Resolução da ficha nº 4 - Funções - 11.º ANO

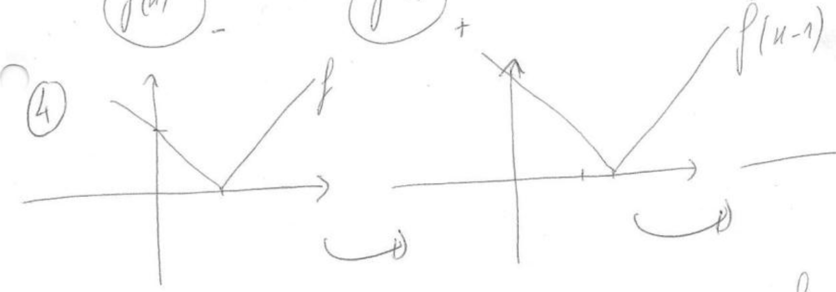
① (A)

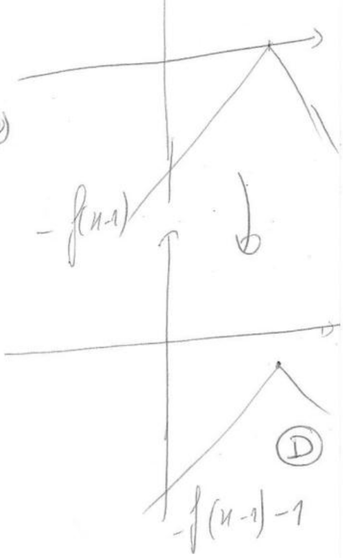
② $(f \circ g)(-3) = f(g(-3)) = f(-4) = |-4| = 4$

③ $\frac{f(x)}{g(x)} < 0 \Leftrightarrow$

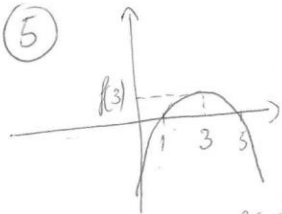
$\Leftrightarrow \frac{f(x)^{+}}{g(x)_{-}}$ ou $\frac{f(x)^{-}}{g(x)_{+}}$ (D) $f(0)$ existe

④





⑤



$D'_f =]-\infty; f(3)]$ (D)

⑥ $a > 0 \wedge b < 0$ (B)

⑦ $h'(1) = m =$ declive

$A(-2,0)$ $\vec{AB} = (2,1)$

$B(0,1)$ $m = \frac{1}{2}$ (C)

⑧ $f^{-1}(2) + (g \circ h)(\sqrt{2}) = 3 + g(h(\sqrt{2})) = 3 + g(1) = 3 + 3 = 6$

(9) $v(t) = t^3 - 15t^2 + 63t$

a) $v'(t) = 3t^2 - 30t + 63$

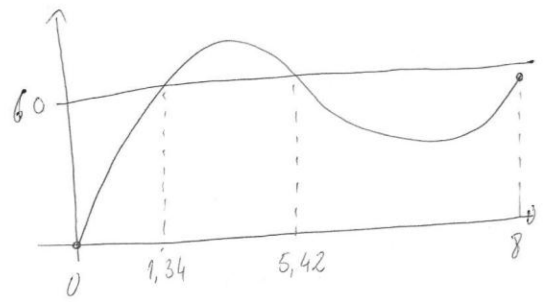
$v'(t) = 0 \Leftrightarrow 3t^2 - 30t + 63 = 0 \Leftrightarrow t = \frac{30 \pm \sqrt{900 - 12 \times 63}}{6} \Leftrightarrow$

$\Leftrightarrow x = 3 \vee x = 7$

	0		3		7		8
$v'(t)$	+	+	0	−	0	+	+
$v(t)$	0	↗	M	↘	m	↗	56

Vel. Máx = $v(3) = 81$ centímetros de rotação por minuto.

b)



$5,42 - 1,34 = 4,08$

R: 4m5s

($0,08 \times 60 \simeq 5$ segundos)

(10) $\frac{x^2+1}{2-x} < 0$

	$-\infty$	2	$+\infty$
x^2+1	+	+	+
$2-x$	+	0	−
$\frac{x^2+1}{2-x}$	+	S.S.	−

$S =]2; +\infty[$

(11)

a)

x	$-\infty$		-3		0		2	$+\infty$
$f(x)$		$+$	$+$	$+$	0	$-$	S.S	$+$
$g(x)$		$-$	0	$+$	$+$	$+$	$+$	$+$
$f(x)\times g(x)$		$-$	0	$+$	0	$-$	S.S	$+$

$f(x)\times g(x) < 0 \Leftrightarrow x \in \left]-\infty;-3\right] \cup \left[0;2\right[$

b) $a=3$ e $c=2$

$f(x) = 3 + \frac{b}{x-c} \rightsquigarrow f(x) = 3 + \frac{b}{x-2}$

Como $f(0)=0$ vem $3 + \frac{b}{0-2} = 0 \Leftrightarrow \frac{b}{-2} = -3 \Leftrightarrow b = 6$

Logo $a=3$; $b=6$ e $c=2$

(12) $f(x) = 2 + \frac{1}{1-x}$

a) $f(x) < -1 \Leftrightarrow 2 + \frac{1}{1-x} < -1 \Leftrightarrow \frac{1}{1-x} < -3 \Leftrightarrow \frac{1}{1-x} + 3 < 0 \Leftrightarrow$

$\Leftrightarrow \frac{1+3-3x}{1-x} < 0 \Leftrightarrow \frac{4-3x}{1-x} < 0$

x	$-\infty$		1		$\frac{4}{3}$	$+\infty$
$4-3x$		$+$	$+$	$+$	0	$-$
$1-x$		$+$	0	$-$	$-$	$-$
$\frac{4-3x}{1-x}$		$+$	S.S	$-$	0	$+$

$S = \left]1; \frac{4}{3}\right[$

b) $x=1$ Assimptote vertical
$y=2$ " horizontal

13

a) $x + c = 6 \Leftrightarrow \boxed{c = 6 - x}$

$V_{\text{pirâmide}} = \frac{1}{3} A_b \times h = \frac{1}{3} \times 2x \times 2x \times (6-x) =$

$= \frac{4x^2(6-x)}{3} = \frac{24x^2 - 4x^3}{3} = 8x^2 - \frac{4}{3}x^3$ c.q.d.

b) $V'(x) = 16x - 4x^2$

$V'(x) = 0 \Leftrightarrow 16x - 4x^2 = 0 \Leftrightarrow x(16 - 4x) = 0 \Leftrightarrow x = 0 \vee x = 4$

	0		4		6
$V'(x)$		+	0	−	
$V(x)$		↗	M	↘	

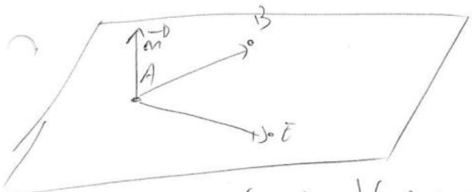
$V(4) = 8 \times 4^2 - \frac{4}{3} \times 4^3 =$

$= \frac{128}{3}$

R: Volume máximo $= \frac{128}{3}$ para $x = 4$.

c) $\boxed{x = 1}$

$A(1,1,0) \quad B(-1,1,0) \quad E(0,0,5)$



$\vec{AB} = B - A = (-2, 0, 0)$

$\vec{AE} = E - A = (-1, -1, 5)$

$\begin{cases} \vec{n} \cdot \vec{AB} = 0 \\ \vec{n} \cdot \vec{AE} = 0 \end{cases} \quad \begin{cases} (a,b,c)(-2,0,0) = 0 \\ (a,b,c)(-1,-1,5) = 0 \end{cases} \Leftrightarrow \begin{cases} -2a = 0 \\ -a - b + 5c = 0 \end{cases} \quad \begin{cases} a = 0 \\ -b + 5c = 0 \end{cases}$

$\begin{cases} a = 0 \\ b = 5c \end{cases} \quad \vec{n} = (0, 5c, c) \; ; c \in \mathbb{R}$

$\vec{n}$ pode ser, por exemplo, $(0,5,1)$ se $c = 1$.

$Ax + By + Cz + D = 0$

$5y + z + D = 0$

$(1,1,0) \rightarrow 5 + 0 + D = 0 \Leftrightarrow D = -5$

$\therefore \; 5y + z - 5 = 0$

(14) $7h40 \to t = 10$

a) $7h55 \to t = 25$

$$d(10) = 45 - \frac{5600}{10^2+300} = 31$$

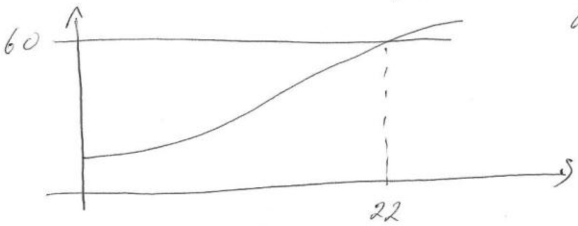
$7h40m + 31m = 8h11m$

$$d(25) = 45 - \frac{5600}{25^2+300} \approx 39$$

$7h55m + 39m = 8h34m$ (A Maria chega atrasada)

b) $t + d(t) \leq 60$

$$t + d(t) = t + 45 - \frac{5600}{t^2+300}$$



A Maria pode sair de casa até 22 m depois das 7h30, ou seja, até às 7h52m.

(15) $(f \circ g)(2) = f[g(2)] = f(-3) = -2$ (A)

(16) a) $4 - \frac{4}{x+2} \geq 3$

$\Leftrightarrow \frac{4}{x+2} - \underset{(x+2)}{1} \leq 0 \Leftrightarrow \frac{4-x-2}{x+2} \leq 0$

$\Leftrightarrow \frac{2-x}{x+2} \leq 0$

Raízes:

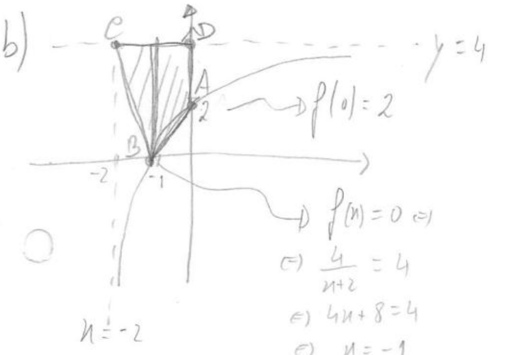
$2-x=0 \Leftrightarrow x=2$

$x+2=0 \Leftrightarrow x=-2$

	$-\infty$	-2		2	$+\infty$
$2-x$	+	+	+	0	−
$x+2$	−	0	+	+	+
$\frac{2-x}{x+2}$	−	ND	+	0	−

$S =]-\infty; -2[\cup [2, +\infty[$

b)



$\to f(0) = 2$

$\to f(x) = 0 \Leftrightarrow$

$\Leftrightarrow \frac{4}{x+2} = 4$

$\Leftrightarrow 4x + 8 = 4$

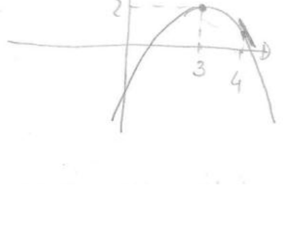
$\Leftrightarrow x = -1$

$A_1 = \frac{1 \times 4}{2} = 2$

$A_2 = \frac{4+2}{2} \times 1 = 3$

$A_{pedida} = 5$

(17)



$f'(4) < 0$ (D)

(18) a) $P(a, a, z)$

$x + 2y + 3z = 9$

$P \in$ plano: $a + 2a + 3z = 9$

$3a + 3z = 9$

$a + z = 3$

$z = 3 - a$

$V = A_b \times h = a \times a \times (3-a) = a^2(3-a) =$

$= 3a^2 - a^3$

b) $V' = 6a - 3a^2$

Raízes:

$6a - 3a^2 = 0 \Leftrightarrow a(6-3a) = 0$

$\Leftrightarrow a = 0 \vee 6 - 3a = 0 \Leftrightarrow a = 0 \vee a = 2$

	0		2		3
V'	ND	+	0	−	ND
V	ND	↗	Max	↘	ND

Volume máx. para $a = 2$

↗ em $]0, 2]$ e ↘ em $[2, 3[$

c) $(x, y, z) = (9, 0, 0) + K(1, 2, 3); K \in \mathbb{R}$

$A(x, 0, 0)$

$A \in$ plano $\quad x + 0 + 0 = 9$

$x = 9$

$A(9, 0, 0)$

(19) $C(t) = \frac{t^2 + 25t}{t+1}$

a) $t = 75$

$C(75) = \frac{75^2 + 25 \times 75}{75 + 1} \simeq 99 \rightsquigarrow 1h39m$

R: Sairá às 13h39m.

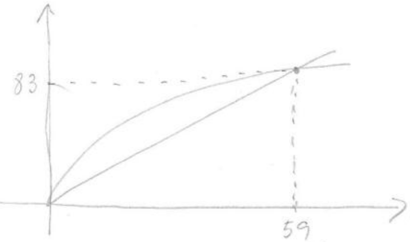
b) $e(t) = 25 \Leftrightarrow$

$\Leftrightarrow \dfrac{t^2+25t}{t+1} = 25 \Leftrightarrow t^2 + \cancel{25t} = \cancel{25t} + 25$

$\Leftrightarrow t^2 = 25 \Leftrightarrow t = \pm 5$

R: 5 cm

e) $p(t) = t + \underbrace{0,4t}_{40\%} = 1,4t$



A proposta do Manuel é mais favorável se o atraso for inferior a 59 m.
Para um atraso de 59 m o tempo de permanência na empresa é de 83 m para ambas as propostas.

(20) $f(-4) + f^{-1}(2) = -3 + 3 = 0$ (B)

(21) a) $3 + \dfrac{6}{x} \leq 5 \Leftrightarrow \dfrac{6}{x} - 2 \leq 0$

$\Leftrightarrow \dfrac{6-2x}{x} \leq 0$

Raízes:
$6-2x=0$
$x = \boxed{3}$

$x = \boxed{0}$

	$-\infty$	0		3	$+\infty$
$6-2x$	+	+	+	0	−
x	−	0	+	+	+
$\frac{6-2x}{x}$	−	ND	+	0	−

$S =]-\infty, 0[\cup [3, +\infty[$

b) $f(2) = 6$

$f'(x) = -\dfrac{6}{x^2}$

$m = f'(2) = -\dfrac{6}{4} = -\dfrac{3}{2}$

$y = mx + b$

$y = -\dfrac{3}{2}x + b$

$(2,6) \to 6 = -\dfrac{3}{\cancel{2}} \times \cancel{2} + b$

$9 = b$

$\therefore y = -\dfrac{3}{2}x + 9$

c) $g(x) = \dfrac{1}{3}x^3 - 3x^2 + 8x - 3$

$g'(x) = x^2 - 6x + 8$

Raízes:
$x^2 - 6x + 8 = 0 \Leftrightarrow x = \dfrac{6 \pm \sqrt{36 - 4 \times 1 \times 8}}{2}$

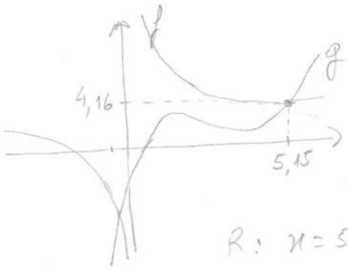
$\Leftrightarrow x = 2 \vee x = 4$

	$-\infty$	2		4	$+\infty$
g'	+	0	−	0	+
g	↗	Máx	↘	mín	↗

Máx: $g(2) = \dfrac{8}{3} - 12 + 16 - 3 = \dfrac{11}{3}$

$A_{\triangle} = \dfrac{4 \times \frac{11}{3}}{2} = \dfrac{44}{6} = \dfrac{22}{3}$

d)



R: $x = 5,15$

(22) $\dfrac{f}{g}$ teria os zeros de f (5 zeros) mas como tem 1 em comum com g, $\dfrac{f}{g}$ simplifica e ficamos com 4 zeros. (C)

(23) $(g \circ f)(3) = g(f(3)) = g(1) = -1 + 3 = 2$ (D)

(24) Nasceram $2500 + 500 = 3000$

a) $a(0) = \dfrac{6}{1} = 6$

$a(1) = \dfrac{11+6}{1+1} = \dfrac{17}{2} = 8,5$

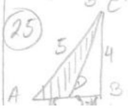
$8,5 - 6 = 2,5$

b) A.H.a: $y = 11$

A.H.b: $y = 1$

Valor pedido $= 11000 - 1000 =$
$= 10.000$

(25)



$\overline{CD}^2 = 4^2 + (3-x)^2$

$\overline{CD}^2 = 16 + 9 - 6x + x^2$

$\overline{CD}^2 = x^2 - 6x + 25$

$\overline{CD} = \sqrt{x^2 - 6x + 25}$

$P = \sqrt{x^2 - 6x + 25} + x + 5$

(D)

26

a) $f(x) = x^3+3x^2-9x-11$

$f'(x) = 3x^2+6x-9$

Zeros: $3x^2+6x-9=0$

$x = \frac{-6 \pm \sqrt{36-4\times3\times(-9)}}{2\times3}$

$x=-3 \ \vee \ x=+1$

	$-\infty$	-3		1	$+\infty$
f'	$+$	0	$-$	0	$+$
f	$\nearrow$	Máx	$\downarrow$	min	$\nearrow$

$\nearrow$ em $]-\infty,-3]$ e em $[1,+\infty[$

$\downarrow$ em $[-3,1]$

Máx $= f(-3) = 16$ para $x=-3$

mín $= f(1) = -16$ para $x = 1$

b)

$$\begin{array}{r|rrrr} & 1 & 3 & -9 & -11 \\ -1 & & -1 & -2 & 11 \\ \hline & 1 & 2 & -11 & \boxed{0} \end{array}$$

$f(x) = (x+1)(x^2+2x-11)$

$(f\times g)(x) = f(x)\times g(x) =$

$= \cancel{(x+1)}(x^2+2x-11)\times\frac{x-1}{\cancel{x+1}} =$

$= x^3+2x^2-11x-x^2-2x+11 =$

$= x^3+x^2-13x+11$

$D_{f\times g} = D_f \cap D_g = \mathbb{R} \cap \mathbb{R}\setminus\{-1\} = \mathbb{R}\setminus\{-1\}$

c) $g(x) = \frac{x-1}{x+1} = 1 - \frac{2}{x+1}$

$$\begin{array}{r|l} x-1 & x+1 \\ \hline -x-1 & 1 \\ \hline -2 & \end{array}$$

$y = 1$ A.H.

$x=-1$ A.V.

$P(-1,1)$

$P \in h$, $h(-1) = 1 \Leftrightarrow f(-1)+K = 1 \Leftrightarrow (-1)^3+3(-1)^2-9(-1)-11+K=1 \Leftrightarrow -1+3+9-11+K=1$

$\Leftrightarrow K = 1$

27 $f'(2) = 9$ logo $m = 9$

Como $b=-15$ então $y = 9x-15$

Se $x=2$ então $y = 9\times2-15 = 3$. Logo $f(2) = 3$

(C)

28

a) $A(t) \geq 0,5$

$\Leftrightarrow \frac{2t}{t^2+3} \geq \frac{1}{2}$

$\Leftrightarrow \frac{2t}{t^2+3} - \frac{1}{2} \geq 0$
$\quad\ _{(2)} \qquad _{(t^2+3)}$

$\Leftrightarrow \frac{4t-t^2-3}{2t^2+6} \geq 0$

$\Leftrightarrow \frac{-t^2+4t-3}{2t^2+6} \geq 0$

Zeros:

$-t^2+4t-3=0$

$\Leftrightarrow t = \frac{-4 \pm \sqrt{4^2-4(-1)(-3)}}{2(-1)}$

$\Leftrightarrow t=1 \quad \vee t=3$

$2t^2+6=0$

$\Leftrightarrow 2t^2=-6$

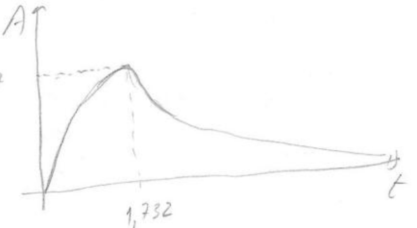
$\Leftrightarrow t^2=-3$ imp.

	0		1		3	$+\infty$
$-t^2+4t-3$		$-$	0	$+$	0	$-$
$2t^2+6$		$+$	$+$	$+$	$+$	$+$
$A(t)$		$-$	0	$+$	0	$-$

$S = [1;3]$

R: Durante 2 anos.

(28b)



$0,732 \times 365 \simeq 267$

R: Ao fim de 1 ano e 267 dias.

(29) $(g \circ h)(a) = \frac{1}{9}$

$\Leftrightarrow g(h(a)) = \frac{1}{9}$

$\Leftrightarrow g(a+1) = \frac{1}{9}$

$\Leftrightarrow \frac{1}{a+1} = \frac{1}{9}$

$\Leftrightarrow a+1 = 9$

$\Leftrightarrow a = 8$ (B)

(30.a.1) $D'_f =]-\infty; -2[\cup]-2; +\infty[$ ou $D'_f = \mathbb{R} \setminus \{-2\}$

(30.a.2) $S =]-\infty; -1] \cup]1; +\infty[$

(30.b) $f(x) = -2 + \frac{b}{x-1}$ e $f(-1) = 0$

Assim, $-2 + \frac{b}{-1-1} = 0 \Leftrightarrow \frac{b}{-2} = 2 \Leftrightarrow b = -4$

Logo $f(x) = -2 - \frac{4}{x-1}$

(31)

$f(x) = y$

$\sqrt{x-1} = y$

$x-1 = y^2$

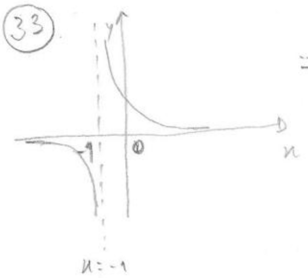
$x = y^2+1$

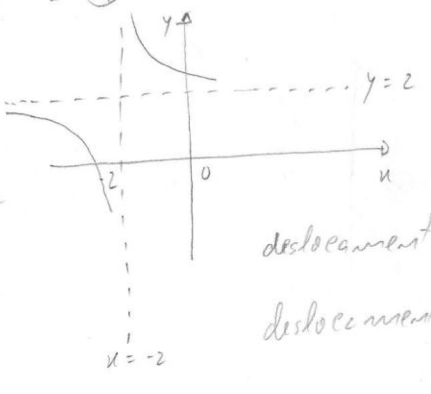
$f^{-1}(x) = x^2+1$

Logo $f^{-1}(3) = 3^2+1 = 9+1 = 10$ (C)

(32) $f(x) - 1$, logo $k = -1$ (A)

(33)



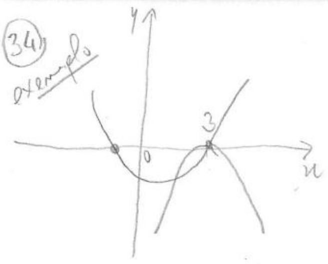


deslocamento na horiz. p/ esquerda $a = 1$

deslocamento na vertic. p/ cima $k = 2$

(B)

(34) exemplo



$$f \times g = 0 \Leftrightarrow f = 0 \vee g = 0$$

2 zeros + 1 zero = 3 zeros
− 1 zero comum
2 zeros

$$\frac{f}{g} = 0 \Leftrightarrow f = 0 \wedge g \neq 0$$

2 zeros
− 1
1 zero

1 zero comum

(A)

(35)

a.1) $k = -1$

a.2) $\lim\limits_{x \to +\infty} f(x) = -1$

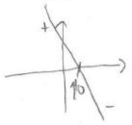
b) $f(x) \leq \frac{4-x}{x+2} \Leftrightarrow \frac{6-x}{x-2} \leq \frac{4-x}{x+2} \Leftrightarrow \frac{6-x}{x-2}_{(x+2)} - \frac{4-x}{x+2}_{(x-2)} \leq 0 \Leftrightarrow$

$\Leftrightarrow \frac{(6-x)(x+2) - (4-x)(x-2)}{x^2-4} \leq 0 \Leftrightarrow \frac{6x+12-\cancel{x^2}-2x-4x+8+\cancel{x^2}-2x}{x^2-4} \leq 0 \Leftrightarrow$

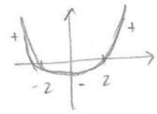
$\Leftrightarrow \frac{-2x+20}{x^2-4} \leq 0$

Raízes:

$-2x+20=0$
$-2x=-20$
$x=10$



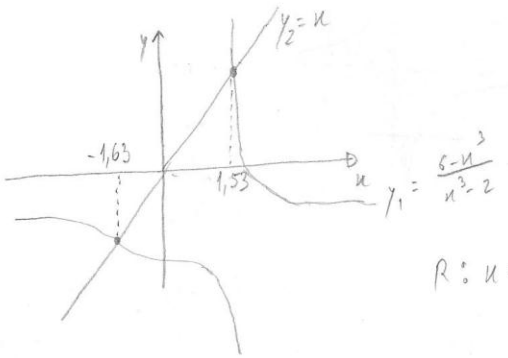
$x^2-4=0$
$x = \pm 2$



	$-\infty$	-2		2		10	$+\infty$
$-2x+20$	+	+	+	+	+	0	−
x^2-4	+	0	−	0	+	+	+
$\frac{-2x+20}{x^2-4}$	+	ND	− P	ND	+	0 P	− P

$S =]-2, 2[\cup [10, +\infty[$

b.2) $(f \circ g)(x) =$
$= f(g(x)) =$
$= f(x^3) =$
$= \frac{6-x^3}{x^3-2}$



R: $x = -1{,}63 \vee x = 1{,}53$

36) $S =]-\infty, -1] \cup]1, +\infty[$ (D)

37) $f \times g = 0 \Leftrightarrow$
$\Leftrightarrow f = 0 \vee g = 0$

Zeros: $\underbrace{-2 \qquad 2 \text{ e } -2}_{-2 \text{ e } 2 \text{ (2 zeros)}}$

$\frac{f}{g} = 0$
$f = 0 \wedge g \neq 0$

$\underbrace{-2 \qquad -2 \text{ e } 2}_{\text{não tem zeros}}$

(C)

38) a) $f(x) < \frac{1}{x-2} \Leftrightarrow \frac{2x-3}{1-x} < \frac{1}{x-2} \Leftrightarrow \underset{(x-2)}{\frac{2x-3}{1-x}} - \underset{(1-x)}{\frac{1}{x-2}} < 0$

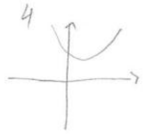
$\Leftrightarrow \frac{(2x-3)(x-2)-(1-x)}{(1-x)(x-2)} < 0 \Leftrightarrow \frac{2x^2-4x-3x+6-1+x}{(1-x)(x-2)} < 0 \Leftrightarrow \frac{2x^2-6x+5}{(1-x)(x-2)} < 0$

Raízes

$2x^2-6x+5=0$

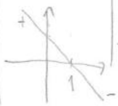
$x = \frac{6 \pm \sqrt{36-40}}{4}$

$x = \frac{6 \pm \sqrt{-4}}{4}$ imp.



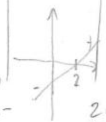
$1-x=0$

$x = 1$



$x-2=0$

$x = 2$



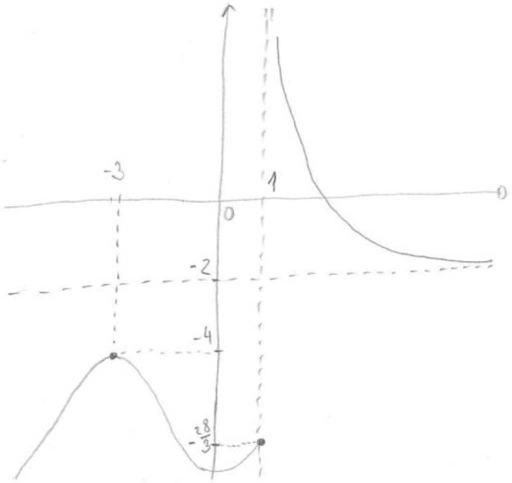
	1		2	$+\infty$
$2x^2-6x+5$		+	+	+
$1-x$		−	−	−
$x-2$		−	0	+
Denom		+	0	−
$\frac{2x^2-6x+5}{(1-x)(x-2)}$		+	ND	−

$S =]2, +\infty[$

b) $(g \circ f)(-3) = 6 \Leftrightarrow g(f(-3)) = 6 \Leftrightarrow g(-4) = 6 \Leftrightarrow -4K+2 = 6 \Leftrightarrow -4K = 4 \Leftrightarrow K = -1$

$f(-3) = \frac{2}{3} \times (-3)^3 + 3 \times (-3)^2 - 13 = -18+27-13 = -4$

c)



$D'_f =]-\infty, -4] \cup]-2, +\infty[$