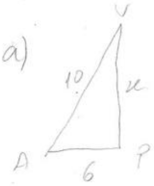
1) a)



$x^2+6^2=10^2$

$x^2=100-36$

$x^2=64$

$x=\pm 8$

$x=8$

logo $V(6,6,8)$

b) $B(12,6,0)$ $\quad M_{[BV]}=\left(\frac{12+6}{2}, \frac{6+6}{2}, \frac{0+8}{2}\right)=(9,6,4)$

$V(6,6,8)$

$C(6,12,0)$ $\quad \vec{CM}=M-C=(3,-6,4)$

$(x,y,z)=(9,6,4)+k(3,-6,4)\ ;\ k\in\mathbb{R}$

c) $P(6,6,0)$

$\left.\begin{matrix} D(0,6,0) \\ V(6,6,8) \end{matrix}\right\} \vec{DV}=(6,0,8)$

$6x+8z+D=0$

$(6,6,0) \rightarrow 6\times 6+8\times 0+D=0$

$D=-36$ Logo a equação do plano será $6x+8z-36=0$

$\Leftrightarrow 3x+4z-18=0$

2) $4x-z+1=0 \perp (4,0,-1) \parallel r$ (D)

3) $A(3,0,0)$ $\quad \vec{HC}=(-3,-1,-3)$ $\quad \cos\alpha=\frac{\vec{HC}\cdot\vec{HA}}{\|\vec{HC}\|\,\|\vec{HA}\|}=\frac{7}{\sqrt{19}\times\sqrt{13}}$

$H(3,-2,3)$ $\quad \vec{HA}=(0,2,-3)$

$C(0,-3,0)$

$\|\vec{HC}\|=\sqrt{9+1+9}=\sqrt{19}$ $\quad \cos^2\alpha=\frac{49}{19\times 13}=\frac{49}{247}$

$\|\vec{HA}\|=\sqrt{4+9}=\sqrt{13}$

$\vec{HC}\cdot\vec{HA}=-3\times 0-1\times 2-3\times(-3)=7$

$\operatorname{sen}^2\alpha=1-\cos^2\alpha=$

$=1-\frac{49}{247}=$

$=\frac{198}{247}$

④ $A(1,0,3)$

$\alpha: 3x+2y-4=0 \quad \vec{\alpha}(3,2,0)$

(A) $3x+2y-3=0 \perp (3,2,0)$ paralelo a α.

(B) $2x-3y-z+1=0 \perp (2,-3,-1)$ perpendicular a α

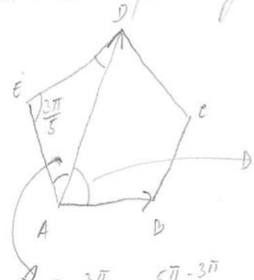
(C) $2x-3y+z=0 \perp (2,-3,1)$ perpendicular a α

(D) $3x+2y=0 \perp (3,2,0)$ paralelo a α

$A(1,0,3) \in$ ao plano $2x-3y-z+1=0$ porque $2\times1-3\times0-3+1=0$

B

⑤ Num pentágono regular, ∢ interno $= \frac{\pi\times 3}{5} = \frac{3\pi}{5} \quad \left(\frac{\pi\times(n-2)}{n}\right)$



$\rightarrow \frac{3\pi}{5} - \frac{\pi}{5} = \frac{2\pi}{5}$

$\rightarrow \frac{\pi - \frac{3\pi}{5}}{2} = \frac{\frac{5\pi-3\pi}{5}}{2} = \frac{\frac{2\pi}{5}}{2} = \frac{2\pi}{10} = \frac{\pi}{5}$

$$\frac{\vec{AB}\cdot\vec{AD}}{\|\vec{AD}\|} = \frac{\overbrace{\|\vec{AB}\|}^{1}\cancel{\|\vec{AD}\|}\cos\left(\frac{2\pi}{5}\right)}{\cancel{\|\vec{AD}\|}} = \cos\left(\frac{2\pi}{5}\right) =$$

$$= \cos\left(2\times\frac{\pi}{5}\right) = \cos^2\left(\frac{\pi}{5}\right) - \operatorname{sen}^2\left(\frac{\pi}{5}\right) =$$

$$= \overbrace{1-\operatorname{sen}^2\left(\frac{\pi}{5}\right)} - \operatorname{sen}^2\left(\frac{\pi}{5}\right)$$

$$= 1 - 2\operatorname{sen}^2\left(\frac{\pi}{5}\right)$$

⑥ $\alpha: x-y-2z=3 \perp (1,-1,-2) \parallel (-1,1,2) \parallel \vec{r}$ B

⑦ reta $AD: (x,y,z)=(3,0,0)+k(3,0,-5),\ k\in\mathbb{R}$

Como a reta AD passa em $D(0,0,z)$, então

$(0,0,z)=(3,0,0)+k(3,0,-5)$

$\Leftrightarrow \begin{cases} 0=3+3k \\ z=-5k \end{cases} \quad \begin{cases} k=-1 \\ z=5 \end{cases}$ Assim $D(0,0,5)$

Sendo $C(0,y,0)$ e $D(0,0,5)$ então $\vec{CD}=(0,-y,5)$

Assim $\|\vec{CD}\|^2 = (-y)^2+5^2 = y^2+25=41$. Logo $y^2=16 \Leftrightarrow y=\pm4$ $C(0,4,0)$

$B(3,-3,0)$ $\quad \vec{BC}=(-3,-1,0)$

$\vec{BD}=(-3,3,5)$

Seja $\vec{n}$ o vetor normal ao plano BCD

$$\begin{cases} \vec{n} \cdot \vec{BC} = 0 \\ \vec{n} \cdot \vec{BD} = 0 \end{cases} \begin{cases} (a, b, c) \cdot (-3, -1, 0) = 0 \\ (a, b, c) \cdot (-3, 3, 5) = 0 \end{cases} \begin{cases} -3a - b = 0 \\ -3a + 3b + 5c = 0 \end{cases} \begin{cases} \boxed{b = -3a} \\ -3a + 3(-3a) + 5c = 0 \end{cases}$$

$\vec{n}(a, b, c)$

$$\begin{cases} \text{———} \\ -3a - 9a + 5c = 0 \end{cases} \begin{cases} \text{———} \\ -12a + 5c = 0 \end{cases} \begin{cases} \\ \boxed{c = \frac{12}{5} a} \end{cases}$$

$$\vec{n}\left(a, -3a, \frac{12}{5} a\right)$$

Se $a = 5$ então $\vec{n}(5, -15, 12)$

(8) $m = tg\, 60° = \sqrt{3}$. Assim a equação da reta AB será $y = \sqrt{3}x + b$

b negativo

(D)

(9) $A(0,0,2)$

a)
$$1(x-0) - 2(y-0) + 1(z-2) = 0$$
$$x - 2y + z - 2 = 0$$

b)
$$M_{[AB]} = \left(\frac{0+4}{2}, \frac{0+0}{2}, \frac{2+0}{2}\right) = (2, 0, 1) = \text{Centro} = C$$

$$\overline{AC} = \text{raio} = \sqrt{(0-2)^2 + (0-0)^2 + (2-1)^2} = \sqrt{5}$$

$$(x-2)^2 + y^2 + (z-1)^2 = 5$$

c) $P(4, y, 0)$ $A(0,0,2)$ $B(4,0,0)$

$$\frac{1}{2} = \cos\left(\frac{\pi}{3}\right) = \frac{\vec{AB} \cdot \vec{AP}}{\|\vec{AB}\| \, \|\vec{AP}\|} = \frac{20}{\sqrt{20} \times \sqrt{y^2 + 20}} = \frac{20}{\sqrt{20y^2 + 400}} \Leftrightarrow \sqrt{20y^2 + 400} = 40$$

$\vec{AB} = (4, 0, -2)$ $\|\vec{AB}\| = \sqrt{16 + 4} = \sqrt{20}$

$\vec{AP} = (4, y, -2)$ $\|\vec{AP}\| = \sqrt{16 + y^2 + 4} = \sqrt{y^2 + 20}$

$\vec{AB} \cdot \vec{AP} = 16 + 4 = 20$

$$\Leftrightarrow 20y^2 + 400 = 1600$$
$$\Leftrightarrow 20y^2 = 1200$$
$$\Leftrightarrow y^2 = 60$$
$$\Leftrightarrow y = \cancel{\pm}\sqrt{60}$$
$$\Leftrightarrow y = \sqrt{60}$$

(10) $x^2 + (y-1)^2 = 2$

$(x,0) \in x^2 + (y-1)^2 = 2$

$x^2 + (0-1)^2 = 2$

$x^2 + 1 = 2$

$x^2 = 1$

$x = \pm 1$

$A(1,0)$

$A(1,0)$

$A(1,0) \quad C(0,1)$ = centro da circunf.

$\vec{AC} = (-1,1) \quad m_{\vec{AC}} = \frac{1}{-1} = -1$

$m_r = -\frac{1}{-1} = 1$

$y - y_0 = 1(x - x_0)$

$y - 0 = 1(x-1)$

$y = x - 1$ (B)

(11) $V(1,1,z) \in$ plano $6x + z - 12 = 0$

a)

$6 \times 1 + z - 12 = 0$

$6 + z - 12 = 0$

$z = 6$

$V(1,1,6)$

b) $P(2,0,0)$

$R(2,2,2)$

$\vec{OR} = (2,2,2)$

$2(x-2) + 2(y-0) + 2(z-0) = 0$

$2x - 4 + 2y + 2z = 0$

$2x + 2y + 2z - 4 = 0$

$x + y + z - 2 = 0$

c) $A(x, 2, x^3)$

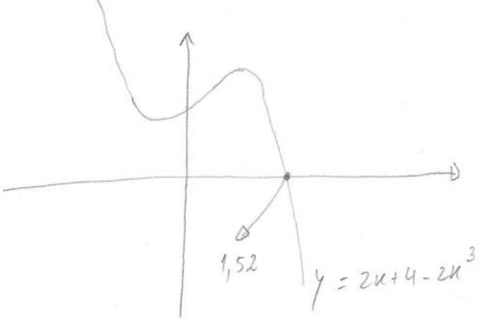
$\vec{OA} = (x, 2, x^3)$

$T(0,0,2)$

$Q(2,2,0)$

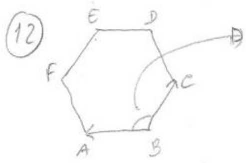
$\vec{TQ} = (2,2,-2)$

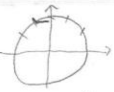
$\vec{OA} \cdot \vec{TQ} = 0 \Leftrightarrow (x,2,x^3) \cdot (2,2,-2) = 0 \Leftrightarrow 2x + 4 - 2x^3 = 0$



R: $x = 1{,}52$

12)





$\frac{180° \times 4}{6} = 120°$

$\vec{BA} \cdot \vec{BC} = \|\vec{BA}\| \, \|\vec{BC}\| \cos 120° = 2 \times 2 \times \left(-\frac{1}{2}\right) = -2$ (B)

13)

a) $\vec{OP} = (-2, 1, 3a) \parallel (2, -1, 1)$

$\frac{-2}{2} = \frac{1}{-1} = \frac{3a}{1} \Leftrightarrow 3a = -1 \Leftrightarrow a = -\frac{1}{3}$

b) $B(x, 0, 0) \in$ plano β, logo $2x - 0 + 0 - 4 = 0 \Leftrightarrow x = 2$ $\quad B(2, 0, 0)$

$\downarrow$

$C(-2, 0, 0)$

$A(1, 2, 3)$

$\vec{AB} = (1, -2, -3) \quad \|\vec{AB}\| = \sqrt{1+4+9} = \sqrt{14}$

$\vec{AC} = (-3, -2, -3) \quad \|\vec{AC}\| = \sqrt{9+4+9} = \sqrt{22}$

$\vec{AB} \cdot \vec{AC} = -3 + 4 + 9 = 10$

$\cos \alpha = \frac{10}{\sqrt{14} \times \sqrt{22}} = \frac{10}{\sqrt{308}} \Leftrightarrow \alpha = \arccos\left(\frac{10}{\sqrt{308}}\right) \simeq 55°$

c) Equação da reta $\perp$ plano β e que passa na origem:

$(x, y, z) = (0, 0, 0) + k(2, -1, 1) \; ; \; k \in \mathbb{R}$

Intersecção da reta com o plano β

$$\begin{cases} x = 2k \\ y = -k \\ z = k \\ 2x - y + z - 4 = 0 \end{cases} \quad \begin{cases} \\ \\ \\ 2(2k) - (-k) + k - 4 = 0 \end{cases} \quad \begin{cases} \\ \\ \\ 4k + k + k - 4 = 0 \end{cases} \quad \begin{cases} \\ \\ \\ 6k - 4 = 0 \end{cases} \quad \begin{cases} x = \frac{4}{3} \\ y = -\frac{2}{3} \\ z = \frac{2}{3} \\ k = \frac{2}{3} \end{cases}$$

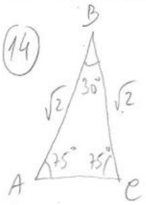
Ponto de tangência da reta c/ o plano β = $T = \left(\frac{4}{3}, -\frac{2}{3}, \frac{2}{3}\right)$

Raio $= \overline{OT} = \sqrt{\frac{16}{9} + \frac{4}{9} + \frac{4}{9}} = \sqrt{\frac{24}{9}}$

Centro $(0, 0, 0)$

$x^2 + y^2 + z^2 = \frac{24}{9}$

(14)



$\overrightarrow{BA} \cdot \overrightarrow{BC} = \sqrt{2} \times \sqrt{2} \times \cos 30° =$

$= 2 \times \frac{\sqrt{3}}{2} =$

$= \sqrt{3}$ (C)

(15)

a) Raio = 1 $\qquad (x+1)^2 + (y-1)^2 + (z-1)^2 = 1$

Centro $(-1, 1, 1)$

b) $M_{[AC]} = \left(\frac{-1-3}{2}, \frac{1+3}{2}, \frac{1+1}{2}\right) = (-2, 2, 1)$ logo $V(-2, 2, z)$

$V(-2, 2, z) \in$ plano $3y + z - 10 = 0$

$3 \times 2 + z - 10 = 0$

$6 + z - 10 = 0$ $\qquad$ logo $V(-2, 2, 4)$

$z = 4$

c) Equação do plano α:

$\overrightarrow{AC} = (-2, 2, 0)$

$A(-1,1,1)$

$C(-3,3,1)$

$P(1, -2, -1) \in \alpha$

$-2(x-1) + 2(y+2) + 0(z+1) = 0$

$-2x + 2 + 2y + 4 = 0$

$-2x + 2y + 6 = 0$

$x - y - 3 = 0$

Intersecção dos planos:

$\begin{cases} x - y - 3 = 0 \\ 3y + z - 10 = 0 \end{cases} \quad \begin{cases} \boxed{y = x - 3} \\ 3y = -z + 10 \end{cases} \quad \begin{cases} \text{---} \\ y = \frac{-z+10}{3} \end{cases} \quad \begin{cases} \\ y = \frac{z-10}{-3} \end{cases}$

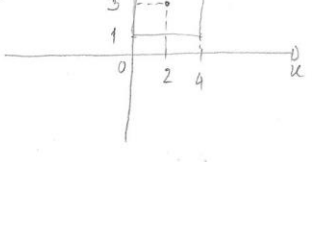
Equação cartesiana da reta

$x - 3 = y = \frac{z-10}{-3}$

Equação vetorial da reta

$(x, y, z) = (3, 0, 10) + k(1, 1, -3)\ ;\ k \in \mathbb{R}$

(16)



(C)

(17)

a) $(x,y,z)=(2,1,4)+k(3,2,4)\ ;\ k\in\mathbb{R}$

b) Reta OD

$O(0,0,0)$ $\quad \overrightarrow{OD}=(4,2,2)$

$D(4,2,2)$

$(x,y,z)=(0,0,0)+k(4,2,2)\ ;\ k\in\mathbb{R}$

Intersecção da reta OD c/ o plano α:

$$\begin{cases} x=4k \\ y=2k \\ z=2k \\ 3x+2y+4z-12=0 \end{cases} \quad \begin{cases} \\ \\ \\ 3\times4k+2\times2k+4\times2k-12=0 \end{cases} \quad \begin{cases} \\ \\ \\ 12k+4k+8k-12=0 \end{cases} \quad \begin{cases} x=2 \\ y=1 \\ z=1 \\ k=\frac{1}{2} \end{cases}$$

Ponto de intersecção $=(2,1,1)$

c) $A(\overset{+}{x},0,0)$ $\quad \overrightarrow{PA}=(x,0,-z)$ $\quad \overrightarrow{PA}\cdot\overrightarrow{PB}=0+0+z^2=z^2>0$

$B(0,\overset{+}{y},0)$ $\quad \overrightarrow{PB}=(0,y,-z)$ $\quad \cos\alpha=\dfrac{\overbrace{\overrightarrow{PA}\cdot\overrightarrow{PB}}^{\oplus}}{\underbrace{\|\overrightarrow{PA}\|}_{\oplus}\underbrace{\|\overrightarrow{PB}\|}_{\oplus}}>0$, logo α é um $\measuredangle$ agudo

$P(0,0,\underset{\neq 0}{z})$

(18) $1+tg^2\alpha = \frac{1}{\cos^2\alpha} \Leftrightarrow 1+tg^2\alpha = \frac{1}{\left(-\frac{1}{\sqrt{5}}\right)^2} \Leftrightarrow 1+tg^2\alpha = \frac{1}{\frac{1}{5}} \Leftrightarrow 1+tg^2\alpha = 5 \Leftrightarrow tg^2\alpha = 4 \Leftrightarrow$

$\Leftrightarrow tg\,\alpha = \pm 2$. Assim, $m = \pm 2$ (C)

Nota $m = tg\,\alpha$

(19.1) $T(0,0,3)$ Centro $(0,0,0)$ $x^2+y^2+z^2=9$

$T'(0,0,-3)$ $r=3$

(19.2) $\overrightarrow{UP} \cdot \overrightarrow{RS} = \|\overrightarrow{UP}\| \|\overrightarrow{RS}\| \cos\pi = 3 \times 3 \times (-1) = -9$

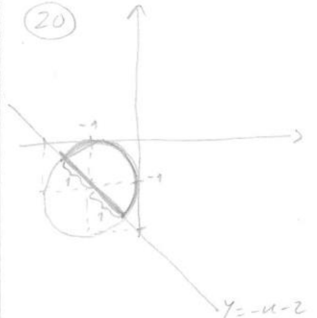
(19.3) $Q(0,y,0) \in$ plano $x+y=2$, logo $0+y=2 \Leftrightarrow y=2$

Assim $Q(0,2,0)$

$\overrightarrow{TQ} = Q-T = (0,2,0)-(0,0,3) = (0,2,-3)$

equação vetorial: $(x,y,z) = (0,0,3)+K(0,2,-3)$; $K \in \mathbb{R}$

(20)



$(x+1)^2+(y+1)^2 \leq 1$ | $x+y+2 \geq 0$

$c(-1,-1)$, $r=1$ | $y \geq -x-2$

$P = \frac{\cancel{2}\pi r}{\cancel{2}} + 2 = \pi \times 1 + 2 = \pi + 2$ (C)

(21.1) $A(x,4,0) \in$ plano $x+y-z-6=0$, logo $x+4-0-6=0 \Leftrightarrow x=2$ c.q.d.

(21.2) $r: (x,y,z) = (1,1,0)+K(1,-1,1)$, $K \in \mathbb{R}$

$\Leftrightarrow \begin{cases} x = 1+K \\ y = 1-K \\ z = K \end{cases}$, $K \in \mathbb{R}$

Intersecção da reta r c/ o plano $x+y-z-6=0$

$1+\cancel{K}+1-\cancel{K}-K-6=0$

$-K=4$

$K=-4$

Ponto de intersecção $(1-4, 1+4, -4) = (-3,5,-4)$

(21.3) $V_{pirâmide} = 4 \Leftrightarrow$

$\frac{1}{3} \times 2 \times 2 \times h = 4 \Leftrightarrow$

$\Leftrightarrow \frac{4}{3}h = 4 \Leftrightarrow$

$\Leftrightarrow h = 3$

$P(1,5,5)$ ← $2+3$

$G(2,6,2)$

$O(0,0,0)$

$\overrightarrow{GO} = (-2,-6,-2)$ $\|\overrightarrow{GO}\| = \sqrt{(-2)^2+(-6)^2+(-2)^2} = \sqrt{44}$

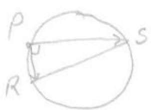
$\overrightarrow{GP} = (-1,-1,3)$ $\|\overrightarrow{GP}\| = \sqrt{(-1)^2+(-1)^2+3^2} = \sqrt{11}$

$\overrightarrow{GO} \cdot \overrightarrow{GP} = 2+6-6 = 2$

$\cos\alpha = \frac{\overrightarrow{GO} \cdot \overrightarrow{GP}}{\|\overrightarrow{GO}\| \|\overrightarrow{GP}\|} = \frac{2}{\sqrt{44} \times \sqrt{11}} = \frac{1}{11} \Leftrightarrow \alpha = \arccos\left(\frac{1}{11}\right)$

$\Leftrightarrow \alpha \simeq 85°$

(22)



$\overrightarrow{PR} \cdot \overrightarrow{PS} = 0$

(D)

(23.1)

$A = 2 \times 3 = 6$

(23.2) $B(1,3,0)$

$C(1,2,3)$

$\overrightarrow{BC} = (0,-1,3)$

reta BC: $(x,y,z) = (1,3,0) + K(0,-1,3)$; $K \in \mathbb{R}$

$$\begin{cases} x = 1 \\ y = 3 - K \\ z = 3K \end{cases} ; K \in \mathbb{R}$$

Intersecção da reta BC c/ o plano xOz $(y=0)$

$3 - K = 0 \Leftrightarrow K = 3$

Ponto de intersecção $(1, 3-3, 3\times3) = (1,0,9)$

(23.3) reta: $\begin{cases} x = K \\ y = K \\ z = 1 - K \end{cases}$; $K \in \mathbb{R}$ $\longrightarrow$ vetor $(1,1,-1) \parallel \alpha$

Plano α:

vetor normal $(1,1,-1)$

Ponto $A(1,2,0)$

$Ax + By + Cz + D = 0$

$x + y - z + D = 0$

$1 + 2 - 0 + D = 0$

$D = -3$

$\alpha: x + y - z - 3 = 0$

$P(2,2,z) \in$ plano $x+y-z-3=0$, logo

$2 + 2 - z - 3 = 0 \Leftrightarrow z = 1$

Assim, $P(2,2,1)$

$O(0,0,0)$

$C(1,2,3)$

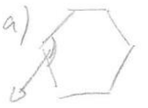
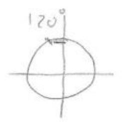
$\overrightarrow{OP} = (2,2,1)$ $\|\overrightarrow{OP}\| = \sqrt{2^2+2^2+1^2} = \sqrt{9} = 3$

$\overrightarrow{OC} = (1,2,3$ $\|\overrightarrow{OC}\| = \sqrt{1^2+2^2+3^2} = \sqrt{14}$

$\overrightarrow{OP} \cdot \overrightarrow{OC} = 2\times1 + 2\times2 + 1\times3 = 2 + 4 + 3 = 9$

Seja $\alpha = \measuredangle POC$, $\cos\alpha = \dfrac{\overrightarrow{OP} \cdot \overrightarrow{OC}}{\|\overrightarrow{OP}\| \|\overrightarrow{OC}\|} = \dfrac{9}{3\sqrt{14}} = \dfrac{3}{\sqrt{14}}$

$\alpha = \arccos\left(\dfrac{3}{\sqrt{14}}\right) \approx 37^\circ$

24.a)  $\overrightarrow{QP}\cdot\overrightarrow{QR} = \|\overrightarrow{QP}\|\,\|\overrightarrow{QR}\|\cos 120° =$ 

$= 4\times 4\times\left(-\frac{1}{2}\right) = -8$

$\frac{(6-2)\times 180°}{6} = 120°$

24.b) Reta PS: $(x,y,z)=(14,5,0)+k(2,3,-1),\ k\in\mathbb{R}$

Interseção da reta PS com o plano PQR:

$\begin{cases} x = 14+2k \\ y = 5+3k \\ z = -k \\ 2x+3y-z-15=0 \end{cases}$ $\begin{cases} \text{—} \\ \text{—} \\ \text{—} \\ 2(14+2k)+3(5+3k)-(-k)-15=0 \end{cases}$ $\begin{cases} \text{—} \\ \text{—} \\ \text{—} \\ 4k+9k+k = \cancel{15}-28-\cancel{15} \end{cases}$

$\begin{cases} x = 10 \\ y = -1 \\ z = 2 \\ k = -2 \end{cases}$ Assim $P(10,-1,2)$. Logo $\overline{PS} = \sqrt{(14-10)^2+(5+1)^2+(0-2)^2} = \sqrt{56}$

Conclui-se que a área lateral $= 4\times\sqrt{56}\times 6 = 24\sqrt{56} \approx 179{,}6$

25.a) $P(1,3,z)$, $z<0$

Como $P \in$ sup. esférica, então $(1-1)^2+(3-2)^2+(z+1)^2 = 10 \Leftrightarrow (z+1)^2 = 9 \Leftrightarrow$

$\Leftrightarrow z+1=3 \vee z+1=-3 \Leftrightarrow z=2 \vee z=-4$. Como $z<0$, logo $P(1,3,-4)$

Plano pedido: $4x+y-2z+d=0$

$(1,3,-4)\in$ $4\times1+3-2\times(-4)+d=0$

$4+3+8+d=0$

$d=-15$ Assim, a equação do plano pedido é $4x+y-2z-15=0$

25.b) $C(1,2,-1)$ $\overrightarrow{OA}=(1,2,1)$ $\overrightarrow{OA}\cdot\overrightarrow{OC} = 1+4-1 = 4$

$A(1,2,1)$ $\overrightarrow{OC}=(1,2,-1)$ $\cos\alpha = \frac{\overrightarrow{OA}\cdot\overrightarrow{OC}}{\|\overrightarrow{OA}\|\|\overrightarrow{OC}\|} = \frac{4}{6} = \frac{2}{3}$

$O(0,0,0)$ $\|\overrightarrow{OA}\| = \sqrt{1^2+2^2+1^2} = \sqrt{6}$

$\|\overrightarrow{OC}\| = \sqrt{1^2+2^2+(-1)^2} = \sqrt{6}$ $\alpha = \arccos\left(\frac{2}{3}\right) \approx 48°$

26) $\overrightarrow{OA}=(2,1)$

$m=\frac{1}{2}$ logo $m_r=-2$

Reta r: $y=-2x+b$

Como $A(2,1)\in r$, $1=-2\times2+b \Leftrightarrow b=5$ (B)

27. a) $\overrightarrow{OP}=(1,1,1)$

$\vec{u}=-2\overrightarrow{OP}=(-2,-2,-2)$

$Q=P+\vec{u}=(1,1,1)+(-2,-2,-2)=(-1,-1,-1)$

Como $Q=P-2\overrightarrow{OP}=P+2\overrightarrow{PO}$, então Q é o simétrico de P, relativ/ à origem. Sendo P um ponto de sup. esf., [PQ] será um diâmetro dessa sup. esf.

27. b) $R(0,y,0)$ com $y<0$

Como $R \in$ sup. esf., então $0^2+y^2+0^2=3 \Leftrightarrow y=\pm\sqrt{3}$. Como $y<0$, $R(0,-\sqrt{3},0)$

$\overrightarrow{OR}=(0,-\sqrt{3},0)$ $\quad \|\overrightarrow{OR}\|=\sqrt{0+(-\sqrt{3})^2+0^2}=\sqrt{3}$

$\overrightarrow{OP}=(1,1,1)$ $\quad \|\overrightarrow{OP}\|=\sqrt{1^2+1^2+1^2}=\sqrt{3}$

$\overrightarrow{OR}\cdot\overrightarrow{OP}=0-\sqrt{3}+0=-\sqrt{3}$

$\cos\alpha=\dfrac{-\sqrt{3}}{\sqrt{3}\times\sqrt{3}}=-\dfrac{\sqrt{3}}{3}$

$\alpha=\arccos\left(-\dfrac{\sqrt{3}}{3}\right)\approx 125°$

28) $m_r=-\dfrac{a}{2}$ $\qquad m_s=\dfrac{2a}{a}=2$

$ax+2y+1=0$

$2y=-ax-1$

$y=\left(-\dfrac{a}{2}\right)x-\dfrac{1}{2}$

Então, $-\dfrac{a}{2}=2 \Leftrightarrow a=-4$ (A)

29 a) $A(2,1,0)$ $\quad C(0,-1,2)$ $\quad V(3,-1,2)$

$\overrightarrow{AV}=V-A=(3,-1,2)-(2,1,0)=(1,-2,2)$

$\overrightarrow{AC}=C-A=(0,-1,2)-(2,1,0)=(-2,-2,2)$

$\cos\alpha=\dfrac{\overrightarrow{AV}\cdot\overrightarrow{AC}}{\|\overrightarrow{AV}\|\,\|\overrightarrow{AC}\|}=\dfrac{(1,-2,2)\cdot(-2,-2,2)}{\sqrt{1^2+(-2)^2+2^2}\times\sqrt{(-2)^2+(-2)^2+2^2}}=\dfrac{-2+4+4}{\sqrt{9}\times\sqrt{12}}$

$\cos\alpha=\dfrac{6}{3\sqrt{12}} \Leftrightarrow \alpha=\arccos\left(\dfrac{6}{3\sqrt{12}}\right)\approx 55°$

b) Seja M o centro da base e ponto médio de [AC]

$M=\left(\dfrac{2+0}{2},\dfrac{1-1}{2},\dfrac{0+2}{2}\right)=(1,0,1)$

$\overrightarrow{VM}=M-V=(1,0,1)-(3,-1,2)=(-2,1,-1)\perp$ plano que contém a base

$-2x+y-z+d=0$

Como $(1,0,1)\in$ plano, logo $-2\times1+0-1+d=0 \Leftrightarrow d=3$

Assim, o plano tem equação $-2x+y-z+3=0$

(30) O ponto $(3,4)$ pertence à reta $y=\frac{4x}{3}$ e a sua distância à origem é $\sqrt{9+16}=5$

O ponto $(5,0)$ pertence ao eixo das abcissas e a sua distância à origem também é 5

Assim, a reta r será a mediatriz do segmento de reta formado por estes 2 pontos

$(x-3)^2+(y-4)^2=(x-5)^2+(y-0)^2 \Leftrightarrow \cancel{x^2}-6x+9+\cancel{y^2}-8y+16=\cancel{x^2}-10x+25+\cancel{y^2}$

$\Leftrightarrow -8y=6x-10x\ \cancel{-9-16+25} \Leftrightarrow y=\frac{1}{2}x$

31.a) $A(x,0,0) \in$ plano $3x+4y-12=0$, logo $3x+0-12=0 \Leftrightarrow x=4$ $A(4,0,0)$

$B(0,y,0) \in$ " " , " $0+4y-12=0 \Leftrightarrow y=3$ $B(0,3,0)$

$A(4,0,0)$ $B(0,3,0)$ $C(0,3,6)$ $G(6,11,0)$

$\overline{AB}=\sqrt{4^2+3^2+0^2}=5$

$\overline{BC}=\sqrt{0^2+0^2+6^2}=6$

$\overline{BG}=\sqrt{6^2+8^2+0^2}=10$

$V=5\times6\times10=300$

31.b) $P(1,-4,3)$

reta r $\hookrightarrow$ $(x,y,z)=(1,-4,3)+K(3,4,0)$, $K\in\mathbb{R}$

$\begin{cases} x=1+3K \\ y=-4+4K \\ z=3 \\ 3x+4y-12=0 \end{cases}$ $\begin{cases} \text{---} \\ \text{---} \\ \text{---} \\ 3+9K-16+16K-12=0 \end{cases}$ $\begin{cases} \text{---} \\ \text{---} \\ \text{---} \\ 25K=25 \end{cases}$ $\begin{cases} x=1+3\times1 \\ y=-4+4\times1 \\ z=3 \\ K=1 \end{cases}$ $\begin{cases} x=4 \\ y=0 \\ z=3 \end{cases}$

Ponto de interseção $(4,0,3)$

(32) $A(x,0) \in$ reta $y=2x+4$, logo $0=2x+4 \Leftrightarrow x=-2$ $A(-2,0)$

$B(0,y) \in$ reta $y=2x+4$, logo $y=2\times0+4 \Leftrightarrow y=4$ $B(0,4)$

$M=\left(\frac{-2+0}{2},\frac{0+4}{2}\right)=(-1,2)$ (B)

(33.a) Ponto $A(x,4,z) \in$ reta r, logo $(x,4,z)=(1,2,1)+K(0,1,5)$, $K\in\mathbb{R}$

$\Leftrightarrow \begin{cases} x=1 \\ 4=2+K \\ z=1+5.K \end{cases} \Leftrightarrow \begin{cases} x=1 \\ K=2 \\ z=1+5\times2 \end{cases} \Leftrightarrow \begin{cases} x=1 \\ \text{---} \\ z=11 \end{cases}$ Assim, $A(1,4,11)$

O plano é da forma $2x+3y-z+d=0$.

Como $A(1,4,11)$ pertence ao plano, então $2\times1+3\times4-11+d=0 \Leftrightarrow$

$\Leftrightarrow 2+12-11+d=0 \Leftrightarrow 3+d=0 \Leftrightarrow d=-3$ Equação do plano: $2x+3y-z-3=0$

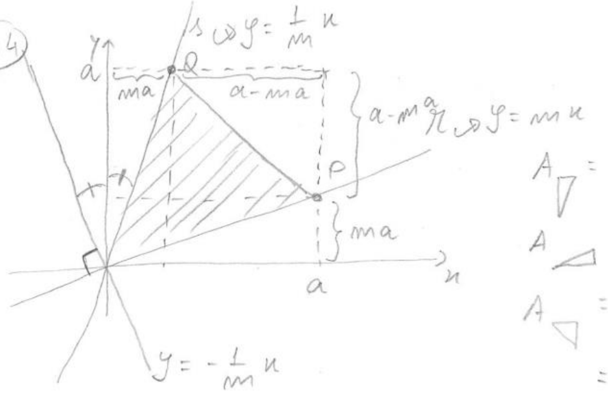
33.5

$$\begin{cases} x=1 \\ y=2+k \\ z=1+5k \\ 2x+3y-z-9=0 \end{cases}, k\in\mathbb{R} \Leftrightarrow \begin{cases} \_\_\_ \\ \_\_\_ \\ \_\_\_ \\ 2\times 1+3(2+k)-(1+5k)-9=0 \end{cases} \begin{cases} \_\_\_ \\ \_\_\_ \\ \_\_\_ \\ 2+6+3k-1-5k-9=0 \end{cases}$$

$$\begin{cases} \_\_\_ \\ \_\_\_ \\ \_\_\_ \\ -2k=2 \end{cases} \begin{cases} x=1 \\ y=1 \\ z=-4 \\ k=-1 \end{cases}$$

Ponto intersecção $(1,1,-4)$

34



$P(a, ma)$

$Q(ma, a)$

$$A_{\triangledown} = \frac{ma\times a}{2} = \frac{ma^2}{2}$$

$$A_{\triangle} = A_{\triangledown} = \frac{ma^2}{2}$$

$$A_{\triangle} = \frac{(a-ma)^2}{2} = \frac{a^2-2ma^2+m^2a^2}{2} = \frac{a^2}{2} - ma^2 + \frac{m^2a^2}{2}$$

$$A_{\square} = a\times a = a^2$$

$$A_{pedida} = a^2 - \frac{a^2}{2} + ma^2 - \frac{m^2a^2}{2} - \frac{ma^2}{2} - \frac{ma^2}{2} =$$

$$= \frac{a^2}{2} + \cancel{ma^2} - \frac{m^2a^2}{2} \cancel{-\frac{2ma^2}{2}} =$$

$$= \frac{a^2}{2} - \frac{m^2a^2}{2} = \frac{a^2}{2}(1-m^2) \text{ c.q.d.}$$

35.a $A(0,y,0) \in$ plano ABC, logo $3\times 0+4y+4\times 0-12=0 \Leftrightarrow y=3$

$A(0,3,0)$

$B(x,0,0) \in$ plano ABC, logo $3x+4\times 0+4\times 0-12=0 \Leftrightarrow x=4$

$B(4,0,0)$

$h =$ altura do cilindro $= \overline{AB} = \sqrt{(0-4)^2+(3-0)^2+(0-0)^2} = 5$

$V_{\text{cil}} = 10\pi \Leftrightarrow \pi\times\overline{BC}^2\times 5 = 10\pi \Leftrightarrow \overline{BC}^2 = 2 \underset{\overline{BC}>0}{\Leftrightarrow} \overline{BC} = \sqrt{2}$

(35.5) Seja r a reta que contém P e que é perpendicular ao plano ABC. Pretende-se obter as coordenadas do ponto de intersecção, I, da reta r com o plano ABC

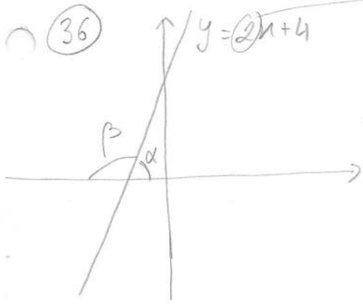
$r: (x, y, z) = (3, 5, 6) + K(3, 4, 4), K \in \mathbb{R}$

$$\begin{cases} x = 3+3K \\ y = 5+4K \quad , K \in \mathbb{R} \\ z = 6+4K \\ 3x+4y+4z-12=0 \end{cases} \Leftrightarrow \begin{cases} \equiv \\ \equiv \\ \equiv \\ 3(3+3K)+4(5+4K)+4(6+4K)-12=0 \end{cases}$$

$$\Leftrightarrow \begin{cases} \equiv \\ 9+9K+20+16K+24+16K-12=0 \end{cases} \Leftrightarrow \begin{cases} \equiv \\ 41K=-41 \end{cases} \Leftrightarrow \begin{cases} x = 3-3 \\ y = 5-4 \\ z = 6-4 \\ K=-1 \end{cases} \Leftrightarrow \begin{cases} x=0 \\ y=1 \\ z=2 \\ - \end{cases}$$

$I(0, 1, 2)$

(36) $y = 2x + 4$



$tg\,\alpha = 2$

$\alpha = arctg(2)$

$\beta = \pi - arctg(2) \simeq 2,03$ (C)

37.a) $\vec{n}(3,-6,2)$

$3x-6y+2z+d=0$

$(5,3,6) \hookrightarrow 3\times5-6\times3+2\times6+d=0 \Leftrightarrow 15-18+12+d=0 \Leftrightarrow d=-9$

eq.ª plano EFG: $3x-6y+2z-9=0$

37.b) $A(7,1,4)$ $M_{[A,G]}=\left(\frac{7+5}{2}, \frac{1+3}{2}, \frac{4+6}{2}\right)=(6,2,5)=$ centro

$G(5,3,6)$

raio $=\overline{AM}=\sqrt{(7-6)^2+(1-2)^2+(4-5)^2}=\sqrt{3}$

equação da sup. esf. $(x-6)^2+(y-2)^2+(z-5)^2=3$

38.a) $\vec{BE}=(2,2,-2) \Leftrightarrow E-B=(2,2,-2) \Leftrightarrow E=(0,2,4)+(2,2,-2) \Leftrightarrow E=(2,4,2)$

$\vec{BO}=(0,-2,-4)$

$\vec{BE}=(2,2,-2)$

$\|\vec{BO}\|=\sqrt{0+(-2)^2+(-4)^2}=\sqrt{20}$

$\|\vec{BE}\|=\sqrt{2^2+2^2+(-2)^2}=\sqrt{12}$

$\vec{BO}\cdot\vec{BE}=0\times2+2\times(-2)+(-2)\times(-4)=4$

$\cos\alpha=\frac{\vec{BO}\cdot\vec{BE}}{\|\vec{BO}\|\,\|\vec{BE}\|}=\frac{4}{\sqrt{20}\times\sqrt{12}}$

$\cos\alpha=\frac{4}{\sqrt{240}}$

$\alpha=\arccos\left(\frac{4}{\sqrt{240}}\right)\approx 75^\circ$

38.b) $\vec{OE}=(2,4,2)$

$2x+4y+2z+d=0$

$G(0,2,2) \hookrightarrow 2\times0+4\times2+2\times2+d=0$

$\Leftrightarrow d=-12$

$\alpha \hookrightarrow 2x+4y+2z-12=0$

$P(x,0,0) \hookrightarrow 2x+4y+2z-12=0$

$2x-12=0 \Leftrightarrow x=6$

$P(6,0,0)$

$Q(0,y,0)$ (...) $Q(0,3,0)$

$R(0,0,z)$ (...) $R(0,0,6)$

$A_{base}=\frac{6\times3}{2}=9$

altura $=6$

$V=\frac{9\times6}{3}=18$

39) ∢ interno do hexágono $=\frac{4\times180^\circ}{6}=120^\circ$

∢ NMQ $=60^\circ \Rightarrow m=\operatorname{tg}120^\circ=-\sqrt{3}$

$y=-\sqrt{3}x+b$

$(1,0) \hookrightarrow 0=-\sqrt{3}\times1+b$

$\sqrt{3}=b$ (A)

40.a) (C) $(x, y, z) = (7, -10, 3) + k(0, 3, 3), k \in \mathbb{R}$

$\hookrightarrow (0, 3, 3) \cdot (-3, -2, 2) = 0 - 6 + 6 = 0$

logo a reta é $\perp$ à reta EF

$E(7, 2, 15) \in$ reta desta opção

$(7, 2, 15) = (7, -10, 3) + k(0, 3, 3)$

$$\begin{cases} 7 = 7 \\ 2 = -10 + 3k \\ 15 = 3 + 3k \end{cases} \quad \begin{cases} - \\ 3k = 12 \\ 3k = 12 \end{cases} \quad \begin{cases} k = 4 \\ k = 4 \end{cases}$$

- Nas restantes opções, a reta não é $\perp$ à reta EF ou não contém o ponto E

40.b) Plano ABF $\perp (-3, -2, 2)$

$-3x - 2y + 2z + d = 0$

$G(6, 10, 13) \hookrightarrow -3 \times 6 - 2 \times 10 + 2 \times 13 + d = 0 \Leftrightarrow \dots \Leftrightarrow d = 12$

Plano ABF: $-3x - 2y + 2z + 12 = 0$

$B(0, y, 0) \in$ plano ABF, logo $-3 \times 0 - 2y + 2 \times 0 + 12 = 0$
$\Leftrightarrow \dots \Leftrightarrow y = 6$

Assim, $B(0, 6, 0)$

Raio $= \overline{BD} = \overline{EG} = \sqrt{(7-6)^2 + (2-10)^2 + (15-13)^2} = \dots = \sqrt{69}$

equação pedida: $x^2 + (y-6)^2 + z^2 = 69$

41.a) $\overline{RQ} = \sqrt{(-2+5)^2 + (1-5)^2 + (1+3)^2} = \sqrt{9 + 16 + 16} = \sqrt{41}$, logo opção C

41.b) $\overrightarrow{RS} \parallel \overrightarrow{PQ}$

$\overrightarrow{PQ} = Q - P = (-2, 1, 1) - (1, -1, 2) = (-3, 2, -1)$

Plano: $-3x + 2y - z + d = 0$

$P(1, -1, 2) \hookrightarrow -3 \times 1 + 2 \times (-1) - 2 + d = 0$

$-3 - 2 - 2 + d = 0$

$d = 7$

Plano pedido: $-3x + 2y - z + 7 = 0$

$\Leftrightarrow 3x - 2y + z - 7 = 0$

(42.a) Opção A: não contém o ponto $(1,0,1)$

Opção B: não porque o vetor diretor do plano $(1,6,2)$ não é paralelo ao vetor diretor da reta $(-1,6,2)$

Opção D: não contém o ponto $(1,0,1)$

Opção C: é a correta pois contém $(1,0,1)$ e é $\perp$ reta BE

(42.b) Seja I o centro da base da pirâmide.

$I(-2,0,2)$

$\overline{EI} = \sqrt{(-2+2)^2+(6-0)^2+(2-2)^2} = \sqrt{0+36+0} = 6$

$\frac{\overline{AB}^2 \times 6}{3} = 20 \Leftrightarrow \overline{AB}^2 = 10 \underset{\overline{AB}>0}{\Leftrightarrow} \overline{AB} = \sqrt{10}$

$B = E + \overrightarrow{EB} = (-2,6,2)+(1,-6,-2) = (-1,0,0)$

$\overline{OA}^2 + 1^2 = (\sqrt{10})^2$

$\Leftrightarrow \overline{OA}^2 = 9$

$\Leftrightarrow \overline{OA} = 3$

Então $A(0,0,3)$ e $B(-1,0,0)$

Assim $\overrightarrow{AB} = B - A = (-1,0,0)-(0,0,3) = (-1,0,-3)$

(43.a) $A(0,y,0) \in$ plano de equação $4y-3z=16$

Assim $4y-3\times 0=16 \Leftrightarrow y=4$. Então $A(0,4,0)$

Opção D

$x+3y+4z=3 \perp$ plano definido por $4y-3z=16$ pois

$(1,3,4).(0,4,-3) = 1\times 0+3\times 4+4\times(-3)=0$

$(1,2,-1) \in$ plano definido por $x+3y+4z=3$ pois

$1+3\times 2+4\times(-1)=3$ Verdade

(43.b) reta AV: $(x,y,z)=(0,4,0)+k(0,4,-3)$, $k \in \mathbb{R}$

$V(0,0,z) \in$ reta AV, logo

$(0,0,z)=(0,4,0)+k(0,4,-3) \Leftrightarrow \begin{cases} 0=0 \\ 0=4+4k \\ z=0-3k \end{cases} \begin{cases} — \\ k=-1 \\ z=3 \end{cases}$

Então $V(0,0,3)$ e $A(0,4,0)$

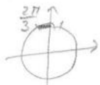
$\overline{VA} = \sqrt{0^2+(-4)^2+3^2} = \sqrt{25} = 5$

Volume $= \frac{\pi\times 3^2\times 5}{3} = 15\pi$

(44) $(x+2)^2+(y-1)^2=9 <$ centro $(-2,1)$; raio $=\sqrt{9}=3$

Comprimento do arco $AB = 2\pi \Leftrightarrow \alpha \times 3 = 2\pi \Leftrightarrow \alpha = \frac{2\pi}{3}$

$\overrightarrow{CA}\cdot\overrightarrow{CB} = \|\overrightarrow{CA}\|\,\|\overrightarrow{CB}\|\cos\left(\frac{2\pi}{3}\right) = 3\times 3\times\left(-\frac{1}{2}\right) = -\frac{9}{2}$



(45.a) $\overrightarrow{AB}\cdot\overrightarrow{HE} = \|\overrightarrow{AB}\|\,\|\overrightarrow{HE}\|\cos 90^\circ = 0$ logo opção B

(45.b) Como $E \in$ reta definida por $(x,y,z)=(0,0,3)+k(1,-1,-1),\ k\in\mathbb{R}$
então $E(k,-k,3-k);\ k\in\mathbb{R}$

Como $\overrightarrow{AB}\perp\overrightarrow{AE}$ então $\overrightarrow{AB}\cdot\overrightarrow{AE}=0 \Leftrightarrow$

$\begin{cases} \overrightarrow{AB} = B-A = (1,-1,2)-(-2,5,0) = (3,-6,2) \\ \overrightarrow{AE} = E-A = (k,-k,3-k)-(-2,5,0) = (k+2,-k-5,3-k) \end{cases}$

$\Leftrightarrow (3,-6,2)\cdot(k+2,-k-5,3-k)=0$

$\Leftrightarrow 3k+6+6k+30+6-2k=0$

$\Leftrightarrow 7k+42=0$

$\Leftrightarrow 7k=-42$

$\Leftrightarrow k=-\frac{42}{7}=-6$ Assim, $E(-6,6,9)$

(46.a) Como a cota de M é 2, então $z=2\times 2$ ou seja $z=4$ (B)

(46.b) plano BCJ // plano LEF

Plano LEF:

$3x-\sqrt{3}y+d=0$

$B(x,0,0)\in$ plano BCJ, logo $3x-6=0 \Leftrightarrow x=2$. Assim $B(2,0,0)$

$\overrightarrow{BM} = M-B = \left(\frac{4}{3},\frac{2\sqrt{3}}{3},2\right)-(2,0,0) = \left(-\frac{2}{3},\frac{2\sqrt{3}}{3},2\right)$

$L = B+2\overrightarrow{BM} = (2,0,0)+2\left(-\frac{2}{3},\frac{2\sqrt{3}}{3},2\right) = (2,0,0)+\left(-\frac{4}{3},\frac{4\sqrt{3}}{3},4\right) =$
$= \left(\frac{2}{3},\frac{4\sqrt{3}}{3},4\right)$

Como $L\in$ plano LEF, logo

$3\times\frac{2}{3}-\sqrt{3}\times\frac{4\sqrt{3}}{3}+d=0$

$\Leftrightarrow 2-4+d=0$

$\Leftrightarrow d=2$

Então a equação do plano LEF será $3x-\sqrt{3}y+2=0$

(47) a) reta OD // reta AC logo rejeitamos C e D visto que $(0,3,-4)$ não é paralelo a $(0,4,3)$

Entre A e B, vejamos qual a reta que contém $(0,0,0)$:

A: $(0,0,0)=(0,6,8)+K(0,2,\frac{3}{2}), K\in\mathbb{R}$

$\begin{cases} 0=0 \\ 0=6+2K \\ 0=8+\frac{3K}{2} \end{cases} \begin{cases} K=-3 \\ 0=16+3K \end{cases} \begin{cases} K=-3 \\ K=-\frac{16}{3} \end{cases}$ imp.

B: $(0,0,0)=(0,-4,-3)+K(0,2,\frac{3}{2}), K\in\mathbb{R}$

$\begin{cases} 0=0 \\ 0=-4+2K \\ 0=-3+\frac{3K}{2} \end{cases} \begin{cases} 2K=4 \\ 0=-6+3K \end{cases} \begin{cases} K=2 \\ K=2 \end{cases}$

Assim, (B)

b) Como $C \in$ reta AC, então C é da forma $(10,4K,3K), K\in\mathbb{R}$ uma vez que

$AC: (x,y,z)=(10,0,0)+K(0,4,3), K\in\mathbb{R}$
$\Leftrightarrow (x,y,z)=(10,0,0)+(0,4K,3K), K\in\mathbb{R}$
$\Leftrightarrow (x,y,z)=(10,4K,3K), K\in\mathbb{R}$

$A(10,0,0)$
$B(10;12{,}5;0)$
$C(10;4K;3K)$

$\vec{CA}=A-C=(10,0,0)-(10,4K,3K)=(0,-4K,-3K)$
$\vec{CB}=B-C=(10;12{,}5;0)-(10,4K,3K)=(0;12{,}5-4K;-3K)$

$\vec{CA}\cdot\vec{CB}=0 \Leftrightarrow (0,-4K,-3K)\cdot(0;12{,}5-4K;-3K)=0 \Leftrightarrow$
$\Leftrightarrow -50K+16K^2+9K^2=0 \Leftrightarrow 25K^2-50K=0 \Leftrightarrow K(25K-50)=0 \Leftrightarrow$
$\Leftrightarrow K=0 \vee 25K-50=0 \Leftrightarrow K=0 \vee 25K=50 \Leftrightarrow \underbrace{K=0}_{\text{cond. imp.}} \vee K=2$

Assim, $C(10,8,6)$

Outra forma...
- obter a equação do plano BCD (que passa por B e é $\perp$ $(0,4,3)$)
- intersetar a reta AC com o plano BCD, para obter C

(48) $A(4,0,0)$. $G(12, \frac{13}{2}, 2)$

a) Centro $\left(\frac{4+12}{2}, \frac{\frac{13}{2}}{2}, \frac{2}{2}\right) = \left(8, \frac{13}{4}, 1\right)$ (A)

$raio = \sqrt{(8-4)^2 + \left(\frac{13}{4}-0\right)^2 + (1-0)^2} = \sqrt{\frac{441}{16}}$

b) Plano ABC:

$3x + 4y + d = 0$

$(4,0,0) \Rightarrow 3\times 4 + 4\times 0 + d = 0$

$\Leftrightarrow d = -12$

$3x + 4y - 12 = 0$

Reta FG:

$(x,y,z) = (12, \frac{13}{2}, 2) + K(3,4,0);\ K \in \mathbb{R}$

Interseção da reta FG com o plano ABC:

$$\begin{cases} x = 12 + 3K \\ y = \frac{13}{2} + 4K \ ; K\in\mathbb{R} \\ z = 2 \\ 3x + 4y - 12 = 0 \end{cases} \Leftrightarrow \begin{cases} \text{---} \\ \text{---} \\ \text{---} \\ 3(12+3K) + 4\left(\frac{13}{2} + 4K\right) - 12 = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} \text{---} \\ \text{---} \\ \text{---} \\ 36 + 9K + 26 + 16K - 12 = 0 \end{cases} \begin{cases} \text{---} \\ \text{---} \\ \text{---} \\ 25K = -50 \end{cases} \Leftrightarrow \begin{cases} x = 12 - 6 \\ y = \frac{13}{2} - 8 \\ z = 2 \\ K = -2 \end{cases} \begin{cases} x = 6 \\ y = -\frac{3}{2} \\ z = 2 \end{cases} \therefore F\left(6, -\frac{3}{2}, 2\right)$$

(49) Seja $\overline{OC} = x$. Logo $\overline{AO} = 4x$ e $\overline{OD} = \frac{4x}{3}$

$C(0, x)$ $D\left(\frac{4x}{3}, 0\right)$ $E\left(\frac{8x}{3}, x\right)$

$\overrightarrow{DC} = C - D = \left(-\frac{4x}{3}, x\right)$; $\overrightarrow{DE} = E - D = \left(\frac{4x}{3}, x\right)$

$\overrightarrow{DC} \cdot \overrightarrow{DE} = -7 \Leftrightarrow \left(-\frac{4x}{3}, x\right)\cdot\left(\frac{4x}{3}, x\right) = -7 \Leftrightarrow -\frac{16x^2}{9} + x^2 = -7 \Leftrightarrow -16x^2 + 9x^2 = -63$

$\Leftrightarrow -7x^2 = -63 \Leftrightarrow x^2 = 9 \Leftrightarrow x = \pm\sqrt{9} \overset{x>0}{\Leftrightarrow} x = 3$. Assim, $\overline{OA} = 4\times 3 = 12$

(50) a) $A(2\sqrt{3}, 6, 0)$

$x = 2\sqrt{3}$ (C)

b) $B \in$ reta AB, logo $B(\sqrt{3}K, 16-5K, 0);\ K\in\mathbb{R}$

$B \in$ plano mediador de $[OA]$, logo $\overline{BO} = \overline{BA} \Leftrightarrow$

$\Leftrightarrow \sqrt{(\sqrt{3}K)^2 + (16-5K)^2} = \sqrt{(\sqrt{3}K - 2\sqrt{3})^2 + (16-5K-6)^2} \Leftrightarrow$

$\Leftrightarrow \cancel{3K^2} + 256 - 160K + \cancel{25K^2} = \cancel{3K^2} - 12K + 12 + 100 - 100K + \cancel{25K^2}$

$\Leftrightarrow -48K = -144 \Leftrightarrow K = 3$ Logo $B(3\sqrt{3}, 1, 0)$

Em alternativa, poder-se-ia fazer $\overrightarrow{BM}\cdot\overrightarrow{OA} = 0$ (...)

Sendo M o ponto médio de $[OA]$, $M(\sqrt{3}, 3, 0)$

$\overline{BM} = \sqrt{(3\sqrt{3}-\sqrt{3})^2 + (1-3)^2} = \sqrt{(2\sqrt{3})^2 + (-2)^2} = \sqrt{12+4} = 4$

$\overline{OA} = \sqrt{(2\sqrt{3}-0)^2 + (6-0)^2} = \sqrt{12+36} = \sqrt{48}$

$A_{base} = \frac{4\times\sqrt{48}}{2} = 2\sqrt{48}$

$V = 2\sqrt{48}\times 5$
$= 10\sqrt{48}$
$= 10\times 4\sqrt{3}$
$= 40\sqrt{3}$