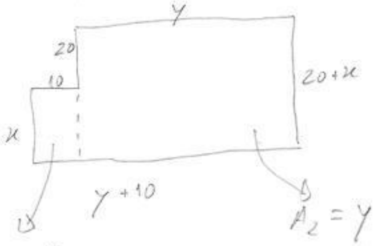
Propostas de resolução da ficha nº 8 - 10 ano - funções I

1

1.1



$x + y + 10 + 20 + x = 100$

$y = 70 - 2x$

$A_1 = 10x$

$A_2 = y(20+x) = (70-2x)(20+x) =$
$= 1400 + 70x - 40x - 2x^2 = -2x^2 + 30x + 1400$

$A_{total} = A_1 + A_2 = -2x^2 + 30x + 1400 + 10x = -2x^2 + 40x + 1400$ c.q.d.

1.2 Vértice = ?

$-2x^2 + 40x = 0$

$x(-2x + 40) = 0$

$x = 0 \lor -2x = -40$

$x = 0 \lor x = 20$

$\frac{20}{2} = 10$

$A(10) = -2 \times 10^2 + 40 \times 10 + 1400 =$
$= 1600$

$V(10; 1600)$

R: $x = 10$ e $A = 1600$

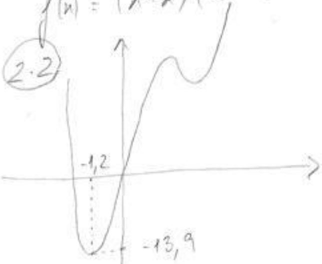
2 $f(x) = x^4 - 3x^3 - 3x^2 + 14x$

2.1

	1	-3	-3	14	0
-2		-2	10	-14	0
	1	-5	7	0	0

$f(x) = (x+2)(x^3 - 5x^2 + 7x) = (x+2) \cdot x \cdot (x^2 - 5x + 7)$

2.2



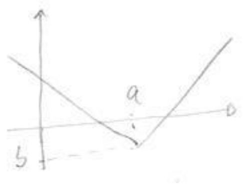
$D' = [-13,9 ; +\infty[$

$a = -13,9$

3 $[-2; -1[\cup]-1; 4[$

4 D

5



$a > 0 \land b < 0$ B

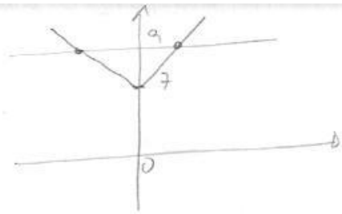
(6) $g(x) = 9$

$|x| + 7 = 9$

$|x| = 2$

$x = 2 \vee x = -2$ (D)

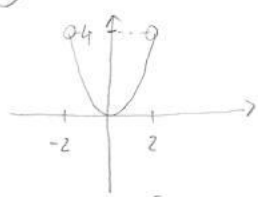
ou



(7) $g(x) = -f(x) + 7$ (A)

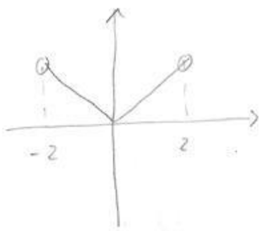
(8) (C)

(9) $x^2 < 4$



$]-2, 2[$

$|x| < 2$



(C)

(10) $A_{\square} = x \times x = x^2$

(10.1) $A_{\triangle} = \dfrac{(14-2x)\times(5-x)}{2} = \dfrac{70 - 14x - 10x + 2x^2}{2} = \dfrac{2x^2 - 24x + 70}{2} =$

$= x^2 - 12x + 35$

$A_{total} = 4x^2 + 2(x^2 - 12x + 35) = 4x^2 + 2x^2 - 24x + 70 = 6x^2 - 24x + 70$

(10.2) $V = ?$

$6x^2 - 24x = 0$

$x(6x - 24) = 0$

$x = 0 \vee 6x = 24$

$x = 0 \vee x = 4$

$x = 0 \vee$

$\frac{4}{2} = 2$

$A(2) = 6 \times 2^2 - 24 \times 2 + 70 = 46$

$V(2, 46)$

R: $x = 2$ e $A = 46$

(10.3) $A_{\square} = 10 \times 14 = 140$

Metade $A_{\square} = 70$

$A(x) = 70$

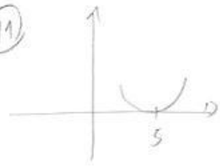
$6x^2 - 24x + \cancel{70} = \cancel{70}$

$x(6x - 24) = 0$

$x = 0 \vee x = 4$

R: $x = 4$

(11)



$D' = [0, +\infty[$ (B)

(12) Tópicos:

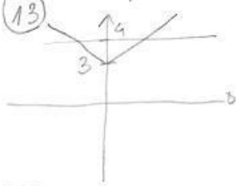
Rejeita A : inicialmente a distância decresce

Rejeita B : a distância nunca chega a ser zero

Rejeita D : a distância inicial deverá ser igual a distância final

Opção (C)

(13)



$g(x) = 4$ (D)

(14.1)

	1	-3	-6	8
4		4	4	-8
	1	1	-2	0

$x^3 - 3x^2 - 6x + 8 < 0$

$(x-4)(x^2 + x - 2) < 0$

Raízes:

$x^2 + x - 2 = 0$

$x = \frac{-1 \pm \sqrt{1^2 - 4 \times 1 \times (-2)}}{2 \times 1}$

$x = 1 \vee x = -2$

	$-\infty$	-2		1		4	$+\infty$
$x-4$	-	-	-	-	-	0	+
x^2+x-2	+	0	-	0	+	+	+
$(x-4)(x^2+x-2)$	-	0	+	0	-	0	+

$S =]-\infty, -2[\cup]1, 4[$

(14.2) $f(-3) = (-3)^3 - 3(-3)^2 - 6(-3) + 8 = -28$

$f(0) = 0^3 - 3 \times 0^2 - 6 \times 0 + 8 = 8$

$A(-3, -28)$

$B(0, 8)$

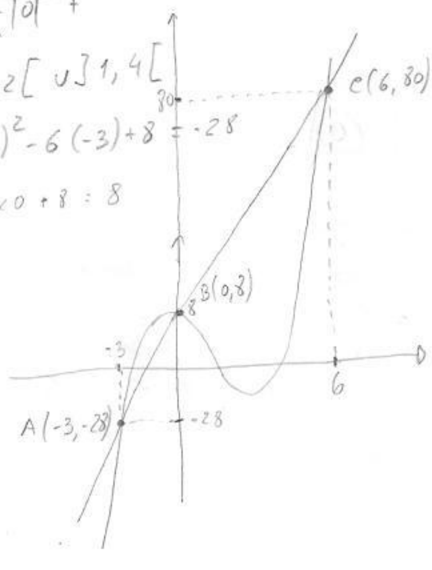
$\overrightarrow{AB} = B - A = (3, 36)$

$m = \frac{36}{3} = 12$

$y = 12x + b$

$B(0, 8)$

$y = 12x + 8$



(15) $f(x) = x^4 + x^3 - 7x^2 - x + 6$

(15.1) $E = ?$

$f(0) = 6$, logo $E(0,6)$

$B = ?$ $D = ?$

Raízes: $\boxed{-3}$, $\boxed{1}$

	1	1	-7	-1	6
-3		-3	6	3	-6
	1	-2	-1	2	0
1		1	-1	-2	
	1	-1	-2	0	

$x^2 - x - 2 = 0$

$$x = \frac{1 \pm \sqrt{(-1)^2 - 4(1)(-2)}}{2 \times 1}$$

$x = \boxed{2} \vee x = \boxed{-1}$

Resumindo,

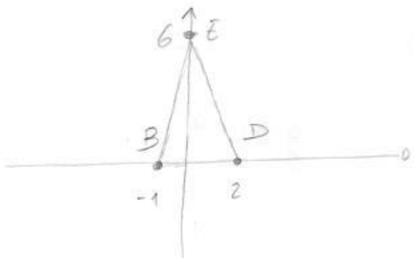
$A(-3, 0)$

$B(-1, 0)$

$C(1, 0)$

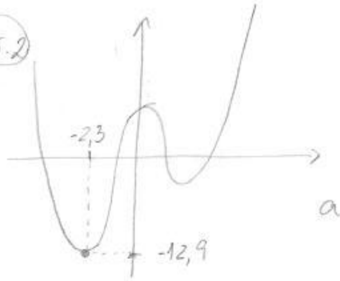
$D(2, 0)$

$E(0, 6)$



$$A_{\Delta} = \frac{3 \times 6}{2} = 9$$

(15.2)



$a = -12,9$

(16) D

(17) $g\left(\frac{2}{3}\right) = \frac{2}{3} + \frac{1}{6} = \frac{5}{6}$ (C)

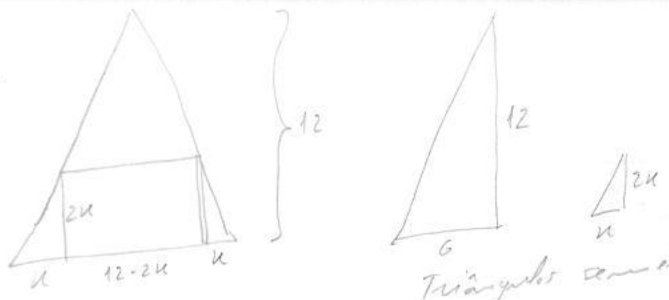
(18) Tópicos para a composição:

Rejeição de B: as 2 irmãs percorrem a mesma distância e esta opção não mostra isso

Rejeição de (C): a distância percorrida por qualquer das irmãs tem que crescer sempre

Opção correta: A

(19) (19.1)



Triângulos semelhantes

$A_{\square} = 2x(12-2x) = 24x - 4x^2$

$A_{\triangle} = \frac{12 \times 12}{2} = 72$

$A_{\text{pretendida}} = 72 - (24x - 4x^2) = 4x^2 - 24x + 72$

(19.2) $S(x) = 4x^2 - 24x + 72$

Vértice = ?

$4x^2 - 24x = 0$

$x(4x - 24) = 0$

$x = 0 \vee x = 6$

$\frac{6}{2} = 3$

$S(3) = 4(3)^2 - 24(3) + 72 = 36 - 72 + 72 = 36$

$V(3, 36)$

R: O valor de x é 3 e a área mínima é 36.

(19.3) $A > 40$

$4x^2 - 24x + 72 > 40$

$4x^2 - 24x + 72 - 40 > 0$

$4x^2 - 24x + 32 > 0$

Zeros:

$4x^2 - 24x + 32 = 0$

(...)

$x = 2 \vee x = 4$

	0		2		4		6
$4x^2-24x+32$	ND	+	0	-	0	+	ND
		↑				↑	

$S:]0, 2[\cup]4, 6[$

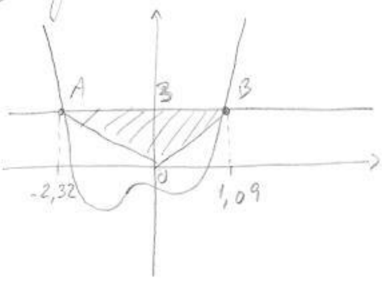
(20) B

(21) Volume $= 10 \times 6 \times 0{,}3t = 18t$

Das 9h — 14h, $t = 5$

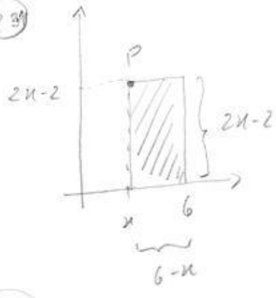
Se $t = 5$, Volume $= 18 \times 5 = 90\ m^3$, ou seja, 90.000 l.

22. $g(x) = x^4 + 2x^3 - 1$



$$A_{\triangle} \approx \frac{(1{,}09 + 2{,}32) \times 3}{2} \approx 5{,}1$$

23.1



$$A_{\square} = (6-x)(2x-2) = 12x - 12 - 2x^2 + 2x = -2x^2 + 14x - 12$$ c.q.d.

23.2

$A < 8$

$-2x^2 + 14x - 12 < 8$

$-2x^2 + 14x - 20 < 0$

Zeros:

$-2x^2 + 14x - 20 = 0$

$(\ldots)$

$x = 5 \vee x = 2$

x	1		2		5		6
$-2x^2+14x-20$	ND	$-$	0	$+$	0	$-$	ND

$S =]1, 2[\cup]5, 6[$

24.

24.1 $D'_f = [-2, 3]$

24.2 $x = -4 \vee x = -2 \vee x = 0$

24.3 $S =]2, 6] \setminus \{4\}$

(25) A

(26) A

(27)

(27.1) $D'_f = [-1, +\infty[$ $\quad D'_g =]-\infty, 1]$ $\updownarrow$

$D'_h = [2, +\infty[$ $\uparrow$ $\quad D'_j = [-1, +\infty[$ $\rightarrow$

(27.2) $f(x) = a(x-h)^2 + k$

Como $V(2,-1)$ logo $\underline{h=2}$ e $\underline{k=-1}$

Assim $f(x) = a(x-2)^2 - 1$

Como $f(0) = 1$, então $a(0-2)^2 - 1 = 1 \Leftrightarrow \underline{a = \frac{1}{2}}$

(28)

(28.1) $A(-1,0)$ $B(4,0)$ $\quad P(x, -2x+8)$

$r: x=-1$ $\quad -2x+8=0 \Leftrightarrow -2x=-8 \Leftrightarrow x=4$

$A = \frac{B+b}{2} \times h = \frac{5+x+1}{2} \times (-2x+8) = \frac{(x+6)(-2x+8)}{2} =$

$= \frac{-2x^2+8x-12x+48}{2} = -x^2+4x-6x+24 = -x^2-2x+24$ c.q.d.

(28.2) $S(x) > 21$

$-x^2-2x+24 > 21$

$-x^2-2x+3 > 0$

	-1		1		4
$-x^2-2x+3$	ND	+	0	-	ND

Raízes

$-x^2-2x+3=0$

$x = \frac{2 \pm \sqrt{4+12}}{-2}$

$x=-3 \vee x=1$

$S =]-1, 1[$

(29) D

$V(-1,0)$

$h = |x+1| = \begin{cases} x+1 & \text{se } x+1 \geq 0 \\ -x-1 & \text{se } x+1 < 0 \end{cases} = \begin{cases} x+1 & \text{se } x \geq -1 \\ -x-1 & \text{se } x < -1 \end{cases}$