1. a) $\begin{cases} u_1 = 2 \\ u_{n+1} = u_n + u_{n-1} \end{cases}$

b) $n = 9$

$\downarrow$

$u_{10} = u_9 + u_8$

$u_{10} = 68 + 42$

$u_{10} = 110$ //

$u_7 = 26$

$u_8 = 42$

$u_9 = 68$

2. $u_n = \dfrac{3n+1}{n+2}$ $\quad v_n = \begin{cases} v_1 = 10 \\ v_{n+1} = v_n - 2 \end{cases}$ $\quad w_n = \begin{cases} w_1 = 5 \\ w_n = 2 w_{n-1} \end{cases}$

a) $u_2 = \dfrac{7}{4}$ $\quad v_2 = v_1 - 2 = 8$ $\quad w_2 = 2 w_1 = 10$

$u_3 = \dfrac{10}{5}$ $\quad v_3 = v_2 - 2 = 6$ $\quad w_3 = 2 w_2 = 20$

b) $u_n = \dfrac{8}{3} \Leftrightarrow \dfrac{3n+1}{n+2} = \dfrac{8}{3} \Leftrightarrow 3(3n+1) = 8(n+2) \Leftrightarrow$

$\Leftrightarrow 9n + 3 = 8n + 16 \Leftrightarrow n = 13$

$\rightarrow \dfrac{8}{3}$ é termo de ordem 13.

d) $u_{n+1} - u_n = \dfrac{3(n+1)+1}{n+1+2} - \dfrac{3n+1}{n+2} = \dfrac{3n+4}{n+3}_{(n+2)} - \dfrac{3n+1}{n+2}_{(n+3)} =$

$= \dfrac{\cancel{3n^2} + \cancel{6n} + 4n + 8 - \cancel{3n^2} - \cancel{9n} - n - 3}{(n+2)(n+3)} = \dfrac{5}{(n+2)(n+3)} > 0$, logo $u_n \uparrow$

c) $u_n > 2 \Leftrightarrow \dfrac{3n+1}{n+2} > 2 \Leftrightarrow \dfrac{3n+1}{n+2} - 2 > 0 \Leftrightarrow \dfrac{3n+1-2n-4}{n+2} > 0$

$\Leftrightarrow \dfrac{n-3}{\underset{+}{n+2}} > 0 \Leftrightarrow n - 3 > 0 \Leftrightarrow n > 3$

$\therefore$ Todos os termos são superiores a 2 a partir de ordem 4.

e) 1º Processo

$$\begin{array}{r|l} 3n+1 & n+2 \\ \hline -3n-6 & 3 \\ -5 & \end{array}$$

$\dfrac{3n+1}{n+2} = 3 - \dfrac{5}{n+2}$

$\dfrac{1}{n+2}$: $\dfrac{1}{3}$; $\dfrac{1}{4}$; $\dfrac{1}{5}$; $\dfrac{1}{6}$ $\rightarrow 0$

$0 < \dfrac{1}{n+2} \leq \dfrac{1}{3}$

$-\dfrac{1}{3} \leq -\dfrac{1}{n+2} < 0$

$-\dfrac{5}{3} \leq -\dfrac{5}{n+2} < 0$

$-\dfrac{5}{3} + 3 \leq 3 - \dfrac{5}{n+2} < 0 + 3$

$\boxed{\dfrac{4}{3} \leq u_n < 3}$

2.º Processo :

Como $u_n \nearrow$, então $u_1 = \frac{4}{3}$ é minorante

Como $u_n \nearrow$ e $u_n \to 3$, então 3 é majorante

f) Sendo u_n monótona e limitada, então u_n é convergente.

g) v_n : 10; 8, 6, 4, 2, ---- w_n : 5, 10, 20, 40, ----

$\boxed{v_n = 12 - 2n}$

5×2, 5×2^2, 5×2^3

$\boxed{w_n = 5 \times 2^{n-1}}$

3) $u_n = \frac{1-n}{2}$

a) $u_{n+1} - u_n = \frac{1-(n+1)}{2} - \frac{1-n}{2} = \frac{\cancel{1}-n-\cancel{1}}{2} - \frac{1-n}{2} =$

$= \frac{-\cancel{n}-1+\cancel{n}}{2} = -\frac{1}{2} \Rightarrow$ prog. aritm. de $r = -\frac{1}{2}$ //

b) Sendo $r < 0$, logo $u_n \searrow$

c) $S_{13} = \frac{u_1 + u_{13}}{2} \times 13 = \frac{0-6}{2} \times 13 = -3 \times 13 = -39$

$u_1 = 0$

$u_{13} = \frac{1-13}{2} = -6$

4) $u_n = 5 \times \left(\frac{1}{2}\right)^{2n-1}$

a) $\frac{u_{n+1}}{u_n} = \frac{5 \times \left(\frac{1}{2}\right)^{2n+2-1}}{5 \times \left(\frac{1}{2}\right)^{2n-1}} = \frac{\cancel{5} \times \left(\frac{1}{2}\right)^{2n+1}}{\cancel{5} \times \left(\frac{1}{2}\right)^{2n-1}} = \left(\frac{1}{2}\right)^{\cancel{2n}+1-\cancel{2n}+1} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

prog. geom. $r = \frac{1}{4}$

$u_1 = 5 \times \left(\frac{1}{2}\right)^1 = \frac{5}{2}$

b) Como $u_1 = \frac{5}{2}$ e $r = \frac{1}{4}$; $u_n \searrow$, u_n é convergente.

e) $S_{13} = u_1 \times \frac{1-r^{13}}{1-r} = \frac{5}{2} \times \frac{1-\left(\frac{1}{4}\right)^{13}}{1-\frac{1}{4}} \simeq 3,3(3)$

$S_n = u_1 \times \frac{1-r^n}{1-r}$

$\lim\limits_{n\to+\infty} S_n = \lim\limits_{n\to+\infty} \frac{5}{2} \times \frac{1-\overbrace{\left(\frac{1}{4}\right)^n}^{\to 0}}{1-\frac{1}{4}} = \frac{5}{2} \times \frac{1}{\frac{3}{4}} = \frac{5}{2} \times \frac{4}{3} = \frac{20}{6} = \frac{10}{3}$

(5) $a_n = 4\left(-\frac{1}{2}\right)^{3-n}$

a) $\frac{a_{n+1}}{a_n} = \frac{4\left(-\frac{1}{2}\right)^{3-(n+1)}}{4\left(-\frac{1}{2}\right)^{3-n}} = \frac{\cancel{4}\left(-\frac{1}{2}\right)^{2-n}}{\cancel{4}\left(-\frac{1}{2}\right)^{3-n}} = \left(-\frac{1}{2}\right)^{2-\cancel{n}-3+\cancel{n}} = \left(-\frac{1}{2}\right)^{-1} = -2$

prog. geom. $r = -2$

$a_1 = 4\left(-\frac{1}{2}\right)^{3-1} = 4\left(-\frac{1}{2}\right)^{2} = 4 \times \frac{1}{4} = 1$

b) $S_{10} = a_1 \times \frac{1-r^{10}}{1-r} = 1 \times \frac{1-(-2)^{10}}{1-(-2)} = \frac{1-2^{10}}{3} = -341$

c) $a_1 = 1$, $r = -2$ > a_n é não monótona e divergente (oscilante)

(6) $u_n = \frac{8n+3}{4n}$ $\quad v_n : \begin{cases} v_1 = 9 \\ v_{n+1} = v_n + \frac{1}{3} \end{cases}$

a) $u_n = 5 \Leftrightarrow$

$\Leftrightarrow \frac{8n+3}{4n} = 5 \Leftrightarrow 8n+3 = 20n \Leftrightarrow 12n = 3 \Leftrightarrow n = \frac{1}{4} \notin \mathbb{N}$

∴ 5 não termo de u_n

b) $u_{n+1} - u_n = \frac{8(n+1)+3}{4(n+1)} - \frac{8n+3}{4n} = \frac{8n+11}{4n+4} - \frac{8n+3}{4n} =$

$= \frac{4n(8n+11) - (4n+4)(8n+3)}{4n(4n+4)} = \frac{\cancel{32n^2} + 44n - \cancel{32n^2} - 12n - 32n - 12}{4n(4n+4)}$

$= \frac{-12}{4n(4n+4)} < 0$, $u_n \searrow$ (Não é prog. aritm.)

c) $u_n = \frac{8n+3}{4n}$

$$\begin{array}{r|l} 8n+3 & 4n+0 \\ \cline{2-2} -8n+0 & 2 \\ \hline +3 & \end{array}$$

$= 2 + \frac{3}{4n}$

$\frac{1}{4n}$: $\frac{1}{4}, \frac{1}{8}, \frac{1}{12}, \dots \to 0$

$0 < \frac{1}{4n} \le \frac{1}{4}$

$0 < \frac{3}{4n} \le \frac{3}{4}$

$2 < 2 + \frac{3}{4n} \le 2 + \frac{3}{4}$

$\boxed{2 < u_n \le \frac{11}{4}}$

d) $v_1 = 9$

$v_2 = v_1 + \frac{1}{3} = 9 + \frac{1}{3} = \frac{28}{3}$

$v_3 = v_2 + \frac{1}{3} = 9 + \frac{1}{3} + \frac{1}{3} = \frac{29}{3}$

1.º 9
2.º $9 + \frac{1}{3}$
3.º $9 + 2 \times \frac{1}{3}$
4.º $9 + 3 \times \frac{1}{3}$
(...)

$v_n = 9 + (n-1) \times \frac{1}{3} = 9 + \frac{n-1}{3} = \frac{27+n-1}{3} = \frac{26+n}{3}$

$v_{n+1} - v_n = \frac{26+n+1}{3} - \frac{26+n}{3} = \frac{\cancel{26}+\cancel{n}+1-\cancel{26}-\cancel{n}}{3} = \frac{1}{3} \to$ prog. aritm. $r = \frac{1}{3}$

ou $\begin{cases} v_1 = 9 \\ v_{n+1} = v_n + \frac{1}{3} \end{cases} \Leftrightarrow \begin{cases} v_1 = 9 \\ v_{n+1} - v_n = \frac{1}{3} \end{cases}$

7 a) 1.º $0,5 + 1$
2.º $0,5 + 2 \times 1$
3.º $0,5 + 3 \times 1$
$n \to 0,5 + n \times 1$

$\boxed{e_n = 0,5 + n}$

b) $e_{n+1} - e_n = \cancel{0,5} + \cancel{n} + 1 - \cancel{0,5} - \cancel{n} = 1$

$r = 1 \to$ prog. aritm. P

c) $1 + 2 + 3 + \dots + 8 = \frac{1+8}{2} \times 8 = \frac{9 \times 8}{2} = \frac{72}{2} = 36$ horas

↑ 1.º dia ↑ 2.º dia ↑ 3.º dia

Total de horas: 36
Taxa total: $8 \times 0,5 = 4$

∴ O indivíduo pagará 4 € + 36 € = 40 €

⑧ $u_n = 2n + 7$ $\qquad v_n = 5 \times 2^{4-2n}$

a) $u_1 = 2+7 = 9$ $\qquad v_1 = 5 \times 2^{2} = 20$

$u_5 = 10+7 = 17$ $\qquad v_5 = 5 \times 2^{-6} = \frac{5}{2^6} = 0,078125$

b) $u_{n+1} - u_n = 2(n+1)+7-2n-7 = \cancel{2n}+2+\cancel{7}-\cancel{2n}-\cancel{7} = 2$

É prog. aritm. de $r = 2$

$$\frac{v_{n+1}}{v_n} = \frac{5 \times 2^{4-2(n+1)}}{5 \times 2^{4-2n}} = \frac{\cancel{5} \times 2^{4-2n-2}}{\cancel{5} \times 2^{4-2n}} = 2^{\cancel{4}-\cancel{2n}-2-\cancel{4}+\cancel{2n}} = \frac{1}{4}$$

É prog. geom. de $r = \frac{1}{4}$

c) $\left.\begin{matrix} u_1 = 9 \\ r = 2 \end{matrix}\right\rangle u_n = 9+(n-1)2$

$\left.\begin{matrix} v_1 = 20 \\ r = \frac{1}{4} \end{matrix}\right\rangle v_n = 20 \times \left(\frac{1}{4}\right)^{n-1}$

d) $\left.\begin{matrix} u_1 = 9 \\ u_{10} = 20+7 = 27 \end{matrix}\right\rangle S_{10} = \frac{9+27}{2} \times 10 = 180$

$\left.\begin{matrix} v_1 = 20 \\ r = \frac{1}{4} \end{matrix}\right\rangle S_{10} = 20 \times \frac{1-\left(\frac{1}{4}\right)^{10}}{1-\frac{1}{4}} \simeq 26,(6)$

e) $$\lim_{n \to +\infty} S_n = \lim_{n \to +\infty} \left[20 \times \frac{1-\overbrace{\left(\frac{1}{4}\right)^{n}}^{0}}{1-\frac{1}{4}} \right] = 20 \times \frac{1}{\frac{3}{4}} = 20 \times \frac{4}{3} = \frac{80}{3}$$

(9) a) 10.º

$\downarrow$

10 alunos $\rightarrow$ $35h + 10\times 12\,m = 35h + 2h = \underline{37h}$

20.º

$\downarrow$

20 alunos $\rightarrow$ $35h + 20\times 12\,m = 35h + 4h = \underline{39h}$

b) $12\,m = \frac{12}{60}\,h = 0,2\,h$

$$\boxed{a_n = 35 + 0,2\,n}$$

c) $n = 4\times 25 = 100$

$n = 100 \rightarrow a_{100} = 35 + 0,2\times 100 = 35 + 20 = 55\,h$

(10) 1.ª hora $\rightarrow$ $1+2+3+1 \rightarrow 6+1$

2.ª hora $\rightarrow$ $1+2+3+2 \rightarrow 6+2$

...

12.ª hora $\rightarrow$ $1+2+3+12 \rightarrow 6+12$

13.ª hora $\rightarrow$ $1+2+3+1 \rightarrow 6+1$

14.ª hora $\rightarrow$ $1+2+3+2 \rightarrow 6+2$

...

24.ª hora $\rightarrow$ $1+2+3+12 \rightarrow 6+12$

$6\times 12 + \frac{1+12}{2}\times 12 =$

$= 72 + 78 = 150$

Total $= 150\times 2 = 300$ //

(11) a) Falso $\rightarrow$ $\frac{(-1)^n}{n}$ é convergente para zero e não monótona.

b) Falso $\rightarrow$ $(-1)^n$ é limitada e divergente

c) Falso $\rightarrow$ $\frac{2n+1}{n} \rightarrow 2$ e é $\downarrow$

d) Falso $\rightarrow$ $a_n = n \rightarrow +\infty$

$b_n = -n \rightarrow -\infty$

$a_n \times b_n = -n^2 \rightarrow -\infty$

(12) Distância percorrida pelo Aquiles: $10, 20, 30, \dots$ ($\uparrow$ 1s, $\uparrow$ 2s, ...)

Distância "percorrida" pela Tartaruga: $101, 102, 103, \dots$ ($\uparrow$ 1s, $\uparrow$ 2s, ...)

Aquiles $\rightarrow$ $10 + (n-1) \times 10 = \cancel{10} + 10n - \cancel{10} = \underline{10n}$

Tartaruga $\rightarrow$ $101 + (n-1) \times 1 = 101 + n - 1 = \underline{n + 100}$

$10n = n + 100 \Leftrightarrow 9n = 100 \Leftrightarrow n = \frac{100}{9} \simeq 11,1$

Aquiles ultrapassa a tartaruga ao fim de 11,1 s.

(13) $a_n = \frac{n^2 - 13}{4}$ (B)

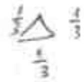
$\frac{n^2-13}{4} = 3 \Leftrightarrow n^2 - 13 = 12 \Leftrightarrow n^2 = 25 \Leftrightarrow n = 5$

(14) $1.500.000 (1 + 0,2)^n = 1,5 \times 10^6 \times 1,2^n$ (A)

(15) $u_7 = u_3 + 4r$

$u_2 = u_3 - 20$ (A)

(16) $u_{n-1} = 2u_n \Leftrightarrow \frac{u_n}{u_{n-1}} = \frac{1}{2}$ prog. geom. (B)

(17)   

$P = 3\times 3 = 9$ $\quad P = 3\times 1 = 3$ $\quad P = 3\times \frac{1}{3} = 1$

$$v_n = 3\times \frac{1}{3^{n-2}} = \frac{3}{3^{n-2}} = 3^{1-n+2} = 3^{3-n}$$ (C)

(18) $\lim\limits_{n\to+\infty} u_n = \lim\limits_{n\to+\infty} \left(\frac{1}{2}\right)^n = 0$ (Infinitésimo) (B)

(19) $20, 23, 26, \ldots$

$n=1 \quad n=2 \quad n=3 \quad \ldots$

$u_n = u_1 + (n-1)r$

$u_n = 20 + (n-1)3$

$u_n = 20 + 3n - 3 = 17 + 3n$ (B)

(20) $P_1 = 4\times 4$

$P_2 = 4\times 2\sqrt{2}$ $\quad \times \frac{\sqrt{2}}{2}$

$P_3 = 4\times 2$ $\quad \times \frac{\sqrt{2}}{2}$

$x^2 = 4+4$

$x = 2\sqrt{2}$

$x^2 = 2+2$

$x = 2$

$\vdots$

$$P_n = 16\times\left(\frac{\sqrt{2}}{2}\right)^{n-1}$$ (B)

(21) $A_1 = 3\times 3 = 9$

$A_2 = \frac{3}{2}\times\frac{3}{2} = \frac{9}{2^2}$

$A_3 = \frac{3}{4}\times\frac{3}{4} = \frac{9}{2^4}$

$A_4 = \frac{3}{8}\times\frac{3}{8} = \frac{9}{2^6}$

$\ldots$

$A_n = \frac{9}{2^{2n-2}}$ (C)

(22) $u_1 = 60$

$r = 0{,}75 = \frac{3}{4}$

$S_{11} = 60\times\frac{1-\left(\frac{3}{4}\right)^n}{1-\frac{3}{4}} =$

$= 60\times\frac{1-\left(\frac{3}{4}\right)^n}{\frac{1}{4}} =$

$= 240\left(1-\left(\frac{3}{4}\right)^{11}\right)$ (C)

(23) D é Falsa. Exemplo:

$$u_n = \begin{cases} (-1)^n, & n < 100 \\ n, & n \geq 100 \end{cases}$$

$\rightarrow u_n$ é não monótona

$\rightarrow u_n \rightarrow +\infty$ (infinitamente grande positivo)