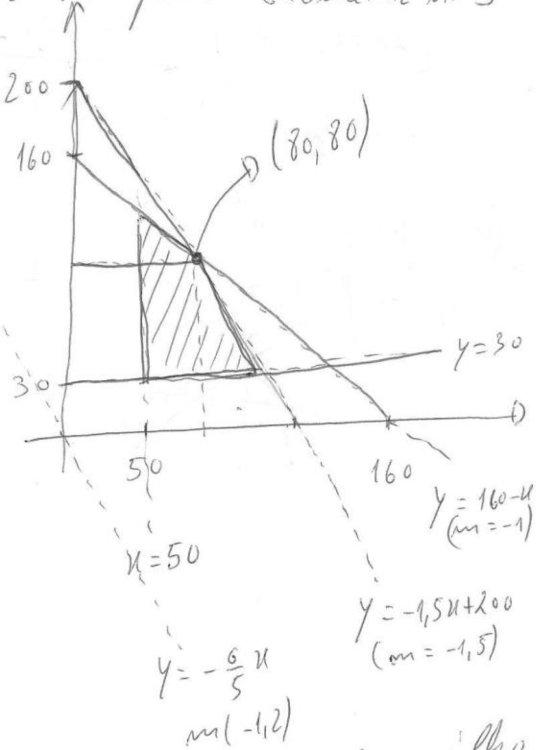
Proposta de resolução da ficha - Geometria n.º 3

① x hectares de trigo
y hectares de milho

$$\begin{cases} x+y \leq 160 \Leftrightarrow y \leq 160-x \\ x \geq 50 \\ y \geq 30 \\ 1500x+1000y \leq 200000 \end{cases}$$

$L(x) = 600x+500y$



$\rightarrow$ $1{,}5x+y \leq 200$

$y = -1{,}5x+200$

$\rightarrow$ $600x+500y=0$

$6x+5y=0$

$y = -\frac{6}{5}x$

R: Deve semear 80 hectares de trigo e 80 de milho.

② $\overrightarrow{AB}\cdot\overrightarrow{AC} = \overrightarrow{AB}\cdot(\overrightarrow{AB}+\overrightarrow{AD}) = \overrightarrow{AB}\cdot\overrightarrow{AB} + \overrightarrow{AB}\cdot\overrightarrow{AD} =$

$= \|\overrightarrow{AB}\|^2 + 0 = \|\overrightarrow{AB}\|^2 = \overline{AB}^2$

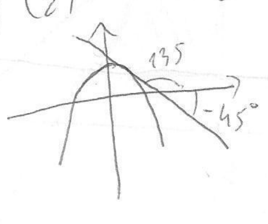
③ $(\times 2)\begin{cases} x+y-z=1 \\ 2x+2y-2z=1 \end{cases}$ $\begin{cases} -2x-2y+2z=-2 \\ 2x+2y-2z=1 \end{cases}$

$0=-1$ imp.

R: (A)

④ $f(x) = 1-x^2$. $f'(x) = -2x$

$m = f'\left(\frac{1}{2}\right) = -2\times\frac{1}{2} = -1 \Rightarrow m=-1 \rightarrow$ Inclinação $= 135°$



R: (C)

(5) 1 litro de X dá lucro de 4

x " " " " " " 4x

1 " " Y " " " 5

y " " " " " " 5y

Função que dá o lucro ~~L(x)~~ = $L = 4x + 5y$

B e D excluídos

Cada litro de X tem $\frac{1}{2}$ laranja e $\frac{1}{2}$ manga

x " " " " $\frac{x}{2}$ " e $\frac{x}{2}$ "

Cada litro de Y tem $\frac{2}{3}$ " e $\frac{1}{3}$ "

y " " " " $\frac{2y}{3}$ " e $\frac{y}{3}$ "

Como existem 12 l de laranja $\frac{x}{2} + \frac{2y}{3} \leq 12$

" " 10 l " " $\frac{x}{2} + \frac{y}{3} \leq 10$

Opção A

(6) $(x,y,z) = (1,2,3) + K(0,0,1)$; $K \in \mathbb{R}$

$(\Leftrightarrow)$ $x = 2 \wedge y = 1$ → reta vertical

(7)

(a.1) $m = \frac{1}{2}$ passa por $(-5,0)$

$y = mx + b$

$y = \frac{1}{2}x + b$

$(-5,0) \rightarrow 0 = \frac{1}{2} \times (-5) + b$

$\frac{5}{2} = b$

$\underset{(2)}{y} = \underset{(1)}{\frac{1}{2}}x + \underset{(1)}{\frac{5}{2}} \Leftrightarrow$

$\Leftrightarrow 2y = x + 5 \Leftrightarrow$

$\Leftrightarrow x - 2y + 5 = 0$ c.q.d.

(a.2) Circunferência / reta

$\begin{cases} x^2 + y^2 = 25 \\ x - 2y + 5 = 0 \end{cases}$ $\begin{cases} (2y-5)^2 + y^2 = 25 \\ x = 2y - 5 \end{cases}$

$\begin{cases} 4y^2 - 20y + \cancel{25} + y^2 = \cancel{25} \\ \_\_\_\_ \end{cases}$ $\begin{cases} 5y^2 - 20y = 0 \\ \_\_\_\_ \end{cases}$ $\begin{cases} y(5y - 20) = 0 \\ \_\_\_\_ \end{cases}$

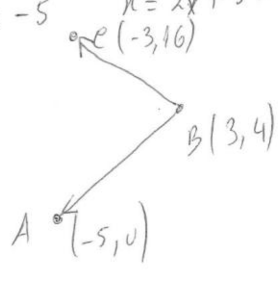
$\begin{cases} y = 0 \vee y = 4 \\ \downarrow \quad\quad \downarrow \end{cases}$

$x = -5$ $\quad$ $x = 2 \times 4 - 5 = 3$

Pontos $A(-5,0)$

$B(3,4)$ c.q.d.

(a.3)



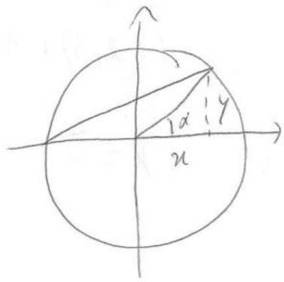
$\overrightarrow{BC} = C - B = (-6, 12)$

$\overrightarrow{BA} = A - B = (-8, -4)$

$\overrightarrow{BA} \cdot \overrightarrow{BC} = (-6,12) \cdot (-8,-4) = 48 - 48 = 0$

logo $\overrightarrow{BA} \perp \overrightarrow{BC}$ c.q.d.

(b.1) $\sin\alpha = \frac{y}{5} \Rightarrow y = 5\sin\alpha$



$\cos\alpha = \frac{x}{5} \rightarrow x = 5\cos\alpha$

Teorema de Pitágoras:

$(5+x)^2 + y^2 = d^2 \Leftrightarrow$

$\Leftrightarrow (5+5\cos\alpha)^2 + (5\sin\alpha)^2 = d^2 \Leftrightarrow$

$\Leftrightarrow 25 + 50\cos\alpha + 25\cos^2\alpha + 25\sin^2\alpha = d^2 \Leftrightarrow$

$\Leftrightarrow 25 + 50\cos\alpha + 25(\underbrace{\cos^2\alpha + \sin^2\alpha}_{1}) = d^2 \Leftrightarrow$

$\Leftrightarrow 25 + 50\cos\alpha + 25 = d^2$

$\Leftrightarrow 50 + 50\cos\alpha = d^2$ c.q.d.

(b.2) $1 + tg^2\alpha = \frac{1}{\cos^2\alpha} \Leftrightarrow 1 + 24 = \frac{1}{\cos^2\alpha} \Leftrightarrow 25 = \frac{1}{\cos^2\alpha}$

$\Leftrightarrow \cos^2\alpha = \frac{1}{25} \Leftrightarrow \cos\alpha = \cancel{\pm}\frac{1}{5} \Leftrightarrow \cos\alpha = \frac{1}{5}$

então $d^2 = 50 + 50 \times \frac{1}{5} \Leftrightarrow d^2 = 50 + 10 \Leftrightarrow d^2 = 60$

$\Leftrightarrow d = \sqrt{60}$

8 a) $U(5,3,5)$ vetor $(10,15,6) \perp$ plano α

$$(x,y,z) = (5,5,5) + k(10,15,6);\ k \in \mathbb{R}$$

b) Coordenadas de M

$$\begin{cases} 10x + 15y + 6z = 125 \\ x = 5 \\ z = 5 \end{cases} \quad \begin{cases} 50 + 15y + 30 = 125 \\ = \\ = \end{cases} \quad \begin{cases} 15y = 45 \\ = \\ = \end{cases}$$

$$\begin{cases} y = 3 \\ x = 5 \\ z = 5 \end{cases} \qquad M(5,3,5)$$

Coordenadas de N

$$\begin{cases} 10x + 15y + 6z = 125 \\ y = 5 \\ z = 5 \end{cases} \quad \begin{cases} 10x + 75 + 30 = 125 \\ = \\ = \end{cases} \quad \begin{cases} 10x = 20 \\ = \\ = \end{cases}$$

$$\begin{cases} x = 2 \\ y = 5 \\ z = 5 \end{cases} \qquad N(2,5,5) \qquad Q(5,3,0)$$

$$\overrightarrow{QN} = N - Q = (-3,0,5) \qquad \overrightarrow{QN} \cdot \overrightarrow{QM} = (-3,0,5)\cdot(0,-2,5) = 0+0+25 = 25$$

$$\overrightarrow{QM} = M - Q = (0,-2,5)$$

$$\cos\beta = \frac{\overrightarrow{QN} \cdot \overrightarrow{QM}}{\|\overrightarrow{QN}\|\,\|\overrightarrow{QM}\|}$$

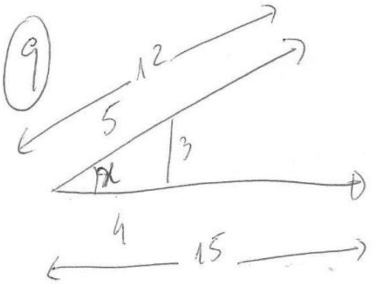
$$\cos\beta = \frac{25}{\sqrt{34}\,\sqrt{29}}$$

$$\|\overrightarrow{QN}\| = \sqrt{9+25} = \sqrt{34}$$

$$\beta = \cos^{-1}\left(\frac{25}{\sqrt{986}}\right) \simeq 37^{\circ}$$

$$\|\overrightarrow{QM}\| = \sqrt{4+25} = \sqrt{29}$$

(9)



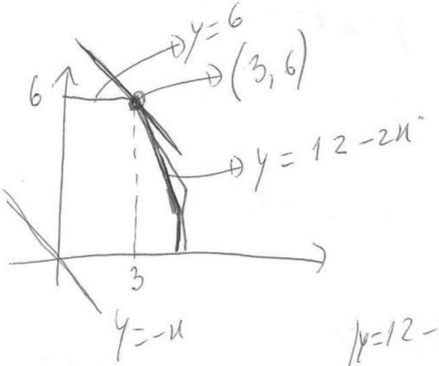
$\cos x = \frac{4}{5}$

$\vec{AD} \cdot \vec{AE} = \|\vec{AD}\| \|\vec{AE}\| \cos x = 12 \times 15 \times \frac{4}{5} = 144$

(10) $z = x + y$

$y + x = 0$

$\boxed{y = -x}$



$\begin{cases} y = 12 - 2x \\ y = 6 \end{cases}$ $\begin{cases} 6 = 12 - 2x \\ \end{cases}$

$\begin{cases} 2x = 6 \end{cases}$ $\begin{cases} x = 3 \\ y = 6 \end{cases}$

Se $x = 3$ e $y = 6$

então $z = x + y = 3 + 6 = 9$

(11) $P(0,4,3)$

a) reta $(x,y,z)=(0,1,-3)+k(1,0,2)\ ;\ k\in\mathbb{R}$

esfera $(x+2)^2+(y-1)^2+(z-4)^2\leq 3$

Plano α

$Ax+By+Cz+D=0$

$1x+0y+2z+D=0$

$P(0,4,3) \to 0+0+6+D=0$

$D=-6$

$\boxed{\text{Plano } \alpha : x+2z-6=0}$

$(-2,1,4)\in\alpha$?

$-2+2\times 4-6=0$

$-2+8-6=0$

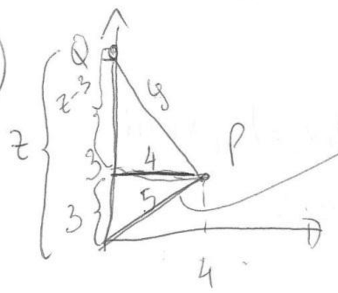
$0=0$ Verdade

(círculo de raio $\sqrt{3}$)

Área da secção

$A=\pi r^2=\pi\times(\sqrt{3})^2=3\pi$

(b.1)



$\to 5 \quad (4^2 + 3^2 = x^2 \Leftrightarrow x = 5)$

$$y^2 = 4^2 + (z-3)^2 \Leftrightarrow$$
$$\Leftrightarrow y^2 = 16 + z^2 - 6z + 9 \Leftrightarrow$$
$$\Leftrightarrow y^2 = 25 + z^2 - 6z$$
$$\Leftrightarrow y = \sqrt{z^2 - 6z + 25}$$

$$\text{Perímetro} = f(z) =$$
$$= z + 5 + y =$$
$$= z + 5 + \sqrt{z^2 - 6z + 25}$$

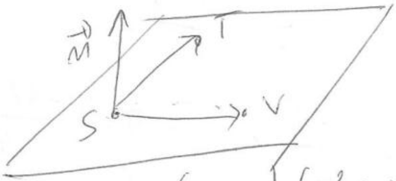
(b.2) Perímetro = 16

$$z + 5 + \sqrt{z^2 - 6z + 25} = 16 \Leftrightarrow$$
$$\Leftrightarrow \sqrt{z^2 - 6z + 25} = 16 - 5 - z \Leftrightarrow$$
$$\Rightarrow \left(\sqrt{z^2 - 6z + 25}\right)^2 = (11 - z)^2 \Leftrightarrow$$
$$= 121 - 22z + \cancel{z^2} \Leftrightarrow$$
$$\Leftrightarrow \cancel{z^2} - 6z + 25 = 121 - 22z + \cancel{z^2}$$

$$16z = 96 \Leftrightarrow$$
$$\Leftrightarrow z = \frac{96}{16} \Leftrightarrow$$
$$\Leftrightarrow z = 6$$

(12) $S(1,1,0)\quad T(-1,1,0)\quad V(0,0,2)$

a)



$\overrightarrow{ST} = (-2,0,0)$

$\overrightarrow{SV} = (-1,-1,2)$

$\begin{cases} \vec{n}\cdot\overrightarrow{ST} = 0 \\ \vec{n}\cdot\overrightarrow{SV} = 0 \end{cases}$ $\begin{cases} (a,b,c)\cdot(-2,0,0) = 0 \\ (a,b,c)\cdot(-1,-1,2) = 0 \end{cases}$ $\begin{cases} -2a = 0 \\ -a-b+2c = 0 \end{cases}$ $\begin{cases} a = 0 \\ b = 2c \end{cases}$

$\vec{n}(0, 2c, c) \underset{c=1}{\rightsquigarrow} (0,2,1)$

Plano STV $\quad Ax + By + Cz + D = 0$

$0x + 2y + z + D = 0$

$V(0,0,2) \rightarrow 0 + 2\times 0 + 2 + D = 0$

$D = -2$

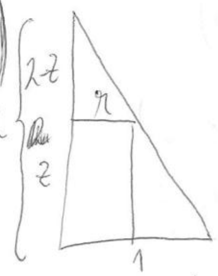
Plano STV $\rightsquigarrow 2y + z - 2 = 0$

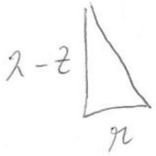
reta $x = 0 \wedge y = 2z$ é paralela a $(0,2,1)$

$(0, 2z, z) \rightsquigarrow (0,2,1)$

Se a reta é $\parallel$ ao vetor $\perp$ ao plano então a reta é $\perp$ ao plano.

5.1





$$\frac{2-z}{2} = \frac{r}{1}$$

$$r = \frac{2-z}{2}$$

$$A_{cilindro} = A_b \times h = \pi r^2 h = \pi \frac{(2-z)^2}{4} \times z = \pi \frac{4-4z+z^2}{4} \times z$$

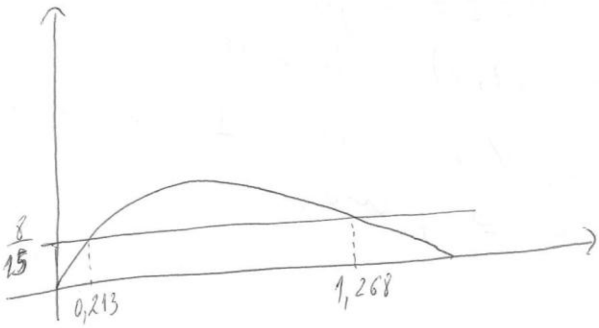
$$= \pi \frac{z^3-4z^2+4z}{4} = \pi\left(\frac{z^3}{4} - z^2 + z\right) \text{ c.q.d.}$$

5.2

$$V_{pirâmide} = \frac{1}{3} A_b \times h = \frac{1}{3} \times 4 \times 2 = \frac{8}{3}$$

$$\frac{1}{5} V_{pirâmide} = \frac{1}{5} \times \frac{8}{3} = \frac{8}{15}$$

$$\pi\left(\frac{z^3}{4} - z^2 + z\right) > \frac{8}{15}$$



(13) $x^2+y^2+(z-2)^2=4$

Sup. esférica de centro $C(0,0,2)$ e $R=2$, logo tangente a xOy. (B)

(14) $y=-\frac{1}{2}x+\frac{3}{5}$

$m=-\frac{1}{2}$

$m'=+2$

$y=2x+b$

$(1,4)$ $4=2\times1+b$

$2=b$

$\therefore y=2x+2$ (A)

(15) $L=3x+y$

x	y	L
0	3	3
4	3	15
6	2	20
6	0	18

(D)

(16) $\alpha: x+2y-2z=11 \perp (1,2,-2)$ $V(1,2,6)$

a) Se é // tem o mesmo vector director

$Ax+By+Cz+D=0$

$x+2y-2z+D=0$

$(1,2,6)$ $1+2(2)-2(6)+D=0$

$1+4-12+D=0$

$D=7$

$\therefore \gamma: x+2y-2z+7=0$

b) $\beta: 2x-y+z=3 \perp (2,-1,1)$

$(1,2,-2)\cdot(2,-1,1)=2-2-2=-2\neq0$. Como os vectores directores não são perpendiculares, os planos t.b. não são.

c) $V(1,2,6) \rightarrow W(1,2,-6)$

$x=1 \wedge y=2 \wedge -6<z<6$

d) Recta: $(x,y,z)=(1,2,6)+k(1,2,-2)$

$(x,y,z)=(1+k,\ 2+2k,\ 6-2k)$

Plano: $x+2y-2z=11$

Intersectando plano c/ recta

$1+k+2(2+2k)-2(6-2k)=11$

$1+k+4+4k-12+4k=11$

$9k=18$

$k=2$

$(x,y,z)=(1+2,\ 2+2(2),\ 6-2(2))=$

$=(3,6,2)$

$C(3,6,2)$ $V(1,2,6)$

altura $=\overline{VC}=\sqrt{(3-1)^2+(6-2)^2+(2-6)^2}$

$=\sqrt{4+16+16}=6$

$V=\frac{1}{3}A_b\times h=$

$=\frac{1}{3}\pi\times3^2\times6=$

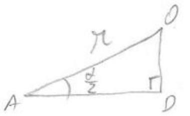
$=18\pi$

(17) [OAC] é isósceles pois $\overline{OA} = \overline{OC}$ = raio

$\overline{AC} = 2\overline{AD}$ pois [OAD] e [OCD] são triângulos iguais

igualdade de triângulos

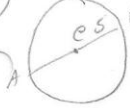
$\sphericalangle CAB = \frac{\alpha}{2}$ pois trata-se de um $\sphericalangle$ inscrito num arco de amplitude α



$\cos\left(\frac{\alpha}{2}\right) = \frac{\overline{AD}}{r}$

$\overline{AD} = r\cos\left(\frac{\alpha}{2}\right)$

$\overrightarrow{AB} \cdot \overrightarrow{AC} = \|\overrightarrow{AB}\| \underbrace{\|\overrightarrow{AC}\|}_{2\overline{AD}} \cos\left(\frac{\alpha}{2}\right) = 2r \times r\cos\left(\frac{\alpha}{2}\right) \times \cos\left(\frac{\alpha}{2}\right) = 4r^2\cos^2\left(\frac{\alpha}{2}\right)$

(18) 

$\overrightarrow{CA} \cdot \overrightarrow{CB} = \|\overrightarrow{CA}\| \|\overrightarrow{CB}\| \cos 180° = 5 \times 5 \times (-1) = -25$ (A)

(19) $r: (x,y) = (1,3) + k(2,0),\ k \in \mathbb{R}$ $s: y = \frac{3}{4}x + 1$

$\vec{u} = (2,0)$ $\vec{v} = (4,3)$

$\cos\alpha = \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\| \|\vec{v}\|} = \frac{(2,0)\cdot(4,3)}{2 \times 5} = \frac{8}{10} = \frac{4}{5}$ $\alpha = \cos^{-1}\left(\frac{4}{5}\right) \simeq 37°$ (A)

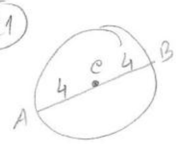
(20) $r: x = \frac{y}{2} = \frac{z}{3}$ ou $(x,y,z) = (0,0,0) + k(1,2,3)$

$(x,y,z) = (k, 2k, 3k)$

Intersectando com plano $3x - z = 0$

$3k - 3k = 0$

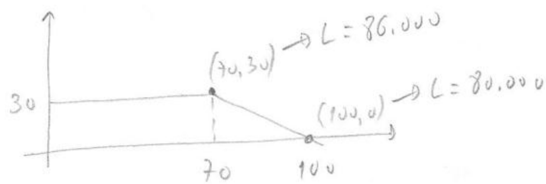
$0 = 0$, logo opção (D)

(21) 

$\overrightarrow{CA} \cdot \overrightarrow{CB} = \|\overrightarrow{CA}\| \|\overrightarrow{CB}\| \cos\pi =$
$= 4 \times 4 \times (-1) = -16$ (B)

(22) $\begin{cases} y \le 30 \\ x + y \le 100 \end{cases}$

$L = 800x + 1000y$



(A)

(23) a) $A(0,-2)$ $C(4,1)$

$\vec{AC} = (4,3)$

$m = \frac{3}{4}$

$m' = -\frac{4}{3}$

$y = -\frac{4}{3}x + b$

$(0,-2) \rightsquigarrow -2 = -\frac{4}{3} \times 0 + b$

$-2 = b$

$t: y = -\frac{4}{3}x - 2$

b) $A_{\bigcirc} = \pi r^2 = \pi \times 5^2 = 25\pi$

$r = \overline{AC} = \sqrt{(0-4)^2 + (-2-1)^2} = 5$

Área sombreada corresponde a $\frac{1}{6}$, logo $\alpha = \frac{2\pi}{6} = \frac{\pi}{3}$

$\vec{CP} \cdot \vec{CQ} = \|\vec{CP}\| \, \|\vec{CQ}\| \cos\left(\frac{\pi}{3}\right)$

$= 5 \times 5 \times \frac{1}{2} = \frac{25}{2}$

(24) Se $P \in$ sup. esf. então $tg^2\alpha + \sin^2\alpha + (2 + \cos\alpha - 2)^2 = 4$, ou seja $tg^2\alpha + \underbrace{\sin^2\alpha + \cos^2\alpha}_{=1} = 4$

$tg^2\alpha + 1 = 4$

$tg^2\alpha = 3$

$tg\alpha = \cancel{\pm}\sqrt{3} \quad (\alpha \in]0; \frac{\pi}{2}[)$

$1 + tg^2\alpha = \frac{1}{\cos^2\alpha}$

$1 + 3 = \frac{1}{\cos^2\alpha}$

$\cos^2\alpha = \frac{1}{4}$

$\cos\alpha = \cancel{\pm}\frac{1}{2}$

$\sin^2\alpha + \cos^2\alpha = 1$

$\sin^2\alpha + \frac{1}{4} = 1$

$\sin^2\alpha = \frac{3}{4}$

$\sin\alpha = \pm\sqrt{\frac{3}{4}} = \cancel{\pm}\frac{\sqrt{3}}{2}$

$P(tg\alpha, \sin\alpha, 2 + \cos\alpha)$

$\hookrightarrow P\left(\sqrt{3}; \frac{\sqrt{3}}{2}; 2 + \frac{1}{2}\right) = \left(\sqrt{3}; \frac{\sqrt{3}}{2}; \frac{5}{2}\right)$

(25) a) $A = ?$

$\begin{cases} 18x - 6y + 5z = 72 \\ y = 0 \\ z = 0 \end{cases} \begin{cases} 18x = 72 \\ = \end{cases} \begin{cases} x = 4 \\ y = 0 \\ z = 0 \end{cases}$ $A(4,0,0)$ $B(5,3,0)$

$\overline{AB} = \sqrt{(4-5)^2 + (0-3)^2 + (0-0)^2} = \sqrt{10}$

$A_b = \sqrt{10} \times \sqrt{10} = 10$

$V = \frac{1}{3} A_b \times h = \frac{1}{3} \times 10 \times 6 = \frac{60}{3} = 20$

b) $\begin{cases} 6x + 18y - 5z = 24 \\ 18x - 6y + 5z = 72 \\ z = 6 \end{cases} \begin{cases} 6x + 18y = 54 \\ 18x - 6y = 42 \\ \_\_ \end{cases} \xrightarrow{:3} \begin{cases} 2x + 6y = 18 \\ 18x - 6y = 42 \end{cases} \begin{cases} 2(3) + 6y = 18 \\ = \end{cases}$

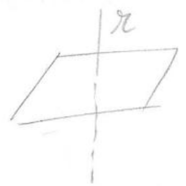
$20x = 60$

$x = 3$

$\begin{cases} y = 2 \\ x = 3 \\ z = 6 \end{cases}$ Então $V(3,2,6)$

e) $S(-1, -15, 5)$

$ADV \perp (6, 18, -5)$



$\uparrow (6, 18, -15)$

$r: \dfrac{x+1}{6} = \dfrac{y+15}{18} = \dfrac{z-5}{-5}$

$B(5,3,0) \in r$?

$\dfrac{5+1}{6} = \dfrac{3+15}{18} = \dfrac{0-5}{-5}$ Verdade, logo $B \in r$.

(26) $ABC: 6x + 3y + 4z = 12$

$A = ?$

$\begin{cases} 6x+3y+4z = 12 \\ y = 0 \\ z = 0 \end{cases} \begin{cases} 6x = 12 \\ - \\ - \end{cases} \begin{cases} x = 2 \\ y = 0 \\ z = 0 \end{cases}$ $A(2,0,0)$

$r: (x, y, z) = (2, 0, 0) + k(6, 3, 4);\ k \in \mathbb{R}$

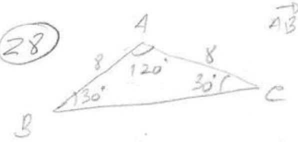
(27) (A) $x = 1$ e $y = 1 \to L = 2\times1 + 1 = 3$

(B) $x = 0$ e $y = 2 \to L = 2\times0 + 2 = 2$ opção (B)

(C) $x = 3$ e $y = 1 \to L = 2\times3 + 1 = 7$

(D) $x = 0$ e $y = 1 \notin$ à região

(28)



$\overrightarrow{AB} \cdot \overrightarrow{AC} = \|\overrightarrow{AB}\| \|\overrightarrow{AC}\| \cos 120° =$

$= 8 \times 8 \times \left(-\frac{1}{2}\right) = -32$ (B)

(29) $\overrightarrow{AI} = \overrightarrow{AD} + \overrightarrow{DI}$

$\overrightarrow{AJ} = \overrightarrow{AB} + \overrightarrow{BJ}$

$\overrightarrow{AI} \cdot \overrightarrow{AJ} = \left(\overrightarrow{AD} + \overrightarrow{DI}\right) \cdot \left(\overrightarrow{AB} + \overrightarrow{BJ}\right) =$

$= \underbrace{\overrightarrow{AD} \cdot \overrightarrow{AB}}_{=0} + \overrightarrow{AD} \cdot \overrightarrow{BJ} + \overrightarrow{DI} \cdot \overrightarrow{AB} + \underbrace{\overrightarrow{DI} \cdot \overrightarrow{BJ}}_{=0} =$

$= 2\, \overrightarrow{BJ} \cdot \overrightarrow{BJ} + 2\overrightarrow{DI} \cdot \overrightarrow{DI} =$

$= 2\|\overrightarrow{BJ}\|^2$

(30)

a.1) $\vec{u}(5,2,2)$ $O(0,0,0)$

$5x+2y+2z=0$

a.2) $x=2$

a.3) $\vec{u}(5,2,2)$ $\frac{x-4}{5} = \frac{y+4}{2} = \frac{z+4}{2}$

$U(4,-4,-4)$

a.4)

$r = \overline{UT} = 4$ $(x-4)^2+(y+4)^2+(z+4)^2=16$

centro $(4,-4,-4)$

b) $A(4,-4,z)$ $\vec{OA}=(4,-4,z)$ $(4,-4,z).(4,0,-4)=8$

$T(4,0,-4)$ $\vec{OT}=(4,0,-4)$ $\Leftrightarrow 16-4z=8$

$\Leftrightarrow z=2$ //

c) $V(2,-2,z)$

$5x+2y+2z=12$

$5\times 2+2(-2)+2z=12$

$10-4+2z=12$

$z=3$

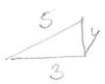
logo altura pirâmide $=3$

$V_{pirâmide} = \frac{1}{3}\times 4\times 4\times 3 = 16$

$V_{cubo} = 4\times 4\times 4 = 64$

$V_{poliedro} = 64+16 = 80$

(31)



$y^2+3^2=5^2$

$y=4$

$P(3,4)$

$\vec{OP}=(3,4)$

$m=\frac{4}{3}$ logo $m'=-\frac{3}{4}$

$(3,4) \rightsquigarrow y-4=-\frac{3}{4}(x-3) \Leftrightarrow$

$\Leftrightarrow y-4=-\frac{3}{4}x+\frac{9}{4}$

$\Leftrightarrow y=-\frac{3}{4}x+\frac{25}{4}$

(32) $(x,y,z)=(3,4,5)+K(1,0,0)$; $K\in\mathbb{R}$ reta // ao eixo dos xx

(A) $y=5 \wedge z=6$ reta // ao eixo dos xx

33) $D(2,1,0)$  $x^2 = 1^2 + 1^2$
$x = \sqrt{2}$

plano DCE $\perp (3,3,1)$

$3x + 3y + z + D = 0$

$(2,1,0) \rightsquigarrow 3(2) + 3(1) + 0 + D = 0$

$6 + 3 + D = 0$

$D = -9$

· plano DCE $\rightsquigarrow 3x + 3y + z - 9 = 0$

$E(1,1,z) \in$ plano DCE, logo

$3(1) + 3(1) + z - 9 = 0$

$z = 3$

Assim a altura da pirâmide = 3

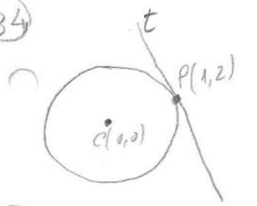
$V_{\text{pirâmide}} =$

$= \frac{1}{3} A_b \times h =$

$= \frac{1}{3} \times \sqrt{2} \times \sqrt{2} \times 3 =$

$= 2$

34)



$\vec{CP} = (1,2)$

$m_{CP} = \frac{2}{1} = 2$

$m_t = -\frac{1}{2}$ (B)

35) $s \rightsquigarrow (x,y,z) = (-1,0,3) + k(1,1,-1) \quad // \ (1,1,-1)$

$\beta \rightsquigarrow 3x + 3y + \alpha z = 1 \quad \perp (3,3,\alpha)$

$s // \beta$ se $(1,1,-1) \cdot (3,3,\alpha) = 0$

$3 + 3 - \alpha = 0$

$\alpha = 6$

36)

a) $\frac{x+3}{1} = \frac{y-3}{-5} = \frac{z-1}{1}$

b) plano ABC $\perp (-1,2,2)$

$F(-2,1,-1) \in$ plano ABC

$Ax + By + Cz + D = 0$

$-x + 2y + 2z + D = 0$

$(-2,1,-1) \rightsquigarrow -(-2) + 2(1) + 2(-1) + D = 0$

$D = -2$

Assim temos

$-x + 2y + 2z - 2 = 0$

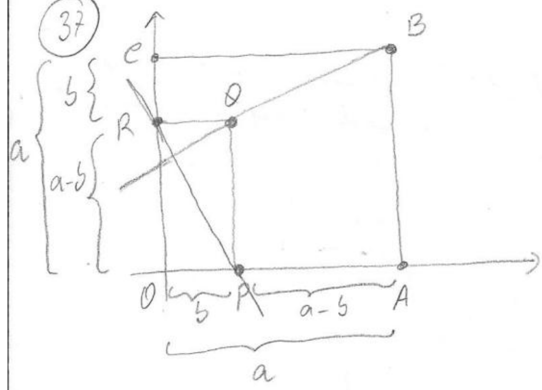
ou $x - 2y - 2z + 2 = 0$

c) D será a intersecção da reta ED com o plano ABC

$\begin{cases} x - y = -6 \\ y - z = 2 \\ x - 2y - 2z + 2 = 0 \end{cases}$ $\begin{cases} x = y - 6 \\ z = y - 2 \\ y - 6 - 2y - 2(y-2) + 2 = 0 \end{cases}$ $\begin{cases} \text{=} \\ \text{=} \\ y - \cancel{6} - 2y - 2y + \cancel{4} + \cancel{2} = 0 \end{cases}$

$\begin{cases} \text{=} \\ -3y = 0 \end{cases}$ $\begin{cases} x = 0 - 6 \\ z = 0 - 2 \\ y = 0 \end{cases}$ $\begin{cases} x = -6 \\ z = -2 \\ y = 0 \end{cases}$ $D \rightsquigarrow (-6, 0, -2)$

(37)



$Q(b; a-b)$ $\quad \overrightarrow{QB} = (a-b; b)$

$B(a; a)$

$R(0; a-b)$ $\quad \overrightarrow{RP} = (b; b-a)$

$P(b; 0)$

$\overrightarrow{QB} \cdot \overrightarrow{RP} =$

$= (a-b, b) \cdot (b, b-a) =$

$= ab - b^2 + b^2 - ab = 0$

logo $\overrightarrow{QB} \perp \overrightarrow{RP}$ e, assim,

as retas QB e RP são $\perp$.

(38)

a.1) Ponto do plano $(1, 3, -4) = F$

vetor diretor do plano $(2, 3, 6) = \overrightarrow{FA}$

$2x + 3y + 6z + D = 0$

$(1, 3, -4) \Leftrightarrow 2 \times 1 + 3 \times 3 + 6 \times (-4) + D = 0$

$\Leftrightarrow 2 + 9 - 24 + D = 0$

$\Leftrightarrow D = 13$

Logo, a equação do plano será $2x + 3y + 6z + 13 = 0$

a.2) ponto da reta $(1, 3, -4) = F$

vetor diretor da reta $(2, 3, 6) = \overrightarrow{FA}$

$$\frac{x-1}{2} = \frac{y-3}{3} = \frac{z+4}{6}$$

a.3) Raio $= \|\overrightarrow{FS}\| = \|\overrightarrow{FA}\| = \sqrt{2^2 + 3^2 + 6^2} = \sqrt{49} = 7$

Centro $(1, 3, -4)$

$(x-1)^2 + (y-3)^2 + (z+4)^2 = 49$

b) • Reta FE

$(x, y, z) = (1, 3, -4) + k(6, 2, -3)$; $k \in \mathbb{R}$ porque a reta FE é perpendicular ao plano HED

$\Leftrightarrow (x, y, z) = (1 + 6k, 3 + 2k, -4 - 3k)$; $k \in \mathbb{R}$

• Interseção da reta FE com o plano HED

$6(1+6k) + 2(3+2k) - 3(-4-3k) + 25 = 0$

$\Leftrightarrow 6 + 36k + 6 + 4k + 12 + 9k + 25 = 0$

$\Leftrightarrow 49k = -49$

$\Leftrightarrow k = -1$

Assim $(1 + 6(-1), 3 + 2(-1), -4 - 3(-1)) = (-5, 1, -1) = E$

(39) $P(x, -4, 1)$

$\vec{OP} = (x, -4, 1)$

$\vec{u} = (2, 3, 6)$

$\vec{OP} \cdot \vec{u} = 0 \Leftrightarrow$

$\Leftrightarrow (x, -4, 1) \cdot (2, 3, 6) = 0 \Leftrightarrow$

$\Leftrightarrow 2x - 12 + 6 = 0 \Leftrightarrow$

$\Leftrightarrow 2x = 6 \Leftrightarrow$

$\Leftrightarrow x = 3$ (C)

(40) Reta $\frac{x}{1} = \frac{y}{1} \wedge z = 2$

vetor diretor da reta $(1, 1, 0)$

se a reta é $\perp$ ao plano então os vetores diretores são paralelos, logo $x + y = 5$ será a opção correta.

↳ vetor diretor $(1, 1, 0)$

(41) $\vec{ED} \cdot \vec{DC} =$

$= (\vec{EA} + \vec{AD}) \cdot \vec{DC} =$

$= \vec{EA} \cdot \vec{DC} + \underbrace{\vec{AD} \cdot \vec{DC}}_{0 \text{ porque } \vec{AD} \perp \vec{DC}} =$

$= \|\vec{EA}\| \|\vec{DC}\| \cos \pi =$

$= 3 \times 4 \times (-1) =$

$= -12$

Área $= 6$

$\frac{\overline{EA} \times \overline{AD}}{2} = 6 \Leftrightarrow$

$\Leftrightarrow \overline{EA} \times \overline{AD} = 12$

$\Leftrightarrow \overline{EA} \times 4 = 12$

$\Leftrightarrow \overline{EA} = 3$

(42)

a) $x + y + 2z + D = 0$

$(1,2,3) \Rightarrow 1 + 2 + 2 \times 3 + D = 0$

$\Leftrightarrow D = -9$

Assim a equação do plano será $x + y + 2z - 9 = 0$

b) Ponto $A(x, 0, 0) \in$ ao plano $x + y + 2z = 12$

logo $x + 0 + 0 = 12 \Leftrightarrow x = 12$ Assim, $A(12,0,0)$

Ponto $C(0, 0, z) \in$ ao plano $x + y + z = 12$

logo $0 + 0 + 2z = 12 \Leftrightarrow z = 6$ Assim, $C(0,0,6)$

$$M = \left(\frac{12+0}{2}, \frac{0+0}{2}, \frac{0+6}{2} \right) = (6, 0, 3)$$

c) Recta $OP \perp$ plano ABC

$(x, y, z) = (0,0,0) + K(1,1,2)$; $K \in \mathbb{R}$

$(x, y, z) = (K, K, 2K)$; $K \in \mathbb{R}$

Intersecção da recta OP com o plano ABC

$K + K + 2(2K) = 12$

$\Leftrightarrow 6K = 12$

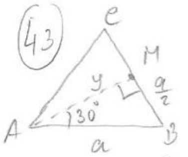
$\Leftrightarrow K = 2$

Assim $P =$ intersecção da recta OP c/ plano $ABC = (2,2,4)$

Raio da esfera $= \|\overrightarrow{OP}\| = \sqrt{4+4+16} = \sqrt{24}$

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \times (\sqrt{24})^3 = \frac{4\pi (\sqrt{24})^2 \times \sqrt{24}}{3} = \frac{4\pi \times 24 \times 2\sqrt{6}}{3} = 64\pi\sqrt{6}$$

(43)



$y^2 + \left(\frac{a}{2}\right)^2 = a^2$

$\Leftrightarrow y^2 + \frac{a^2}{4} = a^2$

$\Leftrightarrow y^2 = a^2 - \frac{a^2}{4}$

$\Leftrightarrow y^2 = \frac{3a^2}{4} \Leftrightarrow y = \frac{\sqrt{3}\,a}{2}$

$\overrightarrow{AB} \cdot \overrightarrow{AM} =$

$= \|\overrightarrow{AB}\| \|\overrightarrow{AM}\| \cos 30° =$

$= a \times \frac{\sqrt{3}\,a}{2} \times \frac{\sqrt{3}}{2} =$

$= \frac{3a^2}{4}$ c.q.d.