = 4^2 + 5^2 - 2 \times 4 \times 5 \times \cos\alpha$

$36 = 41 - 40\cos\alpha \Leftrightarrow \cos\alpha = \frac{5}{40} \Leftrightarrow \boxed{\cos\alpha = \frac{1}{8}}$

$\text{sen}^2\alpha + \cos^2\alpha = 1 \Leftrightarrow \text{sen}^2\alpha + \frac{1}{64} = 1 \Leftrightarrow \text{sen}^2\alpha = \frac{63}{64} \Leftrightarrow$

$\Leftrightarrow \text{sen}\,\alpha = \sqrt{\frac{63}{64}} \Leftrightarrow \text{sen}\,\alpha \approx 0{,}992$ (B)

(53) $f(x) = g(-x) + g\left(\frac{\pi}{2} - x\right) =$

$= \frac{1}{4}\cos(-2x) - \cos x + \frac{1}{4}\cos\left(2\left(\frac{\pi}{2} - x\right)\right) - \cos\left(\frac{\pi}{2} - x\right) =$

$= \frac{1}{4}\cos(2x) - \cos x + \frac{1}{4}\cos(\pi - 2x) - \text{sen}\,x =$

$= \cancel{\frac{1}{4}\cos(2x)} - \cos x - \cancel{\frac{1}{4}\cos(2x)} - \text{sen}\,x = -\cos x - \text{sen}\,x$ (B)

(54) $\text{sen}\left(3 \arccos \frac{1}{2}\right) = \text{sen}\left(\cancel{3} \times \frac{\pi}{\cancel{3}}\right) = \text{sen}(\pi) = 0$