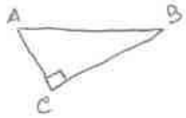
Resolução da ficha n.º 2 - Geometria - 11.º ano

(1) $A(5,0,0)$ $B(0,3,1)$

a) $x+2y-z=5$
$5+0-0=5$ verd.
$0+6-1=5$ verd.

Como $A, B \in$ plano logo a recta $AB \subset$ plano

b) $C(0,0,z)$, $z>0$



$\vec{CA} \cdot \vec{CB} = 0 \Leftrightarrow$

$\Leftrightarrow (5,0,-z)\cdot(0,3,1-z) = 0 \Leftrightarrow 0+0-z+z^2 = 0 \Leftrightarrow$

$\Leftrightarrow z^2 - z = 0 \Leftrightarrow z(z-1) = 0 \Leftrightarrow z=0 \vee \underline{\underline{z=1}}$

$C(0,0,1)$

c) $V = \frac{1}{3} Ab \times h$

$h = 5$
$Ab = \pi r^2$
$r = \overline{OB} = \sqrt{0^2+3^2+1^2} = \sqrt{10}$

$V = \frac{1}{3}\pi \cdot 10 \times 5$

$V = \frac{50\pi}{3}$

(2) a) esfera mais próxima
$C(3,3,3)$ $r=3$

esfera mais afastada
$C(3,9,3)$ $r=3$

Seja $P(1,8,1)$
$\overline{PC} = 3$? $\overline{PC} = \sqrt{(3-1)^2+(9-8)^2+(3-1)^2} = \sqrt{4+1+4} = \sqrt{9} = 3$ ✓

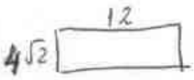
b) 

$\vec{PC} = (2,1,2)$
$2x+y+2z+D=0$
$(1,8,1) \rightarrow 2+8+2+D=0$
$D=-12$

$2x+y+2z-12=0$

c) 

$x^2+1^2=3^2$
$x^2=8$
$x=\sqrt{8}=2\sqrt{2}$



$P = 24+4\sqrt{2}+4\sqrt{2}$
$P = 24+8\sqrt{2}$

(3) Se $\overline{VM} = 2\,\overline{VQ} = 12$, logo $\overline{UQ} = 6$

a) $U(6,6,6)$ $V(3,3,12)$

$\overline{UV} = \sqrt{3^2+3^2+6^2} = \sqrt{9+9+36} = \sqrt{54} = 3\sqrt{6}$

b) Recta UV: $\frac{x-6}{-3} = \frac{y-6}{-3} = \frac{z-6}{6}$ $\quad \overrightarrow{UV} = (-3;-3;6)$

$\begin{cases} x = 4 \\ \frac{x-6}{-3} = \frac{y-6}{-3} = \frac{z-6}{6} \end{cases}$ $\begin{cases} \frac{4-6}{-3} = \frac{z-6}{6} \\ \frac{4-\cancel{6}}{-\cancel{3}} = \frac{y-\cancel{6}}{-\cancel{3}} \\ x = 4 \end{cases}$ $\begin{cases} z = 10 \\ y = 4 \\ x = 4 \end{cases}$ $\quad I \to (4,4,10)$

c) $A(6,6,z)$; $z>0$

$V_{total} = V_{cubo} + V_{pirâmide}$

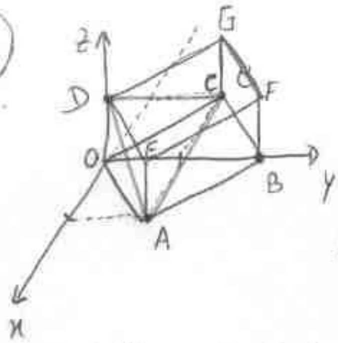
$V_{cubo} = 6 \times 6 \times 6 = 216$

$V_{pirâmide} = \frac{1}{3} Ab \times h = \frac{1}{3} \times 36 \times 6 = 72$

$V_{total} = 288$, logo metade é $V = 144$

$6 \times 6 \times z = 144 \Leftrightarrow z = \frac{144}{36} \Leftrightarrow \underline{\underline{z = 4}}$

(4)

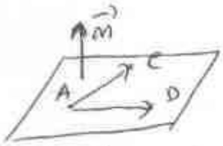


a) $E(1,1,1)$ $\quad A(1,1,0)$

$F(0,2,1)$ $\quad C(-1,1,0)$

$G(-1,1,1)$ $\quad D(0,0,1)$

b)



$\overrightarrow{AC} = (-2;0;0)$

$\overrightarrow{AD} = (-1;-1;1)$

$\begin{cases} \vec{m} \cdot \overrightarrow{AC} = 0 \\ \vec{m} \cdot \overrightarrow{AD} = 0 \end{cases}$ $\begin{cases} (a,b,c) \cdot (-2,0,0) = 0 \\ (a,b,c) \cdot (-1,-1,1) = 0 \end{cases}$ $\begin{cases} -2a = 0 \\ -a-b+c = 0 \end{cases}$ $\begin{cases} a = 0 \\ +b = +c \end{cases}$

$\vec{m} = (0;c;c) \to (0,1,1)$

$y + z + D = 0$

$(0,0,1) \to 0 + 1 + D = 0$

$D = -1$

$\boxed{y + z - 1 = 0}$ //

e) Plano OAED : $y = x \Leftrightarrow x - y = 0$

Recta de intersecção : recta AD

$A(1,1,0)$ $D(0,0,1)$ $\qquad \frac{x}{-1} = \frac{y}{-1} = \frac{z-1}{1}$

$\overrightarrow{AD} = (-1,-1,1)$

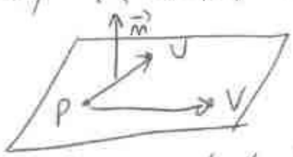
eq. vectorial : $(x,y,z) = (0,0,1) + k(-1,-1,1)$; $k \in \mathbb{R}$

d) $\overline{OA} = \sqrt{1+1+0} + \sqrt{2}$ $\qquad A(1,1,0)$

$\overline{AB} = \sqrt{1+1+0} = \sqrt{2}$ $\qquad B(0,2,0)$

$V = \sqrt{2} \times \sqrt{2} \times 1 = 2 \text{ cm}^2$

5 a) $P(0,2,0)$ $U(2,2,2)$ $V(2,0,2)$



$\overrightarrow{PU} = (2,0,2)$

$\overrightarrow{PV} = (2,-2,2)$

$$\begin{cases} \vec{m} \cdot \overrightarrow{PU} = 0 \\ \vec{m} \cdot \overrightarrow{PV} = 0 \end{cases} \begin{cases} (a,b,c)\cdot(2,0,2) = 0 \\ (a,b,c)\cdot(2,-2,2) = 0 \end{cases} \begin{cases} 2a + 2c = 0 \\ 2a - 2b + 2c = 0 \end{cases} \begin{cases} a + c = 0 \\ a - b + c = 0 \end{cases}$$

$$\begin{cases} a = -c \\ -\cancel{c} - b + \cancel{c} = 0 \end{cases} \begin{cases} a = -c \\ b = 0 \end{cases} \quad \vec{m} = (-c; 0; c) \rightarrow (-1, 0, 1)$$

$-x + z + D = 0$

$(0,2,0) \rightarrow 0 + D = 0$

$D = 0$

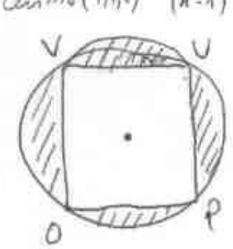
$-x + z = 0$

b) raio $= \frac{\overline{RT}}{2} = \frac{\sqrt{12}}{2} = \frac{\cancel{2}\sqrt{3}}{\cancel{2}} = \sqrt{3}$ $\qquad R(2,0,0)$

Centro $(1,1,1)$ $\quad (x-1)^2 + (y-1)^2 + (z-1)^2 = 3$ $\qquad T(0,2,2)$

$\overline{RT} = \sqrt{4+4+4} = \sqrt{12}$

c)



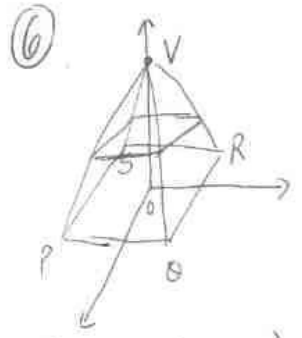
$r = \sqrt{3}$

$A_{\text{círculo}} = \pi r^2 = 3\pi$ $\qquad \overline{PU} = \sqrt{4+0+4} = \sqrt{8}$

$A_{\square} = 2 \times \sqrt{8} = 2 \times 2\sqrt{2} = 4\sqrt{2}$

$A_{\text{pretendida}} = 3\pi - 4\sqrt{2}$

(6)



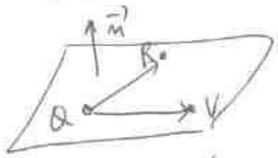
a) $Q(2,2,0)$

$A_b = 4 \times 4 = 16$

$V_{\text{pirâmide}} = \frac{1}{3} A_b \times h \Leftrightarrow 32 = \frac{1}{3} \times 16 \times h \Leftrightarrow$

$\Leftrightarrow 32 = \frac{16}{3} h \Leftrightarrow h = 6$, logo $V(0,0,6)$

b) $Q(2,2,0)$ $R(-2,2,0)$ $V(0,0,6)$



$\overrightarrow{QR} = (-4;0;0)$

$\overrightarrow{QV} = (-2;-2;6)$

$\begin{cases} \vec{n} \cdot \overrightarrow{QR} = 0 \\ \vec{n} \cdot \overrightarrow{QV} = 0 \end{cases} \begin{cases} (a,b,c).(-4,0,0) = 0 \\ (a,b,c).(-2,-2,6) = 0 \end{cases} \begin{cases} -4a = 0 \\ -2a - 2b + 6c = 0 \end{cases} \begin{cases} a = 0 \\ -2b + 6c = 0 \end{cases}$

$\begin{cases} \overline{} \\ -b + 3c = 0 \end{cases} \begin{cases} \\ b = 3c \end{cases}$ $\vec{n} = (0;3c;c) \rightarrow (0,3,1)$

$3y + z + D = 0$

$(0,0,6) \rightarrow 0 + 6 + D = 0$

$D = -6$

$\boxed{3y + z - 6 = 0}$ //

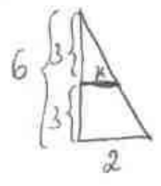
c) reta $\perp$ QRV logo reta // $\vec{n}(0,3,1)$

$(x,y,z) = (0,0,0) + k(0,3,1); k \in \mathbb{R}$

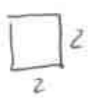
$(x,y,z) = k(0,3,1); k \in \mathbb{R}$

d) aresta QV: $\overrightarrow{QV} = (-2,-2,6)$ $V(0,0,6)$

$\begin{cases} \frac{x}{-2} = \frac{y}{-2} = \frac{z-6}{6} \\ z = 3 \end{cases} \begin{cases} \frac{x}{-2} = -\frac{1}{2} \\ \frac{y}{-2} = -\frac{1}{2} \\ z = 3 \end{cases} \begin{cases} x = 1 \\ y = 1 \\ z = 3 \end{cases}$ $M \rightarrow (1,1,3)$



$\frac{6}{2} = \frac{3}{x} \Leftrightarrow x = 1$



Área $= 4$

7 a) $A(4,3,0)$ $C(0,-5,0)$ $B(0,5,0)$

$\vec{AC} = (-4,-8,0)$ $\vec{AB} = (-4,2,0)$

$\vec{AC} \cdot \vec{AB} = 16 - 16 = 0$ logo $AC \perp AB$

b) Vector director, por exemplo, $(0,0,1)$

$r \equiv (x,y,z) = (0,5,0) + K(0,0,1)$; $K \in \mathbb{R}$

c) Supondo $D = (0,5,1)$

$\vec{AD} = (-4,2,1)$ $\vec{AC} = (-4,-8,0)$

$\vec{AB} = (-4,2,0)$ $\vec{AC} \cdot \vec{AD} = 0$ logo

Ora, $\vec{AC} \cdot \vec{AB} = 0$ e

$\vec{AC} \perp$ plano ABD

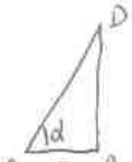
$-4x - 8y + D = 0$

$(0,+5,0) \to 0 - 40 + D = 0$

$D = +40$

$\Rightarrow -4x - 8y + 40 = 0 \iff \boxed{x + 2y - 10 = 0}$ //

d)



$tg\alpha = \dfrac{\overline{BD}}{5} \iff \overline{BD} = 5\,tg\alpha$

$V_{cilindro} = A_b \times h = \pi \times 5^2 \times 5\,tg\alpha$

$\boxed{V(\alpha) = 125\pi\, tg\alpha}$ //

8 a) $\vec{OA} = (0,4,3) \perp$ plano

$4y + 3z + D = 0$

$(0,4,3) \to 16 + 9 + D = 0$

$D = -25$

$\Rightarrow \boxed{4y + 3z - 25 = 0}$ //

b) Se $r = 5$ logo $C(0,0,-5)$

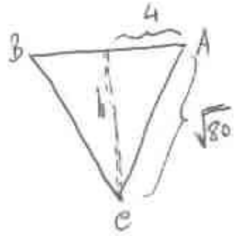
$A(0,4,3)$
$B(0,-4,3)$
$C(0,0,-5)$

Recta BC: $\vec{BC} = (0,4,-8)$

$\begin{cases} (x,y,z) = (0,0,-5) + K(0,4,-8);\ K \in \mathbb{R} \\ 4y + 3z - 25 = 0 \end{cases}$ $\begin{cases} x = 0 \\ y = 4K \\ z = -5 - 8K \\ 4(4K) + 3(-5-8K) - 25 = 0 \end{cases}$

$\begin{cases} = \\ 16K - 15 - 24K - 25 = 0 \end{cases}$ $\begin{cases} x = 0 \\ y = -20 \\ z = -5 + 40 \\ K = -5 \end{cases}$ $\begin{cases} x = 0 \\ y = -20 \\ z = 35 \end{cases}$ $I \equiv (0; -20; 35)$

e)



$\overline{AB} = \sqrt{0+64+0} = 8$

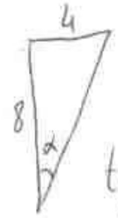
$\overline{AC} = \sqrt{0+16+64} = \sqrt{80}$

$h^2 + 4^2 = (\sqrt{80})^2$

$h^2 = 80 - 16$

$h^2 = 64$

$h = 8$

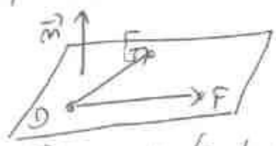


$tg\,\alpha = \frac{4}{8} = \frac{1}{2} \Rightarrow \alpha \simeq 26{,}565^\circ$

$2\alpha \cong 53{,}13^\circ$

$tg \simeq 1{,}33$

9 a) $D(2,-2,0)$ $G(-2,-2,4)$ $F(-2,2,4)$



$\vec{DG} = (-4, 0, 4)$

$\vec{DF} = (-4, 4, 4)$

$\begin{cases} \vec{n} \cdot \vec{DG} = 0 \\ \vec{n} \cdot \vec{DF} = 0 \end{cases}$ $\begin{cases} (a,b,c) \cdot (-4,0,4) = 0 \\ (a,b,c) \cdot (-4,4,4) = 0 \end{cases}$ $\begin{cases} -4a + 4c = 0 \\ -4a + 4b + 4c = 0 \end{cases}$ $\begin{cases} -a + c = 0 \\ -a + b + c = 0 \end{cases}$

$\begin{cases} a = c \\ -\cancel{c} + b + \cancel{c} = 0 \end{cases}$ $\begin{cases} a = c \\ b = 0 \end{cases}$ $\vec{n} = (c; 0; c) \Rightarrow (1, 0, 1)$

$x + z + D = 0$

$(2,-2,0) \Rightarrow 2 + 0 + D = 0$

$D = -2$

$\boxed{x + z - 2 = 0}$ //

b) Centro $(0,0,2)$
$r = 2$

$x^2 + y^2 + (z-2)^2 = 4$

c) $C(-2,-2,0)$ $H(2,-2,4)$ $\vec{u} = (k^2 - 2k;\ k^2 - 1;\ 3)$

$\vec{CH} = (4; 0; 4)$

$\frac{4}{k^2-2k} = \frac{0}{k^2-1} = \frac{4}{3}$ imp. $\Rightarrow$ Não existe k.

d) $P(2;y;4)$ $\quad M(0;2;0)$ $\quad N(0;2;4)$

$\overline{NM} = 4$

$\overline{PN} = 4 \Leftrightarrow \sqrt{4+(y-2)^2+0} = 4 \Leftrightarrow \sqrt{4+y^2-4y+4} = 4 \Leftrightarrow$

$\Leftrightarrow y^2-4y+8 = 16 \Leftrightarrow y^2-4y-8=0 \Leftrightarrow y = \dfrac{4 \pm \sqrt{16+32}}{2}$

$\Leftrightarrow y = \dfrac{4 \pm \sqrt{48}}{2} \Leftrightarrow y = \dfrac{4 \pm 4\sqrt{3}}{2} \Leftrightarrow y = 2 \pm 2\sqrt{3}$

Atendendo à figura, $y = 2 - 2\sqrt{3}$

logo $P(2;\, 2-2\sqrt{3};\, 4)$

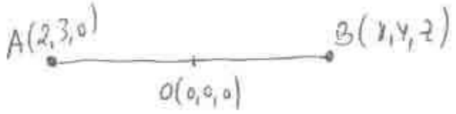
(10) a) $P = 2\pi r = 2\pi \Leftrightarrow r = 1$

$h^2+1 = 13$
$h^2 = 12$
$h = \sqrt{12}$

logo β é $z = \sqrt{12}$ ou $z = 2\sqrt{3}$ //

b) $A(2,3,0)$

$\overline{OA} = \sqrt{4+9+0} = \sqrt{13} = r$, logo $A \in$ Sup. esférica



$$\begin{cases} \dfrac{2+x}{2} = 0 \\ \dfrac{3+y}{2} = 0 \\ \dfrac{0+z}{2} = 0 \end{cases} \quad \begin{cases} x = -2 \\ y = -3 \\ z = 0 \end{cases} \quad B(-2;-3;0)$$

c) $\alpha \perp \overrightarrow{AB}$ $\qquad \overrightarrow{AB} = (-4;-6;0)$

$\alpha \perp (-4;-6;0)$

$Ky - 2x = z \Leftrightarrow -2x + Ky - z = 0 \perp (-2;K;-1)$

$(-4;-6;0)\cdot(-2;K;-1) = 0 \Leftrightarrow 8 - 6K + 0 = 0 \Leftrightarrow 6K = 8 \Leftrightarrow$

$\Leftrightarrow K = \dfrac{4}{3}$

(11) $\vec{u}(2,5,0)$

a) $(-5,2,0)$ e $(5;-2;1)$ por exemplo

b) $\vec{e_1}(1,0,0)$

$\vec{u}\cdot\vec{e_1} = \|\vec{u}\|\|\vec{e_1}\| \cos\alpha \Leftrightarrow \cos\alpha = \dfrac{2}{\sqrt{29}\times 1} \Leftrightarrow \alpha \simeq 1{,}19 \text{ rad}$

$\|\vec{u}\| = \sqrt{4+25} = \sqrt{29}$

c) $\vec{u}(2,5,0)$

$$\left.\begin{array}{l} 2x+5y+D=0 \\ (0,1,0) \to 0+5+D=0 \\ D=-5 \end{array}\right\rangle \boxed{2x+5y-5=0}\ //$$

...

d) $$\begin{cases} 2x+5y-5=0 \ (3) \\ x+y+z=1 \ (15) \\ 3y-2z=1 \ (-5) \end{cases} \begin{cases} 2x+5y = 5 \\ x+y+z=1 \\ 3y-2z=1 \end{cases} \begin{cases} 6x+15y = 15 \\ 15x+15y+15z=15 \\ -15y+10z=-5 \end{cases}$$

$$\begin{cases} 6x+10z=10 \\ 15x+25z=10 \end{cases} \begin{cases} 3x+5z=5 \\ 3x+5z=2 \end{cases} \begin{cases} -3x-5z=-5 \\ 3x+5z=2 \end{cases} \begin{cases} 0=-3 \\ \text{imp.} \end{cases}$$

Não existe qualquer ponto comum aos três planos e eles não são paralelos. $\bigstar$.

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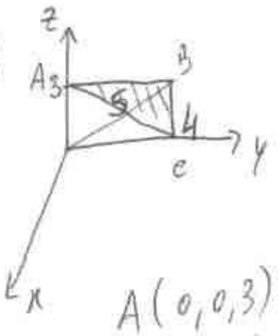
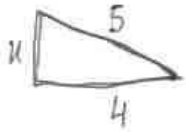
a)





$x^2+4^2=5^2$
$x^2+16=25$
$x^2=9$
$x=3$

$A(0,0,3)$ $C(0,4,0)$

$\overrightarrow{AC}=(0,4,-3)$

$(x,y,z)=(0,0,3)+K(0,4,-3)\ ;\ K\in\mathbb{R}$

$$\begin{cases} x=0 \\ y=4K \\ z=3-3K \end{cases} \quad \begin{cases} x=0 \\ K=\frac{y}{4} \\ 3K=-z+3 \end{cases} \quad \begin{cases} x=0 \\ K=\frac{y}{4} \\ K=\frac{-z+3}{3} \end{cases}$$

equação

$$\begin{cases} x=0 \\ \frac{y}{4}=\frac{z-3}{-3} \end{cases} \quad \begin{cases} x=0 \\ -3y=4z-12 \end{cases} \quad \begin{cases} x=0 \\ 3y+4z-12=0 \end{cases}$$

b.1) Circunferência no plano $y=0$

Centro $(0,0,0)$ $r=3$

$x^2+y^2+z^2=9 \wedge y=0$

$x^2+z^2=9 \wedge y=0$

b.2) Cilindro

$V=Ab\times h$ $\quad \| \quad Ab=\pi r^2=9\pi$, $h=4$

$V=36\pi$