

① $A(x) = 3x^3 - 9x^2 - 30x + 72$

①.1 $A(-3) = 3(-3)^3 - 9(-3)^2 - 30(-3) + 72 = -81 - 81 + 90 + 72 = 0$
 logo, como $A(-3) = 0$ então $A(x)$ é divisível por $x+3$.

①.2 $d(x) = 3x - 6 = 3(x-2)$ $\alpha = 2$

②
$$\begin{array}{r|rrrr} 3 & -9 & -30 & 72 \\ & 6 & -6 & -72 \\ \hline 3 & -3 & -36 & 0 \end{array}$$

resto = 0

$Q(x) = \frac{3x^2 - 3x - 36}{3} \quad (*)$

$\Rightarrow Q(x) = x^2 - x - 12$

①.3

$3x^2 - 3x - 36 = 0$

$\Rightarrow x = \frac{3 \pm \sqrt{9 + 432}}{6}$

$\Rightarrow x = \frac{3 \pm 21}{6} \quad \begin{cases} x = 4 \\ x = -3 \end{cases}$

Decomposição:

$A(x) = 3(x-2)(x-4)(x+3)$

$A(x) = 0 \quad \Leftrightarrow$

$\Leftrightarrow 3(x-2)(x-4)(x+3) = 0 \quad \Leftrightarrow$

$\Leftrightarrow x-2=0 \vee x-4=0 \vee x+3=0$

$\Leftrightarrow x=2 \vee x=4 \vee x=-3$

$S = \{-3; 2; 4\}$

①.4 $A(x) \cdot (15-3x) > 0 \quad \Leftrightarrow 3(x-2)(x-4)(x+3)(15-3x) > 0$

	$-\infty$	-3		2		4		5	$+\infty$
3	+	+	+	+	+	+	+	+	+
$x-2$	-	-	-	0	+	+	+	+	+
$x-4$	-	-	-	-	-	0	+	+	+
$x+3$	-	0	+	+	+	+	+	+	+
$15-3x$	+	+	+	+	+	+	+	0	-
P	-	0	+	0	-	0	+	0	-
		$\uparrow$	$\uparrow$	$\uparrow$		$\uparrow$	$\uparrow$	$\uparrow$	

$15-3x = 0$

$3x = 15$

$x = 5$

$S = [-3; 2] \cup [4; 5]$

② $f(x) = x^3 - 7x - 6$

②.1 $f(2) = 2^3 - 7 \times 2 - 6 = 8 - 14 - 6 = -12 \rightarrow 2$ não é zero
 $f(3) = 3^3 - 7 \times 3 - 6 = 27 - 21 - 6 = 0 \rightarrow 3$ é zero

②.2 $f(x) > 0 \Leftrightarrow x^3 - 7x - 6 > 0$

③
$$\begin{array}{r|rrrr} 1 & 1 & 0 & -7 & -6 \\ & & 3 & 9 & 6 \\ \hline & 1 & 3 & 2 & 0 \end{array}$$

$x^2 + 3x + 2 = 0$
 $\Leftrightarrow x = \frac{-3 \pm \sqrt{9-8}}{2}$

$\Leftrightarrow x = \frac{-3 \pm 1}{2}$

$\Leftrightarrow x = (-1) \vee x = (-2)$

$x^3 - 7x - 6 > 0 \Leftrightarrow$

$\Leftrightarrow (x-3)(x+1)(x+2) > 0$

	$-\infty$	-2		-1		3	$+\infty$
$x-3$	-	-	-	-	-	0	+
$x+1$	-	-	-	0	+	+	+
$x+2$	-	0	+	+	+	+	+
P	-	0	+	0	-	0	+

$S =]-2; -1[\cup]3; +\infty[$

③ $P(x) = (a+1)x^3 - 2x^2 + x - a$

③.1 $d(x) = 2x - 4 = 2(x-2) \rightarrow d=2$

$P(2) = -12 \Leftrightarrow$

$\Leftrightarrow (a+1)2^3 - 2 \times 2^2 + 2 - a = -12 \Leftrightarrow$

$\Leftrightarrow 8(a+1) - 8 + 2 - a = -12 \Leftrightarrow$

$\Leftrightarrow 8a + 8 - 6 - a = -12$

$\Leftrightarrow 7a = -12 - 2$

$\Leftrightarrow 7a = -14$

$\Leftrightarrow a = -\frac{14}{7}$

$\Leftrightarrow a = -2$

③.2 Se $a = -2$ então $P(x) = (-2+1)x^3 - 2x^2 + x - 2 = -x^3 - 2x^2 + x - 2$

$P(x) = -x^3 - 2x^2 + x - 2$

③.1
$$\begin{array}{r|rrrr} -1 & -1 & -2 & 1 & -2 \\ & & 1 & 1 & -2 \\ \hline & -1 & -1 & 2 & 0 \end{array}$$

$-x^2 - x + 2 = 0$

$x = \frac{1 \pm \sqrt{1+8}}{-2}$

$x = \frac{1 \pm 3}{-2} \quad \begin{cases} x = (-2) \\ x = (1) \end{cases}$

Decomposição:

$P(x) = -(x+1)(x+2)(x-1)$

4) $f(x) = x^3 + 6x^2 + 5x - 12$

4.1)
$$\begin{array}{r|rrrr} -3 & 1 & 6 & 5 & -12 \\ & & -3 & -9 & 12 \\ \hline & 1 & 3 & -4 & 0 \end{array}$$

$x^2 + 3x - 4 = 0$

$x = \frac{-3 \pm \sqrt{9+16}}{2}$

$x = \frac{-3 \pm 5}{2} \begin{cases} x = 1 \\ x = -4 \end{cases}$

Decomposição

$f(x) = (x+3)(x-1)(x+4)$

$f(x) = 0 \Leftrightarrow$

$\Leftrightarrow (x+3)(x-1)(x+4) = 0$

$\Leftrightarrow x+3=0 \vee x-1=0 \vee x+4=0$

$\Leftrightarrow x = -3 \vee x = 1 \vee x = -4$

$S = \{-4; -3; 1\}$

4.2) $f(x) < 0 \Leftrightarrow$

$\Leftrightarrow (x+3)(x-1)(x+4) < 0$

	$-\infty$	-4	-3	1	$+\infty$
$x+3$	-	-	-	+	+
$x-1$	-	-	-	-	+
$x+4$	-	0	+	+	+
f	-	0	+	0	+

$S =]-\infty, -4[\cup]-3; 1[$

4.3) $f(x)$ é não injectiva, pois $f(-4) = f(-3) = f(1) = 0$ (3 os pontos diferentes com a mesma imagem).

5) 5.1) $f(x) = x^5 - 5x^3 + 4x$

$$\begin{array}{r|rrrrrr} 1 & 1 & 0 & -5 & 0 & 4 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ \hline 1 & 1 & 0 & -5 & 0 & 4 & 0 \\ 1 & 1 & 1 & 1 & -4 & -4 & \\ \hline 1 & 1 & 1 & -4 & -4 & 0 & \\ -1 & 1 & -1 & 0 & 4 & & \\ \hline 1 & 1 & 0 & -4 & 0 & & \end{array}$$

$x^2 - 4 = 0 \Leftrightarrow$

$\Leftrightarrow x^2 = 4$

$\Leftrightarrow x = \pm \sqrt{4}$

$\Leftrightarrow x = \pm 2$

Decomp.

$f(x) = x(x-1)(x+1)(x-2)(x+2)$

	$-\infty$	-2	-1	0	1	2	$+\infty$
x	-	-	-	-	+	+	+
$x-1$	-	-	-	-	-	+	+
$x+1$	-	-	-	+	+	+	+
$x-2$	-	-	-	-	-	-	+
$x+2$	-	0	+	+	+	+	+
f	-	0	+	0	-	0	+

$\oplus \forall x \text{ em }]-2; -1[\cup]0; 1[\cup]2; +\infty[$

$\ominus \forall x \text{ em }]-\infty; -2[\cup]-1; 0[\cup]1; 2[$

5.2 $P(x) = x^4 - 4x^3 + \frac{15}{4}x^2 + x - 1$

$$\begin{array}{r|rrrrr} & 1 & -4 & \frac{15}{4} & 1 & -1 \\ 2 & & 2 & -4 & -\frac{1}{2} & 1 \\ \hline & 1 & -2 & -\frac{1}{4} & \frac{1}{2} & 0 \\ 2 & & 2 & 0 & -\frac{1}{2} & \\ \hline & 1 & 0 & -\frac{1}{4} & 0 & \end{array}$$

$$x^2 - \frac{1}{4} = 0$$

$$\Leftrightarrow x^2 = \frac{1}{4}$$

$$\Leftrightarrow x = \pm \frac{1}{2}$$

$$P(x) = (x-2)(x-2)(x-\frac{1}{2})(x+\frac{1}{2})$$

Estudar o sinal ...

5.3 $P(x) = x^4 - 9x^3 + 29x^2 - 39x + 18$

$$\begin{array}{r|rrrrr} & 1 & -9 & 29 & -39 & 18 \\ 1 & & 1 & -8 & 21 & -18 \\ \hline & 1 & -8 & 21 & -18 & 0 \\ 2 & & 2 & -12 & 18 & \\ \hline & 1 & -6 & 9 & 0 & \end{array}$$

$$x^2 - 6x + 9 = 0$$

$$\Leftrightarrow x = \frac{6 \pm \sqrt{36-36}}{2} \begin{cases} x=3 \\ x=3 \end{cases}$$

$$P(x) = (x-1)(x-2)(x-3)(x-3)$$

Estudar o sinal ...

5.4 $P(x) = -4x^3 + 3x + 1$

$$\begin{array}{r|rrrr} & -4 & 0 & 3 & 1 \\ 1 & & -4 & -4 & -1 \\ \hline & -4 & -4 & -1 & 0 \end{array}$$

$$-4x^2 - 4x - 1 = 0$$

$$x = \frac{4 \pm \sqrt{16-16}}{-8} \begin{cases} x = -\frac{1}{2} \\ x = -\frac{1}{2} \end{cases}$$

$$P(x) = -4(x-1)(x+\frac{1}{2})(x+\frac{1}{2})$$

Estudar o sinal ...

5.5 $P(x) = x^4 - 4x^3 - x^2 + 4x$

$$\begin{array}{r|rrrrr} & 1 & -4 & -1 & 4 & 0 \\ 0 & & 0 & 0 & 0 & 0 \\ \hline & 1 & -4 & -1 & 4 & 0 \\ 1 & & 1 & -3 & -4 & \\ \hline & 1 & -3 & -4 & 0 & \end{array}$$

$$x^2 - 3x - 4 = 0$$

$$\Leftrightarrow x = \frac{3 \pm \sqrt{9+16}}{2}$$

$$\Leftrightarrow x = \frac{3 \pm 5}{2} \begin{cases} x=4 \\ x=-1 \end{cases}$$

$$P(x) = x(x-1)(x-4)(x+1)$$

Estudar o sinal ...

⑥ $P(x) = 2x^4 - ax^3 + bx^2 + x - 6$

⑥.1 $\begin{cases} P(-3) = 0 \\ P(-2) = 0 \end{cases} \Leftrightarrow \begin{cases} 2(-3)^4 - a(-3)^3 + b(-3)^2 - 3 - 6 = 0 \\ 2(-2)^4 - a(-2)^3 + b(-2)^2 - 2 - 6 = 0 \end{cases} \Leftrightarrow$

$\Leftrightarrow \begin{cases} 162 + 27a + 9b - 9 = 0 \\ 32 + 8a + 4b - 8 = 0 \end{cases} \Leftrightarrow \begin{cases} 27a + 9b + 153 = 0 \\ 8a + 4b + 24 = 0 \end{cases} \Leftrightarrow$

$\Leftrightarrow \begin{cases} 3a + b + 17 = 0 \\ 2a + b + 6 = 0 \end{cases} \Leftrightarrow \begin{cases} 3a + b + 17 = 0 \\ -2a - b - 6 = 0 \end{cases} \Leftrightarrow \begin{cases} 3a + b + 17 = 0 \\ a + 11 = 0 \end{cases} \Leftrightarrow$

$\Leftrightarrow \begin{cases} a = -11 \\ -22 + b + 6 = 0 \end{cases} \Rightarrow \begin{cases} a = -11 \\ b = 16 \end{cases}$

⑥.2 $P(x) = 2x^4 + 11x^3 + 16x^2 + x - 6$

$$\begin{array}{r|rrrrr} 2 & 11 & 16 & 1 & -6 \\ -3 & & -6 & -15 & -3 & 6 \\ \hline 2 & 5 & 1 & -2 & 0 \\ -2 & & -4 & -2 & 2 \\ \hline 2 & 1 & -1 & 0 \end{array}$$

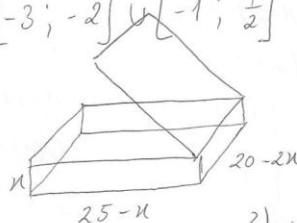
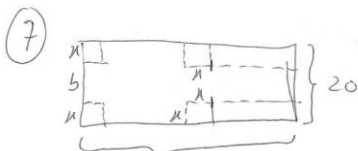
$2x^2 + x - 1 = 0 \Rightarrow x = \frac{-1 \pm \sqrt{1+8}}{4} \Rightarrow \begin{cases} x = \frac{1}{2} \\ x = -1 \end{cases}$

$P(x) = 2(x+3)(x+2)(x-\frac{1}{2})(x+1)$

$P(x) < 0 \Leftrightarrow 2(x+3)(x+2)(x-\frac{1}{2})(x+1) < 0$

	$-\infty$	-3	-2	$-\frac{1}{2}$	0	1	$+\infty$
2	+	+	+	+	+	+	+
$x+3$	-	0	+	+	+	+	+
$x+2$	-	-	0	+	+	+	+
$x-\frac{1}{2}$	-	-	-	0	+	+	+
$x+1$	-	-	-	-	0	+	+
P	+	0	-	0	+	0	+

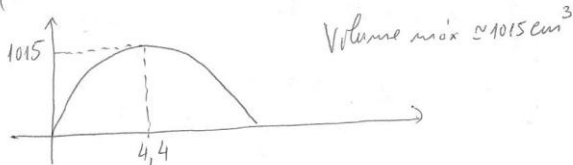
$S = [-3; -2] \cup [-1; \frac{1}{2}]$



⑦.1 $P(x) = (25-x)(20-2x) \cdot x = (500 - 50x - 20x^2 + 2x^2) \cdot x = (500 - 70x + 2x^2) \cdot x$

⑦.2 $P(x) = 2x^3 - 70x^2 + 500x$

Calculadora gráfica



Volumen max $\approx 1015 \text{ cm}^3$

73) $f(x) < 0 \Leftrightarrow x(25-x)(20-2x) < 0$

raíces: 0; 25; 10

	$-\infty$	0		10		25	$+\infty$
x	-	0	+	+	+	+	+
$25-x$	+	+	+	+	+	0	-
$20-2x$	+	+	+	0	-	-	-
P	-	0	+	0	-	0	+

$S =]-\infty, 0] \cup [10; 25]$

8) 4.º grau
1.ª raíz de multiplicidad 3
-2.ª raíz
 $P(0) = 2$

$P(x) = a(x-1)(x-1)(x-1)(x+2)$

Como $P(0) = 2 \Leftrightarrow a(-1)(-1)(-1)(2) = 2 \Leftrightarrow -2a = 2 \Leftrightarrow$

$\Leftrightarrow a = -1$

$P(x) = -(x-1)(x-1)(x-1)(x+2)$

9) $P(-2) = 1$

$P(5) = 3$

$P(x) = \frac{(x+2)(x-5)}{Q(x)}$

$R(x) = ax + b$

$P(x) = (x+2)(x-5) \cdot Q(x) + R(x) \Leftrightarrow$

$\Leftrightarrow P(x) = (x+2)(x-5) \cdot Q(x) + ax + b$

$\begin{cases} P(-2) = 1 \\ P(5) = 3 \end{cases} \Leftrightarrow \begin{cases} (-2+2)(x-5) \cdot Q(x) + a(-2) + b = 1 \\ (5+2)(5-5) \cdot Q(x) + a(5) + b = 3 \end{cases} \Leftrightarrow$

$\Leftrightarrow \begin{cases} -2a + b = 1 \\ 5a + b = 3 \end{cases} \Leftrightarrow \begin{cases} 2a - b = -1 \\ 5a + b = 3 \end{cases} \Leftrightarrow \begin{cases} b = 1 + 2a \\ 7a = 2 \end{cases}$

$\Leftrightarrow \begin{cases} b = 1 + \frac{4}{7} \\ a = \frac{2}{7} \end{cases} \quad R(x) = \frac{2}{7}x + \frac{11}{7}$

2ª Parte

① $P_{usto} = P(3) = 3 \times 3^2 - 6 \times 3 - 9 = 27 - 18 - 9 = 0$
 Operar (D)

② $3u+3 = 3(u+1) \rightarrow d = -1$

$P(-1) = (-1)^{2m} - 1 = +1 - 1 = 0$, logo $P(u)$ é divisível por $3u+3$

Operar (B)

③ $A(u) = \frac{3u^4 - u^3 - 3u + 1}{3u - 1}$ $div = 3u - 1 = 3(u - \frac{1}{3})$

	3	-1	0	-3	1
$\frac{1}{3}$		1	0	0	-1
	3	0	0	-3	0

$A(u) = \frac{3u^3 - 3}{3} = u^3 - 1$

Operar (B)

④ $P(-3) = 0$

Operar (B)

⑤ $P(-2) = 0 \Leftrightarrow (-2)^3 - 2 \times (-2)^2 - 2 - a = 0 \Leftrightarrow$

$\Leftrightarrow -8 - 8 - 2 - a = 0 \Leftrightarrow a = -18$

Operar (A)

⑥ $P(-1) = 4 \Leftrightarrow (-1)^4 - m(-1)^2 + 2(-1) - 1 = 4 \Leftrightarrow$

$\Leftrightarrow 1 - m - 2 - 1 = 4 \Leftrightarrow m = -6$

Operar (B)