① P.A. $2; 4; 6; 8; \ldots$ P.A. de $r = 2$ e $u_1 = 2$

$u_n = u_1 + (n-1)r = 2 + (n-1)\times 2 = 2 + 2n - 2 = 2n$

$u_n = 2n$

10 anos $\rightarrow u_{10} = 2\times 10 = 20$ cm

1000 " $\rightarrow u_{1000} = 2\times 1000 = 2000$ cm $= 20$ m

10.000 " $\rightarrow u_{10.000} = 20.000$ cm $= 200$ m

500 Km $= 50\,000\,000$ cm

$u_n = 50\,000\,000 \Leftrightarrow 2n = 50\,000\,000 \Leftrightarrow n = 25.000.000$

R: Atingirá 6000 Km depois de 25.000.000 anos.

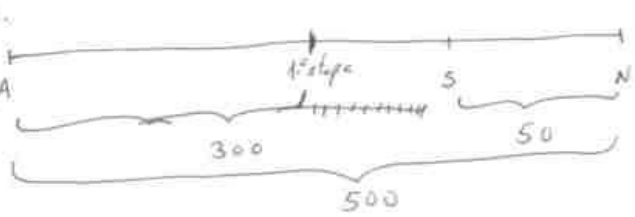
② P.A. $\begin{cases} u_{12} = 42 \\ u_{26} = 70 \end{cases}$ $u_{26} = u_{12} + 14r \Leftrightarrow 70 = 42 + 14r \Leftrightarrow 28 = 14r \Leftrightarrow$

$\Leftrightarrow \boxed{r = 2}$

$u_1 = u_{12} - 11r = 42 - 11\times 2 = 20 \rightarrow$ 1.º filé

$S_{26} = \frac{u_1 + u_{26}}{2}\times 26 = \frac{20+70}{2}\times 26 = 1170 \rightarrow$ plateia

③ P.G.



R $u_1 = 300$, $r = \frac{1}{3}$ $\Big|$ $u_n = u_1 \times r^{n-1} = 300\times\left(\frac{1}{3}\right)^{n-1}$

$u_4 = 300\times\left(\frac{1}{3}\right)^{4-1} = 300\times\frac{1}{27} \approx 11{,}1$ Km $\rightarrow$ 4.ª etapa

$S = \lim S_n = \lim\left(300\times\frac{1-\left(\frac{1}{3}\right)^n}{1-\frac{1}{3}}\right) \rightarrow 0 = \frac{300}{\frac{2}{3}} = \frac{900}{2} = 450$ Km

④ $r = 1 \rightarrow A = \pi$

$r = 1{,}5 \rightarrow A = 2{,}25\pi$

$r = 2 \rightarrow A = 4\pi$

$r = 2{,}5 \rightarrow A = 6{,}25\pi$

$r = 3 \rightarrow A = 9\pi$

$A_1 = 2{,}25\pi - \pi = 1{,}25\pi$ $\Big)$ $+0{,}5\pi$

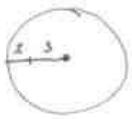
$A_2 = 4\pi - 2{,}25\pi = 1{,}75\pi$ $\Big)$ $+0{,}5\pi$

$A_3 = 6{,}25\pi - 4\pi = 2{,}25\pi$

$(\ldots)$

P.A. de $r = 0{,}5\pi$

⑤ P.G.



$u_1 = 2 \quad e \quad r = \frac{1}{2}$

$$S = \lim S_n = \lim \left(2 \times \frac{1-\left(\frac{1}{2}\right)^n}{1-\frac{1}{2}} \right) = \frac{2}{\frac{1}{2}} = 4 \text{ m}$$

R: A rã não chega ao centro (apenas anda 4 m)

⑥ P.G.

A) $u_1 = 500$, $r = \frac{1}{2}$ $\Big\}$ $S_{20} = 500 \times \frac{1-\left(\frac{1}{2}\right)^{20}}{1-\frac{1}{2}} \approx 999,999 \text{ €}$

B) $u_1 = \frac{1000}{3}$, $r = \frac{1}{3}$ $\Big\}$ $S_{20} = \frac{1000}{3} \times \frac{1-\left(\frac{1}{3}\right)^{20}}{1-\frac{1}{3}} \approx 499,(9) \text{ €}$

C) $u_1 = 250$, $r = \frac{1}{4}$ $\Big\}$ $S_{20} = 250 \times \frac{1-\left(\frac{1}{4}\right)^{20}}{1-\frac{1}{4}} \approx 333,(3) \text{ €}$

⑦ P.A.

$20 + 30 + 40 + \dots$

$u_1 = 20$, $r = 10$ $\Big\}$ $u_n = 20 + (n-1) \times 10 = 20 + 10n - 10 = 10 + 10n$

$u_n = 10 + 10n$

$u_{20} = 10 + 10 \times 20 = 210$ $\qquad S_{21} = \frac{20 + 210}{2} \times 20 = 2300 \text{ m}$

⑧ P.A.

$r = 0,25$

$u_{30} = 10$

$u_1 = u_{30} - 29r = 10 - 29 \times 0,25 = 2,75$ ✍ 2h45 min

⑨ P.G.

$u_1 = 8$

$r = \frac{1}{2}$

$$S = \lim S_n = \lim 8 \times \left(\frac{1-\left(\frac{1}{2}\right)^n}{1-\frac{1}{2}} \right) = \frac{8}{\frac{1}{2}} = 16 \text{ cm}$$

⑩ P.A.

$r = 2$

$u_n = 38$

$u_1 = 4$

$u_n = 4 + (n-1)(2) =$

$= 4 + 2n - 2 =$

$= 2 + 2n$

$2 + 2n = 38$

$2n = 36$

$n = 18$

$S_{18} = \frac{4 + 38}{2} \times 18 = 378$

(11) P.G.

1.ª descarga $\rightarrow u_1 = 0,4$

2.ª " $\rightarrow u_2 = 0,8$

$r = 2$ $\quad u_n = 0,4 \times 2^{n-1}$

$u_n = 384.000\,K_{mm} \Rightarrow$

$\Rightarrow 0,4 \times 2^{n-1} = 384.000.000.000_{mm} \Rightarrow$

$\Rightarrow 2^{n-1} = 9,6 \times 10^{11}$

$n-1 \simeq 40$

$\boxed{n \simeq 41}$

R: 41 descargas

(12) P.A.

$10 + 14 = 24$ €

$S_6 = 24 \Rightarrow \frac{u_1 + u_6}{2} \times 6 = 24 \Rightarrow \frac{u_1 + u_1 + 3,5}{2} \times 6 = 24 \Rightarrow \frac{2u_1 + 3,5}{2} \times 6 = 24$

$r = 0,7$

$\Downarrow$

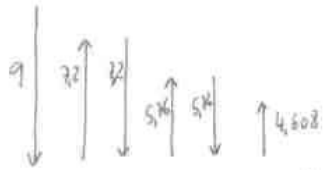
$u_6 = u_1 + 5r = u_1 + 5(0,7) = u_1 + 3,5$

$\Rightarrow (u_1 + 1,75) \times 6 = 24 \Rightarrow u_1 + 1,75 = 4 \Rightarrow u_1 = \boxed{2,25} \Rightarrow u_2 = 2,25 + 0,7 = \boxed{2,95} \Rightarrow$

$\Rightarrow u_3 = 2,95 + 0,7 = \boxed{3,65} \Rightarrow u_4 = 3,65 + 0,7 = \boxed{4,35} \Rightarrow u_5 = 4,35 + 0,7 = \boxed{5,05} \Rightarrow$

$\Rightarrow u_6 = 5,05 + 0,7 = \boxed{5,75}$

(13) P.G.



No 3.º salto chega 4,608 m

Após 1.º salto $\rightarrow u_1 = 7,2$

$r = 0,8$

$u_n = 7,2 \times (0,8)^{n-1}$

$u_n < 1,8 \Rightarrow 7,2 \times (0,8)^{n-1} < 1,8 \Rightarrow$

$\Rightarrow (0,8)^{n-1} < 0,25$

$\Rightarrow n - 1 = 7 \Rightarrow \boxed{n = 8}$

(14) P.A.

Seja n = nº de ordem do degrau exceto o 1.º.

$u_{19} = u_4 + 6$

$\Downarrow$

$0,5 + 18r = 0,5 + 3r + 6 \Rightarrow$

$\Rightarrow r = 0,4$ m

$u_1 = 0,5$ e $r = 0,4$

$u_n = 0,5 + (n-1)(0,4)$

$= 0,5 + 0,4n - 0,4$

$u_n = 0,1 + 0,4n$

$0,1 + 0,4n = 18,5$

$0,4n = 18,4$

$n = 46$

R: Nº de degraus $= 46 \times 2 = 92$.

(15) P.A.

1.° degrau: $\overset{(\cdots)}{\times\ \times\ \times\ \times\ \times\ \times\ \times\ \times\ \times\ \times}$ $\quad 10+9+\cdots+1 = \frac{10+1}{2} \times 10 = 55$

2.° degrau: $9+8+\cdots+1 = \frac{9+1}{2} \times 9 = 45$

3.° " : 36

4.° " : 28

5.° " : 21

6.° " : 15

7.° " : 10

8.° " : 6

9.° " : 3

10.° " : 1

$Total = 55+45+\cdots+1 = 220.$