Resolução da ficha n.º 2 Trigonometria 11.º ano

(1) Raciocínio baseado na semelhança de triângulos.

(2)

$P = 2\pi r$, $r = 50$ } $P = 2\pi \times 50 = 100\pi$

Para 1 volta (360°) a cama avança 100π cm.

$360° \text{ — } 100\pi$
$42° \text{ — } x$

$x \simeq 36{,}7$ cm //

Para 42° a cama avança ≈ 36,7 cm.

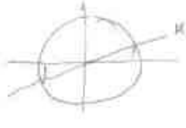
$1 \text{ volta — } 100\pi \text{ cm}$
$x \text{ — } 20000 \text{ cm}$

$x \cong 63{,}7$ voltas //

(3) a) Deslocamento na horizontal
ou também,
Simetria em relação ao eixo dos xx.

b) $\operatorname{sen}(\pi + x) = -\operatorname{sen} x$

c)



Senos com o mesmo valor absoluto mas simétricos.

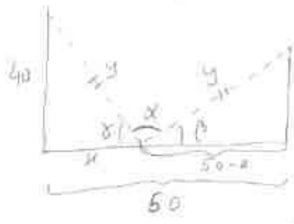
(4)

$\operatorname{tg} x = \frac{20}{15} \Rightarrow x \simeq 53{,}13°$

$\operatorname{tg} y = \frac{30}{15} \Rightarrow x \simeq 63{,}43°$

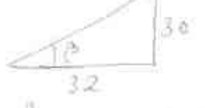
$y - x \cong 10{,}3°$

(5)

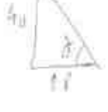


$40^2 + x^2 = y^2$
$30^2 + (50-x)^2 = y^2$

$1600 + x^2 = 900 + 2500 - 100x + x^2$
$100x = 1800$
$x = 18$



$\operatorname{tg} \beta = \frac{30}{32} \Rightarrow \beta \simeq 43{,}15°$

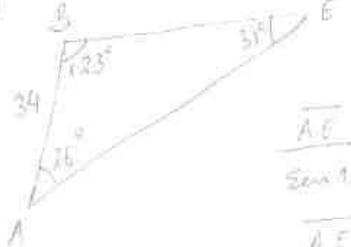


$\operatorname{tg} \gamma = \frac{40}{18} \Rightarrow \gamma \simeq 65{,}77°$

$\alpha = 180° - 43{,}15° - 65{,}77°$

$\alpha \simeq 71{,}08°$

(6)



$180 - 123 - 26 = 31$

$$\frac{\overline{AE}}{\operatorname{sen} 123°} = \frac{\overline{BE}}{\operatorname{sen} 26°} = \frac{\overline{AB}}{\operatorname{sen} 31°} \Leftrightarrow$$

$$\Leftrightarrow \frac{\overline{AE}}{\operatorname{sen} 123°} = \frac{34}{\operatorname{sen} 31°} \Leftrightarrow \overline{AE} \approx 55{,}36 \text{ m} //$$

(7)

$$R = \frac{v_0^2 \sqrt{2}}{32}\left(\operatorname{sen}(2\theta) - \cos(2\theta) - 1\right)$$

$$\left.\begin{array}{l} v_0 = 32 \\ \theta = 60° \end{array}\right\} \quad R = \frac{32^2 \times \sqrt{2}}{32}\left(\operatorname{sen} 120° - \cos 120° - 1\right) =$$

$$= 32\sqrt{2}\left(\frac{\sqrt{3}}{2} + \frac{1}{2} - 1\right) =$$

$$= \frac{32\sqrt{6}}{2} + \frac{32\sqrt{2}}{2} - 32\sqrt{2} =$$

$$= 16\sqrt{6} + 16\sqrt{2} - 32\sqrt{2} = 16\sqrt{6} - 16\sqrt{2} \simeq 16{,}56$$

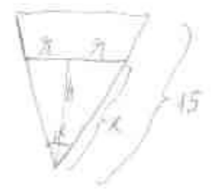
Colocando $y = 32\sqrt{2}\,(\operatorname{sen} 2x - \cos 2x - 1)$ na cal. gráfica

Janela:

Máx $\approx 18{,}75$ para $\theta \simeq 67{,}5°$

(8)

a)

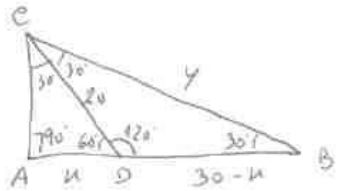


$\operatorname{sen}\left(\frac{\alpha}{2}\right) = \frac{r}{x} \Rightarrow r = x \operatorname{sen}\left(\frac{\alpha}{2}\right) \to$ Raio

$\cos\left(\frac{\alpha}{2}\right) = \frac{h}{x} \Rightarrow h = x \cos\left(\frac{\alpha}{2}\right) \to$ Altura

b) $V(x) = \frac{1}{3} A_b \times h = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\, x^2 \operatorname{sen}^2\left(\frac{\alpha}{2}\right) x \cos\left(\frac{\alpha}{2}\right) = x^3 \frac{\pi}{3} \operatorname{sen}^2\left(\frac{\alpha}{2}\right)\cos\left(\frac{\alpha}{2}\right)$

(9)

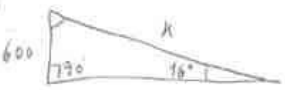


$\cos 60^\circ = \frac{x}{20} \Leftrightarrow x = 10 \rightarrow \overline{DB} = 20$

$\cos 30^\circ = \frac{30}{y} \Leftrightarrow y \simeq 34,64$

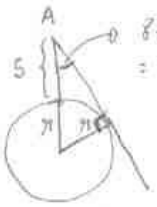
Distância percorrida $= 20 + 34,64 = 54,64$

(10)



$\sin 16^\circ = \frac{600}{x} \Leftrightarrow x \simeq 2176,8$ m

(11)



$87^\circ 40' = 87,67^\circ$

$60' \text{ — } 1^\circ$

$40' \text{ — } x$

$x = 0,67^\circ$

$\sin 87,67^\circ = \frac{r}{5+r} \Leftrightarrow$

$\Leftrightarrow (\sin 87,67)(5+r) = r \Leftrightarrow$

$\Leftrightarrow 5 \sin 87,67 + r \sin 87,67 = r \Leftrightarrow$

$\Leftrightarrow r(1 - \sin 87,67) = 5 \sin 87,67$

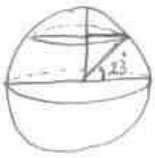
$\Leftrightarrow r = \frac{5 \sin 87,67}{1 - \sin 87,67} \simeq 6042$ Km

(12)

$\text{tg } 30^\circ = \frac{1,5}{x} \Leftrightarrow x \simeq 2,598$

Altura $\simeq 5,196$ m

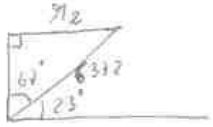
(13)



a) Eq. $\rightarrow P_0 = 2\pi r = 2\pi \times 6372 \simeq 40036,45678$

$v = \frac{40036,45678}{24} \simeq 1668,2$ Km/h

b)



$\sin 67^\circ = \frac{r_2}{6372}$

$r_2 \simeq 5865,46$ Km

$v = \frac{25865,46}{24} \simeq 1535,57$ Km/h

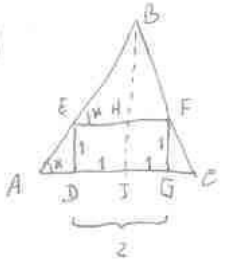
c) $\sin(90-x) = \frac{r}{6372} \Leftrightarrow \cos x = \frac{r}{6372} \Leftrightarrow$

$\Leftrightarrow r = 6372 \cos x$

d) $v = 531\pi \cos 90^\circ = 0$

$v = \frac{2\pi \times 6372 \cos x}{24} = 531\pi \cos x$

(14)



$$tg\,x = \frac{1}{\overline{AD}} \Rightarrow \overline{AD} = \frac{1}{tg\,x}$$

$$tg\,x = \frac{\overline{BH}}{1} \Rightarrow \overline{BH} = tg\,x$$

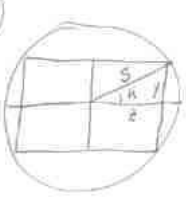
$$\overline{AC} = \frac{1}{tg\,x} + \frac{1}{tg\,x} + 2 = 2 + \frac{2}{tg\,x}$$

$$\overline{BI} = 1 + tg\,x$$

$$A = \frac{\left(2 + \frac{2}{tg\,x}\right)(1 + tg\,x)}{2} = \frac{2 + 2\,tg\,x + \frac{2}{tg\,x} + 2}{2} = \frac{4 + 2\,tg\,x + \frac{2}{tg\,x}}{2}$$

$$A = 2 + tg\,x + \frac{1}{tg\,x}$$

(15)



$$\cos x = \frac{x}{5} \Rightarrow x = 5\cos x$$

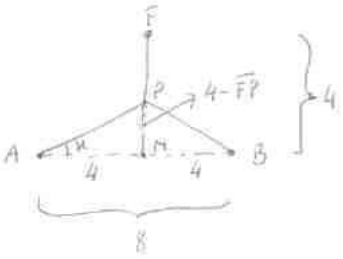
$$\sin x = \frac{y}{5} \Rightarrow y = 5\sin x$$

$$A_{\text{▨}} = 2x \cdot 2y = 4xy = 4 \cdot 5\cos x \cdot 5\sin x = 100\,\sin x \cos x$$

$$A_{\bigcirc} = \pi r^2 = \pi 5^2 = 25\pi$$

$$A_{\text{semicírculo}} = 25\pi - 100\sin x \cos x = 25\pi - 50\sin(2x)$$

(16)



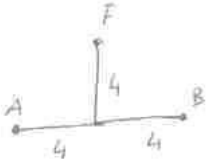
$$\cos x = \frac{4}{\overline{AP}} \Rightarrow \boxed{\overline{AP} = \frac{4}{\cos x}}$$

$$tg\,x = \frac{\overline{FH}}{4} = \frac{4 - \overline{FP}}{4} = 1 - \frac{\overline{FP}}{4}$$

$$\Rightarrow \frac{\overline{FP}}{4} = 1 - tg\,x \Rightarrow \boxed{\overline{FP} = 4 - 4\,tg\,x}$$

a) $g(x) = 2 \times \frac{4}{\cos x} + 4 - 4\,tg\,x = 4 + \frac{8}{\cos x} - 4\,tg\,x =$

$= 4 + \frac{8}{\cos x} - 4\frac{\sin x}{\cos x} = 4 + \frac{8 - 4\sin x}{\cos x}$



b) $g(0) = 4 + \frac{8 - 4\sin 0}{\cos 0} = 4 + \frac{8 - 4 \times 0}{1} = 4 + 8 = 12$

(17) $g(x) = \sin x + \sin(2x)$

$g(x) = 0 \Leftrightarrow \sin x + \sin(2x) = 0 \Leftrightarrow$

$\Leftrightarrow \sin x = -\sin(2x) \Leftrightarrow$

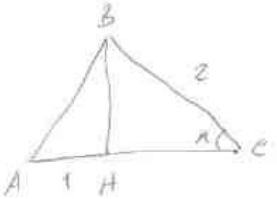
$\Leftrightarrow \sin x = \sin(-2x) \Leftrightarrow$

$\Leftrightarrow x = -2x + 2k\pi \ \cup \ x = \pi + 2x + 2k\pi$

$\Leftrightarrow 3x = 2k\pi \ \vee \ -x = \pi + 2k\pi$

$\Leftrightarrow x = \frac{2k\pi}{3} \ \vee \ x = -\pi - 2k\pi$

$\Leftrightarrow x = \frac{2k\pi}{3} \ \vee \ x = \pi + 2k\pi \quad | \ k \in \mathbb{Z}$



$\sin x = \frac{\overline{BH}}{2} \Rightarrow \overline{BH} = 2\sin x$

$\cos x = \frac{\overline{CH}}{2} \Rightarrow \overline{CH} = 2\cos x$

$A = \frac{(1 + 2\cos x)\, 2\sin x}{2} =$

$= \frac{2\sin x + 4\sin x \cos x}{2} = \sin x + 2\sin x \cos x =$

$= \sin x + \sin(2x)$

(18)

$\sin\theta = \frac{h}{a} \Rightarrow h = a\sin\theta$

$\cos\theta = \frac{x}{a} \Rightarrow x = a\cos\theta$

$A = \frac{2a\cos\theta \times a\sin\theta}{2} = a^2 \sin\theta\cos\theta = \frac{a^2}{2}\sin(2\theta)$