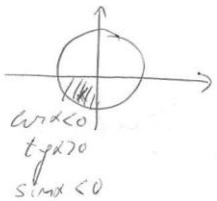
Resolução da ficha nº 3 - Trigonometria - 11.º ano

① $\sin^2\alpha + \cos^2\alpha = 1$

$\sin^2\alpha = 1 - \cos^2\alpha$

$\sin\alpha = \pm\sqrt{1-\cos^2\alpha}$

$\sin\alpha = -\sqrt{1-\cos^2\alpha}$ (B)



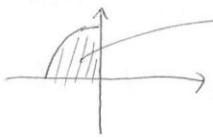
② $\cos\left(\frac{\pi}{2}+\beta\right) = -\sin\beta = -\frac{1}{5}$ (B)

③ $\beta + 2\alpha = \pi$

$\beta = \pi - 2\alpha$

$\cos\beta = \cos(\pi - 2\alpha) = -\cos(2\alpha)$ (D)

④ $\theta \in \left]\frac{\pi}{2}, \pi\right[$



$\sin\theta > 0$

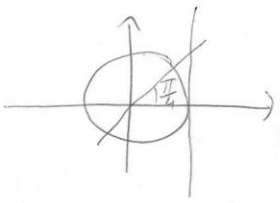
$\operatorname{tg}\theta < 0$

$\underbrace{\sin\theta}_{+} \cdot \underbrace{\operatorname{tg}\theta}_{+} > 0$ (D)

⑤ $1 + 3\operatorname{tg}(2x) = 4$

$3\operatorname{tg}(2x) = 3$

$\operatorname{tg}(2x) = 1$



$2x = \frac{\pi}{4} \vee 2x = \frac{5\pi}{4}$

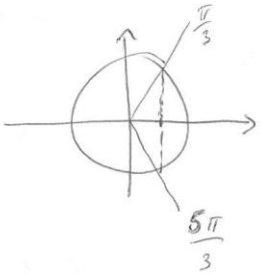
$x = \frac{\pi}{8} \vee x = \frac{5\pi}{8}$

(C)

⑥ $5 + 2\cos x = 6$

$2\cos x = 1$

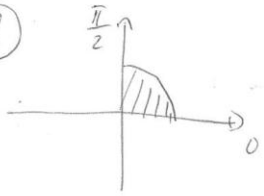
$\cos x = \frac{1}{2}$



$x = \frac{\pi}{3} \vee x = \frac{5\pi}{3}$ (B)

(7) $\sin\alpha = \frac{y}{1}$ $\quad d = 1 - y = 1 - \sin\alpha$ (B)

$\sin\alpha = y$

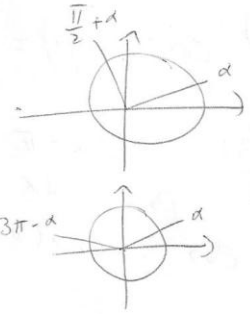
(8) 

$\sin(\pi - \alpha) = \sin\alpha > 0$ (B)

(9) a) $5\sin\left(\frac{\pi}{2}+\alpha\right) + 2\cos(3\pi - \alpha) =$

$= 5\cos\alpha - 2\cos\alpha =$

$= 3\cos\alpha$



$tg\alpha = \sqrt{8}$

$1 + tg^2\alpha = \frac{1}{\cos^2\alpha}$

$1 + (\sqrt{8})^2 = \frac{1}{\cos^2\alpha}$

$9 = \frac{1}{\cos^2\alpha}$

$\cos^2\alpha = \frac{1}{9}$

$\cos\alpha = \pm\frac{1}{3}$

Se $\cos\alpha = -\frac{1}{3}$ então,

$3\cos\alpha = 3\times\left(-\frac{1}{3}\right) = -1$

(9.5) $P(r, s)$ $Q(u, 0)$

$\overrightarrow{OP} \cdot \overrightarrow{PQ} = 0 \Leftrightarrow$

$\Leftrightarrow (r, s)\cdot(u - r, -s) = 0 \Leftrightarrow$

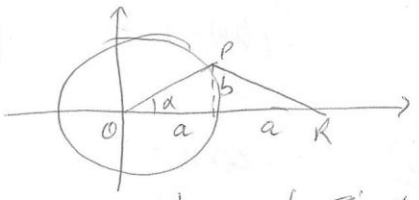
$\Leftrightarrow ru - r^2 - s^2 = 0 \Leftrightarrow$

$\Leftrightarrow ru - \underbrace{(r^2 + s^2)}_{1} = 0 \Leftrightarrow$

$\Leftrightarrow ru - 1 = 0$

$\Leftrightarrow ru = 1$

$\Leftrightarrow u = \frac{1}{r}$

(10) 

$\sin\alpha = \frac{b}{1} \Leftrightarrow b = \sin\alpha$

$\cos\alpha = \frac{a}{1} \Leftrightarrow a = \cos\alpha$

$Á_{rea} = \frac{2a \times b}{2} = \cos\alpha\sin\alpha$ (A)

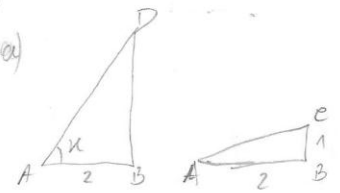
(11) (B)



(12) $\overline{OE} = \cos\theta$ } abcissa de A é $\cos\theta + \sin\theta$ (E)
$\overline{EB} = \sin\theta$ }

(13)

13.a)



$tg\,x = \frac{\overline{BD}}{2}$

$\overline{BD} = 2\,tg\,x$

$A = \frac{2 \times 2\,tg\,x}{2}$

$= 2\,tg\,x$

$A = \frac{2 \times 1}{2}$

$= 1$

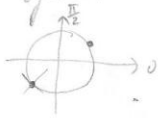
$A_{sombreada} =$

$= 2\,tg\,x - 1$

13.b) $2\,tg\,x - 1 = 1$ em $]0, \frac{\pi}{2}[$

$2\,tg\,x = 2$

$tg\,x = 1$



$x = \frac{\pi}{4}$

13.c) $\sin\left(\frac{\pi}{2} + x\right) = \frac{5}{13} \Leftrightarrow \boxed{\cos x = \frac{5}{13}}$

$1 + tg^2 x = \frac{1}{\cos^2 x}$

$1 + tg^2 a = \frac{1}{\left(\frac{5}{13}\right)^2}$

$1 + tg^2 x = \frac{169}{25}$

$tg^2 x = \frac{144}{25}$

$tg\,x = \overset{\frac{1}{}}{\times} \frac{12}{5} \left(]0, \frac{\pi}{2}[\right)$

$\boxed{tg\,x = \frac{12}{5}}$

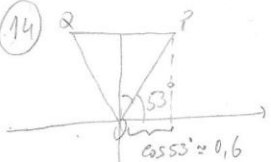
2: $2\,tg\,x - 1 =$

$= 2 \times \frac{12}{5} - 1 =$

$= \frac{24}{5} - 1 =$

$= \frac{19}{5}$

(14)



$\cos 53° \approx 0{,}6$

$p \approx 0{,}6 \times 2 + 1 + 1 = 3{,}2$ (A)

(15)

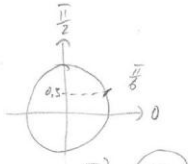
π — 180°

3 — x

$x = \frac{3 \times 180}{\pi}$

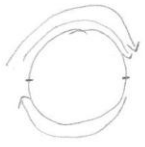
$x \simeq 172°$ (B)

(16)



$\left[\frac{\pi}{6}, \frac{\pi}{2}\right]$ (D)

(17)

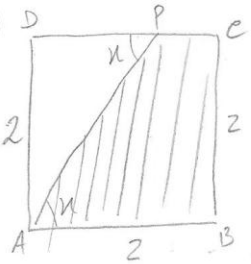


10h45 + 1h30 m =

= 12h15 m (C)

(18)

a)



$tg\, x = \frac{2}{\overline{DP}}$

$\boxed{\overline{DP} = \frac{2}{tg\, x}}$

$A_{\triangle} = \frac{b \times h}{2} = \frac{2 \times \frac{2}{tg\, x}}{2} = \frac{2}{tg\, x}$

$A_{\square} = \frac{B+b}{2} \times h = \frac{2 + 2 - \frac{2}{tg\, x}}{2} \times 2$

$= 4 - \frac{2}{tg\, x}$ c.q.d.

b) $4 - \frac{2}{tg\, x} = \frac{12 - 2\sqrt{3}}{3} \Leftrightarrow$

$\Leftrightarrow 4 - \frac{2}{tg\, x} = 4 - \frac{2\sqrt{3}}{3}$

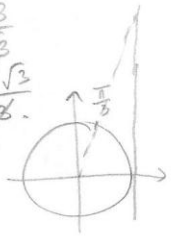
$\Leftrightarrow \frac{2}{tg\, x} = \frac{2\sqrt{3}}{3}$

$\Leftrightarrow 2\sqrt{3}\, tg\, x = 6$

$\Leftrightarrow tg\, x = \frac{3}{\sqrt{3}}$

$\Leftrightarrow tg\, x = \frac{3\sqrt{3}}{3}$

$\Leftrightarrow tg\, x = \sqrt{3}$



em $]\frac{\pi}{4}, \frac{\pi}{2}[$

$x = \frac{\pi}{3}$

c) $\cos\left(x + \frac{\pi}{2}\right) = -\frac{15}{17}$

$\Leftrightarrow -\sin x = -\frac{15}{17}$

$\Leftrightarrow \boxed{\sin x = \frac{15}{17}}$

$\sin^2 x + \cos^2 x = 1$

$\left(\frac{15}{17}\right)^2 + \cos^2 x = 1$

$\cos^2 x = \frac{64}{289}$

$\cos x = \pm \frac{8}{17}$

$tg\, x = \frac{\sin x}{\cos x} = \frac{\frac{15}{17}}{\frac{8}{17}} = \frac{15}{8}$

Então, $A = 4 - \frac{2}{tg\, x} = 4 - \frac{2}{\frac{15}{8}} =$

$= 4 - \frac{16}{15} = \frac{44}{15}$

(19) $tg\, \alpha = 2$

Como $tg^{-1}(2) \simeq 1{,}107$ então $\alpha = 1{,}107 + \pi \simeq 4{,}25$ (B)

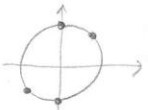
(20)

a) $\sin \alpha = \frac{\overline{PR}}{5} \Leftrightarrow \overline{PR} = 5 \sin \alpha$ Logo $A_{\triangle} = \frac{b \times h}{2} = \frac{10 \cos \alpha \times 5 \sin \alpha}{2} = 25 \sin \alpha \cos \alpha$ c.q.d.

$\cos \alpha = \frac{\overline{OR}}{5} \Leftrightarrow \overline{OR} = 5 \cos \alpha$

b) $f(x) = 25 \cos^2 \alpha \Leftrightarrow 25 \sin \alpha \cos \alpha = 25 \cos^2 \alpha \Leftrightarrow \cos^2 \alpha - \sin \alpha \cos \alpha = 0 \Leftrightarrow$

$\Leftrightarrow \cos \alpha (\cos \alpha - \sin \alpha) = 0 \Leftrightarrow \cos \alpha = 0 \vee \cos \alpha - \sin \alpha = 0 \Leftrightarrow \cos \alpha = 0 \vee \cos \alpha = \sin \alpha$

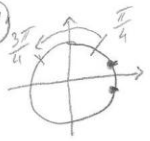


Ora, em $]0, \frac{\pi}{2}[$, $\alpha = \frac{\pi}{4}$.

c) Se $f(\theta) = 5$ então $25 \sin \theta \cos \theta = 5 \Leftrightarrow \sin \theta \cos \theta = \frac{1}{5}$

Assim, $(\sin \theta + \cos \theta)^2 = \sin^2 \theta + 2 \sin \theta \cos \theta + \cos^2 \theta = 1 + 2 \times \frac{1}{5} = \frac{7}{5}$

(1 e $\frac{1}{5}$)

(21) 

(C)

(22) $\alpha+\beta=\frac{\pi}{2} \Leftrightarrow \beta=\frac{\pi}{2}-\alpha$ Onde $\sin\alpha+\sin\beta+\sin\theta =$

$\alpha+\theta=2\pi \Leftrightarrow \theta=2\pi-\alpha$ $= \sin\alpha+\sin\left(\frac{\pi}{2}-\alpha\right)+\sin(2\pi-\alpha) =$

$= \sin\alpha+\cos\alpha-\sin\alpha = \cos\alpha$ (D)

(23) $\cos\left(\frac{3\pi}{2}-\alpha\right)=-\frac{4}{5}$

$\Leftrightarrow -\sin\alpha=-\frac{4}{5}$

$\Rightarrow \boxed{\sin\alpha=\frac{4}{5}}$

$\sin^2\alpha+\cos^2\alpha=1$

$\frac{16}{25}+\cos^2\alpha=1$

$\cos^2\alpha=\frac{9}{25}$

$\boxed{\cos\alpha=\overset{+}{\times}\frac{3}{5}}$

$\alpha\in\left]0,\frac{\pi}{2}\right[$

$tg\,\alpha=\frac{\sin\alpha}{\cos\alpha}=\frac{\frac{4}{5}}{\frac{3}{5}}=\frac{4}{3}$

Assim, $3-\frac{1}{tg\alpha}=3-\frac{1}{\frac{4}{3}}=3-\frac{3}{4}=\frac{9}{4}$

(24) $\sin\theta=-\frac{1}{3}$

$\sin(\pi-\theta)=\dots$

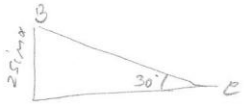
$\sin(\pi+\theta)=-\sin\theta=-\left(-\frac{1}{3}\right)=\frac{1}{3}$ (B)

$\sin\left(\frac{\pi}{2}-\theta\right)=\dots$

$\sin\left(\frac{\pi}{2}+\theta\right)=\dots$

(25) $\sin\alpha=\frac{h}{2}$

$h=2\sin\alpha$



$\sin 30^\circ=\frac{2\sin\alpha}{\overline{BC}}$

$\overline{BC}=\frac{2\sin\alpha}{\frac{1}{2}}$

$\overline{BC}=4\sin\alpha$ (A)

(26) a) $P(\cos\alpha;\sin\alpha)$

$B(3,0)$

$d=\overline{PB}=\sqrt{(3-\cos\alpha)^2+(0-\sin\alpha)^2}=\sqrt{9-6\cos\alpha+\underbrace{\cos^2\alpha+\sin^2\alpha}_{1}}=\sqrt{10-6\cos\alpha}$

c.q.d.

26.5.1

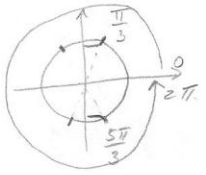
$d^2 = 7$

$10 - 6\cos\alpha = 7$

$-6\cos\alpha = -3$

$\cos\alpha = \frac{1}{2}$

Em $[0, 2\pi[$, $\alpha = \frac{\pi}{3} \vee \alpha = \frac{5\pi}{3}$



ou

$\cos\alpha = \frac{1}{2}$

$\cos\alpha = \cos\frac{\pi}{3}$

$\alpha = \frac{\pi}{3} + 2k\pi \vee \alpha = -\frac{\pi}{3} + 2k\pi, k \in \mathbb{Z}$

$k = 0 \rightsquigarrow \alpha = \frac{\pi}{3} \vee \alpha = \cancel{-\frac{\pi}{3}}$

$k = 1 \rightsquigarrow \alpha = \cancel{\frac{7\pi}{3}} \vee \alpha = \frac{5\pi}{3}$

$k = 2 \rightsquigarrow \alpha = \ldots \vee \alpha = \cancel{\frac{11\pi}{3}}$

$k = -1 \rightsquigarrow \alpha = \cancel{-\frac{5\pi}{3}} \vee \alpha = \cancel{-\frac{7\pi}{3}}$

26.5.2

$tg\,\alpha = -\sqrt{35}$

$1 + tg^2\alpha = \frac{1}{\cos^2\alpha}$

$1 + (-\sqrt{35})^2 = \frac{1}{\cos^2\alpha}$

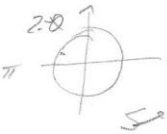
$1 + 35 = \frac{1}{\cos^2\alpha}$

$36 = \frac{1}{\cos^2\alpha}$

$\cos^2\alpha = \frac{1}{36}$

$\cos\alpha = \cancel{\pm}\ \frac{1}{6}$

$\cos\alpha = -\frac{1}{6}$



$\alpha \in [0, \pi[$ e $tg\alpha < 0$ então $\alpha \in 2.º Q$

Assim, $d^2 = 10 - 6\cos\alpha =$

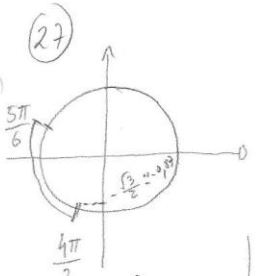
$= 10 - 6\left(-\frac{1}{6}\right) =$

$= 10 + 1$

$= 11$

Logo $d = \sqrt{11}$

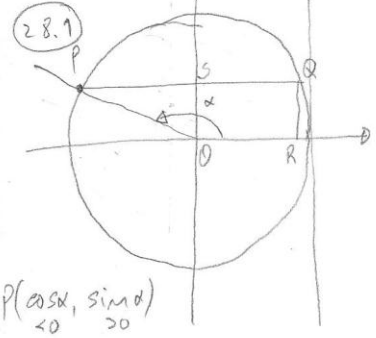
27



$\sin\alpha = -0,9$ não tem solução

(D)

28.1



$\overline{PS} = -\cos\alpha$

$\overline{PQ} = 2 \times \overline{PS} = -2\cos\alpha$

$\overline{OR} = \overline{SQ} = \overline{PS} = -\cos\alpha$

$\overline{QR} = \sin\alpha$

$A_{PQRO} = \frac{B+b}{2} \times h = \frac{\overline{PQ} + \overline{OR}}{2} \times \overline{QR} =$

$= \frac{-2\cos\alpha + (-\cos\alpha)}{2} \times \sin\alpha =$

$= \frac{-3\cos\alpha}{2} \times \sin\alpha = -\frac{3}{2}\cos\alpha\sin\alpha$ c.q.d.

$P(\underset{<0}{\cos\alpha}, \underset{>0}{\sin\alpha})$

(28.2) Se a reta OP interseta a reta $u=1$ num ponto de ordenada $-\frac{7}{24}$, então $\boxed{\operatorname{tg}\alpha = -\frac{7}{24}}$

Pretende-se o valor de $-\frac{3}{2}\operatorname{sen}\alpha\cos\alpha$

$$1+\operatorname{tg}^2\alpha = \frac{1}{\cos^2\alpha}$$

$$\Leftrightarrow 1+\left(-\frac{7}{24}\right)^2 = \frac{1}{\cos^2\alpha} \Leftrightarrow$$

$$\Leftrightarrow 1+\frac{49}{576} = \frac{1}{\cos^2\alpha}$$

$$\Leftrightarrow \frac{625}{576} = \frac{1}{\cos^2\alpha}$$

$$\Leftrightarrow \cos^2\alpha = \frac{576}{625}$$

$$\Leftrightarrow \cos\alpha = -\sqrt{\frac{576}{625}}$$

$$\Leftrightarrow \cos\alpha = -\frac{24}{25}$$

Como $\alpha \in \left]\frac{\pi}{2}, \pi\right[$, $\boxed{\cos\alpha = -\frac{24}{25}}$

$$\operatorname{tg}\alpha = \frac{\operatorname{sen}\alpha}{\cos\alpha}$$

$$\Leftrightarrow -\frac{7}{24} = \frac{\operatorname{sen}\alpha}{-\frac{24}{25}}$$

$$\Leftrightarrow \operatorname{sen}\alpha = -\frac{7}{24}\times\left(-\frac{24}{25}\right)$$

$$\Leftrightarrow \boxed{\operatorname{sen}\alpha = \frac{7}{25}}$$

Conclusão,

$$-\frac{3}{2}\operatorname{sen}\alpha\cos\alpha =$$

$$= -\frac{3}{2}\times\frac{7}{25}\times\left(-\frac{24}{25}\right) = \frac{504}{1250} = \frac{252}{625}$$