1.ª parte

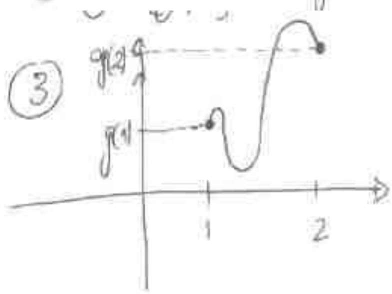
(1) $m = tg\,\alpha = 1$

$y = mx + b$

$y = x + b$

$(2,3) \to 3 = 2 + b \Leftrightarrow b = 1 \longrightarrow \boxed{y = x+1}$ // (A)

(2) $h'(a) = tg\,\alpha = \frac{3}{6} = \frac{1}{2}$ (B)

(3) 

(C) $g(1) < g(2)$

$\underset{[1;2]}{t.v.m} = \frac{g(2) - g(1)}{2 - 1} > 0 \Leftrightarrow g(2) - g(1) > 0 \Leftrightarrow$

$\Leftrightarrow g(2) > g(1)$

(4) Não tem derivada para $x = -1$ e $x = 1$. Em $]-1, 1[$ a derivada é negativa (f-ção decrescente) (C)
para $x < -1$ derivada nula (f-ção constante)

(5) $h(x) = 4x^3$ $\quad h'(2) = 12 \times 2^2 = 48$ (A)

$h'(x) = 12x^2$

(6) Gráfico (A) pois para $x = a$ a derivada é nula;
à esquerda a derivada é negativa (f-ção $\searrow$)
" direita " " " positiva (" $\nearrow$)

(7) $\lim\limits_{h \to 0} \frac{f(1+h) - f(1)}{h} = f'(1) = m = 3$ (B)

$\boxed{2.^{a}\ \text{Parte}}$

(1) $e(t) = \dfrac{8t+6}{t+6} \quad ; \ t \neq -6$

a) $e(t) = 4 \Leftrightarrow \dfrac{8t+6}{t+6} = 4 \Leftrightarrow \dfrac{8t+6}{t+6} - 4 = 0 \Leftrightarrow$

$\Leftrightarrow \dfrac{8t+6-4t-24}{t+6} = 0 \Leftrightarrow \dfrac{4t-18}{t+6} = 0 \Leftrightarrow 4t-18=0 \Leftrightarrow$

$\Leftrightarrow 4t = 18 \Leftrightarrow t = \dfrac{18}{4} \Leftrightarrow t = \dfrac{9}{2} \Leftrightarrow t = 4{,}5$

↳ 4 anos e 6 meses

b) $e'(1) = ?$

$e'(t) = \dfrac{(8t+6)'(t+6)-(8t+6)(t+6)'}{(t+6)^2} = \dfrac{8(t+6)-(8t+6)\times 1}{(t+6)^2}$

$= \dfrac{\cancel{8t}+48-\cancel{8t}-6}{(t+6)^2} = \dfrac{42}{(t+6)^2}$

Logo $e'(1) = \dfrac{42}{7^2} = \dfrac{42}{49} = \dfrac{6}{7}$

ou

$e'(1) = \lim\limits_{x\to 1} \dfrac{e(x)-e(1)}{x-1} = \lim\limits_{x\to 1} \dfrac{\frac{8x+6}{x+6} - 2}{x-1} =$

$= \lim\limits_{x\to 1} \dfrac{8x+6-2x-12}{(x+6)(x-1)} = \lim\limits_{x\to 1} \dfrac{6x-6}{(x+6)(x-1)} =$

$= \lim\limits_{x\to 1} \dfrac{6\cancel{(x-1)}}{(x+6)\cancel{(x-1)}} = \dfrac{6}{1+6} = \dfrac{6}{7}$

ou

$$\begin{array}{r|l} 8\cancel{t}+6 & t+6 \\ \hline -\cancel{8}t-48 & 8 \\ \hline -42 & \end{array}$$

$e(t) = 8 - \dfrac{42}{t+6}$

$e'(t) = \dfrac{42}{(t+6)^2}$

$e'(1) = \dfrac{42}{49} = \dfrac{6}{7}$

c) $D(t) > C(t)$:

$$\frac{5t+4}{t+2} > \frac{8t+6}{t+6} \Leftrightarrow \underset{(t+6)}{\frac{5t+4}{t+2}} - \underset{(t+2)}{\frac{8t+6}{t+6}} > 0 \Leftrightarrow$$

$$\Leftrightarrow \frac{(5t+4)(t+6)-(8t+6)(t+2)}{(t+6)(t+2)} > 0 \Leftrightarrow$$

$$\Leftrightarrow \frac{5t^2+30t+4t+24-8t^2-16t-6t-12}{(t+6)(t+2)} > 0 \Leftrightarrow$$

$$\Leftrightarrow \frac{-3t^2+12t+12}{(t+6)(t+2)} > 0$$

$-3t^2+12t+12=0$
$-t^2+4t+4=0$
$t^2-4t-4=0$
$t = \frac{4 \pm \sqrt{16+16}}{2}$
$t = \frac{4 \pm \sqrt{32}}{2}$
$t = 4,8 \vee \cancel{t = -0,8}$

	0		4,8		$+\infty$
$-3t^2+12t+12$		+	0	−	
$t+6$		+	+	+	
$t+2$		+	+	+	
$\frac{-3t^2+12t+12}{(t+6)(t+2)}$		+	0	−	

$D(t) > C(t)$ para $t < 4,8$

A partir de 4,8 anos $D(t) < C(t)$.

d) $P(t) = -8t^2 + 64t + 27$

$P'(t) = -16t + 64$

$-16t+64=0$
$16t = 64$
$t = 4$

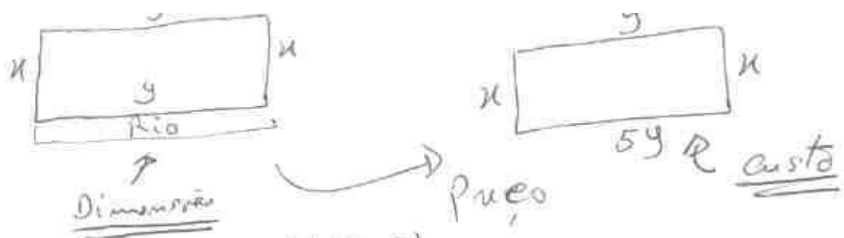
	0		4		$+\infty$
$P'(t) = -16t+64$		+	0	−	
$P(t)$		↗	M	↘	

$P(4) = -8 \times 16 + 64 \times 4 + 27 = 155$

R.: As árvores atingem a produção máxima de ... para $t = 4$ anos.

(2)

a)



$$x + y + x + 5y = 2400 \Leftrightarrow$$
$$\Leftrightarrow 6y + 2x = 2400 \Leftrightarrow$$
$$\Leftrightarrow 3y + x = 1200 \Leftrightarrow$$
$$\Leftrightarrow 3y = 1200 - x \Leftrightarrow$$
$$\Leftrightarrow y = 400 - \frac{1}{3}x$$

b) $A = x \cdot y = x\left(400 - \frac{1}{3}x\right) = 400x - \frac{1}{3}x^2$

$A'(x) = 400 - \frac{2}{3}x$

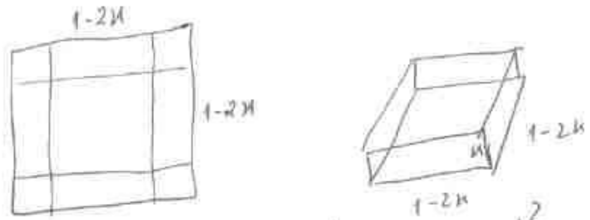
$400 - \frac{2}{3}x = 0 \Leftrightarrow$
$1200 - 2x = 0$
$1200 = 2x$
$x = 600$

	0		600		$+\infty$
$A'(x)$		+	0	−	
$A(x)$		↗	M	↘	

$A(600) = 400 \times 600 - \frac{1}{3}(600)^2 = 120.000$

(3)

a)



$V(x) = x(1-2x)(1-2x) = x(1-2x)^2 = x(1-4x+4x^2) \Leftrightarrow$
$V(x) = 4x^3 - 4x^2 + x$

b) $V'(x) = 12x^2 - 8x + 1$

$12x^2 - 8x + 1 = 0$
$x = \frac{8 \pm \sqrt{64-48}}{24}$
$x = \frac{8 \pm 4}{24}$; $x = 0{,}5$; $x \simeq 0{,}167$

	0		$\frac{1}{6}$ 0,167		0,5	$+\infty$
$V'(x)$		+	0	−	0	+
$V(x)$		↗	M	↘	m	↗

Vol. máx para $x = \frac{1}{6}$

c) $V\left(\frac{1}{6}\right) \simeq 0{,}074$

(1)

a) $C(x) = 0{,}2 \Leftrightarrow \dfrac{0{,}1x + 1{,}2}{x+1} = 0{,}2 \Leftrightarrow$

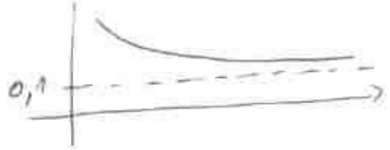
$\Leftrightarrow 0{,}1x + 1{,}2 = 0{,}2x + 0{,}2 \Leftrightarrow$

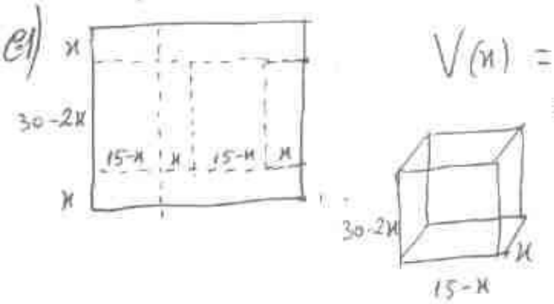
$\Leftrightarrow -0{,}1x = -1 \Leftrightarrow 0{,}1x = 1 \Leftrightarrow x = 10$

Temos de comprar 10 iogurtes

b) $\lim\limits_{x \to +\infty} C(x) = \lim\limits_{x \to +\infty} \dfrac{0{,}1x + 1{,}2}{x+1} = 0{,}1$

Significado: Por mais iogurtes que se comprem, o seu preço nunca atingirá 0,1 €.





e1) $V(x) = x(15-x)(30-2x) =$

$= (15x - x^2)(30-2x) =$

$= 450x - 30x^2 - 30x^2 + 2x^3$

$= 2x^3 - 60x^2 + 450x$

e.2) $V(x) = 2x^3 - 60x^2 + 450x$

$V'(x) = 6x^2 - 120x + 450$

$6x^2 - 120x + 450 = 0$

$x^2 - 20x + 75 = 0$

$x = \dfrac{20 \pm \sqrt{400 - 300}}{2}$

$x = \dfrac{20 \pm 10}{2}$ $\begin{cases} x = 15 \\ x = 5 \end{cases}$

	0		5		15	$+\infty$
$V'(x)$		+	0	−	0	+
$V(x)$		↗	M	↘	m	↗

$V(5) = 2 \times 5^3 - 60 \times 5^2 + 450 \times 5 =$

$= 1000$

Volume máximo $= 1000$ cm^3

5) $V(t) = \dfrac{100t + 7000}{t+2}$

5.1) $V(0) = \dfrac{7000}{2} = 3500$ euros

5.2) $V(t) = 900 \Leftrightarrow \dfrac{100t + 7000}{t+2} = 900 \Leftrightarrow$

$\Leftrightarrow \dfrac{100t+7000}{t+2} - 900 = 0 \Leftrightarrow 100t + 7000 - 900t - 1800$

$\Leftrightarrow -800t + 5200 = 0 \Leftrightarrow 8\cancel{0}\cancel{0}t = 52\cancel{0}\cancel{0} \Leftrightarrow t = 6{,}5$

R: Ao fim de 6 anos e 6 meses.

5.3) $\lim\limits_{t \to +\infty} V(t) = 100$. Logo o preço do carro nunca será inferior a 100 euros, pois o Joaquim nunca conseguirá comprá-lo.

5.4) $V'(3) = ?$

$$V'(t) = \frac{(100t+7000)'(t+2) - (100t+7000)(t+2)'}{(t+2)^2} =$$

$$= \frac{100(t+2) - (100t+7000)\times 1}{(t+2)^2} =$$

$$= \frac{\cancel{100t} + 200 - \cancel{100t} - 7000}{(t+2)^2} = -\frac{6800}{(t+2)^2}$$

$$V'(3) = -\frac{6800}{25} = -272$$

ou

$$\begin{array}{r|l} 100t + 7000 & t+2 \\ \hline -100t - 200 & 100 \\ \hline 6800 & \end{array}$$

$V(t) = 100 + \dfrac{6800}{t+2}$

$V'(t) = -\dfrac{6800}{(t+2)^2}$

$V'(3) = -\dfrac{6800}{25} = -272$

$$V = Ab \times h = 1000$$

$$\Leftrightarrow \pi r^2 \cdot h = 1000 \Leftrightarrow$$

$$\Leftrightarrow \boxed{h = \frac{1000}{\pi r^2}}$$

Para que o material utilizado seja mínimo a área total tem de ser mínima.

$$A_{total} = 2\cancel{\pi} \cancel{r} \times \frac{1000}{\cancel{\pi} r^2} + 2\pi r^2 = \frac{2000}{r} + 2\pi r^2$$

$$A(r) = \frac{2000 + 2\pi r^3}{r}$$

$$A'(r) = \frac{(2000 + 2\pi r^3)' r - (2000 + 2\pi r^3) r'}{r^2}$$

$$A'(r) = \frac{6\pi r^2 \cdot r - 2000 - 2\pi r^3}{r^2}$$

$$A'(r) = \frac{6\pi r^3 - 2\pi r^3 - 2000}{r^2} = \frac{4\pi r^3 - 2000}{r^2}$$

$$4\pi r^3 - 2000 = 0$$

$$r^3 = \frac{2000}{4\pi}$$

$$r \simeq 5{,}41926$$

Se $r = 5{,}419$

$$h = \frac{1000}{\pi (5{,}419)^2} = 10{,}84$$

$$\begin{cases} r \simeq 5{,}42 \\ h \simeq 10{,}84 \end{cases}$$

ou

$$A(r) = \frac{2\pi r^3 + 2000}{r} = 2\pi r^2 + \frac{2000}{r} \Rightarrow A'(r) = 4\pi r - \frac{2000}{r^2}$$

$$\begin{array}{l|l} 2\pi r^3 + 0r^2 + 0r + 2000 & r + 0 \\ \hline -2\pi r^3 + 0 & 2\pi r^2 + 0r + 0 \\ \quad 0r^2 + 0r & \\ \quad -0r^2 - 0r & \\ \qquad 0r + 2000 & \\ \qquad -0r - 0 & \\ \qquad\quad +2000 & \end{array}$$