Resolução da ficha de geometria nº1

① $A_O = \pi r^2 \quad P_O = 2\pi r$

$\cancel{\pi} r^2 = \cancel{\pi} \Leftrightarrow r^2 = 1 \Leftrightarrow r = \cancel{\pm} 1 \Leftrightarrow r = 1$

$P = 2\pi \times 1$

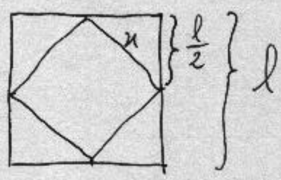
$P = 2\pi$

② $\cancel{\pi} r^2 = 2\cancel{\pi} \Leftrightarrow$

$\Leftrightarrow r = \cancel{\pm}\sqrt{2}$

$\Leftrightarrow r = \sqrt{2}$

③



Área quadrado maior $= l \times l = l^2$

$x^2 = \left(\frac{l}{2}\right)^2 + \left(\frac{l}{2}\right)^2$

$x^2 = \frac{l^2}{4} + \frac{l^2}{4}$

$x^2 = \frac{2l^2}{4}$

$x^2 = \frac{l^2}{2} \Leftrightarrow x = \sqrt{\frac{l^2}{2}} = \frac{\sqrt{l^2}}{\sqrt{2}} = \frac{l}{\sqrt{2}} = \frac{\sqrt{2}\,l}{2}$

Área do quadrado menor $= \frac{l^2}{2}$

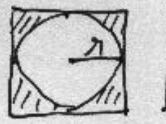
Razão entre áreas $= \frac{l^2}{\frac{l^2}{2}} = \frac{2\cancel{l^2}}{\cancel{l^2}} = 2$

Perímetro do quadrado maior $= 4l$

Perímetro do quadrado menor $= 4 \times \frac{\sqrt{2}\,l}{2} = 2\sqrt{2}\,l$

Razão entre os perímetros $= \frac{4\cancel{l}}{2\sqrt{2}\,\cancel{l}} = \frac{2}{\sqrt{2}} = \frac{\cancel{2}\sqrt{2}}{\cancel{2}} = \sqrt{2}$

④



$\} \ell$

raio $= \frac{\ell}{2}$

a) $A_{\square} = \ell^2$

$A_{\bigcirc} = \pi r^2 = \pi \frac{\ell^2}{4}$

$A_{sombreada} = \ell^2 - \pi \cdot \frac{\ell^2}{4}$

b) $3{,}14 < \pi < 3{,}15$

$\ell = 10 \rightarrow A = 100 - 25\pi$

$3{,}14 < \pi < 3{,}15$

$78{,}5 < 25\pi < 78{,}75$

$-78{,}75 < -25\pi < -78{,}5$

$21{,}25 < 100 - 25\pi < 21{,}5$

$\} \ell$

a) $A_{quadrado\ menor} = \frac{\ell^2}{2}$ (ver ex.º 3)

raio $= \frac{\sqrt{\frac{\ell^2}{2}}}{2} = \frac{\frac{\ell}{\sqrt{2}}}{2} = \frac{\ell}{2\sqrt{2}}$

$A_{\bigcirc} = \pi r^2 = \frac{\pi \ell^2}{4 \times 2} = \frac{\pi \ell^2}{8}$

$A_{sombreada} = \frac{\ell^2}{2} - \pi \frac{\ell^2}{8}$

$\ell = 10 \rightarrow A = 50 - 12{,}5\pi$

b) $3{,}14 < \pi < 3{,}15$

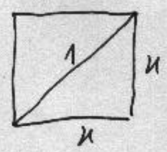
$39{,}25 < 12{,}5\pi < 39{,}375$

$-39{,}375 < -12{,}5\pi < -39{,}25$

$10{,}625 < 50 - 12{,}5\pi < 10{,}75$

Para as figuras seguintes, resoluções semelhantes.

(5)



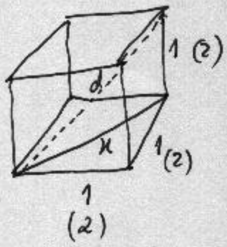
$$x^2 + x^2 = 1$$
$$2x^2 = 1$$
$$x^2 = \frac{1}{2}$$
$$x = \sqrt{\frac{1}{2}} = \frac{\sqrt{1}}{\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

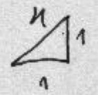
(6) $V_1 = 1 \Rightarrow V_2 = 2$

$$\Downarrow \qquad\qquad \Downarrow$$

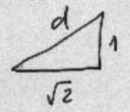
$a = \sqrt[3]{1} = 1 \qquad a = \sqrt[3]{2} \qquad$ Aresta $= \sqrt[3]{2}$

(7)

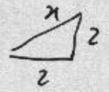




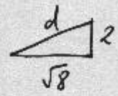
$$x^2 = 1^2 + 1^2$$
$$x = \sqrt{2}$$



$$d^2 = 1 + 2$$
$$\underline{\underline{d = \sqrt{3}}}$$



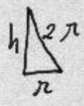
$$x^2 = 4 + 4$$
$$x^2 = 8$$
$$x = \sqrt{8} = \sqrt{4 \times 2} = \sqrt{4} \times \sqrt{2} = 2\sqrt{2}$$



$$d^2 = 4 + 8$$
$$d^2 = 12$$
$$d = \sqrt{12} = \sqrt{4 \times 3} = \sqrt{4} \times \sqrt{3} = \underline{\underline{2\sqrt{3}}}$$

aresta $a \Rightarrow \underline{\underline{d = a\sqrt{3}}}$

(8)

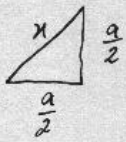


$$h^2 + r^2 = 4r^2 \Leftrightarrow h^2 = 3r^2 \Leftrightarrow h = \sqrt{3}\, r$$
$$A_\Delta = \frac{b \times h}{2} = \frac{\cancel{2} r \sqrt{3}\, r}{\cancel{2}} = \sqrt{3}\, r^2$$
$$A_O = \pi r^2$$
$$A_{\text{Sombreada}} = A_\Delta - 3 \times \frac{1}{6} A_O = \sqrt{3}\, r^2 - \frac{1}{2}\pi r^2 =$$
$$= \left(\sqrt{3} - \frac{\pi}{2}\right) r^2$$

(9)

Poliedros	N.º lados por face	Nº de faces	N.º de vértices	Nº de arestas	Faces + vértices	N.º arestas por vértices	Dual
Tetraedro	3	4	4	6	8	3	Tetraedro
Cubo	4	6	8	12	14	3	Octaedro
Octaedro	3	8	6	12	14	4	Cubo
Dodecaedro	5	12	20	30	32	3	Icosaedro
Icosaedro	3	20	12	30	32	5	Dodecaedro

(10)



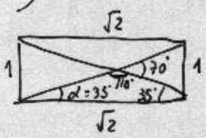
Cubo de aresta a

$$x^2 = \frac{a^2}{4} + \frac{a^2}{4} \Leftrightarrow x^2 = \frac{2a^2}{4} \Leftrightarrow x^2 = \frac{a^2}{2} \Leftrightarrow x = \frac{a}{\sqrt{2}}$$

$$\text{Razão} = \frac{a}{\frac{a}{\sqrt{2}}} = \frac{\sqrt{2}\,\cancel{a}}{\cancel{a}} = \sqrt{2}$$

Raio da esfera circunscrita = metade da aresta = $\frac{a}{2}$

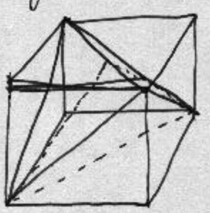
(11) Supondo um cubo de aresta 1.
Fazendo um corte no cubo passando por vértices opostos:

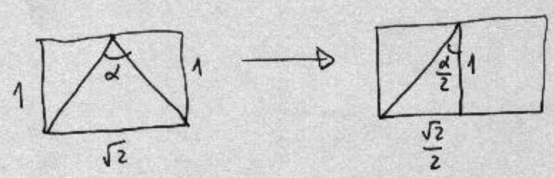


$$tg\,\alpha = \frac{1}{\sqrt{2}} \Rightarrow \alpha \simeq 35^\circ$$

Ângulo $\simeq 70^\circ$

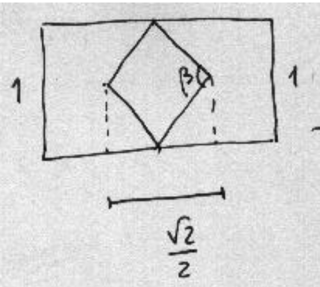
(12)
a)



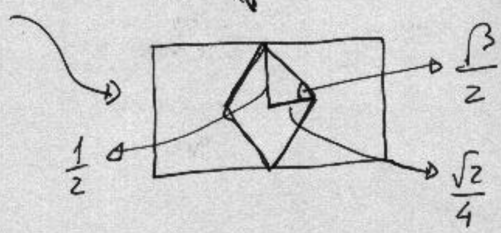


$$tg\,\frac{\alpha}{2} = \frac{\sqrt{2}}{2} \Rightarrow \frac{\alpha}{2} \simeq 35^\circ \Rightarrow \alpha \simeq 70^\circ$$

b)



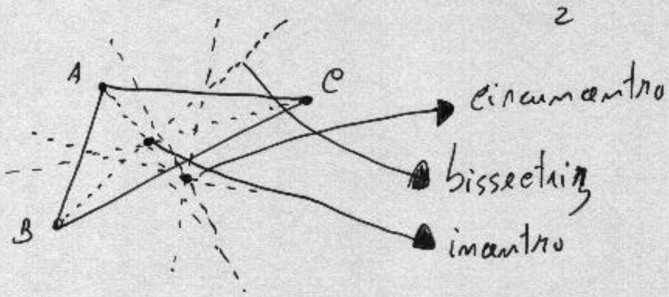
(Ver ex.º 10)



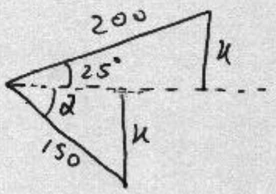
$$\operatorname{tg}\frac{\beta}{2} = \frac{\frac{1}{2}}{\frac{\sqrt{2}}{4}} = \frac{4}{2\sqrt{2}} = \frac{2}{\sqrt{2}}$$

$$\frac{\beta}{2} \simeq 55^\circ \Rightarrow \beta \simeq 110^\circ$$

(13)



(14)



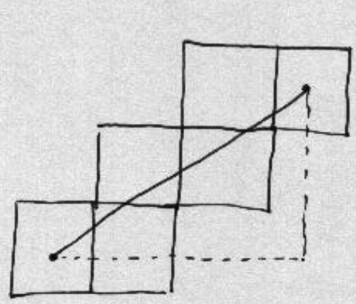
$$\operatorname{sen} 25^\circ = \frac{x}{200} \Rightarrow x \simeq 84{,}5$$

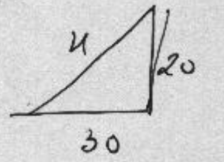
$$\operatorname{sen}\alpha = \frac{x}{150} \Rightarrow \operatorname{sen}\alpha = \frac{84{,}5}{150}$$

$$\Rightarrow \alpha \simeq 34{,}3^\circ$$

(15)

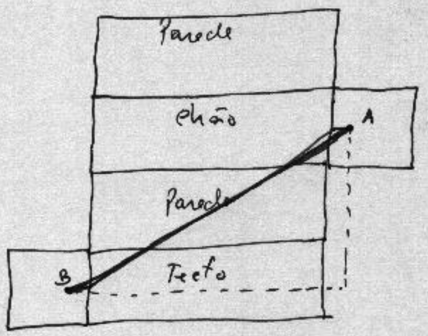
Planificando o cubo:

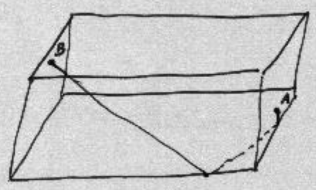


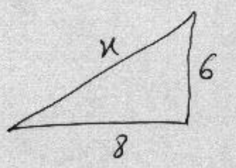


$$x^2 = 400 + 900$$
$$x^2 = 1300$$
$$x \simeq 36$$

(16) Planificando a figura:





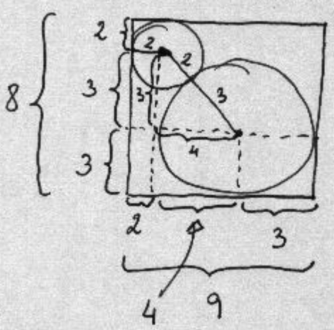


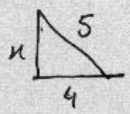
$x^2 = 8^2 + 6^2$

$x^2 = 100$

$x = \underline{\underline{10}}$

(17)





$x^2 + 16 = 25$

$x^2 = 9$

$x = 3$

Volume esfer. maior $= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi 3^3 = 36\pi$

Volume esfera menor $= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi 2^3 = \frac{32}{3}\pi$

Volume das 2 esferas $= 36\pi + \frac{32}{3}\pi \simeq$

Volume do cilindro $= Ab \times h = \pi r^2 \times 8 = \pi \times (4,5)^2 \times 8 = 162\pi$

Volume pretendido $\simeq 162\pi - 46,7\pi \simeq 115,3\pi\,cm^3$

→ Se o líquido cobrir exactamente a esfera maior fica de fora metade da esfera menor.

(18) Triláteros; quadriláteros; pentágonos e hexágonos.
→ Porque um pentágono regular não tem lados paralelos.