Proposta de resolução da ficha n.º 6 - Geometria - 10.º

(1)

$x^2 = 3^2 + 4^2$ Raio $= 5$

$\dots$

$x = 5$ Logo resposta (B)

(2)

$A(2,0)$ $\overrightarrow{AB} = (-2, 6)$ $y - y_0 = m(x - x_0)$

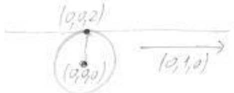
$B(0,6)$ $m = \frac{6}{-2} = -3$ $A(2,0) \rightsquigarrow y - 0 = -3(x-2)$

$y = -3x + 6$

opção (A)

(3) $x^2 + y^2 + z^2 \leq 4 \rightsquigarrow$ esfera de centro $(0,0,0)$ e $R = 2$

$(x,y,z) = (0,0,2) + K(0,1,0)$; $K \in \mathbb{R} \rightsquigarrow$ reta paralela a $(0,1,0)$ e q passa em $(0,0,2)$



A intersecção é um ponto

(4)

(4.1) $x = 2 \land y = 2 \land 0 \leq z \leq 2$

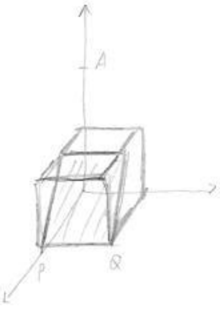
(4.2) $\overline{TA} = \overline{TV}$?

$T(2,0,2)$ $\overline{TA} = \sqrt{(2-0)^2 + (0-0)^2 + (2-4)^2} = \sqrt{8}$

$A(0,0,4)$ $\overline{TV} = \sqrt{(2-0)^2 + (0-2)^2 + (2-2)^2} = \sqrt{8}$ $\Bigg\}$ então T pertence.

$V(0,2,2)$

(4.3)



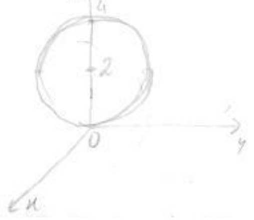
$P = 4 + 2\sqrt{5}$

$x^2 = 2^2 + 1^2$

$x = \sqrt{5}$

(5) (B) $(x,y,z) = (1,1,0) + K\underbrace{(0,0,7)}_{\text{vetor paralelo ao eixo Oz}}$; $K \in \mathbb{R}$

(6)



(A) $x = 0$

(7) (C) BDH

(8)

N.º faces : $n+1+1 = n+2$

N.º arestas : $n+n+n = 3n$ (D)

(9)

(9.1) $B(6,3)$

$C(3,0)$

$(x-3)^2+(y-0)^2 = (x-6)^2+(y-3)^2$

$\cancel{x^2}-6x+\cancel{9}+\cancel{y^2} = \cancel{x^2}-12x+36+\cancel{y^2}-6y+\cancel{9}$

$6y = -12x+6x+36$

$6y = -6x+36$

$y = -x+6$ c.q.d.

(9.2) Reta BD:

$C(3,0)$ $\overrightarrow{CD} = (-3,-3)$

$D(0,-3)$ $m = \frac{-3}{-3} = 1$

$C(3,0) \rightsquigarrow y-0 = 1(x-3)$

$y = x-3$

circunf.

$x^2+y^2 = 9$

Condição: $x^2+y^2 \leq 9 \wedge y \leq x-3$

(9.3)

$A_{\bigcirc} = \pi r^2$

$= \pi \times 3^2$

$= 9\pi$

$A_{◗} = \frac{9\pi}{4}$

$A_{▱} = \frac{B+b}{2} \times h =$

$= \frac{6+3}{2} \times 3$

$= \frac{27}{2}$

$A_{pedida} = A_{▱} - A_{◗} = \frac{27}{2} - \frac{9\pi}{4} \simeq 6,43$

(10) $A(2,0)$

$B(0,2)$

$\overrightarrow{AB} = (-2,2)$

$m = \frac{2}{-2} = -1$

$(2,0) \rightsquigarrow y-0 = -1(x-2)$

$y = -x+2$ (C)

(11)

(11.1)

$x^2 = 1^2+1^2$

$x = \sqrt{2}$

$A_{\square} = \sqrt{2} \times \sqrt{2} = 2$

$V_{sólido} = V_{cubo} + V_{pirâmide}$

$10 = 2 \times 2 \times 2 + \frac{1}{3} Ab \times h$

$10 = 8 + \frac{1}{3} \times \sqrt{2} \times \sqrt{2} \times h$

$\underset{(3)}{10} = \underset{(3)}{8} + \frac{2h}{3}_{(1)} \Leftrightarrow 30 = 24+2h \Leftrightarrow 6 = 2h$

$\Leftrightarrow 3 = h$

cota de E = 3

(11.2) $T(2,0,-2)$ $\quad r = \overline{TC} = \sqrt{1+4+4} = 3$

$C(1,2,0)$ $\quad (x-2)^2 + y^2 + (z+2)^2 = 9$

(11.3) trapézio

(12) $(x-2)^2 + (y+1)^2 \leq 4 \wedge y \geq 0$ (B)

(13) $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} =$ hipótese

$= 2\overrightarrow{DB} + 2\overrightarrow{BE} =$

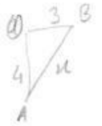
$= 2(\overrightarrow{DB} + \overrightarrow{BE}) =$

$= 2\overrightarrow{DE}$ $\quad$ Como $\overrightarrow{AC} = 2\overrightarrow{DE}$ as retas AC e DE são paralelas.

(14) (14.1) $D(4,0,8)$ $\quad \overrightarrow{DF} = F - D = (-4,-3,0)$

$F(0,-3,8)$ $\quad (x,y,z) = (4,0,8) + k(-4,-3,0);\ k \in \mathbb{R}$

(14.2)



$x^2 = 4^2 + 3^2$

$x = 5$

$A_{DABE} = 5 \times 8 = 40$ $\quad A_{FECB} = 6 \times 8 = 48$

$A_{lateral} = 40 \times 2 + 48$

$= 128$

(15) $(x-1)^2 + (y-3)^2 = 16$

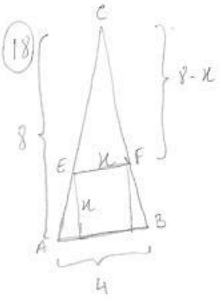
$C(1,3)$ $\quad r = 4$

$1 - 4 = -3$ $\quad x = -3$ é uma reta tangente (A)

(16) $30 + 30 = 60$ arestas (D)

(17) (A)

(18)



$\Delta[ABC]$ e $\Delta[EFC]$ são semelhantes

Como 8 é o dobro de 4, também

$8-x$ é o dobro de x

Assim $8 - x = 2x$

$x = \frac{8}{3}$ (C)

19

19.1 $A = \frac{B+b}{2} \times h = \frac{8+4}{2} \times 3 = 18$

19.2 $A(4,7)$ $\quad (x-4)^2+(y-7)^2=(x-8)^2+(y-10)^2$

$D(8,10)$ $\quad \cancel{x^2}-8x+16+\cancel{y^2}-14y+49=\cancel{x^2}-16x+64+\cancel{y^2}-20y+100$

$$-14y+20y=-16x+8x+64+100-16-49$$

$$6y=-8x+99$$

$$y=-\frac{8}{6}x+\frac{99}{6}$$

$$y=-\frac{4}{3}x+\frac{33}{2}$$

19.3 $(x-4)^2+(y-7)^2 \leq 25 \wedge x \leq 4 \wedge x \geq 0 \wedge y \leq 7$

20 $V = \frac{1}{3} A_b \times h = \frac{1}{3} \times \frac{3 \times 3}{2} \times 6 = 9$ (D)

21.1 $\underline{\overrightarrow{AB}} + \overrightarrow{FG} = \overrightarrow{AC}$

$F + \overrightarrow{CD} = \underline{E}$

$D + 2\overrightarrow{AB} + \overrightarrow{CE} = \underline{F}$

21.2.1 $A(11,-1,2)$ $\quad B(13,2,8)$

$\overline{AB} = \sqrt{(11-13)^2+(-1-2)^2+(2-8)^2} = \sqrt{4+9+36} = \sqrt{49} = 7$

$\overline{BG} = ?$ $\quad x^2 = 7^2+7^2$ $\quad A = 7 \times \sqrt{98}$

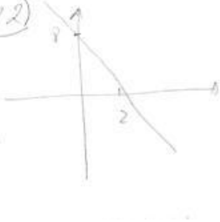
$x = \sqrt{98}$

21.2.2 $F = E + \overrightarrow{AB} = (8,5,0)+(2,3,6)=(10,8,6)$

$\overrightarrow{AB} = (2,3,6)$

$(x,y,z) = (10,8,6) + K(0,0,1)$; $K \in \mathbb{R}$

22



inclinado para a esquerda $\rightarrow$ elimina B e D

$b = 8 \rightarrow$ tem que ser (A)

23.1 $O(0,0,0)$ $Q(5,5,0)$

$r = \overline{OQ} = \sqrt{5^2+5^2+0^2} = \sqrt{50}$ $\therefore$ $(x-5)^2+(y-5)^2+z^2 = 50$

Centro $(5,5,0)$

23.2 $V = 75$

$V = \frac{1}{3} A_b \times h = \frac{1}{3} \times 25 \times h = \frac{25h}{3}$

Então $\frac{25h}{3} = 75 \Leftrightarrow 25h = 225 \Leftrightarrow h = \frac{225}{25} \Leftrightarrow h = 9$

Logo $V\left(\frac{5}{2}, \frac{5}{2}, 9\right)$

24 altura do cone de cima α

" " " " baixo $h-\alpha$

$V_{\text{cone de cima}} = \frac{1}{3}\pi r^2 \cdot \alpha$

$V_{\text{cone de baixo}} = \frac{1}{3}\pi r^2 (h-\alpha)$

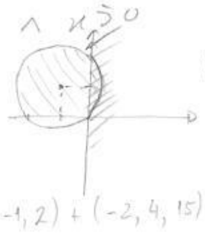
$V_{\text{2 cones}} = \frac{1}{3}\pi r^2\alpha + \frac{1}{3}\pi r^2(h-\alpha) = \cancel{\frac{1}{3}\pi r^2\alpha} + \frac{1}{3}\pi r^2 h - \cancel{\frac{1}{3}\pi r^2\alpha} = \frac{1}{3}\pi r^2 h$

Assim, independentemente da altura de P, α, a soma dos volumes é constante.

25  $x^2 = 3^2+4^2$ $x = 5$ $A_{\boxed{\text{////}}} = 4 \times 5 = 20$ (C)

26 $(x+1)^2+(y-1)^2 \leq 2 \wedge x \geq 0$

círculo de $C(-1,1)$ e $r = \sqrt{2}$



(C)

28.1 $H = A + \overrightarrow{BG} = (11,-1,2) + (-2,4,15) = (9,3,17)$

28.2 Centro $(11,-1,2)$

raio $= \overline{AB} = \sqrt{(11-8)^2+(-1-5)^2+(2-0)^2} = \sqrt{9+36+4} = \sqrt{49} = 7$

$(x-11)^2+(y+1)^2+(z-2)^2 = 49$

28.3 $(x,y,z) = (6,9,15) + k(0,1,0)$; $k \in \mathbb{R}$

(27.1) As retas DQ e VF são concorrentes
As retas EH e VF são não complanares
A reta PQ e o plano HSB são estritamente paralelos
A reta FQ e o plano ADH são concorrentes
Os planos BQV e PQR são perpendiculares

(27.2)

$$P=?\quad \begin{cases} x^2+y^2+z^2-2x-2y-8z=0 \\ y=0 \\ z=0 \end{cases} \quad \begin{cases} x^2-2x=0 \\ — \\ — \end{cases} \quad \begin{cases} x(x-2)=0 \\ — \\ — \end{cases} \quad \begin{cases} x=0 \vee x=2 \\ \\ \end{cases}$$

Então $P(2,0,0)$

$$V=?\quad x^2+y^2+z^2-2x-2y-8z=0$$
$$x^2-2x+y^2-2y+z^2-8z=0$$
$$x^2-2x+\underline{1}+y^2-2y+\underline{1}+z^2-8z+\underline{16}=\underline{1}+\underline{1}+\underline{16}$$
$$(x-1)^2+(y-1)^2+(z-4)^2=18$$

Então $V(1,1,4)$ = centro

$$V_{\text{pirâmide}}=\frac{1}{3}A_b\times h=\frac{1}{3}\times 2\times 2\times 4=\frac{16}{3}$$

(29) B

(30) $(x,y,z)=(-1,2,3)+k(0,1,0)\ ;\ k\in\mathbb{R}$ // $(0,1,0)$ // $x=-1 \wedge z=3$ (C)

(31) a) $y=2x-1$ $\quad (x,y)=(0,-1)+k(1,2)\ ;\ k\in\mathbb{R}$

$m=\frac{2}{1}$

$\vec{u}(1,2)$

b) $y-y_0=m(x-x_0)$

$m=2 \quad P(0,-2)$

$y-(-2)=2(x-0)$

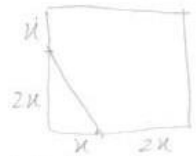
$y+2=2x$

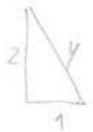
$y=2x-2$

c) $x^2+(y+2)^2\leq 4 \wedge x\geq 0 \wedge y\leq 2x-1$

(32)

a)





$y^2=2^2+1^2$

$y=\sqrt{5}$

$x+2x=3$

$x=1$

$A_{\square}=\sqrt{5}\times\sqrt{5}=5\ dm^2$

b) $\frac{45}{5}=h^2 \Leftrightarrow h=3$. Então sendo a altura da pirâmide [IJKLV], 12, a altura da pirâmide [EFGH] será $\frac{12}{3}=4$. Assim $d=12-4=8\ dm$.