Atividade de diagnóstico

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1.1. $\sin\alpha = \frac{b}{a}$, $\cos\alpha = \frac{c}{a}$ e $\tan\alpha = \frac{b}{c}$

1.2. **a)** $\sin\alpha = \frac{30}{50} = \frac{3}{5}$; $\cos\alpha = \frac{40}{50} = \frac{4}{5}$ e $\tan\alpha = \frac{30}{40} = \frac{3}{4}$

b) $\sin\alpha = \frac{4,5}{7,5} = \frac{3}{5}$; $\cos\alpha = \frac{6}{7,5} = \frac{4}{5}$ e $\tan\alpha = \frac{4,5}{6} = \frac{3}{4}$

2.1. $\sin 25° = \cos(90° - 25°) = \cos 65°$

2.2. $\cos(a - 30°) = \sin\left[90° - (a - 30°)\right] = \sin(90° - a + 30°) =$
$= \sin(120° - a)$

3. $\sin^2\alpha + \cos^2\alpha = 1$

$\sin^2\alpha + \left(\frac{2}{3}\right)^2 = 1 \Leftrightarrow \sin^2\alpha = 1 - \frac{4}{9} \Leftrightarrow \sin^2\alpha = \frac{5}{9}$

Como $\sin\alpha > 0$, $\sin\alpha = \frac{\sqrt{5}}{3}$.

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{\frac{\sqrt{5}}{3}}{\frac{2}{3}} = \frac{\sqrt{5}}{2}$$

4.1. $\tan^2\alpha = 25 \Leftrightarrow \frac{\sin^2\alpha}{\cos^2\alpha} = 25 \Leftrightarrow \sin^2\alpha = 25\cos^2\alpha$

$\sin^2\alpha + \cos^2\alpha = 1 \Leftrightarrow$
$\Leftrightarrow 25\cos^2\alpha + \cos^2\alpha = 1 \Leftrightarrow$
$\Leftrightarrow 26\cos^2\alpha = 1 \Leftrightarrow$
$\Leftrightarrow \cos^2\alpha = \frac{1}{26}$

Como $\cos\alpha > 0$, então $\cos\alpha = \frac{1}{\sqrt{26}} = \frac{\sqrt{26}}{26}$.

4.2. $\sin^2\alpha = 25\cos^2\alpha$

$\sin^2\alpha = 25 \times \frac{1}{26} = \frac{25}{26}$

Como $\sin\alpha > 0$, então $\sin\alpha = \frac{5}{\sqrt{26}} = \frac{5\sqrt{26}}{26}$.

4.3. $2\sin\alpha \times \cos\alpha = 2 \times \frac{5\sqrt{26}}{26} \times \frac{\sqrt{26}}{26} = \frac{2\times5\times26}{26\times26} = \frac{10}{26} = \frac{5}{13}$

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5.1. $\sin 60° - 2\tan 45° + \cos^2 45° =$

$= \frac{\sqrt{3}}{2} - 2\times1 + \left(\frac{\sqrt{2}}{2}\right)^2 =$

$= \frac{\sqrt{3}}{2} - 2 + \frac{1}{2} = \frac{\sqrt{3}}{2} - \frac{3}{2} = \frac{\sqrt{3}-3}{2}$

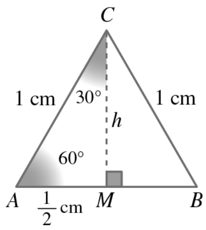
5.2. $\sin^2 45° + \cos^2 60° + \tan^3 45° =$

$= \left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + 1^3 =$

$= \frac{1}{2} + \frac{1}{4} + 1 = \frac{7}{4}$

6. $h^2 + \left(\frac{1}{2}\right)^2 = 1^2 \Leftrightarrow h^2 = 1 - \frac{1}{4} \Leftrightarrow h^2 = \frac{3}{4}$

Logo, $h = \frac{\sqrt{3}}{2}$.



6.1. $\sin 30° = \frac{\overline{AM}}{\overline{AC}} = \frac{\frac{1}{2}}{1} = \frac{1}{2}$

6.2. $\tan 30° = \frac{\overline{AM}}{\overline{MC}} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{2}{2\sqrt{3}} = \frac{\sqrt{3}}{3}$

7. $\overline{BC}^2 = 2,3^2 + 5,2^2 \overset{\overline{BC}>0}{\Leftrightarrow} \overline{BC} = \sqrt{32,33}$

$\tan\alpha = \frac{2,3}{5,2} \approx 0,4$; $\sin\alpha = \frac{2,3}{\sqrt{32,33}} \approx 0,4$ e

$\cos\alpha = \frac{5,2}{\sqrt{32,33}} \approx 0,9$

8. $\alpha \approx \tan^{-1}(2) \approx 63,4°$

9.1. $\overline{BC}^2 = 4^2 + 3^2$
$\overline{BC} = \sqrt{16+9} \Leftrightarrow \overline{BC} = 5$
$\overline{BC} = 5$ cm

9.2. **a)** $\tan\left(C\hat{B}A\right) = \frac{3}{4}$

Logo, $C\hat{B}A = \tan^{-1}\left(\frac{3}{4}\right) \approx 36,9°$.

b) $A\hat{C}B = 90° - C\hat{B}A \approx 90° - 36,87° \approx 53,1°$

10.1. $A\hat{C}B = 90° - 20° = 70°$

10.2. $\frac{\overline{AC}}{\overline{AB}} = \tan 20° \Leftrightarrow \frac{\overline{AC}}{6} = \tan 20° \Leftrightarrow \overline{AC} = 6 \times \tan 20°$

$\overline{AC} \approx 2,2$ cm

$\frac{\overline{AB}}{\overline{BC}} = \cos 20° \Leftrightarrow 6 = \overline{BC} \times \cos 20° \Leftrightarrow \overline{BC} = \frac{6}{\cos 20°}$

$\overline{BC} \approx 6,4$ cm

Atividade inicial 1

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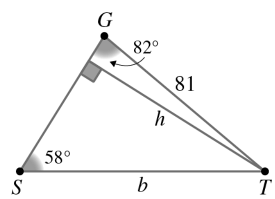
$\frac{h}{81} = \sin 82^{\circ} \Leftrightarrow h = 81\sin 82^{\circ}$

$\frac{h}{a} = \sin 58^{\circ} \Leftrightarrow h = a\sin 58^{\circ}$

$a\sin 58^{\circ} = 81\sin 82^{\circ} \Leftrightarrow$

$\Leftrightarrow a = \frac{81\sin 82^{\circ}}{\sin 58^{\circ}} \Rightarrow a \approx 94,6$



A distância a percorrer entre S. Jorge e a Ilha Terceira é de 94,6 km, aproximadamente.

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1.1. $A = 180° - 60° - 45° = 75°$

$\frac{\sin 75°}{4} = \frac{\sin 45°}{b} = \frac{\sin 60°}{c}$

$$b=\frac{4\times\sin 45°}{\sin 75°}=\frac{4\times\frac{\sqrt{2}}{2}}{\sin 75°}=\frac{2\sqrt{2}}{\sin 75°}\approx 2{,}928$$

$\overline{AC}\approx 2{,}9$ cm

$$c=\frac{4\times\sin 60°}{\sin 75°}=\frac{4\times\frac{\sqrt{3}}{2}}{\sin 75°}=\frac{2\sqrt{3}}{\sin 75°}\approx 3{,}586$$

$\overline{AB}\approx 3{,}6$ cm

1.2. $B=180°-63°-82°=35°$

$$\frac{\sin 82°}{16}=\frac{\sin 63°}{a}=\frac{\sin 35°}{b}$$

$$a=\frac{16\sin 35°}{\sin 82°}\approx 14{,}396$$

$\overline{BC}\approx 14{,}4$ cm

$$b=\frac{16\sin 35°}{\sin 82°}\approx 9{,}267$$

$\overline{AC}\approx 9{,}3$ cm

1.3. $A=180°-85°-15°=80°$

$$\frac{\sin 85°}{20}=\frac{\sin 80°}{a}=\frac{\sin 15°}{c}$$

$$a=\frac{20\times\sin 80°}{\sin 85°}\approx 19{,}771$$

$\overline{BC}\approx 19{,}8$ cm

$$c=\frac{20\times\sin 15°}{\sin 85°}\approx 5{,}196$$

$\overline{AB}\approx 5{,}2$ cm

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2. $\sin 150°+\sin 30°+\sin 90°=\sin(180°-30°)+\frac{1}{2}+1=$

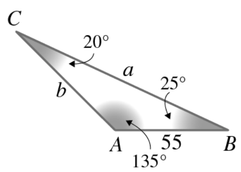
$$=\sin 30°+\frac{3}{2}=\frac{1}{2}+\frac{3}{2}=2$$

3. $180°-20°-25°=135°$

$$\frac{\sin 20°}{55}=\frac{\sin 135°}{a}=\frac{\sin 25°}{b}$$

$$a=\frac{55\times\sin 135°}{\sin 20°}\approx 113{,}7$$

$$b=\frac{55\times\sin 25°}{\sin 20°}\approx 68{,}0$$



Logo, $\overline{AC}\approx 68{,}0$ cm e $\overline{BC}\approx 113{,}7$ cm.

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4.1. $a^2=b^2+c^2-2bc\cos A$

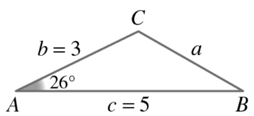
$a^2=3^2+5^2-2\times 3\times 5\cos 26°\Leftrightarrow$

$\Leftrightarrow a^2=34-30\cos 26°\Rightarrow$

$\Rightarrow a^2\approx 7{,}0362$

$\overline{BC}=a\approx\sqrt{7{,}0362}\approx 2{,}7$

$\overline{BC}\approx 2{,}7$ cm



4.2. $$\cos B=\frac{a^2+c^2-b^2}{2ac}\approx\frac{2{,}7^2+5^2-3^2}{2\times 2{,}7\times 5}\approx 0{,}8626$$

$B\approx\cos^{-1}(0{,}8626)\approx 30{,}4°$

5.1. $a=4{,}8$; $b=6{,}5$ e $c=8$

$$\cos A=\frac{b^2+c^2-a^2}{2bc}=\frac{6{,}5^2+8^2-4{,}8^2}{2\times 6{,}5\times 8}=\frac{83{,}21}{104}$$

$$A=\cos^{-1}\left(\frac{83{,}21}{104}\right)\approx 36{,}86°$$

$$\cos B=\frac{a^2+c^2-b^2}{2ac}=\frac{4{,}8^2+8^2-6{,}5^2}{2\times 4{,}8\times 8}=\frac{44{,}79}{76{,}8}$$

$$B=\cos^{-1}\left(\frac{44{,}79}{76{,}8}\right)\approx 54{,}32°$$

$$\cos c=\frac{a^2+b^2-c^2}{2ab}=\frac{4{,}8+6{,}5^2-8^2}{2\times 4{,}8\times 6{,}5}=\frac{1{,}29}{62{,}4}$$

$$c=\cos^{-1}\left(\frac{1{,}29}{62{,}4}\right)\approx 88{,}82°$$

$A\approx 36{,}86°$, $B\approx 54{,}32°$ e $C\approx 88{,}82°$

5.2. $a=3$, $b=c=5$

$$\cos A=\frac{b^2+c^2-a^2}{2bc}=\frac{5^2+5^2-3^2}{2\times 5\times 5}=\frac{41}{50}$$

$$A\approx\cos^{-1}\left(\frac{41}{50}\right)\approx 34{,}92°$$

Como $b=c$, temos $B=C$.

$$B=C=\frac{180°-34{,}92°}{2}=72{,}54°$$

$A\approx 34{,}92°$, $B\approx 72{,}54°$ e $C\approx 72{,}54°$

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6. $A=101°$, $a=\overline{BC}$, $b=7$ cm, $c=12$ cm

6.1. $a^2=b^2+c^2-2bc\cos A=7^2+12^2-2\times 7\times 12\times\cos 101°\approx$

$\approx 225{,}056$

$a\approx\sqrt{225{,}056}\approx 15{,}0$

$\overline{BC}\approx 15{,}0$ cm

6.2. $$\cos B=\frac{a^2+c^2-b^2}{2ac}\approx\frac{225{,}0559+12^2-7^2}{2\times\sqrt{225{,}0559}\times 12}\approx 0{,}8889$$

$B\approx\cos^{-1}(0{,}8889)\approx 27{,}3°$

$C\hat{B}A\approx 27{,}3°$

7. $\cos 120°-\sin 150°+\cos 90°=$

$=\cos(180°-60°)-\sin(180°-30°)+0=$

$$=-\cos 60°-\sin 30°=-\frac{1}{2}-\frac{1}{2}=-1$$

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8.1. $a=7$ cm, $b=8$ cm e $c=9$ cm

$$\cos A=\frac{b^2+c^2-a^2}{2bc}=\frac{8^2+9^2-7^2}{2\times 8\times 9}=\frac{96}{144}=\frac{2}{3}$$

$$A=\cos^{-1}\left(\frac{2}{3}\right)\approx 48{,}2°$$

$$\cos B=\frac{a^2+c^2-b^2}{2ac}=\frac{7^2+9^2-8^2}{2\times 7\times 9}=\frac{66}{126}=\frac{11}{21}$$

$$B=\cos^{-1}\left(\frac{11}{21}\right)\approx 58{,}4°$$

$$\cos C=\frac{a^2+b^2-c^2}{2ab}=\frac{7^2+8^2-9^2}{2\times 7\times 8}=\frac{32}{112}=\frac{2}{7}$$

$C = \cos^{-1}\left(\frac{2}{7}\right) \approx 73,4°$

$A \approx 48,2°;\ B \approx 58,4°$ e $C \approx 73,4°$

8.2. a = 5 cm, b = 8 cm e c = 5 cm

Como $a = c$, temos $A = C$.

$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{8^2 + 5^2 - 5^2}{2 \times 8 \times 5} = \frac{64}{80} = \frac{4}{5}$

$A = \cos^{-1}\left(\frac{4}{5}\right) \approx 36,9°$

$C \approx 36,9°$

$B \approx 180° - 2 \times 36,9° = 106,2°$

$A \approx 36,9°;\ B \approx 106,2°$ e $C \approx 36,9°$

8.3. a = 2,8 cm; $b \approx 4,5$ cm e $c \approx 5,3$ cm

$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{4,5^2 + 5,3^2 - 2,8^2}{2 \times 4,5 \times 5,3} = \frac{40,5}{47,7}$

$A = \cos^{-1}\left(\frac{40,5}{47,7}\right) \approx 31,9°$

$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{2,8^2 + 5,3^2 - 4,5^2}{2 \times 2,8 \times 5,3} = \frac{15,68}{29,68}$

$B = \cos^{-1}\left(\frac{15,68}{29,68}\right) \approx 58,1°$

$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{2,8^2 + 4,5^2 - 5,3^2}{2 \times 2,8 \times 4,5} = 0$

$C = \cos^{-1}(0) = 90°$

$A \approx 31,9°;\ B \approx 58,1°$ e $C = 90°$

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9.1. $A = 35°,\ b = 6$ cm e c = 7 cm

$a^2 = b^2 + c^2 - 2bc\cos A =$

$= 6^2 + 7^2 - 2 \times 6 \times 7 \times \cos 35° \approx$

$\approx 16,1912$

$a \approx \sqrt{16,1912} \approx 4,0$

$\cos B = \frac{a^2 + c^2 - b^2}{2ac} \approx \frac{16,1912 + 7^2 - 6^2}{2\sqrt{16,1912} \times 7} \approx 0,5182$

$B = \cos^{-1}(0,5182) \approx 58,8°$

$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{16,1912 + 6^2 - 7^2}{2 \times \sqrt{16,1912} \times 6} \approx 0,0661$

$C \approx \cos^{-1}(0,0661) \approx 86,2°$

$\overline{BC} \approx 4,0$ cm; $B \approx 58,8°$ e $C \approx 86,2°$

9.2. $B = 15°$, a = 5 cm e c = 8 cm

$b^2 = a^2 + c^2 - 2ac\cos B =$

$= 5^2 + 8^2 - 2 \times 5 \times 8 \times \cos 15° \approx 11,7259$

$b \approx \sqrt{11,7259} \approx 3,424$

$\overline{AC} \approx 3,4$ cm

$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{11,7259 + 8^2 - 5^2}{2\sqrt{11,7259} \times 8} \approx 0,9258$

$A \approx \cos^{-1}(0,9258) \approx 22,2°$

$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{5^2 + 11,7259 - 8^2}{2 \times 5 \times \sqrt{11,7259}} \approx -0,7965$

$C \approx \cos^{-1}(-0,7965) \approx 142,8°$

$\overline{AC} \approx 3,4$ cm; $A \approx 22,2°$ e $C \approx 142,8°$

9.3. $A = 40°$ e $a = b$ = 5 cm

O triângulo [ABC] é isósceles.

$B = A = 40°$

$C = 180° - 2 \times 40° = 100°$

$c^2 = a^2 + b^2 - 2ab\cos C =$

$= 5^2 + 5^2 - 2 \times 5 \times 5 \times \cos 100° \approx 58,6824$

$c \approx \sqrt{58,6824} \approx 7,660$

$\overline{AB} \approx 7,7$ cm

$\overline{AB} \approx 7,7$ cm, $B = 40°$ e $C = 100°$

10. $A = 30°,\ b = 6\sqrt{3}$ e $c = 9$

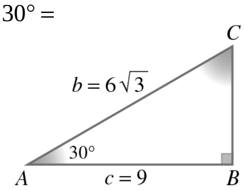
$a^2 = b^2 + c^2 - 2bc\cos A =$

$= \left(6\sqrt{3}\right)^2 + 9^2 - 2 \times 6\sqrt{3} \times 9 \times \cos 30° =$

$= 108 + 81 - 2 \times 54\sqrt{3} \times \frac{\sqrt{3}}{2} =$

$= 27$

$a = \sqrt{27} = 3\sqrt{3}$



$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{\left(\sqrt{27}\right)^2 + 9^2 - \left(6\sqrt{3}\right)^2}{2 \times \sqrt{27} \times 9} = \frac{27 + 81 - 108}{18\sqrt{27}} = 0$

$B = \cos^{-1}(0) = 90°$ (O triângulo [ABC] é retângulo em B.)

$C = 90° - 30° = 60°$

$\overline{BC} = 3\sqrt{3}$ cm, $B = 90°$ e $C = 60°$

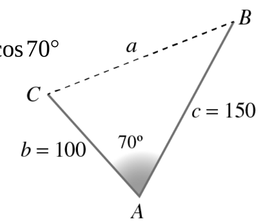
11. $a^2 = b^2 + c^2 - 2bc\cos A =$

$= 100^2 + 150^2 - 2 \times 100 \times 150 \times \cos 70°$

$\approx 22\ 239,396$

$a \approx \sqrt{22\ 239,396} \approx 149,1$

A distância entre os dois balões é de 149 m, aproximadamente.



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12.1. $A = 72°;\ C = 48°$ e $c = 50$ cm

$B = 180° - 72° - 48° = 60°$

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

$\frac{\sin 72°}{a} = \frac{\sin 60°}{b} = \frac{\sin 48°}{50}$

$a = \frac{50 \times \sin 72°}{\sin 48°} \approx 63,989$

$b = \frac{50 \sin 60°}{\sin 48°} \approx 58,268$

$P_{[ABC]} \approx (63,989 + 58,268 + 50)$ cm $\approx 172,3$ cm

12.2. $A = 25°,\ C = 130°$ e $b = 12$ cm

$B = 180° - 25° - 130° = 25°$

O triângulo [ABC] é isósceles: $\overline{BC} = \overline{AC} = 12$ cm

a = 12 cm

$\frac{\sin 130°}{c} = \frac{\sin 25°}{12} \Leftrightarrow c = \frac{12 \times \sin 130°}{\sin 25°} \approx 21,751$

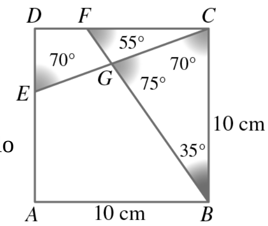
$P_{[ABC]} \approx (12 + 12 + 21,751) \approx 45,8$ cm

13. $C\hat{B}F = 180° - 90° - 55° = 35°$

$B\hat{C}G = C\hat{E}D = 70°$

(ângulos alternos internos)

$B\hat{G}C = 180° - 70° - 35° = 75°$



Aplicando a lei dos senos ao triângulo [BCG], temos:

$$\frac{\sin 75°}{10} = \frac{\sin 70°}{\overline{BG}} = \frac{\sin 35°}{\overline{CG}}$$

$$\overline{BG} = \frac{10\sin 70°}{\sin 75°} \approx 9{,}728$$

$$\overline{CG} = \frac{10\sin 35°}{\sin 75°} \approx 5{,}938$$

$\overline{BG} \approx 9{,}7$ cm e $\overline{CG} \approx 5{,}9$ cm

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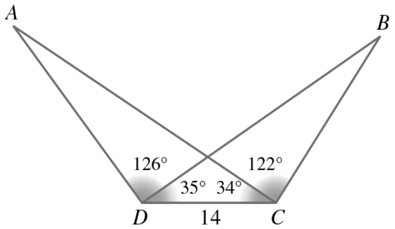
14.



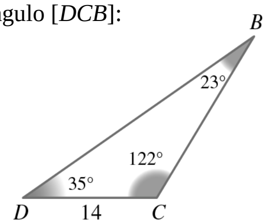
Aplicando a lei dos senos ao triângulo [DCB]:

$C\hat{B}D = 180° - 122° - 35° = 23°$

$$\frac{\sin 23°}{14} = \frac{\sin 35°}{\overline{BC}}$$

$$\overline{BC} = \frac{14 \times \sin 35°}{\sin 23°} \approx$$

$\approx 20{,}5514$

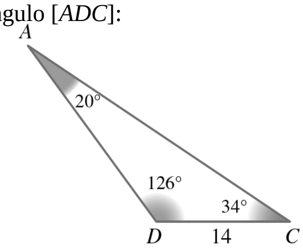


Aplicando a lei dos senos ao triângulo [ADC]:

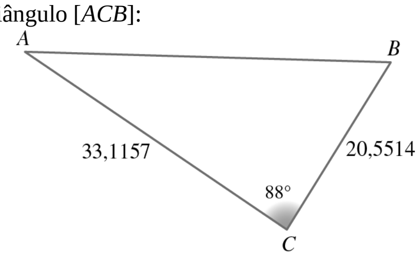
$D\hat{A}C = 180° - 126° - 34° = 20°$

$$\frac{\sin 20°}{14} = \frac{\sin 126°}{\overline{AC}}$$

$$\overline{AC} = \frac{14 \times \sin 126°}{\sin 20°} \approx 33{,}1157$$



Aplicando o Teorema de Carnot ao triângulo [ACB]:



$B\hat{D}A = 122° - 34° = 88°$

$$\overline{AB}^2 = \overline{AC}^2 + \overline{CB}^2 - 2\overline{AC} \times \overline{CB} \times \cos 88° =$$

$$\approx 33{,}1157^2 + 20{,}5514^2 - 2 \times 33{,}1157 \times 20{,}5514 \times \cos 88°$$

$$\approx 1471{,}506$$

$$\overline{AB} \approx \sqrt{1471{,}506} \approx 38{,}4 \text{ m}$$

A distância entre A e B é aproximadamente igual a 38,4 m.

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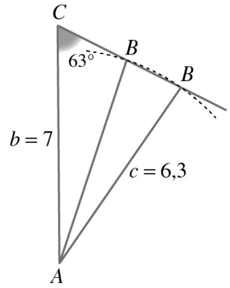
15.1. $\dfrac{\sin 63°}{6{,}3} = \dfrac{\sin B}{7}$



$$\sin B = \frac{7 \times \sin 63°}{6{,}3} \approx 0{,}9900$$

Se B é um ângulo agudo,

$B \approx \sin^{-1}(0{,}9900) \approx 81{,}89$

Se B é um ângulo obtuso,

$B \approx 180° - 81{,}89 \approx 98{,}11$

Se $B \approx 81{,}89$, $A \approx 180° - 63° - 81{,}89° \approx 35{,}11°$

Se $B \approx 98{,}11$, $A \approx 180° - 63° - 98{,}11° \approx 18{,}89°$

Determinação de $a = \overline{BC}$:

$$\frac{\sin A}{a} = \frac{\sin 63°}{6{,}3}$$

$$\frac{\sin 35{,}11°}{a} = \frac{\sin 63°}{6{,}3} \quad \text{ou} \quad \frac{\sin 18{,}89°}{a} = \frac{\sin 63°}{6{,}3}$$

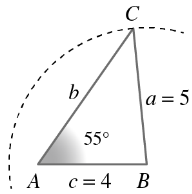
$$a \approx \frac{6{,}3 \times \sin 35{,}11°}{\sin 63°} \approx 4{,}1 \quad \text{ou} \quad a = \frac{6{,}3 \times \sin 18{,}89°}{\sin 63°} \approx 2{,}3$$

Conclusão:

$\overline{BC} = 4{,}1$, $A \approx 35{,}1°$ e $B \approx 81{,}9°$ ou

$\overline{BC} = 2{,}3$, $A \approx 18{,}9°$ e $B \approx 98{,}1°$

15.2. $\dfrac{\sin 55°}{5} = \dfrac{\sin C}{4}$



$$\sin C = \frac{4 \times \sin 55°}{5} \approx 0{,}6553$$

Se C é um ângulo agudo,

$C \approx \sin^{-1}(0{,}6553) \approx 40{,}94°$.

Se C é um ângulo obtuso,

$C = 180° - 40{,}94° = 139{,}06°$.

Se $C \approx 40{,}94°$:

$B = 180° - (55° + 40{,}94°) \approx 84{,}06°$

Se $C \approx 139{,}06°$:

$B = 180° - (55° + 139{,}06°) \approx 180° - 194{,}06° < 0$

B não pode ser negativo. Logo, o problema tem apenas uma solução: $B \approx 84{,}06°$

Determinemos $b = \overline{AC}$:

$$\frac{\sin 55°}{5} = \frac{\sin(84{,}06°)}{b}$$

$$b = \frac{5 \times \sin(84{,}06°)}{\sin 55°} \approx 6{,}1$$

$b \approx 6{,}1$; $B \approx 84{,}1°$ e $C \approx 40{,}9°$

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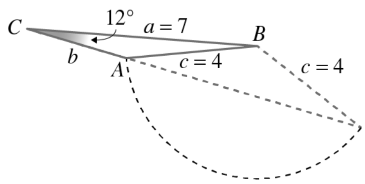
16.1. $a = 7$, $c = 4$ e $C = 12°$



$$\frac{\sin 12°}{4} = \frac{\sin A}{7}$$

$$\sin A = \frac{7 \times \sin 12°}{4}$$

$\approx 0{,}36385$

Se A é agudo:

$A \approx \sin^{-1}(0{,}36385) \approx 21{,}337$

$B \approx 180° - 12° - 21{,}337° \approx 146{,}663°$

$$\frac{\sin 146,663º}{b} = \frac{\sin 12°}{4}$$

$$b = \frac{4\sin 146,663º}{\sin 12°} \approx 10,6$$

Se A é obtuso:

$$A \approx 180° - 21,337 \approx 158,663$$

$$B \approx 180° - 12° - 158,663° \approx 9,337$$

$$\frac{\sin 9,337°}{b} \approx \frac{\sin 12°}{4}$$

$$b \approx \frac{4\sin 9,337°}{\sin 12°} \approx 3,1$$

$A = 21,3°$; $B = 146,7°$ e b = 10,6 ou

$A = 158,7°$; $B = 9,3°$ e b = 3,1

Também se pode usar o Teorema de Carnot.

$$c^2 = a^2 + b^2 - 2ab\cos C$$

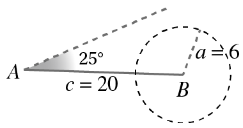
$$16 = 49 + b^2 - 2\times 7\times b\times \cos 12° \Leftrightarrow$$

$$\Leftrightarrow b^2 - 14\cos 12°\times b + 33 = 0 \Rightarrow$$

$$\Rightarrow b = 10,573 \vee b \approx 3,121$$

Para estes valores de b, determinam-se A e B.

16.2. a = 6, c = 20 e $A = 25°$



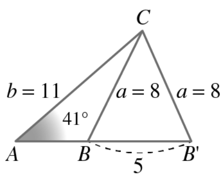
$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

$$\frac{\sin 25}{6} = \frac{\sin C}{20}$$

$$\sin C = \frac{20\times \sin 25}{6} \approx 1,4$$

Como $\sin C > 1$, o triângulo [ABC] não existe.

16.3. a = 8, b = 11 e $A = 41°$



$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin 41°}{8} = \frac{\sin B}{11}$$

$$\sin B = \frac{11\times \sin 41°}{8} \approx 0,90208$$

Se B é agudo:

$$B \approx \sin^{-1}(0,90208) \approx 64,433°$$

$$C \approx 180° - 41° - 64,433° \approx 74,567$$

$$\frac{\sin 74,567°}{c} = \frac{\sin 41°}{8}$$

$$c \approx \frac{8\times \sin 74,567º}{\sin 41} \approx 11,8$$

Se B é obtuso:

$$B = 180° - 64,433° = 115,567°$$

$$C = 180º – 41º – 115,567º = 23,433°$$

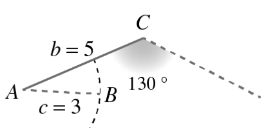
$$\frac{\sin 23,433°}{c} = \frac{\sin 41°}{8}$$

$$c \approx \frac{8\times \sin 23,433º}{\sin 41°} \approx 4,8$$

$B \approx 64,4°$; $C \approx 74,6°$ e c = 11,8 ou

$B \approx 115,6$; $C \approx 23,4°$ e $c \approx 4,8$

16.4. b = 5, c = 3 e C = 130º



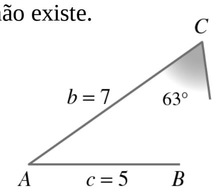
$$\frac{\sin C}{c} = \frac{\sin B}{b}$$

$$\frac{\sin 130°}{3} = \frac{\sin B}{5}$$

$$\sin B = \frac{5\times \sin 130°}{3} \approx 1,28$$

Como $\sin B > 1$, o triângulo [ABC] não existe.

16.5. b = 7, c = 5 e C = 63º



$$\frac{\sin 63°}{5} = \frac{\sin B}{7}$$

$$\sin B = \frac{7\sin 63°}{5} \approx 1,25$$

O seno de um ângulo é sempre não superior a 1.

Logo, o triângulo [ABC] não existe.

16.6. a = 3, b = 7 e c = 11

$c \geq a + b$ **.** Logo, o triângulo [ABC] não existe.

16.7. A = 110º, B = 70º e c = 10

$A + B \geq 180º$. Logo, o triângulo [ABC] não existe.

Atividades complementares

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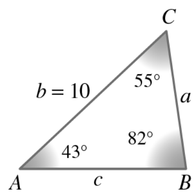
17.1. $B = 180° - 43° - 55° = 82°$



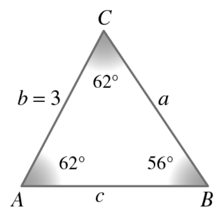
$$\frac{\sin 82°}{10} = \frac{\sin 43°}{a} = \frac{\sin 55°}{c}$$

$$a = \frac{10\sin 43°}{\sin 82°} \approx 6,9$$

$$c = \frac{10\sin 55°}{\sin 82°} \approx 8,3$$

$a \approx 6,9$ cm e $c \approx 8,3$ cm

17.2. $C = 180° - 62° - 56° = 62°$

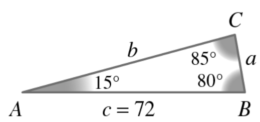


$$\frac{\sin 56°}{3} = \frac{\sin 62°}{a} = \frac{\sin 62°}{c}$$

$$a = c = \frac{3\sin 62°}{\sin 56°} \approx 3,2$$

$a = c \approx 3,2$ cm

17.3. $A = 180° - 80° - 85° = 15°$



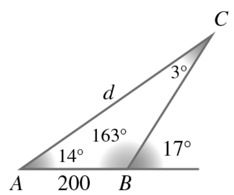
$$\frac{\sin 85°}{72} = \frac{\sin 80°}{b} = \frac{\sin 15°}{a}$$

$$a = \frac{72\sin 15°}{\sin 85°} \approx 18,7$$

$$b = \frac{72\sin 80°}{\sin 85°} \approx 71,2$$

$a \approx 18,7$ cm e $b \approx 71,2$ cm

18. $A\hat{C}B = 17° - 14° = 3°$



O desenho não está feito à escala.

$$C\hat{B}A = 180° - 17° = 163°$$

$$d = \overline{AC}$$

$$\frac{\sin 3°}{200} = \frac{\sin 163°}{d}$$

$$d = \frac{200\sin 163°}{\sin 3°} \approx 1117$$

A distância entre A e C é aproximadamente 1117 m.

19. $c = \overline{AB} = 10$ m

$$C = 105°$$

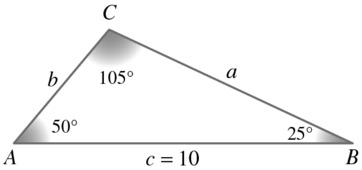
$$A = 2B$$

$$A + B + C = 180°$$

$$2B + B + 105° = 180°$$

$$3B = 75°$$

$B = 25°$ e $A = 50°$



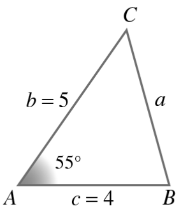
$$\frac{\sin 105°}{10} = \frac{\sin 50°}{a} = \frac{\sin 25°}{b}$$

$$a = \frac{10 \sin 50°}{\sin 105°} \approx 7,93$$

$$b = \frac{10 \sin 25°}{\sin 105°} \approx 4,38$$

$$P_{[ABC]} \approx (7,93 + 4,38 + 10) \text{ m } \approx 22,3 \text{ m}$$

20.1. $A = 55°$, $b = 5$ cm e $c = 4$ cm



$$a^2 = b^2 + c^2 - 2bc\cos A =$$
$$= 5^2 + 4^2 - 2 \times 5 \times 4 \times \cos 55° \approx$$
$$\approx 18,0569$$

$$a \approx \sqrt{18,0569} \approx 4,249$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} \approx \frac{18,0569 + 4^2 - 5^2}{2 \times \sqrt{18,0569} \times 4} \approx 0,2664$$

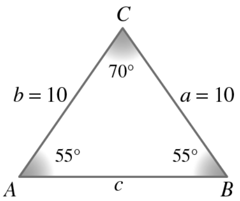
$$B \approx \cos^{-1}(0,2664) \approx 74,5°$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{18,0569 + 5^2 - 4^2}{2\sqrt{18,0569} \times 5} \approx 0,6367$$

$$C \approx \cos^{-1}(0,6367) \approx 50,5°$$

$a \approx 4,2$ cm; $B \approx 74,5°$ e $C \approx 50,5°$

20.2. O triângulo [*ABC*] é isósceles.



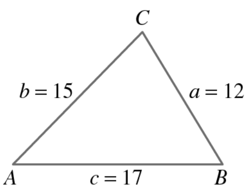
$$A = B = \frac{180° - 70°}{2} = 55°$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$= 10^2 + 10^2 - 2 \times 100 \times \cos 70°$$
$$\approx 131,5960$$

$$c \approx \sqrt{131,5960} \approx 11,5$$

$c \approx 11,5$ cm e $A = B = 55°$

20.3. $a = 12$ cm, $b = 15$ cm e $c = 17$ cm



$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} =$$
$$= \frac{15^2 + 17^2 - 12^2}{2 \times 15 \times 17} =$$
$$= \frac{370}{510} = \frac{37}{51}$$

$$A = \cos^{-1}\left(\frac{37}{51}\right) \approx 43,5°$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{12^2 + 17^2 - 15^2}{2 \times 12 \times 17} = \frac{208}{408} = \frac{26}{51}$$

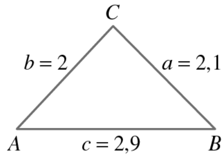
$$B = \cos^{-1}\left(\frac{26}{51}\right) = 59,3°$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{12^2 + 15^2 - 17^2}{2 \times 12 \times 15} = \frac{80}{360} = \frac{2}{9}$$

$$C = \cos^{-1}\left(\frac{2}{9}\right) \approx 77,2°$$

$A \approx 43,5°$; $B \approx 59,3°$ e $C \approx 77,2°$

20.4. $a = 2,1$ cm; $b = 2$ cm e $c = 2,9$ cm



$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} =$$
$$= \frac{2^2 + 2,9^2 - 2,1^2}{2 \times 2 \times 2,9} =$$
$$= \frac{8}{11,6} = \frac{20}{29}$$

$$A = \cos^{-1}\left(\frac{20}{29}\right) \approx 46,4°$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{2,1^2 + 2,9^2 - 2^2}{2 \times 2,1 \times 2,9} = \frac{8,82}{12,18} = \frac{21}{29}$$

$$B = \cos^{-1}\left(\frac{21}{29}\right) \approx 43,6°$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{2,1^2 + 2^2 - 2,9^2}{2 \times 2,1 \times 2} = 0$$

$$C = \cos^{-1}(0) = 90°$$

$A \approx 46,4°$; $B = 43,6°$ e $C = 90°$

21. $d^2 = 250^2 + 300^2 - 2 \times 250 \times 300 \times \cos 85° \approx 139\ 429,639$

$d \approx \sqrt{139\ 426,639} \approx 373,4$ km

A distância entre os dois aviões é de 373,4 km, aproximadamente.

22. Diagonal maior:

$$d_1^2 = 2^2 + 4^2 - 2 \times 2 \times 4 \times \cos 120° = 20 - 2 \times 8 \times \left(-\frac{1}{2}\right) = 28$$

$$d_1 = \sqrt{28} = 2\sqrt{7}$$

Diagonal menor

$180° - 120° = 60°$

$$d_2^2 = 2^2 + 4^2 - 2 \times 2 \times 4 \times \cos 60° = 20 - 2 \times 8 \times \frac{1}{2} = 12$$

$$d_2 = \sqrt{12} = 2\sqrt{3}$$

$d_1 = 2\sqrt{7}$ cm e $d_2 = 2\sqrt{3}$ cm

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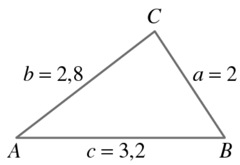
23.1. $a = 2$, $b = 2,8$ e $c = 3,2$



$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} =$$
$$= \frac{2,8^2 + 3,2^2 - 2^2}{2 \times 2,8 \times 3,2} =$$
$$= \frac{14,08}{17,92} = \frac{11}{14}$$

$$A = \cos^{-1}\left(\frac{11}{14}\right) \approx 38,21°$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{2^2 + 3,2^2 - 2,8^2}{2 \times 2 \times 3,2} = \frac{6,4}{12,8} = \frac{1}{2}$$

$$B = \cos^{-1}\left(\frac{1}{2}\right) = 60°$$

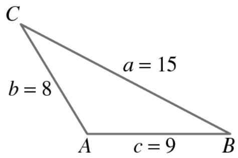
$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{2^2 + 2,8^2 - 3,2^2}{2 \times 2 \times 2,8} = \frac{1,6}{11,2} = \frac{1}{7}$$

$$C = \cos^{-1}\left(\frac{1}{7}\right) \approx 81,79°$$

$A \approx 38,21°$; $B = 60°$ e $C \approx 81,79°$

23.2. a = 15 cm, b = 8 cm e c = 9 cm

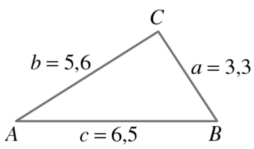
$$\cos A=\frac{b^2+c^2-a^2}{2bc}=$$
$$=\frac{8^2+9^2-15^2}{2\times8\times9}=$$
$$=\frac{-80}{144}=-\frac{5}{9}$$



$$A=\cos^{-1}\left(-\frac{5}{9}\right)\approx123,75°$$
$$\cos B=\frac{a^2+c^2-b^2}{2ac}=\frac{15^2+9^2-8^2}{2\times15\times9}=\frac{242}{270}=\frac{121}{135}$$
$$B=\cos^{-1}\left(\frac{121}{135}\right)\approx26,32°$$
$$\cos C=\frac{a^2+b^2-c^2}{2ab}=\frac{15^2+8^2-9^2}{2\times15\times8}=\frac{208}{240}=\frac{13}{15}$$
$$C=\cos^{-1}\left(\frac{13}{15}\right)\approx29,93°$$
$$A\approx123,75°\ ;\ B\approx26,32°\ \text{ e }\ C\approx29,93°$$

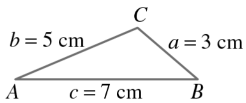
23.3. a = 3,3 cm ; b = 5,6 cm e c = 6,5 cm

$$\cos A=\frac{b^2+c^2-a^2}{2\times b\times c}=$$
$$=\frac{5,6^2+6,5^2-3,3^2}{2\times5,6\times6,5}=$$
$$=\frac{62,72}{72,8}=\frac{56}{65}$$



$$A=\cos^{-1}\left(\frac{56}{65}\right)\approx30,51°$$
$$\cos B=\frac{a^2+c^2-b^2}{2ac}=\frac{3,3^2+6,5^2+5,6^2}{2\times3,3\times6,5}=\frac{21,78}{42,9}=\frac{33}{65}$$
$$B=\cos^{-1}\left(\frac{33}{65}\right)\approx59,49°$$
$$\cos C=\frac{a^2+b^2-c^2}{2ab}=\frac{3,3^2+5,6^2-6,5^2}{2\times3,3\times5,6}=0$$
$$C=\cos^{-1}(0)=90°$$
$$A\approx30,51°\ ;\ B\approx59,49°\ \text{ e }\ C=90°$$

23.4. a = 3 cm, b = 5 cm e c = 7 cm



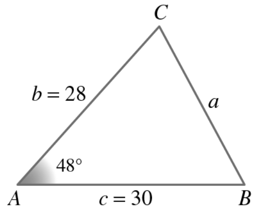
$$\cos A=\frac{b^2+c^2-a^2}{2bc}=$$
$$=\frac{5^2+7^2-3^2}{2\times5\times7}=\frac{65}{70}=\frac{13}{14}$$
$$A=\cos^{-1}\left(\frac{13}{14}\right)=21,79°$$
$$\cos B=\frac{a^2+c^2-b^2}{2ac}=\frac{3^2+7^2-5^2}{2\times3\times7}=\frac{33}{42}=\frac{11}{14}$$
$$B=\cos^{-1}\left(\frac{11}{14}\right)=38,21°$$
$$\cos C=\frac{a^2+b^2-c^2}{2ab}=\frac{3^2+5^2-7^2}{2\times3\times5}=\frac{-15}{30}=-\frac{1}{2}$$
$$C=\cos^{-1}\left(-\frac{1}{2}\right)=180°-60°=120°$$
$$A\approx21,79°\ ;\ B\approx38,21°\text{ e }C=120°$$

24.1. a = 5,2 ; b = 3,2 e c = 6

$$\cos A=\frac{b^2+c^2-a^2}{2bc}=\frac{3,2^2+6^2-5,2^2}{2\times3,2\times6}=\frac{19,2}{38,4}=\frac{1}{2}$$
$$A=\cos^{-1}\left(\frac{1}{2}\right)=60°$$
$$B\hat{A}C=60°$$

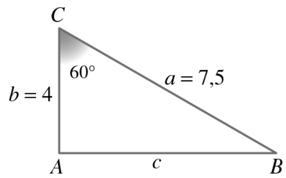
24.2. $\frac{h}{3,2}=\sin60°\Leftrightarrow h=3,2\times\frac{\sqrt{3}}{2}\Leftrightarrow h=\frac{8\sqrt{3}}{5}$

25.1. $A=48°,\ b=28$ cm e c = 30 cm



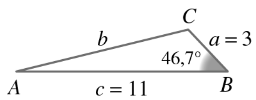
$$a^2=b^2+c^2-2bc\cos A=$$
$$=28^2+30^2-2\times28\times30\times\cos48°$$
$$\approx559,861$$
$$a\approx\sqrt{559,861}\approx23,7$$
$$\cos B=\frac{a^2+c^2-b^2}{2ac}\approx\frac{559,861+30^2-28^2}{2\sqrt{559,861}\times30}\approx0,476$$
$$B\approx\cos^{-1}(0,476)\approx61,6°$$
$$\cos C=\frac{a^2+b^2-c^2}{2ab}\approx\frac{559,861+28^2-30^2}{2\sqrt{559,861}\times28}\approx0,335$$
$$C\approx\cos^{-1}(0,335)\approx70,4°$$
$$a\approx23,7\ \text{ cm, }B\approx61,6°\ \text{ e }\ C\approx70,4°$$

25.2. $C=60°$, a = 7,5 cm e b = 4 cm



$$c^2=a^2+b^2-2ab\cos60°=$$
$$=7,5^2+4^2-2\times7,5\times4\times\frac{1}{2}=42,25$$
$$c=\sqrt{42,25}=6,5$$
$$\cos A=\frac{b^2+c^2-a^2}{2bc}\approx\frac{4^2+6,5^2-7,5^2}{2\times4\times6,5}=\frac{2}{52}=\frac{1}{26}$$
$$A=\cos^{-1}\left(\frac{1}{26}\right)\approx87,8°$$
$$\cos B=\frac{a^2+c^2-b^2}{2ac}=\frac{7,5^2+6,5^2-4^2}{2\times7,5\times6,5}=\frac{82,5}{97,5}=\frac{11}{13}$$
$$B=\cos^{-1}\left(\frac{11}{13}\right)\approx32,2°$$
$$c=6,5\text{ cm ; }\ A\approx87,8°\ \text{ e }\ B=32,2°$$

25.3. $B=46,7°$; $a=3$ cm e c = 11 cm



$$b^2=a^2+c^2-2ac\times\cos B=$$
$$=3^2+11^2-2\times3\times11\times\cos46,7°\approx84,7360$$

$b \approx \sqrt{84,7360} \approx 9,2$

$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc} \approx \dfrac{84,7360 + 11^2 - 3^2}{2\sqrt{84,7660} \times 11} \approx 0,9715$

$A \approx \cos^{-1}(0,9715) \approx 13,7°$

$\cos C = \dfrac{a^2 + b^2 - c^2}{2ab} = \dfrac{3^2 + 84,7360 - 11^2}{2 \times 3 \times \sqrt{84,7360}} \approx -0,4936$

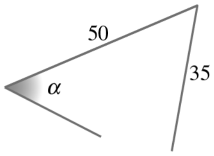
$C \approx \cos^{-1}(-0,4936) \approx 119,6°$

b = 9,2 cm ; $A \approx 13,7°$ e $C \approx 119,6°$

26. $\cos\alpha = \dfrac{50^2 + 41^2 - 35^2}{2 \times 50 \times 41} =$

$= \dfrac{2956}{4100}$

$\alpha = \cos^{-1}\left(\dfrac{2956}{4100}\right) \approx 44°$



27.1. $A = 52°$, $B = 68°$ e c = 15 cm

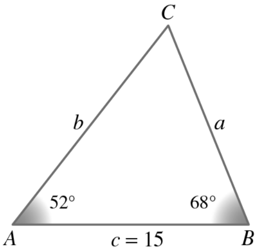
$C = 180° - 52° - 68° \approx 60°$

$\dfrac{\sin 60°}{15} = \dfrac{\sin 52°}{a} = \dfrac{\sin 68°}{b}$

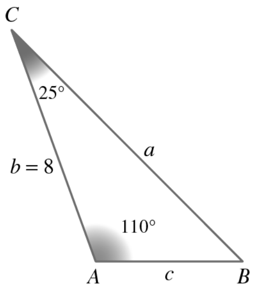
$a = \dfrac{15 \sin 52°}{\sin 60°} \approx 13,6$

$b = \dfrac{15 \sin 68°}{\sin 60°} \approx 16,1$

$C = 60°$; $a = 13,6$ cm e b = 16,1 cm



27.2. $B = 180° - 110° - 25° = 45°$



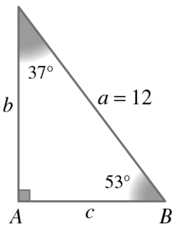
$\dfrac{\sin 45°}{8} = \dfrac{\sin 110°}{a} = \dfrac{\sin 25°}{c}$

$a = \dfrac{8 \sin 110°}{\sin 45°} \approx 10,6$

$c = \dfrac{8 \sin 25°}{\sin 45°} \approx 4,8$

$B = 45°$, $a \approx 10,6$ cm e $c \approx 4,8$ cm

27.3. $A = 180° - 37° - 53° = 90°$



O triângulo [ABC] é retângulo em A.

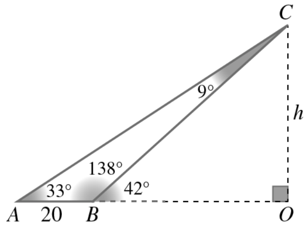
$\dfrac{b}{12} = \sin 53° \Leftrightarrow b = 12 \sin 53° \Rightarrow b \approx 9,6$

$\dfrac{c}{12} = \cos(53°) \Leftrightarrow c = 12\cos(53°) \Rightarrow c \approx 7,2$

$A = 90°$; $b = 9,6$ cm e $c \approx 7,2$ cm

28. $C\hat{B}A = 180° - 42° = 138°$

$A\hat{C}B = 42° - 33° = 9°$



$\dfrac{\sin 9°}{20} = \dfrac{\sin 33°}{\overline{BC}}$

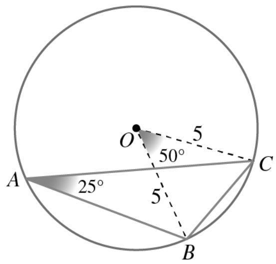
$\overline{BC} = \dfrac{20 \sin 33°}{\sin 9°}$

$\dfrac{h}{\overline{BC}} = \sin 42°$

$h = \overline{BC} \times \sin 42° = \dfrac{20 \sin 33°}{\sin 9°} \times \sin 42° \approx 46,6$

O prédio tem 46,6 m de altura.

29.



$\overset{\frown}{BC} = 2 \times 25° = 50°$

$B\hat{O}C = 50°$

$\overline{BO} = \overline{OC} = 5$ cm

$\overline{BC}^2 = 5^2 + 5^2 - 2 \times 5 \times 5 \times \cos 50° \approx 17,8606$

$\overline{BC} \approx \sqrt{17,8606} \approx 4,2$

$\overline{BC} \approx 4,2$ cm

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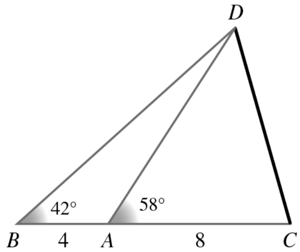
30. $A\hat{D}B = 58° - 42° = 16°$

Pela lei dos senos no triângulo [BAD]:

$\dfrac{\sin 16°}{4} = \dfrac{\sin 42°}{\overline{AD}}$

$\overline{AD} = \dfrac{4 \sin 42°}{\sin 16°} \approx 9,7103$



Aplicando a lei dos cossenos ao triângulo [ACD]:

$\overline{DC}^2 \approx 8^2 + (9,7103)^2 - 2 \times 8 \times 9,7103 \times \cos 58° \approx 75,9591$

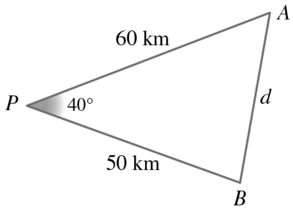
$\overline{DC} \approx \sqrt{75,9591} \approx 8,72$

O poste mede 8,72 m, aproximadamente.

31. 2 h 30 min = 2,5 h

$2{,}5\times 24 = 60 \;\to$ O barco A percorreu 60 km.

$2{,}5\times 20 = 50 \;\to$ O barco B percorreu 50 km.



$d^2 = 60^2 + 50^2 - 2\times 60\times 50\times \cos 40° \approx 1503{,}733$

$d \approx \sqrt{1503{,}733} \approx 38{,}778$

O barco percorreu 38,778 km a uma velocidade de 24 km/h.

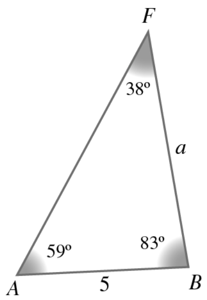
$V = \frac{e}{t}$, logo $24 = \frac{38{,}778}{t} \Leftrightarrow t = \frac{38{,}778}{24}$

$t \approx 1{,}616$, pelo que $t \approx 1$ h 37 min $\quad 0{,}616\times 60 = 36{,}96$

O barco demorou 1h 37 min.

32. $\overline{FB} = a$

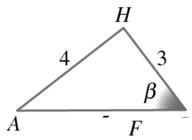
$A\hat{F}B = 180° - 59° - 83° = 38°$



$\frac{\sin 59°}{a} = \frac{\sin 38°}{5}$

$a = \frac{5\sin 59°}{\sin 38°} \approx 6{,}9614$

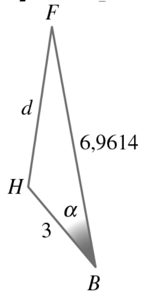
Seja $\beta = H\hat{B}A$.



$\cos\beta = \frac{3^2+5^2-4^2}{2\times 3\times 5} = \frac{3}{5}$

$\beta = \cos^{-1}\left(\frac{3}{5}\right) \approx 53{,}1301°$

Seja $\alpha = F\hat{B}H$.



$\alpha = 83° - 53{,}1301° \approx 29{,}8699°$

$d = \overline{HF}$

$d^2 \approx 3^2 + 6{,}9614^2 - 2\times 3\times$

$\times 6{,}9614\times \cos 29{,}8699° \approx$

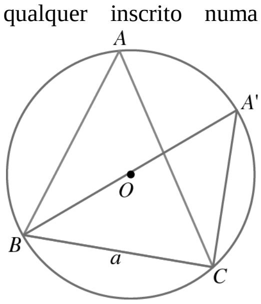
$\approx 21{,}2413$

$d \approx \sqrt{21{,}2413} \approx 4{,}6$

O helicóptero está a cerca de 4,6 km do incêndio.

33. Seja $[ABC]$ um triângulo qualquer inscrito numa circunferência de centro O e raio r e seja A' o ponto da circunferência tal que $[BA']$ é um diâmetro.

Então, o triângulo $[A'BC]$ é um retângulo em C pelo que $\frac{\overline{BC}}{\overline{BA'}} = \sin\left(B\hat{A'}C\right)$.



Como $\overline{BC} = a$, $\overline{BA'} = 2r$ e $B\hat{A'}C = B\hat{A}C = \frac{1}{2}\widehat{BC}$, temos que $\frac{a}{2r} = \sin A$, ou seja, $\frac{a}{\sin A} = 2r$.

Avaliação 1

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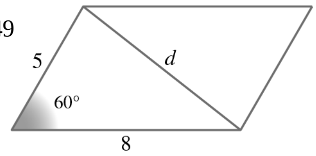
1. $d^2 = 8^2 + 5^2 - 2\times 8\times 5\times \cos 60° =$

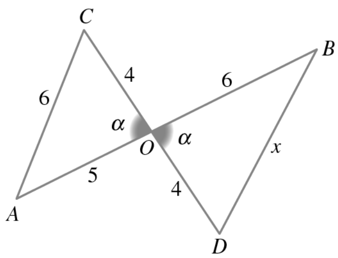
$= 64 + 25 - 2\times 40\times \frac{1}{2} = 49$

$d = \sqrt{49} = 7$



Resposta: (B)

2. Se $\alpha = C\hat{O}A$, então $D\hat{O}B = \alpha$ (ângulos verticalmente opostos). Seja $x = \overline{BD}$.



$\cos\alpha = \frac{4^2+5^2-6^2}{2\times 4\times 5} = \frac{5}{40} = \frac{1}{8}$

$x^2 = 6^2 + 4^2 - 2\times 6\times 4\times \cos\alpha = 36 + 16 - 48\times \frac{1}{8} = 46$

$\overline{DB} = x = \sqrt{46}$ cm

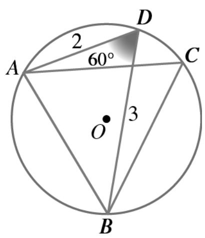
Resposta: (C)

3. $\frac{\sin 30°}{2\sqrt{2}} = \frac{\sin 45°}{x}$

$x = \frac{2\sqrt{2}\times \sin 45°}{\sin 30°} = \frac{2\sqrt{2}\times \frac{\sqrt{2}}{2}}{\frac{1}{2}} = 2\times 2 = 4$

Resposta: (A)

4.



$[ABC]$ é um triângulo equilátero, logo $\widehat{AB} = \frac{360°}{3} = 120°$.

Assim, $A\hat{D}C = \frac{120°}{2} = 60°$.

Pela lei dos cossenos:

$\overline{AB}^2 = 2^2 + 3^2 - 2\times 2\times 3\times \cos 60° = 4 + 9 - 6 = 7$

Logo, $\overline{AB} = \sqrt{7}$ cm.

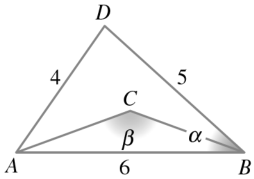
Resposta: (B)

5. $\frac{\sin 30°}{2} = \frac{\sin x}{3}$

$\sin x = \frac{3\sin 30º}{2} = \frac{3\times\frac{1}{2}}{2} = \frac{3}{2}\times\frac{1}{2} = \frac{3}{4}$

Resposta: (D)

6.



$B\hat{A}C = C\hat{B}A = \frac{\alpha}{2}$

$\frac{\alpha}{2} + \frac{\alpha}{2} + \beta = \pi \Leftrightarrow \alpha + \beta = \pi \Leftrightarrow \beta = \pi - \alpha$

$\cos\alpha = \frac{6^2 + 5^2 - 4^2}{2\times 6\times 5} = \frac{45}{60} = \frac{3}{4}$

$\cos\beta = \cos(\pi - \alpha) = -\cos\alpha = -\frac{3}{4}$

Resposta: (D)

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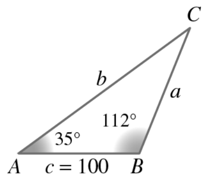
7. $C = 180° - 35° - 112° = 33°$



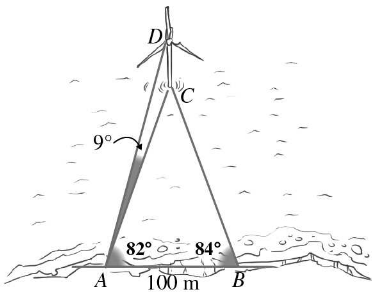
$\frac{\sin C}{c} = \frac{\sin A}{a}$

$\frac{\sin 33°}{100} = \frac{\sin 35°}{a}$

$a = \frac{100\times\sin 35°}{\sin 33°} \approx 105,3$

A distância de B a C é de 105,3 m, aproximadamente.

8.



$B\hat{C}A = 180° - 82° - 84° = 14°$

$\frac{\sin 14°}{100} = \frac{\sin 84}{\overline{AC}}$ (Lei dos senos no triângulo [ABC])

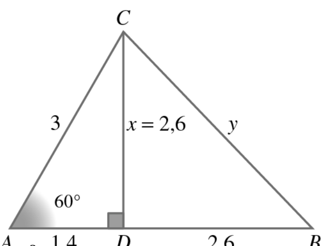
$\overline{AC} = \frac{100\sin 84}{\sin 14} \approx 411,092$

$\frac{\overline{DC}}{\overline{AC}} = \tan 9°$ (O triângulo [ACD] é retângulo em C.)

$\frac{\overline{DC}}{411,092} \approx \tan 9° \Rightarrow \overline{DC} \approx 411,092\times\tan 9° \Rightarrow \overline{DC} \approx 65$

A torre tem 65 m de altura, aproximadamente.

9. Seja $x = \overline{CD}$ e $y = \overline{CB}$.



$x^2 = 3^2 + 1,4^2 - 2\times 3\times 1,4\times\cos 60° = 6,76$

$x = \sqrt{6,76} = 2,6$

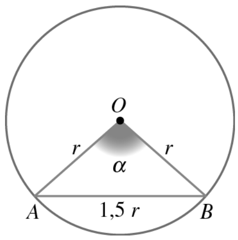
$\overline{AB} = 1,4 + 2,6 = 4$

$y^2 = 3^2 + 4^2 - 2\times 3\times 4\times\cos 60° = 9 + 16 - 12 = 13$

$y = \sqrt{13}$

$\overline{CB} = \sqrt{13}$ cm

10.



$(1,5r)^2 = r^2 + r^2 - 2\times r\times r\times\cos\alpha$

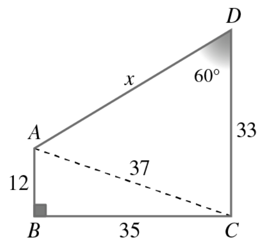
$2,25r^2 = 2r^2 - 2r^2\cos\alpha$

$2r^2\cos\alpha = 2r^2 - 2,25r^2$

$\cos\alpha = \frac{-0,25r^2}{2r^2} \Leftrightarrow$

$\Leftrightarrow \cos\alpha = -\frac{0,25}{2} \Leftrightarrow \cos\alpha = -\frac{1}{8}$

11. O triângulo [ABC] é retângulo em C.



$\overline{AC}^2 = 12^2 + 35^2$

$\overline{AC} = \sqrt{144 + 1225} = \sqrt{1369} = 37$

Pela lei dos cossenos, se $\overline{AD} = x$ cm:

$37^2 = x^2 + 33^2 - 2x + 33\times\cos 60° \Leftrightarrow$

$\Leftrightarrow x^2 - 33x - 280 = 0 \Leftrightarrow$

$\Leftrightarrow x = \frac{33 \pm \sqrt{(-33)^2 - 4\times(-280)}}{2} \Leftrightarrow$

$\Leftrightarrow x = 40 \vee x = -7$

Como $x > 0$, temos $\overline{AD} = 40$ cm.

Atividade inicial 2

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1.1. *C* **1.2.** *E* **1.3.** *F*

1.4. *B* **1.5.** *D* **1.6.** *D*

2. Rotação de centro *O* e amplitude 60° de sentido positivo **ou** rotação de centro *O* e amplitude 300° de sentido negativo

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1.1. *C*

1.2. $\alpha = 90°$ ou $\alpha = -270°$

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2.1. **a)** $225° : 45° = 5$

Lado extremidade: $\dot{O}F$

b) $135° : 45° = 3$

Lado extremidade: $\dot{O}F$

2.2. **a)** $1575° = 135° + 4 \times 360°$

$\begin{array}{r|l} 1575 & 360° \\ \hline 135 & 4 \end{array}$

Lado extremidade: $\dot{O}D$

b) $-1170° = -90° - 3 \times 360°$

$\begin{array}{r|l} 1170° & 360° \\ \hline 90° & 3 \end{array}$

Lado extremidade: $\dot{O}G$

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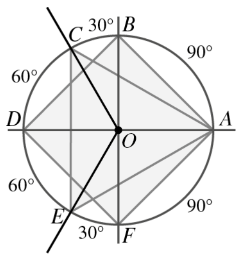
3. $120° - 90° = 30°$



3.1. $1170° = 90° + 3 \times 360°$

O transformado do ponto *A* é o ponto *B*.

$\begin{array}{r|l} 1170 & 360 \\ \hline 90 & 3 \end{array}$

3.2. $900° = 180° + 2 \times 360°$

O transformado do ponto *A* é o ponto *D*.

$\begin{array}{r|l} 900 & 360 \\ \hline 180 & 2 \end{array}$

3.3. $600° = 240° + 360°$

$240° = 180° + 60°$

O transformado do ponto *A* é o ponto *E*.

$\begin{array}{r|l} 600 & 360 \\ \hline 240 & 1 \end{array}$

3.4. $-1530° = -90° - 4 \times 360°$

O transformado do ponto *A* é o ponto *F*.

$\begin{array}{r|l} 1530 & 360 \\ \hline 90 & 4 \end{array}$

3.5. $-840° = -120° - 2 \times 360°$

O transformado do ponto *A* é o ponto *E*.

$\begin{array}{r|l} 840 & 360 \\ \hline 120 & 2 \end{array}$

3.6. $-1080° = -3 \times 360°$

O transformado do ponto *A* é o ponto *A*.

$\begin{array}{r|l} 1080 & 360 \\ \hline 0 & 3 \end{array}$

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4. $\alpha = B\hat{A}C$

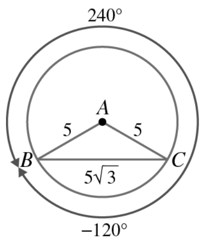
$$\cos\alpha = \frac{5^2 + 5^2 - \left(5\sqrt{3}\right)^2}{2 \times 5 \times 5} =$$

$$= \frac{50 - 75}{50} = -\frac{25}{50} = -\frac{1}{2}$$

$$\cos^{-1}\left(-\frac{1}{2}\right) = 180° - 60° = 120°$$

$360° - 120° = 240°$

$\alpha = -120°$ e $\alpha = 240°$



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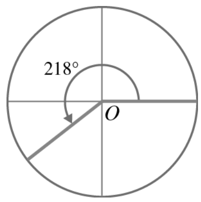
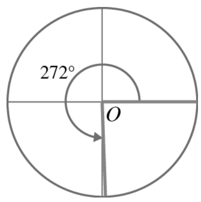
5.1.



α é do 3.º quadrante

$\sin\alpha < 0$ e $\cos\alpha < 0$

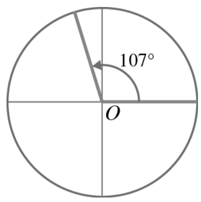
5.2.



α é do 4.º quadrante

$\sin\alpha < 0$ e $\cos\alpha > 0$

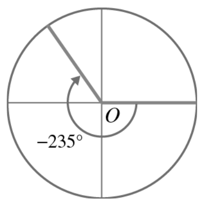
5.3.



α é do 2.º quadrante

$\sin\alpha > 0$ e $\cos\alpha < 0$

5.4.



α é do 2.º quadrante

$\sin\alpha > 0$ e $\cos\alpha < 0$

6. α é do 1.º quadrante

6.1. $\alpha \in$ 1.º Q; $\sin\alpha > 0$

$\alpha + 180° \in$ 3.º Q; $\cos(\alpha + 180°) < 0$

$\sin\alpha + \cos(\alpha + 180°) < 0$

O sinal é negativo.

6.2. $\alpha + 90° \in$ 2.º Q; $\cos(\alpha + 90°) < 0$

$\alpha - 90° \in$ 4.º Q; $\tan(\alpha - 90°) < 0$

$\cos(\alpha + 90°) \times \tan(\alpha - 90°) > 0$

O sinal é positivo.

6.3. $\alpha-90°\in 4.°\ Q;\ \cos(\alpha-90°)>0$

$\alpha+90°\in 2.°\ Q;\ \sin(\alpha+90°)>0$

$\cos(\alpha-90°)+\sin(\alpha+90°)>0$.

O sinal é positivo.

6.4. $\alpha+180°\in 3.°\ Q;\ \tan(\alpha+180°)>0$

$\alpha-90°\in 4.°\ Q;\ -\sin(\alpha-90°)>0$

$\tan(\alpha+180°)-\sin(\alpha-90°)>0$

O sinal é positivo.

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7.1. $P(\cos 30°,\ \sin 30°)$ ou $P\left(\frac{\sqrt{3}}{2}\ ,\ \frac{1}{2}\right)$

$M(1\ ,\ \tan 30°)$ ou $M\left(1\ ,\ \frac{\sqrt{3}}{3}\right)$

Q é a imagem de P pela reflexão central de centro O. Logo, $Q\left(-\frac{\sqrt{3}}{2}\ ,\ -\frac{1}{2}\right)$

7.2. $A\hat{O}Q=180°+30°=210°$

Logo, como Q tem coordenadas $\left(-\frac{\sqrt{3}}{2}\ ,\ -\frac{1}{2}\right)$, então $\cos 210°=-\frac{\sqrt{3}}{2}$ e $\sin 210°=-\frac{1}{2}$.

7.3. $R(\cos 150°\ ,\ \sin 150°)$

$\cos 150°=\cos(180°-30°)=-\cos 30°=-\frac{\sqrt{3}}{2}$

$\sin 150°=\sin(180°-30°)=\sin 30°=\frac{1}{2}$

$R\left(-\frac{\sqrt{3}}{2}\ ,\ \frac{1}{2}\right)$

7.4. S é a imagem de R pela reflexão central de centro O.

Logo, S tem coordenadas $\left(\frac{\sqrt{3}}{2}\ ,\ -\frac{1}{2}\right)$.

Portanto, como $P\hat{O}S=360°-30°=330°$, temos $\sin 330°=-\frac{1}{2}$ e $\cos 330°=\frac{\sqrt{3}}{2}$.

7.5. N é a imagem de M pela reflexão de eixo Ox.

Logo, $N\left(1\ ,\ -\frac{\sqrt{3}}{3}\right)$.

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8.1. $765°=45°+2\times 360$ $\qquad \begin{array}{r|l} 765 & 360 \\ \hline 45 & 2 \end{array}$

$765°=(45°\ ,\ 2)$

$\sin 765°=\sin 45°=\frac{\sqrt{2}}{2}$

$\cos 765°=\cos 45°=\frac{\sqrt{2}}{2}$

$\tan 765°=\tan 45°=1$

8.2. $780°=60°+2\times 360°$ $\qquad \begin{array}{r|l} 780 & 360 \\ \hline 60 & 2 \end{array}$

$780°=(60°\ ,\ 2)$

$\sin 780°=\sin 60°=\frac{\sqrt{3}}{2}$

$\cos 780°=\cos 60°=\frac{1}{2}$

$\tan 780°=\tan 60°=\sqrt{3}$

8.3. $1470°=30°+4\times 360°$ $\qquad \begin{array}{r|l} 1470 & 360 \\ \hline 30 & 4 \end{array}$

$1470°=(30°,\ 4)$

$\sin 1470°=\sin 30°=\frac{1}{2}$

$\cos 1470°=\cos 30°=\frac{\sqrt{3}}{2}$

$\tan 1470°=\tan 30°=\frac{\sqrt{3}}{3}$

9.1. $495°=135°+360°$

$495=(135°\ ,\ 1)$

$\sin 495°=\sin 135°=\sin(180°-45°)=$

$=\sin 45°=\frac{\sqrt{2}}{2}$

$\cos 495°=\cos 135°=\cos(180°-45°)=$

$=-\cos 45°=-\frac{\sqrt{2}}{2}$

9.2. $840°=120°+2\times 360°$ $\qquad \begin{array}{r|l} 840 & 360 \\ \hline 120 & 2 \end{array}$

$840°=(120°\ ,\ 2)$

$\sin 840°=\sin 120°=\sin(180°-60°)=$

$=\sin 60°=\frac{\sqrt{3}}{2}$

$\cos 840°=\cos 120°=\cos(180°-60°)=$

$=-\cos 60°=-\frac{1}{2}$

9.3. $1230°=150°+3\times 360°$ $\qquad \begin{array}{r|l} 1230 & 360 \\ \hline 150 & 3 \end{array}$

$1230°=(150°\ ,\ 3)$

$\sin 1230°=\sin 150°=\sin(180°-30°)=$

$=\sin 30°=\frac{1}{2}$

$\cos 1230°=\cos 150°=\cos(180°-30°)=$

$=-\cos 30°=-\frac{\sqrt{3}}{2}$

9.4. $1170°=90°+3\times 360°$ $\qquad \begin{array}{r|l} 1170 & 360 \\ \hline 90 & 3 \end{array}$

$1170°=(90°,\ 3)$

$\sin 1170°=\sin 90°=1$

$\cos 1170°=\cos 90°=0$

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10.1.

Graus		Rad
180	——	π
135	——	x

$x=\frac{135\pi}{180}=\frac{3\pi}{4}$ $\qquad 135°=\frac{3\pi}{4}$ rad

10.2.

Graus		Rad
180	——	π
270	——	x

$x=\dfrac{270\pi}{180}=\dfrac{3\pi}{2}$

$180°=\dfrac{3\pi}{2}$ rad

10.3.

Graus		Rad
180	——	π
150	——	x

$x=\dfrac{150\pi}{180}=\dfrac{5\pi}{6}$

$150°=\dfrac{5\pi}{6}$ rad

10.4.

Graus		Rad
180	——	π
330	——	x

$x=\dfrac{330\pi}{180}=\dfrac{11\pi}{6}$

$330°=\dfrac{11\pi}{6}$

10.5.

Graus		Rad
180	——	π
450	——	x

$x=\dfrac{450\pi}{180}=\dfrac{5\pi}{2}$

$450°=\dfrac{5\pi}{2}$ rad

10.6.

Graus		Rad
180	——	π
252	——	x

$x=\dfrac{252\pi}{180}=\dfrac{7\pi}{5}$

$252°=\dfrac{7\pi}{5}$

10.7.

Graus		Rad
180	——	π
202,5	——	x

$x=\dfrac{202{,}5\pi}{180}=\dfrac{9\pi}{8}$; $202{,}5°=\dfrac{9\pi}{8}$ rad

10.8. $337°\ 30'=337{,}5°$

Graus		Rad
180	——	π
337,5	——	x

$x=\dfrac{337{,}5\pi}{180}=\dfrac{15\pi}{8}$

$337{,}5°=\dfrac{15\pi}{8}$

10.9. $39°\ 22'\ 30''=\left(39+\dfrac{22}{60}+\dfrac{30}{60^2}\right)^{\circ}=39{,}375°$

Graus		Rad
180	——	π
39,375	——	x

$x=\dfrac{39{,}375\pi}{180}=\dfrac{7\pi}{32}$

$39°\ 22'\ 30''=\dfrac{7\pi}{32}$ rad

11.1.

Graus		Rad
180	——	π
x	——	$\dfrac{2\pi}{3}$

$x=\dfrac{180\times\dfrac{2\pi}{3}}{\pi}=120$

$\dfrac{2\pi}{3}$ rad $=120°$

11.2.

Graus		Rad
180	——	π
x	——	$\dfrac{5\pi}{6}$

$x=\dfrac{180\times\dfrac{5\pi}{6}}{\pi}=150$

$\dfrac{5\pi}{6}$ rad $=150°$

11.3.

Graus		Rad
180	——	π
x	——	$\dfrac{5\pi}{4}$

$x=\dfrac{180\times\dfrac{5\pi}{4}}{\pi}=225$

$\dfrac{5\pi}{4}$ rad $=225°$

11.4.

Graus		Rad
180	——	π
x	——	$\dfrac{5\pi}{8}$

$x=\dfrac{180\times\dfrac{5\pi}{8}}{\pi}=112{,}5$; $\dfrac{5\pi}{8}$ rad $=112{,}5°$

12.1.

Graus		Rad
180	——	π
x	——	8

$x=\dfrac{180\times 8}{\pi}\approx 458{,}366\ 24$

$0{,}366\ 24\times 60\approx 21{,}9744$

$0{,}9744\times 60\approx 58$

8 rad $\approx 458°\ 21'\ 58''$

12.2.

Graus		Rad
180	——	π
x	——	$\frac{8\pi}{7}$

$x = \frac{180 \times \frac{8\pi}{7}}{\pi} \approx 205{,}714\ 29$

$0{,}714\ 29 \times 60 \approx 42{,}8574$

$0{,}857 \times 60 \approx 51$

$\frac{8\pi}{7}$ rad $\approx 205°\ 42'\ 51''$

12.3.

Graus		Rad
180	——	π
x	——	$\frac{5\pi}{11}$

$x = \frac{180 \times \frac{5\pi}{11}}{\pi} \approx 81{,}818\ 18$

$0{,}818\ 18 \times 60 \approx 49{,}0908$

$0{,}0908 \times 60 \approx 5$

$\frac{5\pi}{11}$ rad $\approx 81°\ 49'\ 5''$

12.4.

Graus		Rad
180	——	π
x	——	2,4

$x = \frac{180 \times 2{,}4}{\pi} \approx 137{,}509\ 87$

$0{,}509\ 87 \times 60 \approx 30{,}5922$

$0{,}5922 \times 60 \approx 36$

2,4 rad $\approx 137°\ 30'\ 36''$

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13.1. $\sin\alpha = -\frac{1}{3}$ e $\alpha \in 3.^{\text{o}}\ Q$

$\sin^2\alpha + \cos^2\alpha = 1$

$\left(-\frac{1}{3}\right)^2 + \cos^2\alpha = 1 \Leftrightarrow$

$\Leftrightarrow \cos^2\alpha = 1 - \frac{1}{9} \Leftrightarrow \cos^2\alpha = \frac{8}{9} \Leftrightarrow$

$\Leftrightarrow \cos\alpha = \pm\frac{\sqrt{8}}{3}$

Como $\alpha \in 3.^{\text{o}}\ Q$, $\cos\alpha < 0$. Logo, $\cos\alpha = -\frac{2\sqrt{2}}{3}$.

$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{-\frac{1}{3}}{-\frac{2\sqrt{2}}{3}} = \frac{3}{3 \times 2\sqrt{2}} = \frac{\sqrt{2}}{4}$

$\cos\alpha = -\frac{2\sqrt{2}}{3}$ e $\tan\alpha = \frac{\sqrt{2}}{4}$

13.2. $\cos\alpha = \frac{1}{5}$ e $\alpha \in 4.^{\text{o}}\ Q$

$\sin^2\alpha + \cos^2\alpha = 1$

$\sin^2\alpha + \left(\frac{1}{5}\right)^2 = 1 \Leftrightarrow \sin^2\alpha = 1 - \frac{1}{25} \Leftrightarrow$

$\Leftrightarrow \sin^2\alpha = \frac{24}{25} \Leftrightarrow \sin\alpha = \pm\frac{\sqrt{24}}{5}$

Como $\alpha \in 4.^{\text{o}}\ Q$, $\sin\alpha < 0$. Logo, $\sin\alpha = -\frac{2\sqrt{6}}{5}$.

$\tan\alpha = \frac{-\frac{2\sqrt{6}}{5}}{\frac{1}{5}} = -2\sqrt{6}$

$\sin\alpha = -\frac{2\sqrt{6}}{5}$ e $\tan\alpha = -2\sqrt{6}$

13.3. $\tan\alpha = -\frac{24}{7}$ e $\alpha \in 2.^{\text{o}}\ Q$

$1 + \tan^2\alpha = \frac{1}{\cos^2\alpha}$

$1 + \left(-\frac{24}{7}\right)^2 = \frac{1}{\cos^2\alpha} \Leftrightarrow 1 + \frac{576}{49} = \frac{1}{\cos^2\alpha} \Leftrightarrow$

$\Leftrightarrow \frac{1}{\cos^2\alpha} = \frac{625}{49} \Leftrightarrow \cos^2\alpha = \frac{49}{625} \Leftrightarrow$

$\Leftrightarrow \cos\alpha = \pm\frac{7}{25}$

Como $\alpha \in 2.^{\text{o}}\ Q$, $\cos\alpha < 0$. Logo, $\cos\alpha = -\frac{7}{25}$.

$\sin^2\alpha + \cos^2\alpha = 1$

$\sin^2\alpha + \frac{49}{625} = 1 \Leftrightarrow \sin^2\alpha = 1 - \frac{49}{625} \Leftrightarrow$

$\Leftrightarrow \sin^2\alpha = \frac{576}{625} \Leftrightarrow \sin\alpha = \pm\frac{24}{25}$

Como $\alpha \in 2.^{\text{o}}\ Q$, $\sin\alpha > 0$. Logo, $\sin\alpha = \frac{24}{25}$.

$\cos\alpha = -\frac{7}{25}$ e $\sin\alpha = \frac{24}{25}$.

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14. Seja $\alpha = B\hat{O}P$.

$P(\cos\alpha\ ,\ \sin\alpha)$ e $A(1\ ,\ \tan\alpha)$

Portanto, $\sin\alpha = -\frac{1}{3}$ e $\alpha \in 4.^{\text{o}}\ Q$.

$\sin^2\alpha + \cos^2\alpha = 1$

$\left(-\frac{1}{3}\right)^2 + \cos^2\alpha = 1 \Leftrightarrow \cos^2\alpha = 1 - \frac{1}{9} \Leftrightarrow \cos^2\alpha = \frac{8}{9} \Leftrightarrow$

$\Leftrightarrow \cos\alpha = \pm\frac{2\sqrt{2}}{3}$

Como $\alpha \in 4.^{\text{o}}\ Q$, então $\cos\alpha = \frac{2\sqrt{2}}{3}$.

$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{-\frac{1}{3}}{\frac{2\sqrt{2}}{3}} = -\frac{3}{3 \times 2\sqrt{2}} = -\frac{\sqrt{2}}{4}$

$A\left(1\ ,\ -\frac{\sqrt{2}}{4}\right)$; $\overline{OC} = \overline{BA} = \frac{\sqrt{2}}{4}$

O ponto C tem coordenadas $\left(0\ ,\ \frac{\sqrt{2}}{4}\right)$.

15.1. $\dfrac{\sin^2 x}{1-\cos x} = \dfrac{1-\cos^2 x}{1-\cos x} =$

$= \dfrac{(1-\cos x)(1+\cos x)}{(1-\cos x)} =$

$= 1+\cos x$

15.2. $\dfrac{1+\sin x}{\cos x} = \dfrac{(1+\sin x)(1-\sin x)}{\cos x(1-\sin x)} =$

$= \dfrac{1-\sin^2 x}{\cos x(1-\sin x)} =$

$= \dfrac{\cos^2 x}{\cos x(1-\sin x)} = \dfrac{\cos x}{1-\sin x}$

15.3. $\dfrac{1-\tan^2 x}{1+\tan^2 x} = \dfrac{1-\dfrac{\sin^2 x}{\cos^2 x}}{1+\dfrac{\sin^2 x}{\cos^2 x}} = \dfrac{\dfrac{\cos^2 x-\sin^2 x}{\cos^2 x}}{\dfrac{\cos^2 x+\sin^2 x}{\cos^2 x}} =$

$= \dfrac{\dfrac{\cos^2 x-\sin^2 x}{\cos^2 x}}{\dfrac{1}{\cos^2 x}} = \dfrac{\cos^2 x-\sin^2 x}{\cos^2 x}\times\cos^2 x =$

$= \cos^2 x-\sin^2 x$

15.4. $\dfrac{\tan^3 x}{\cos^2 x} - \tan^3 x = \tan^3 x\left(\dfrac{1}{\cos^2 x}-1\right) =$

$= \tan^3 x\times\dfrac{1-\cos^2 x}{\cos^2 x} = \tan^3 x\times\dfrac{\sin^2 x}{\cos^2 x} =$

$= \tan^3 x\times\tan^2 x =$

$= \tan^5 x$

15.5. $\dfrac{1-\cos x}{\sin x} + \dfrac{\sin x}{1-\cos x} = \dfrac{(1-\cos x)^2+\sin^2 x}{\sin x(1-\cos x)} =$

$= \dfrac{1-2\cos x+\cos^2 x+\sin^2 x}{\sin x(1-\cos x)} =$

$= \dfrac{1-2\cos x+1}{\sin x(1-\cos x)} =$

$= \dfrac{2-2\cos x}{\sin x(1-\cos x)} =$

$= \dfrac{2(1-\cos x)}{\sin x(1-\cos x)} = \dfrac{2}{\sin x}$

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16.1. $\sin\dfrac{7\pi}{6} = \sin\left(\dfrac{6\pi}{6}+\dfrac{\pi}{6}\right) = \sin\left(\pi+\dfrac{\pi}{6}\right) = -\sin\dfrac{\pi}{6} = -\dfrac{1}{2}$

16.2. $\cos\left(-\dfrac{5\pi}{3}\right) = \cos\left(\dfrac{5\pi}{3}\right) = \cos\left(\dfrac{6\pi}{3}-\dfrac{\pi}{3}\right) =$

$= \cos\left(2\pi-\dfrac{\pi}{3}\right) = \cos\left(-\dfrac{\pi}{3}\right) = \cos\dfrac{\pi}{3} = \dfrac{1}{2}$

16.3. $\tan\left(\dfrac{4\pi}{3}\right) = \tan\left(\dfrac{3\pi}{3}+\dfrac{\pi}{3}\right) = \tan\left(\pi+\dfrac{\pi}{3}\right) =$

$= \tan\dfrac{\pi}{3} = \sqrt{3}$

16.4. $\sin\left(-\dfrac{5\pi}{4}\right) = -\sin\dfrac{5\pi}{4} = -\sin\left(\dfrac{4\pi}{4}+\dfrac{\pi}{4}\right) =$

$= -\sin\left(\pi+\dfrac{\pi}{4}\right) = \sin\dfrac{\pi}{4} = \dfrac{\sqrt{2}}{2}$

16.5. $\cos\left(\dfrac{3\pi}{4}\right) = \cos\left(\dfrac{4\pi}{4}-\dfrac{\pi}{4}\right) = \cos\left(\pi-\dfrac{\pi}{4}\right) = -\cos\dfrac{\pi}{4} = -\dfrac{\sqrt{2}}{2}$

16.6. $\tan\left(-\dfrac{11\pi}{6}\right) = -\tan\dfrac{11\pi}{6} = -\tan\left(\dfrac{12\pi}{6}-\dfrac{\pi}{6}\right) =$

$= -\tan\left(2\pi-\dfrac{\pi}{6}\right) = -\tan\left(-\dfrac{\pi}{6}\right) = \tan\left(\dfrac{\pi}{6}\right) = \dfrac{\sqrt{3}}{3}$

16.7. $\sin\left(-\dfrac{7\pi}{4}\right) = -\sin\dfrac{7\pi}{4} = -\sin\left(\dfrac{8\pi}{4}-\dfrac{\pi}{4}\right) =$

$= -\sin\left(2\pi-\dfrac{\pi}{4}\right) = -\sin\left(-\dfrac{\pi}{4}\right) =$

$= \sin\left(\dfrac{\pi}{4}\right) = \dfrac{\sqrt{2}}{2}$

16.8. $\cos\dfrac{5\pi}{6} = \cos\left(\dfrac{6\pi}{6}-\dfrac{\pi}{6}\right) = -\cos\left(\dfrac{\pi}{6}\right) = -\dfrac{\sqrt{3}}{2}$

16.9. $\tan\left(-\dfrac{\pi}{6}\right) = -\tan\left(\dfrac{\pi}{6}\right) = -\dfrac{\sqrt{3}}{3}$

16.10. $\sin\left(\dfrac{5\pi}{3}\right) = \sin\left(\dfrac{6\pi}{3}-\dfrac{\pi}{3}\right) = \sin\left(2\pi-\dfrac{\pi}{3}\right) =$

$= \sin\left(-\dfrac{\pi}{3}\right) = -\sin\left(\dfrac{\pi}{3}\right) = -\dfrac{\sqrt{3}}{2}$

16.11. $\cos\left(-\dfrac{\pi}{3}\right) = \cos\left(\dfrac{\pi}{3}\right) = \dfrac{1}{2}$

16.12. $\tan\left(-\dfrac{2\pi}{3}\right) = -\tan\left(\dfrac{2\pi}{3}\right) = -\tan\left(\dfrac{3\pi}{3}-\dfrac{\pi}{3}\right) =$

$= -\tan\left(\pi-\dfrac{\pi}{3}\right) = \tan\left(\dfrac{\pi}{3}\right) = \sqrt{3}$

16.13. $\sin\left(-\dfrac{5\pi}{6}\right) = -\sin\left(\dfrac{5\pi}{6}\right) = -\sin\left(\dfrac{6\pi}{6}-\dfrac{\pi}{6}\right) =$

$= -\sin\left(\pi-\dfrac{\pi}{6}\right) = -\sin\left(\dfrac{\pi}{6}\right) = -\dfrac{1}{2}$

16.14. $\cos\left(\dfrac{11\pi}{6}\right) = \cos\left(\dfrac{12\pi}{6}-\dfrac{\pi}{6}\right) = \cos\left(2\pi-\dfrac{\pi}{6}\right) =$

$= \cos\left(-\dfrac{\pi}{6}\right) = \cos\left(\dfrac{\pi}{6}\right) = \dfrac{\sqrt{3}}{2}$

16.15. $\tan\left(-\dfrac{\pi}{4}\right) = -\tan\left(\dfrac{\pi}{4}\right) = -1$

16.16. $\sin\left(-\dfrac{4\pi}{3}\right) = -\sin\left(\dfrac{4\pi}{3}\right) = -\sin\left(\dfrac{3\pi}{3}+\dfrac{\pi}{3}\right) =$

$= -\sin\left(\pi+\dfrac{\pi}{3}\right) = \sin\left(\dfrac{\pi}{3}\right) = \dfrac{\sqrt{3}}{2}$

17.1. $\cos\left(\dfrac{5\pi}{4}\right) - \sin\left(-\dfrac{3\pi}{4}\right) - 2\tan\left(\dfrac{4\pi}{3}\right) =$

$= \cos\left(\dfrac{4\pi}{4}+\dfrac{\pi}{4}\right) + \sin\left(\dfrac{3\pi}{4}\right) - 2\tan\left(\dfrac{3\pi}{3}+\dfrac{\pi}{3}\right) =$

$= \cos\left(\pi+\dfrac{\pi}{4}\right) + \sin\left(\dfrac{4\pi}{4}+\dfrac{\pi}{4}\right) - 2\tan\left(\pi+\dfrac{\pi}{3}\right) =$

$= -\cos\dfrac{\pi}{4} + \sin\left(\pi-\dfrac{\pi}{4}\right) - 2\tan\left(\dfrac{\pi}{3}\right) =$

$= -\dfrac{\sqrt{2}}{2} + \sin\left(\dfrac{\pi}{4}\right) - 2\sqrt{3} =$

$= -\dfrac{\sqrt{2}}{2} + \dfrac{\sqrt{2}}{2} - 2\sqrt{3} = -2\sqrt{3}$

17.2. $$\frac{\cos\left(\frac{7\pi}{6}\right)-\sin\left(-\frac{5\pi}{3}\right)}{\tan\left(\frac{13\pi}{6}\right)}=\frac{\cos\left(\frac{6\pi}{6}+\frac{\pi}{6}\right)+\sin\left(\frac{6\pi}{3}-\frac{\pi}{3}\right)}{\tan\left(\frac{12\pi}{6}+\frac{\pi}{6}\right)}=$$

$$=\frac{\cos\left(\pi+\frac{\pi}{6}\right)+\sin\left(2\pi-\frac{\pi}{3}\right)}{\tan\left(2\pi+\frac{\pi}{6}\right)}=$$

$$=\frac{\cos\left(\pi+\frac{\pi}{6}\right)+\sin\left(2\pi-\frac{\pi}{3}\right)}{\tan\left(2\pi+\frac{\pi}{6}\right)}=$$

$$=\frac{-\cos\left(\frac{\pi}{6}\right)+\sin\left(-\frac{\pi}{3}\right)}{\tan\left(\frac{\pi}{6}\right)}=\frac{-\frac{\sqrt{3}}{2}-\sin\left(\frac{\pi}{3}\right)}{\frac{\sqrt{3}}{3}}=$$

$$=\frac{-\frac{\sqrt{3}}{2}-\frac{\sqrt{3}}{2}}{\frac{\sqrt{3}}{3}}=-\frac{\sqrt{3}}{\frac{\sqrt{3}}{3}}=$$

$$=-\frac{3\sqrt{3}}{\sqrt{3}}=-3$$

17.3. $$\cos\left(\frac{11\pi}{6}\right)+\sin\left(-\frac{5\pi}{3}\right)=$$

$$=\cos\left(\frac{12\pi}{6}-\frac{\pi}{6}\right)-\sin\left(\frac{6\pi}{3}-\frac{\pi}{3}\right)=$$

$$=\cos\left(2\pi-\frac{\pi}{6}\right)-\sin\left(2\pi-\frac{\pi}{3}\right)=$$

$$=\cos\left(-\frac{\pi}{6}\right)-\sin\left(-\frac{\pi}{3}\right)=$$

$$=\cos\left(\frac{\pi}{6}\right)+\sin\left(\frac{\pi}{3}\right)=$$

$$=\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{2}=\sqrt{3}$$

17.4. $$\sqrt{2}\sin\left(\frac{7\pi}{4}\right)+\sqrt{3}\tan\left(\frac{4\pi}{3}\right)=$$

$$=\sqrt{2}\sin\left(\frac{8\pi}{4}-\frac{\pi}{4}\right)+\sqrt{3}\tan\left(\frac{3\pi}{3}+\frac{\pi}{3}\right)=$$

$$=\sqrt{2}\sin\left(2\pi-\frac{\pi}{4}\right)+\sqrt{3}\tan\left(\pi+\frac{\pi}{3}\right)=$$

$$=\sqrt{2}\sin\left(-\frac{\pi}{4}\right)+\sqrt{3}\tan\frac{\pi}{3}=$$

$$=-\sqrt{2}\sin\frac{\pi}{4}+\sqrt{3}\times\sqrt{3}=$$

$$=-\sqrt{2}\times\frac{\sqrt{2}}{2}+3=-1+3=2$$

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18.1. $$\tan\left(\frac{\pi}{2}-\theta\right)\times\tan\theta=\frac{\sin\left(\frac{\pi}{2}-\theta\right)}{\cos\left(\frac{\pi}{2}-\theta\right)}\times\frac{\sin\theta}{\cos\theta}=$$

$$=\frac{\cos\theta}{\sin\theta}\times\frac{\sin\theta}{\cos\theta}=1$$

18.2. $$\frac{\tan(\pi-\alpha)\times\sin\left(\frac{\pi}{2}+\alpha\right)}{\cos\left(\frac{3\pi}{2}-\alpha\right)}=$$

$$=\frac{\tan(-\alpha)\times\cos\alpha}{\cos\left[\pi+\left(\frac{\pi}{2}-\alpha\right)\right]}=\frac{-\tan\alpha\times\cos\alpha}{-\cos\left(\frac{\pi}{2}-\alpha\right)}=$$

$$=\frac{-\frac{\sin\alpha}{\cos\alpha}\times\cos\alpha}{-\sin\alpha}=\frac{\sin\alpha}{\sin\alpha}=1$$

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19.1. $$\sin(\pi-\alpha)+\cos\left(\frac{\pi}{2}-\alpha\right)+2\sin(-\alpha)=$$

$$=\sin\alpha+\sin\alpha-2\sin\alpha=$$

$$=0$$

19.2. $$\tan(-\alpha)\times\sin\left(\frac{\pi}{2}+\alpha\right)+\sin(\pi+\alpha)=$$

$$=-\tan\alpha\times\cos\alpha-\sin\alpha=$$

$$=-\frac{\sin\alpha}{\cos\alpha}\times\cos\alpha-\sin\alpha=$$

$$=-\sin\alpha-\sin\alpha=$$

$$=-2\sin\alpha$$

19.3. $$\sin^2\left(\frac{\pi}{2}+\alpha\right)+\sin^2(5\pi-\alpha)=$$

$$=\left[\sin\left(\frac{\pi}{2}+\alpha\right)\right]^2+\left[\sin(4\pi+\pi-\alpha)\right]^2=$$

$$=\cos^2\alpha+\left[\sin(\pi-\alpha)\right]^2=$$

$$=\cos^2\alpha+\sin^2\alpha=1$$

20.

- $$\cos\left(\frac{\pi}{2}+x\right)=\frac{12}{13}\wedge x\in\left]\pi,\frac{3\pi}{2}\right[\Leftrightarrow$$

  $$\Leftrightarrow -\sin x=\frac{12}{13}\wedge x\in\left]\pi,\frac{3\pi}{2}\right[\Leftrightarrow$$

  $$\Leftrightarrow \sin x=-\frac{12}{13}\wedge x\in\left]\pi,\frac{3\pi}{2}\right[$$

- $$13\sin\left(x-\frac{\pi}{2}\right)-5\tan(2\pi+x)=$$

  $$=13\sin\left[-\left(\frac{\pi}{2}-x\right)\right]-5\tan x=$$

  $$=-13\sin\left(\frac{\pi}{2}-x\right)-5\tan x=$$

  $$=-13\cos x-5\tan x$$

- $$\sin^2 x+\cos^2 x=1$$

  $$\left(-\frac{12}{13}\right)^2+\cos^2 x=1\Leftrightarrow\cos^2 x=1-\frac{144}{169}\Leftrightarrow$$

  $$\Leftrightarrow\cos^2 x=\frac{25}{169}\Leftrightarrow\cos x=\pm\sqrt{\frac{25}{169}}$$

  Como $x\in$ 3.º Q, então $\cos x=-\frac{5}{13}$.

  $$\tan x=\frac{\sin x}{\cos x}=\frac{-\frac{12}{13}}{-\frac{5}{13}}=\frac{12}{5}$$

- $-13\cos x - 5\tan x =$

  $= -13\times\left(-\dfrac{5}{13}\right) - 5\times\dfrac{12}{5} = 5 - 12 = -7$

21. • $3\sin\left(x-\dfrac{\pi}{2}\right) - 2 = 0 \wedge x \in \left]0\ ,\ \pi\right[\Leftrightarrow$

$\Leftrightarrow \sin\left(x-\dfrac{\pi}{2}\right) = \dfrac{2}{3} \wedge x \in \left]0\ ,\ \pi\right[\Leftrightarrow$

$\Leftrightarrow -\sin\left(\dfrac{\pi}{2}-x\right) = \dfrac{2}{3} \wedge x \in \left]0\ ,\ \pi\right[\Leftrightarrow$

$\Leftrightarrow -\cos x = \dfrac{2}{3} \wedge x \in \left]0\ ,\ \pi\right[\Leftrightarrow$

$\Leftrightarrow \cos x = -\dfrac{2}{3} \wedge x \in \left]\dfrac{\pi}{2}\ ,\ \pi\right[$

- $\tan(\pi - x) + \cos\left(\dfrac{\pi}{2}+x\right) =$

  $= -\tan x - \sin x$

- $\sin^2 x + \cos^2 x = 1$

  $\sin^2 x + \left(-\dfrac{2}{3}\right)^2 = 1 \Leftrightarrow \sin^2 x = 1 - \dfrac{4}{9} \Leftrightarrow$

  $\Leftrightarrow \sin^2 x = \dfrac{5}{9} \Leftrightarrow \sin x = \pm\dfrac{\sqrt{5}}{3}$

  Como $x \in$ 2.º Q, temos $\sin x = \dfrac{\sqrt{5}}{3}$.

  $\tan x = \dfrac{\frac{\sqrt{5}}{3}}{-\frac{2}{3}} = -\dfrac{\sqrt{5}}{2}$

- $-\tan x - \sin x =$

  $= \dfrac{\sqrt{5}}{2} - \dfrac{\sqrt{5}}{3} = \dfrac{3\sqrt{5}-2\sqrt{5}}{6} = \dfrac{\sqrt{5}}{6}$

22. Sendo $A\hat{O}B = \alpha$, $A\hat{O}C = \dfrac{\pi}{2} + \alpha$.

A ordenada de T é igual a $\tan\left(\dfrac{\pi}{2}+\alpha\right)$.

Logo, $\tan\left(\dfrac{\pi}{2}+\alpha\right) = -2$.

$\tan\left(\dfrac{\pi}{2}+\alpha\right) = -2 \Leftrightarrow \dfrac{\sin\left(\frac{\pi}{2}+\alpha\right)}{\cos\left(\frac{\pi}{2}+\alpha\right)} = -2 \Leftrightarrow$

$\Leftrightarrow \dfrac{\cos\alpha}{-\sin\alpha} = -2 \Leftrightarrow$

$\Leftrightarrow -\dfrac{1}{\tan\alpha} = -2 \Leftrightarrow$

$\Leftrightarrow \tan\alpha = \dfrac{1}{2}$

No triângulo [OAB], considerando [OA] para base, temos que a altura é igual a $\sin\alpha$.

Logo, a área de [OAB] é $A = \dfrac{\sin\alpha}{2}$ pois $\overline{OA} = 1$.

Determinemos $\sin\alpha$ sabendo que $\tan\alpha = \dfrac{1}{2}$.

$1 + \tan^2\alpha = \dfrac{1}{\cos^2\alpha}$

$1 + \left(\dfrac{1}{2}\right)^2 = \dfrac{1}{\cos^2\alpha} \Leftrightarrow \dfrac{1}{\cos^2\alpha} = \dfrac{5}{4} \Leftrightarrow \cos^2\alpha = \dfrac{4}{5}$

$\sin^2\alpha = 1 - \cos^2\alpha = 1 - \dfrac{4}{5} = \dfrac{1}{5}$

Como $\alpha \in$ 1.º Q, $\sin\alpha = \sqrt{\dfrac{1}{5}} = \dfrac{1}{\sqrt{5}} = \dfrac{\sqrt{5}}{5}$.

$A_{[ABC]} = \dfrac{\sin\alpha}{2} = \dfrac{1}{2}\times\dfrac{\sqrt{5}}{5} = \dfrac{\sqrt{5}}{10}$

Atividades complementares

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23. $\overset{\frown}{BC} = 2\times 20° = 40°$

$\overset{\frown}{ADB} = 2\times 130° = 260°$

$\overset{\frown}{AC} = 360° - (260° + 40°) = 60°$

$\overset{\frown}{AB} = 40° + 60° = 100°$

23.1. a) $B\hat{O}C = 40°$

$360° - 40° = 320°$

$\alpha = 40°$ ou $\alpha = -320°$

b) $B\hat{O}A = 100°$

$360° - 100° = 260°$

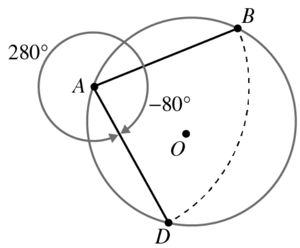
$\alpha = -100°$ ou $\alpha = 260°$

23.2. Se D é imagem de B numa rotação de centro A, então $\overline{AD} = \overline{AB}$. Logo, $\overset{\frown}{AD} = \overset{\frown}{AB} = 100°$.

Assim, $\overset{\frown}{BD} = 360° - (100° + 100°) = 160°$ pelo que:

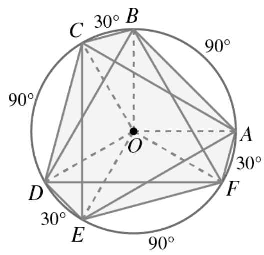
$D\hat{A}B = 160° : 2 = 80°$

$360° - 80° = 280°$



Portanto, $\beta = -80°$ ou $\beta = 280°$.

24.1. a) $A\hat{O}D = 90° + 30° + 90° = 210°$



Lado extremidade: $\dot{O}D$

b) $A\hat{O}E = -30° - 90° = -120°$

Lado extremidade: $\dot{O}E$

c) $330° = 360° - 30°$

$A\hat{O}F = 330°$

Lado extremidade: $\dot{O}F$

d) $A\hat{O}D = -30° - 90° - 30° = -150°$

Lado extremidade: $\dot{O}D$

24.2. a) $960° = 240° + 2 \times 360°$ $\quad \begin{array}{r|l} 960 & 360 \\ \cline{2-2} 240 & 2 \end{array}$

$960° = (240°\ ,\ 2)$

$A\hat{O}E = 90° + 30° + 90° + 30° = 240°$

Lado extremidade: $\dot{O}E$

b) $-1470° = -30° - 4 \times 360°$ $\quad \begin{array}{r|l} 1470 & 360 \\ \cline{2-2} 30 & 4 \end{array}$

$-1470° = (-30°\ ,\ 4)$

$A\hat{O}F = -30°$

Lado extremidade: $\dot{O}F$

c) $1530° = 90° + 4 \times 360°$ $\quad \begin{array}{r|l} 1530 & 360 \\ \cline{2-2} 90 & 4 \end{array}$

$1530° = (90°\ ,\ 4)$

$A\hat{O}B = 90°$

Lado extremidade: $\dot{O}B$

d) $-1320° = -240 - 3 \times 360$ $\quad \begin{array}{r|l} 1320 & 360 \\ \cline{2-2} 240 & 3 \end{array}$

$-1320° = (-240°\ ,\ -3)$

$A\hat{O}C = -30° - 90° - 30° - 90° = -240°$

Lado extremidade: $\dot{O}C$

e) $1200° = 120° + 3 \times 360°$ $\quad \begin{array}{r|l} 1200 & 360 \\ \cline{2-2} 120 & 3 \end{array}$

$1200° = (120°\ ,\ 3)$

$A\hat{O}C = 90° + 30° = 120°$

Lado extremidade: $\dot{O}C$

f) $-990° = -270° - 2 \times 360°$ $\quad \begin{array}{r|l} 990 & 360 \\ \cline{2-2} 270 & 2 \end{array}$

$990° = (-270°\ ,\ -2)$

$A\hat{O}B = -(360° - 90°) = -270°$

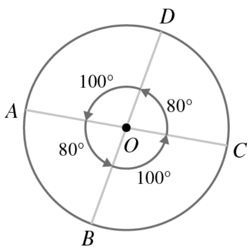
Lado extremidade: $\dot{O}B$

25. $\overset{\frown}{CD} = 40° \times 2 = 80°$; $\overset{\frown}{AB} = \overset{\frown}{CD} = 80°$

$\overset{\frown}{AD} = 180° - 80° = 100°$; $\overset{\frown}{BC} = \overset{\frown}{AD} = 100°$

25.1. $\overset{\frown}{AB} = 80°$, $\overset{\frown}{BC} = 100°$, $\overset{\frown}{CD} = 80°$ e $\overset{\frown}{DA} = 100°$

25.2.



a) $540° = 180° + 360°$

O transformado do ponto *A* é o ponto *C*.

b) $-640° = -280° - 360°$

$A\hat{O}B = -280°$

O transformado do ponto *A* é o ponto *B*.

c) $980° = 260° + 2 \times 360°$ $\quad \begin{array}{r|l} 980 & 360 \\ \cline{2-2} 260 & 2 \end{array}$

$A\hat{O}D = 260°$

O transformado do ponto *A* é o ponto *D*.

d) $-900° = -180° - 2 \times 360°$ $\quad \begin{array}{r|l} 900 & 360 \\ \cline{2-2} 180 & 2 \end{array}$

O transformado do ponto *A* é o ponto *C*.

e) $1080° = 0° + 3 \times 360°$ $\quad \begin{array}{r|l} 1080 & 360 \\ \cline{2-2} 0 & 3 \end{array}$

O transformado do ponto *A* é o ponto *A*.

f) $-1180° = -100° - 3 \times 360°$ $\quad \begin{array}{r|l} 1180 & 360 \\ \cline{2-2} 100 & 3 \end{array}$

$A\hat{O}D = -100°$

O transformado do ponto *A* é o ponto *D*.

g) $1520° = 80° + 4 \times 360°$ $\quad \begin{array}{r|l} 1520 & 360 \\ \cline{2-2} 80 & 4 \end{array}$

$A\hat{O}B = 80°$

O transformado do ponto *A* é o ponto *B*.

h) $-1440° = 0 - 4 \times 360°$

O transformado do ponto *A* é o ponto *A*.

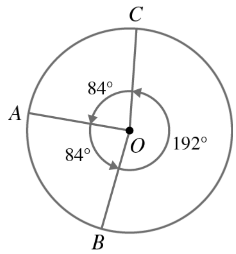
26.1. $360° - 96° = 264°$

$\alpha = -96°$ ou $\alpha = 264°$

26.2. $96° \times 2 = 192°$

$360° - 192° = 168°$

$\dfrac{180° - 96°}{2} = 42°$; $42° \times 2 = 84°$

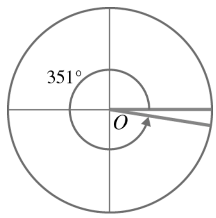


a) $\beta = -192°$ ou $\beta = 168°$

b) $\beta = 84°$ ou $\beta = -276°$

c) $\beta = -84°$ ou $\beta = 276°$

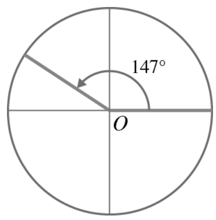
27.1. $351° = 360° - 9°$



α pertence ao 4.º quadrante

$\sin\alpha < 0$ e $\cos\alpha > 0$

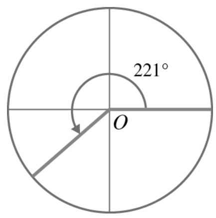
27.2. $147° = 180° - 33°$



α pertence ao 2.º quadrante

$\sin\alpha > 0$ e $\cos\alpha < 0$

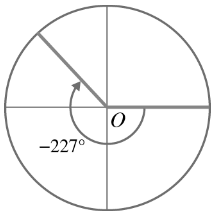
27.3. $221° = 180° + 41°$



α pertence ao 3.º quadrante

$\sin\alpha < 0$ e $\cos\alpha < 0$

27.4. $-227° = -180° - 47°$



α pertence ao 2.º quadrante

$\sin\alpha > 0$ e $\cos\alpha < 0$

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28.1. $A(\cos 45°\ ,\ \sin 45°)$ ou $A\left(\frac{\sqrt{2}}{2}\ ,\ \frac{\sqrt{2}}{2}\right)$

$M(1\ ,\ \tan 45°)$ ou $M(1\ ,\ 1)$

C é a imagem de A na reflexão central de centro O.

Logo, $C\left(-\frac{\sqrt{2}}{2}\ ,\ -\frac{\sqrt{2}}{2}\right)$

28.2. $P\hat{O}C = 180° + 45° = 225°$. Logo, $C(\cos 225°\ ,\ \sin 225°)$.

Temos, portanto, $\sin 225° = -\frac{\sqrt{2}}{2}$ e $\cos 225° = -\frac{\sqrt{2}}{2}$.

28.3. $P\hat{O}B = 180° - 45° = 135°$; $B(\cos 135°\ ,\ \sin 135°)$

$\cos 135° = \cos(180° - 45°) = -\cos 45° = -\frac{\sqrt{2}}{2}$

$\sin 135° = \sin(180° - 45°) = \sin 45° = \frac{\sqrt{2}}{2}$

$B\left(-\frac{\sqrt{2}}{2}\ ,\ \frac{\sqrt{2}}{2}\right)$

28.4. $P\hat{O}D = 360° - 45° = 315°$; $D(\cos 315°\ ,\ \sin 315°)$

Por outro lado, D é a imagem do ponto B pela reflexão central de centro O. Assim, $D\left(\frac{\sqrt{2}}{2}\ ,\ -\frac{\sqrt{2}}{2}\right)$.

Logo, $\sin 315° = -\frac{\sqrt{2}}{2}$ e $\cos 315° = \frac{\sqrt{2}}{2}$.

28.5. N é a imagem de M pela reflexão de eixo Ox.

Como $M(1\ ,\ 1)$, então N tem coordenadas $(1\ ,\ -1)$.

29.1. $750° = 30° + 2 \times 360$

$750° = (30°\ ,\ 2)$

$\begin{array}{r|l} 750 & 360 \\ \hline 30 & 2 \end{array}$

$\sin 750° = \sin 30° = \frac{1}{2}$

$\cos 750° = \cos 30° = \frac{\sqrt{3}}{2}$

$\tan 750° = \tan 30° = \frac{\sqrt{3}}{3}$

29.2. $1140° = 60° + 3 \times 360$

$1140° = (60°\ ,\ 3)$

$\begin{array}{r|l} 1140 & 360 \\ \hline 60 & 3 \end{array}$

$\sin 1140° = \sin 60° = \frac{\sqrt{3}}{2}$

$\cos 1140° = \cos 60° = \frac{1}{2}$

$\tan 1140° = \tan 60° = \sqrt{3}$

29.3. $405° = 45° + 360°$

$405° = (45°\ ,\ 1)$

$\sin 405° = \sin 45° = \frac{\sqrt{2}}{2}$

$\cos 405° = \cos 45° = \frac{\sqrt{2}}{2}$

$\tan 405° = \tan 45° = 1$

30.1. $480° = 120° + 360°$; $480° = (120°\ ,\ 1)$

$\sin 480° = \sin 120° = \sin(180° - 60°) = \sin 60° = \frac{\sqrt{3}}{2}$

$\cos 480° = \cos 120° = \cos(180° - 60°) = -\cos 60° = -\frac{1}{2}$

30.2. $870° = 150° + 2 \times 360°$

$\begin{array}{r|l} 870 & 360 \\ \hline 150 & 2 \end{array}$

$870° = (150°\ ,\ 2)$

$\sin 870° = \sin 150° = \sin(180° - 30°) = \sin 30° = \frac{1}{2}$

$\cos 870° = \cos 150° = \cos(180° - 30°) = -\cos 30° = -\frac{\sqrt{3}}{2}$

30.3. $1215° = 135° + 3 \times 360°$

$\begin{array}{r|l} 1215 & 360 \\ \hline 135 & 3 \end{array}$

$1215° = (135°\ ,\ 3)$

$\sin 1215° = \sin 135° = \sin(180° - 45°) = \sin 45° = \frac{\sqrt{2}}{2}$

$\cos 1215° = \cos 135° = \cos(180° - 45°) = -\cos 45° = -\frac{\sqrt{2}}{2}$

30.4. $810° = 90° + 2 \times 360°$

$\begin{array}{r|l} 810 & 360 \\ \hline 90 & 2 \end{array}$

$810° = (90°\ ,\ 2)$

$\sin 810° = \sin 90° = 1$

$\cos 810° = \cos 90° = 0$

31.1.

Graus		Rad
180	——	π
240	——	x

$x = \frac{240\pi}{180} = \frac{4\pi}{3}$

$240° = \frac{4\pi}{3}$ rad

31.2.

Graus		Rad
180	——	π
210	——	x

$x = \frac{210\pi}{180} = \frac{7\pi}{6}$

$210° = \frac{7\pi}{6}$ rad

31.3.

Graus		Rad
180	——	π
300	——	x

$x = \frac{300\pi}{180} = \frac{5\pi}{3}$

$300° = \frac{5\pi}{3}$ rad

31.4.

Graus		Rad
180	——	π
315	——	x

$x=\frac{315\pi}{180}=\frac{7\pi}{4}$

$315°=\frac{7\pi}{4}$ rad

31.5.

Graus		Rad
180	——	π
–288	——	x

$x=\frac{-288\pi}{180}=-\frac{8\pi}{5}$

$-288°=-\frac{8\pi}{5}$ rad

31.6.

Graus		Rad
180	——	π
420	——	x

$x=\frac{420\pi}{180}=\frac{7\pi}{3}$

$420°=\frac{7\pi}{3}$ rad

32.1.

Graus		Rad
180	——	π
x	——	$\frac{7\pi}{5}$

$\frac{7\pi}{5}$ rad = 252°

32.2.

Graus		Rad
180	——	π
x	——	$-\frac{5\pi}{2}$

$x=\frac{-\frac{5\pi}{2}\times 180}{\pi}=-450$, logo $-\frac{5\pi}{2}$ rad = –450°

32.3.

Graus		Rad
180	——	π
x	——	$\frac{3\pi}{8}$

$x=\frac{180\times\frac{3\pi}{8}}{\pi}=67{,}5$, logo $\frac{3\pi}{8}$ rad = 67,5°

32.4.

Graus		Rad
180	——	π
x	——	$-\frac{5\pi}{16}$

$x=\frac{-\frac{5\pi}{16}\times 180}{\pi}=56{,}25°$, logo $-\frac{5\pi}{16}$ rad = – 56,25°

32.5.

Graus		Rad
180	——	π
x	——	$\frac{3\pi}{32}$

$x=\frac{180\times\frac{3\pi}{32}}{\pi}=16{,}875°$, logo $\frac{3\pi}{32}$ rad = 16,875°

32.6.

Graus		Rad
180	——	π
x	——	$-\frac{6\pi}{25}$

$x=\frac{-\frac{6\pi}{25}\times 180}{\pi}=-43{,}2°$, logo $-\frac{6\pi}{25}$ rad = -43,2°

33. $A\hat{C}B=\pi-\frac{7\pi}{12}-\frac{\pi}{6}=\frac{12\pi-7\pi-2\pi}{12}=\frac{3\pi}{12}=\frac{\pi}{4}$

$\frac{\pi}{4}$ rad = 45°

$A\hat{C}B=\frac{\pi}{4}$ rad = 45°

34. Se [*AC*] é o lado de um triângulo equilátero inscrito na circunferência, então $\overset{\frown}{AC}=\frac{2\pi}{3}$ rad.

Como $\overset{\frown}{BC}=2\times\frac{2\pi}{15}=\frac{4\pi}{15}$ rad, temos que:

$\overset{\frown}{AB}=\frac{2\pi}{3}-\frac{4\pi}{15}=\frac{10\pi-4\pi}{15}=\frac{6\pi}{15}=\frac{2\pi}{5}$

$\frac{2\pi}{\frac{2\pi}{5}}=2\pi\times\frac{5}{2\pi}=5$

[*AB*] é o lado de um pentágono regular inscrito na circunferência.

35.1. $T_1(1\ ,\ \tan\alpha)$

$\tan\alpha=2$

$1+\tan^2\alpha=\frac{1}{\cos^2\alpha}$

$1+2^2=\frac{1}{\cos^2\alpha}\Leftrightarrow\cos^2\alpha=\frac{1}{5}\Leftrightarrow\cos\alpha=\pm\sqrt{\frac{1}{5}}$

Como $\alpha\in$ 1.º Q, $\cos\alpha=\frac{1}{\sqrt{5}}=\frac{\sqrt{5}}{5}$.

$\sin^2\alpha+\cos^2\alpha=1$

$\sin^2\alpha+\frac{1}{5}=1\Leftrightarrow\sin^2\alpha=1-\frac{1}{5}\Leftrightarrow\sin\alpha=\pm\sqrt{\frac{4}{5}}$

Como $\alpha\in$ 1.º *Q*, temos $\sin\alpha=\frac{2}{\sqrt{5}}=\frac{2\sqrt{5}}{5}$.

$\sin\alpha=\frac{2\sqrt{5}}{5}$, $\cos\alpha=\frac{\sqrt{5}}{5}$ e $\tan\alpha=2$

35.2. $P_2(\cos\beta\ ,\ \sin\beta)$

$\sin\beta=\frac{3}{4}$

$\sin^2\beta+\cos^2\beta=1$

$$\frac{9}{16}+\cos^2\beta=1 \Leftrightarrow \cos^2\beta=1-\frac{9}{16} \Leftrightarrow \cos^2\beta=\frac{7}{16}$$

Como $\beta\in$ 2.º Q, então $\cos\beta=-\frac{\sqrt{7}}{4}$.

$$\tan\beta=\frac{\sin\beta}{\cos\beta}=\frac{\frac{3}{4}}{-\frac{\sqrt{7}}{4}}=-\frac{3}{\sqrt{7}}=-\frac{3\sqrt{7}}{7}$$

$\sin\beta=\frac{3}{4}$, $\cos\beta=-\frac{\sqrt{7}}{4}$ e $\tan\beta=-\frac{3\sqrt{7}}{4}$

35.3. $P_3(\cos\theta\ ,\ \sin\theta)$

$\cos\theta=-\frac{7}{9}$

$\sin^2\theta+\cos^2\theta=1$

$$\sin^2\theta+\left(-\frac{7}{9}\right)^2=1 \Leftrightarrow \sin^2\theta=1-\frac{49}{81} \Leftrightarrow \sin^2\theta=\frac{32}{81}$$

Como $\theta\in$ 3.º Q, $\sin\theta=-\frac{\sqrt{32}}{9}=-\frac{4\sqrt{2}}{9}$.

$$\tan\theta=\frac{\sin\theta}{\cos\theta}=\frac{-\frac{4\sqrt{2}}{9}}{-\frac{7}{9}}=\frac{4\sqrt{2}}{7}$$

$\sin\theta=-\frac{4\sqrt{2}}{9}$, $\cos\theta=-\frac{7}{9}$ e $\tan\theta=\frac{4\sqrt{2}}{7}$

35.4. $T_4(1\ ,\ \tan\varphi)$

$\tan\varphi=-\frac{3}{4}$

$$1+\tan^2\varphi=\frac{1}{\cos^2\varphi}$$

$$1+\left(-\frac{3}{4}\right)^2=\frac{1}{\cos^2\varphi} \Leftrightarrow 1+\frac{9}{16}=\frac{1}{\cos^2\varphi} \Leftrightarrow$$

$$\Leftrightarrow \frac{1}{\cos^2\varphi}=\frac{25}{16} \Leftrightarrow \cos^2\varphi=\frac{16}{25}$$

Como $\varphi\in$ 4.º Q, então $\cos\varphi=\frac{4}{5}$.

$\sin^2\varphi+\cos^2\varphi=1$

$$\sin^2\varphi+\left(\frac{4}{5}\right)^2=1 \Leftrightarrow \sin^2\varphi=1-\frac{16}{25} \Leftrightarrow \sin^2\varphi=\frac{9}{25}$$

Como $\varphi\in$ 4.º Q, temos $\sin\varphi=-\frac{3}{5}$.

$\sin\varphi=-\frac{3}{5}$, $\cos\varphi=\frac{4}{5}$ e $\tan\varphi=-\frac{3}{4}$

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36. $\overline{BD}^2=\overline{AB}^2+\overline{AD}^2 \Leftrightarrow \overline{BD}^2=2^2+1^2$

$\overline{BD}=\sqrt{5}$ cm

$\overline{BC}^2=\overline{BD}^2+\overline{DC}^2-2\overline{BD}\times\overline{DC}\times\cos\left(B\hat{D}C\right)$

Determinemos $\cos\left(B\hat{D}C\right)$, sabendo que $\tan\left(B\hat{D}C\right)=2$.

$$1+\tan^2\left(B\hat{D}C\right)=\frac{1}{\cos^2\left(B\hat{D}C\right)}$$

$$1+2^2=\frac{1}{\cos^2\left(B\hat{D}C\right)} \Leftrightarrow \cos^2 B\hat{D}C=\frac{1}{5}$$

Como $0<B\hat{D}C<\pi$ e $\tan\left(B\hat{D}C\right)>0$, então $\cos\left(B\hat{D}C\right)>0$.

$$\cos\left(B\hat{D}C\right)=\frac{1}{\sqrt{5}}=\frac{\sqrt{5}}{5}$$

$$\overline{BC}^2=\left(\sqrt{5}\right)^2+3^2-2\sqrt{5}\times3\times\frac{\sqrt{5}}{5}=5+9-6\times\frac{\sqrt{5}\times\sqrt{5}}{5}=8$$

$\overline{BC}=\sqrt{8}$ cm $=2\sqrt{2}$ cm

37.1. $\tan x+\frac{\cos x}{1+\sin x}=\frac{\sin x}{\cos x}+\frac{\cos x}{1+\sin x}=$

$$=\frac{\sin x(1+\sin x)+\cos^2 x}{\cos x(1+\sin x)}=\frac{\sin x+\sin^2 x+\cos^2 x}{\cos x(1+\sin x)}=$$

$$=\frac{\sin x+1}{\cos x(\sin x+1)}=\frac{1}{\cos x}$$

37.2. $\sin^2 x-\sin^4 x=\sin^2 x\left(1-\sin^2 x\right)=$

$$=\left(1-\cos^2 x\right)\times\cos^2 x=\cos^2 x-\cos^4 x$$

37.3. $\frac{\sin x\cos x}{\sin x-\cos x}+\frac{\cos x}{1-\tan x}=$

$$=\frac{\sin x\cos x}{\sin x-\cos x}+\frac{\cos x}{1-\frac{\sin x}{\cos x}}=$$

$$=\frac{\sin x\cos x}{\sin x-\cos x}+\frac{\cos x}{\frac{\cos x-\sin x}{\cos x}}=$$

$$=\frac{\sin x\cos x}{\sin x-\cos x}-\frac{\cos^2 x}{\sin x-\cos x}=$$

$$=\frac{\sin x\cos x-\cos^2 x}{\sin x-\cos x}=\frac{\cos x(\sin x-\cos x)}{\sin x-\cos x}=\cos x$$

37.4. $\frac{1}{1-\sin x}-\frac{1}{1+\sin x}=$

$$=\frac{1+\sin x-(1-\sin x)}{(1-\sin x)(1+\sin x)}=$$

$$=\frac{1+\sin x-1+\sin x}{1-\sin^2 x}=$$

$$=\frac{2\sin x}{\cos^2 x}=\frac{2\sin x}{\cos x}\times\frac{1}{\cos x}=$$

$$=2\tan x\times\frac{1}{\cos x}=\frac{2\tan x}{\cos x}$$

38.1. $\left(\sin\frac{4\pi}{3}-\cos\frac{11\pi}{6}\right)\tan\left(-\frac{5\pi}{6}\right)=$

$$=\left[\sin\left(\frac{3\pi}{3}+\frac{\pi}{3}\right)-\cos\left(\frac{12\pi}{6}-\frac{\pi}{6}\right)\right]\times\left(-\tan\frac{5\pi}{6}\right)=$$

$$=\left[-\sin\left(\pi+\frac{\pi}{3}\right)-\cos\left(2\pi-\frac{\pi}{6}\right)\right]\times\tan\left(\frac{6\pi}{6}-\frac{\pi}{6}\right)=$$

$$=-\left(-\sin\frac{\pi}{3}-\cos\left(-\frac{\pi}{6}\right)\times\tan\left(\pi-\frac{\pi}{6}\right)\right)=$$

$$=-\left(-\frac{\sqrt{3}}{2}-\cos\frac{\pi}{6}\right)\times\tan\left(-\frac{\pi}{6}\right)=$$

$$=-\left(-\frac{\sqrt{3}}{2}-\frac{\sqrt{3}}{2}\right)\times\left(-\tan\frac{\pi}{6}\right)=$$

$$=-\left(-\sqrt{3}\right)\times\left(-\frac{\sqrt{3}}{3}\right)=-\frac{\sqrt{3}\times\sqrt{3}}{3}=-\frac{3}{3}=-1$$

38.2. $\sqrt{2}\sin\frac{7\pi}{4}+\tan\frac{9\pi}{4}+\cos\frac{5\pi}{2}=$

$=\sqrt{2}\sin\left(\frac{8\pi}{4}-\frac{\pi}{4}\right)+\tan\left(\frac{8\pi}{4}+\frac{\pi}{4}\right)+\cos\left(\frac{4\pi}{2}+\frac{\pi}{2}\right)=$

$=\sqrt{2}\sin\left(2\pi-\frac{\pi}{4}\right)+\tan\left(2\pi+\frac{\pi}{4}\right)+\cos\left(2\pi+\frac{\pi}{2}\right)=$

$=\sqrt{2}\sin\left(-\frac{\pi}{4}\right)+\tan\frac{\pi}{4}+\cos\frac{\pi}{2}=$

$=-\sqrt{2}\times\frac{\sqrt{2}}{2}+1=$

$=-\frac{2}{2}+1=$

$=-1+1=0$

38.3. $\dfrac{\cos\frac{11\pi}{4}\times\sin\left(-\frac{7\pi}{6}\right)}{\sqrt{2}\sin^2\frac{\pi}{5}+\sqrt{2}\cos^2\frac{\pi}{5}}=$

$=\dfrac{-\cos\left(\frac{8\pi}{4}+\frac{3\pi}{4}\right)\sin\left(\frac{6\pi}{6}+\frac{\pi}{6}\right)}{\sqrt{2}\left(\sin^2\frac{\pi}{5}+\cos^2\frac{\pi}{5}\right)}=$

$=\dfrac{-\cos\left(2\pi+\frac{3\pi}{4}\right)\times\sin\left(\pi+\frac{\pi}{6}\right)}{\sqrt{2}\times1}=$

$=\dfrac{-\cos\left(\frac{4\pi}{4}-\frac{\pi}{4}\right)\times\left(-\sin\frac{\pi}{6}\right)}{\sqrt{2}}=$

$=\dfrac{-\cos\left(\pi-\frac{\pi}{4}\right)\times\sin\frac{\pi}{6}}{\sqrt{2}}=$

$=\dfrac{-\cos\frac{\pi}{4}\times\frac{1}{2}}{\sqrt{2}}=\dfrac{-\frac{\sqrt{2}}{2}\times\frac{1}{2}}{\sqrt{2}}=$

$=\dfrac{-\sqrt{2}\times\frac{1}{2}\times\frac{1}{2}}{\sqrt{2}}=-\frac{1}{4}$

38.4. $\dfrac{2\cos\frac{11\pi}{3}+\tan\frac{4\pi}{3}}{3\tan\left(-\frac{11\pi}{6}\right)+2\sin\left(\frac{11\pi}{6}\right)}=$

$=\dfrac{2\cos\left(\frac{12\pi}{3}-\frac{\pi}{3}\right)+\tan\left(\frac{3\pi}{3}+\frac{\pi}{3}\right)}{-3\tan\frac{11\pi}{6}+2\sin\left(\frac{12\pi}{6}-\frac{\pi}{6}\right)}=$

$=\dfrac{2\cos\left(4\pi-\frac{\pi}{3}\right)+\tan\left(\pi+\frac{\pi}{3}\right)}{-3\tan\left(\frac{12\pi}{6}-\frac{\pi}{6}\right)+2\sin\left(2\pi-\frac{\pi}{6}\right)}=$

$=\dfrac{2\cos\left(-\frac{\pi}{3}\right)+\tan\frac{\pi}{3}}{-3\tan\left(2\pi-\frac{\pi}{6}\right)+2\sin\left(-\frac{\pi}{6}\right)}=$

$=\dfrac{2\cos\frac{\pi}{3}+\sqrt{3}}{-3\tan\left(-\frac{\pi}{6}\right)-2\sin\frac{\pi}{6}}=$

$=\dfrac{2\times\frac{1}{2}+\sqrt{3}}{3\tan\frac{\pi}{6}-2\times\frac{1}{2}}=\dfrac{1+\sqrt{3}}{3\times\frac{\sqrt{3}}{3}-1}=$

$=\dfrac{\sqrt{3}+1}{\sqrt{3}-1}=\dfrac{\left(\sqrt{3}+1\right)\left(\sqrt{3}+1\right)}{\left(\sqrt{3}-1\right)\left(\sqrt{3}-1\right)}=\dfrac{3+2\sqrt{3}+1}{3-1}=$

$=\sqrt{3}+2$

39.1. $\left[\sin\left(\frac{\pi}{2}+x\right)-\sin\left(x-\pi\right)\right]^2-1=$

$=\left(\cos x+\sin\left(\pi-x\right)\right)^2-1=$

$=\left(\cos x+\sin x\right)^2-1=$

$=\cos^2x+2\sin x\cos x+\sin^2x-1=$

$=\left(\sin^2x+\cos^2x\right)+2\sin x\cos x-1=$

$=1+2\sin\cos x-1=2\sin x\cos x$

39.2. $\cos\left(\frac{5\pi}{2}+x\right)\times\sin\left(\pi+x\right)\times\tan\left(\frac{\pi}{2}-x\right)=$

$=\cos\left(\frac{4\pi}{2}+\frac{\pi}{2}+x\right)\times\left(-\sin x\right)\times\dfrac{\sin\left(\frac{\pi}{2}-x\right)}{\cos\left(\frac{\pi}{2}-x\right)}=$

$=-\cos\left(2\pi+\frac{\pi}{2}+x\right)\times\sin x\times\dfrac{\cos x}{\sin x}=$

$=-\cos\left(\frac{\pi}{2}+x\right)\times\cos x=$

$=-\left(-\sin x\right)\times\cos x=\sin x\cos x$

40.1. $A(1\,,\,0)$, $P\left(\cos x\,,\,\sin x\right)$, $Q\left(1\,,\,\tan x\right)$

Área de $[AQP]$ = Área de $[OAQ]$ – Área $[OAP]$ =

$=\dfrac{\overline{OA}\times\overline{AQ}}{2}-\dfrac{\overline{OA}\times\text{ordenada de }P}{2}=$

$=\dfrac{1\times\tan x}{2}-\dfrac{1\times\sin x}{2}=\dfrac{\tan x-\sin x}{2}$

40.2. Como $\overline{OP}=\overline{OA}=1$, se $\overline{AP}=\overline{OP}$ então $\overline{OA}=\overline{AP}=\overline{OP}=1$, ou seja, o triângulo $[OAP]$ é equilátero, pelo que $x=\frac{\pi}{3}$.

a) $A_{[AQP]}=\dfrac{\tan\frac{\pi}{3}-\sin\frac{\pi}{3}}{2}=\frac{1}{2}\left(\sqrt{3}-\frac{\sqrt{3}}{2}\right)=$

$=\frac{1}{2}\times\frac{\sqrt{3}}{2}=\frac{\sqrt{3}}{4}$

b) $\overline{OA}=1$, $\overline{AP}=1$ e $\overline{AQ}=\tan\frac{\pi}{3}=\sqrt{3}$

$\overline{OQ}^2=\overline{OA}^2+\overline{AQ}^2$

$\overline{OQ}^2=1^2+\left(\sqrt{3}\right)^2\Leftrightarrow\overline{OQ}=\sqrt{1+3}\Leftrightarrow\overline{OQ}=2$

$\overline{PQ}=\overline{OQ}-\overline{PQ}=2-1=1$

Perímetro de $\left[AQP\right]=\overline{AQ}+\overline{PQ}+\overline{AP}=$

$=\sqrt{3}+1+1=\sqrt{3}+2$

40.3. $\sin\left(\frac{\pi}{2}+x\right)-\cos\left(\pi-x\right)=\frac{8}{5}\Leftrightarrow$

$\Leftrightarrow \cos x+\cos x=\frac{8}{5}\Leftrightarrow 2\cos x=\frac{8}{5}\Leftrightarrow$

$\Leftrightarrow \cos x=\frac{4}{5}$

$\sin^2 x+\cos^2 x=1$

$\sin^2 x+\left(\frac{4}{5}\right)^2=1\Leftrightarrow \sin^2 x=1-\frac{16}{25}\Leftrightarrow \sin^2 x=\frac{9}{25}$

Como $x\in$ 1.º Q, então $\sin x=\frac{3}{5}$.

$\tan x=\frac{\sin x}{\cos x}=\frac{\frac{3}{5}}{\frac{4}{5}}=\frac{3}{4}$

$A_{[AQP]}=\frac{\frac{3}{4}\times\frac{3}{5}}{2}=\frac{\frac{3}{20}}{2}=\frac{3}{40}$

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41.1. $\sin\frac{11\pi}{12}=\sin\left(\frac{12\pi}{12}-\frac{\pi}{12}\right)=\sin\left(\pi-\frac{\pi}{12}\right)=$

$=\sin\frac{\pi}{12}=a$

41.2. $\sin\frac{13\pi}{12}=\sin\left(\frac{12\pi}{12}+\frac{\pi}{12}\right)=\sin\left(\pi+\frac{\pi}{12}\right)=$

$=-\sin\frac{\pi}{12}=-a$

41.3. $\cos\frac{5\pi}{12}=\cos\left(\frac{6\pi}{12}-\frac{\pi}{12}\right)=\cos\left(\frac{\pi}{2}-\frac{\pi}{12}\right)=$

$=\sin\frac{\pi}{12}=a$

41.4. $\cos\frac{7\pi}{12}=\cos\left(\frac{6\pi}{12}+\frac{\pi}{12}\right)=\cos\left(\frac{\pi}{2}+\frac{\pi}{12}\right)=$

$=-\sin\frac{\pi}{12}=-a$

42.1. $A_{[PQB]}=A_{[OQB]}-A_{[OPB]}=$

$=\frac{\overline{OB}\times\overline{OA}}{2}-\frac{\overline{OB}\times\text{abcissa de }P}{2}=$

$=\frac{1\times 1}{2}-\frac{1\times\cos x}{2}=$

$=\frac{1}{2}-\frac{\cos x}{2}=\frac{1-\cos x}{2}$

42.2. $9\sin\left(\pi+x\right)+8\cos\left(\frac{\pi}{2}+x\right)+15=0\Leftrightarrow$

$\Leftrightarrow -9\sin x-8\sin x=-15\Leftrightarrow$

$\Leftrightarrow -17\sin x=-15\Leftrightarrow \sin x=\frac{15}{17}$

$\sin^2 x+\cos^2 x=1$

$\left(\frac{15}{17}\right)^2+\cos^2 x=1\Leftrightarrow \cos^2 x=1-\frac{225}{289}\Leftrightarrow \cos^2 x=\frac{64}{289}$

Como $x\in\left]0\,,\,\frac{\pi}{2}\right[$, então $\cos x=\sqrt{\frac{64}{289}}=\frac{8}{17}$.

$A_{[PQB]}=\frac{1-\frac{8}{17}}{2}=\frac{\frac{9}{17}}{2}=\frac{9}{34}$

42.3. Se $x=\frac{\pi}{3}$:

$P\left(\cos\frac{\pi}{3}\,,\,\sin\frac{\pi}{3}\right)$ ou $P\left(\frac{1}{2}\,,\,\frac{\sqrt{3}}{2}\right)$

$Q\left(1\,,\,\tan\frac{\pi}{3}\right)$ ou $Q\left(1\,,\,\sqrt{3}\right)$

$\overline{PQ}=\sqrt{\left(1-\frac{1}{2}\right)^2+\left(\sqrt{3}-\frac{\sqrt{3}}{2}\right)^2}=\sqrt{\left(\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}=$

$=\sqrt{\frac{1}{4}+\frac{3}{4}}=\sqrt{\frac{4}{4}}=1$

43.1. $810°=90°+2\times 360°$

$$\begin{array}{r|l} 810 & 360 \\ \hline 90 & 2 \end{array}$$

O ponteiro dos minutos deu 2 voltas + $\frac{1}{4}$ de volta. Logo, passaram 2 h 15 min após as 4 horas.

O relógio marca 6 h 15 min.

43.2. $360°:12=30$

Em cada hora o ponteiro das horas descreve um ângulo de 30° de amplitude.

60 min	——	30°
15 min	——	x

$x=\frac{15\times 30}{60}=7{,}5$; $90°+7{,}5°=97{,}5°$

Graus		Rad
180	——	π
97,5	——	x

$x=\frac{97{,}5\pi}{180}=\frac{13\pi}{24}$

Às 6 h 15 min os ponteiros formam um ângulo de $\frac{13\pi}{24}$ rad de amplitude.

44.1. $\cos^2\frac{5\pi}{11}+\cos^2\frac{\pi}{22}+\cos^2\frac{7\pi}{12}+\cos^2\frac{\pi}{12}=$

$=\cos^2\left(\frac{10\pi}{22}\right)+\cos^2\frac{\pi}{22}+\cos^2\left(\frac{6\pi}{12}+\frac{\pi}{12}\right)+\cos^2\frac{\pi}{12}$

$=\cos^2\left(\frac{11\pi}{22}-\frac{\pi}{22}\right)+\cos^2\frac{\pi}{22}+\left[\cos\left(\frac{\pi}{2}+\frac{\pi}{12}\right)\right]^2+\cos^2\frac{\pi}{12}$

$=\left[\cos\left(\frac{\pi}{2}-\frac{\pi}{22}\right)\right]^2+\cos^2\frac{\pi}{22}+\left(-\sin\frac{\pi}{12}\right)^2+\cos^2\frac{\pi}{12}$

$=\left(\sin^2\frac{\pi}{22}+\cos^2\frac{\pi}{22}\right)+\left(\sin^2\frac{\pi}{12}+\cos^2\frac{\pi}{12}\right)=1+1=2$

44.2. $\sin\frac{4\pi}{9}+\sin\frac{5\pi}{9}+\sin\frac{14\pi}{9}+\sin\frac{13\pi}{9}=$

$=\sin\frac{4\pi}{9}+\sin\frac{5\pi}{9}+\sin\left(\frac{9\pi}{9}+\frac{5\pi}{9}\right)+\sin\left(\frac{9\pi}{9}+\frac{4\pi}{9}\right)=$

$=\sin\frac{4\pi}{9}+\sin\frac{5\pi}{9}+\sin\left(\pi+\frac{5\pi}{9}\right)+\sin\left(\pi+\frac{4\pi}{9}\right)=$

$=\sin\frac{4\pi}{9}+\sin\frac{5\pi}{9}-\sin\frac{5\pi}{9}-\sin\frac{4\pi}{9}=0$

44.3. $\sin\frac{\pi}{13}+\cos\frac{11\pi}{25}+\cos\frac{14\pi}{25}+\cos\frac{15\pi}{26}=$

$=\sin\frac{\pi}{13}+\cos\frac{11\pi}{25}+\cos\left(\frac{25\pi}{25}-\frac{11\pi}{25}\right)+\cos\left(\frac{13\pi}{26}+\frac{2\pi}{26}\right)=$

$=\sin\frac{\pi}{13}+\cos\frac{11\pi}{25}+\cos\left(\pi-\frac{11\pi}{25}\right)+\cos\left(\frac{\pi}{2}+\frac{\pi}{13}\right)=$

$=\sin\frac{\pi}{13}+\cos\frac{11\pi}{25}-\cos\frac{11\pi}{25}-\sin\frac{\pi}{13}=0$

45.1.

Amplitude		Área
2π	——	$\pi\times 8^2$
x	——	$\frac{64\pi}{3}$

$x=\frac{2\pi\times\frac{64}{3}\pi}{64\pi}=\frac{2\pi}{3}$, logo $A\hat{O}B=\frac{2\pi}{3}$ rad.

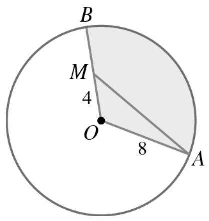
45.2. $\overline{MA}^2=8^2+4^2-2\times8\times4\times\cos\frac{2\pi}{3}=$

$=64+16-2\times32\cos\left(\pi-\frac{\pi}{3}\right)$

$=80+2\times32\cos\frac{\pi}{3}$

$=80+2\times32\times\frac{1}{2}=112$

$\overline{MA}=\sqrt{112}$ cm $=4\sqrt{7}$ cm



Avaliação 2

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1. Se $x\in\left]\pi,\frac{3\pi}{2}\right[$, $\sin x<0$, $\cos x<0$ e $\tan x>0$.

- $\sin x\times\tan(-x)=-\underset{-}{\sin x}\times\underset{+}{\tan x}>0$
- $\cos(-x)+\sin(\pi-x)=\underset{-}{\cos x}+\underset{-}{\sin x}<0$
- $\sin(-x)\times\cos x=-\underset{-}{\sin x}\times\underset{-}{\cos x}<0$
- $\sin\left(\frac{\pi}{2}-x\right)\times\tan x=\underset{-}{\cos x}\times\underset{+}{\tan x}<0$

Resposta: **(A)**

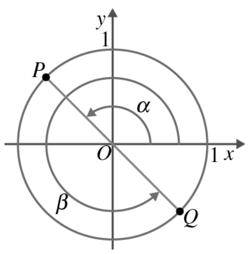
2. $d=\frac{4\pi}{5}$

$a=c=\pi-\frac{4\pi}{5}=\frac{\pi}{5}$

$a+c=\frac{\pi}{5}+\frac{\pi}{5}=\frac{2\pi}{5}$

Resposta: **(A)**

3. $P(x\,,\,y)$ e $Q(-x\,,\,-y)$



P e Q têm coordenadas simétricas

(A) $-\frac{7\pi}{6}=-\frac{6\pi}{6}-\frac{\pi}{6}=-\pi-\frac{\pi}{6}$

P é do 2.º quadrante.

$\frac{5\pi}{6}=\frac{6\pi}{6}-\frac{\pi}{6}=\pi-\frac{\pi}{6}$

Q é do 2.º quadrante.

$[PQ]$ não é um diâmetro da circunferência.

(B) $-\frac{11\pi}{6}=-\frac{12\pi}{6}+\frac{\pi}{6}=-2\pi+\frac{\pi}{6}$

P é do 1.º quadrante

$\frac{19\pi}{6}=\frac{12\pi}{6}+\frac{7\pi}{6}=2\pi+\frac{6\pi}{6}+\frac{\pi}{6}=$

$=2\pi+\pi+\frac{\pi}{6}$

Q é do 3.º quadrante

$\cos\left(-\frac{11\pi}{6}\right)=\cos\left(-2\pi+\frac{\pi}{6}\right)=\cos\frac{\pi}{6}=\frac{\sqrt{3}}{2}$

$\sin\left(-\frac{11\pi}{6}\right)=\sin\left(-2\pi+\frac{\pi}{6}\right)=\sin\frac{\pi}{6}=\frac{1}{2}$

$P\left(\frac{\sqrt{3}}{2}\,,\,\frac{1}{2}\right)$

$\cos\left(\frac{19\pi}{6}\right)=\cos\left(2\pi+\pi+\frac{\pi}{6}\right)=$

$=\cos\left(\pi+\frac{\pi}{6}\right)=-\cos\frac{\pi}{6}=-\frac{\sqrt{3}}{2}$

$\sin\left(\frac{19\pi}{6}\right)=\sin\left(2\pi+\pi+\frac{\pi}{6}\right)=$

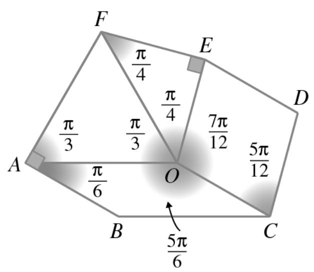
$=\sin\left(\pi+\frac{\pi}{6}\right)=-\sin\frac{\pi}{6}=-\frac{1}{2}$

$Q\left(-\frac{\sqrt{3}}{2}\,,\,-\frac{1}{2}\right)$

Logo, $[PQ]$ é um diâmetro da circunferência.

Resposta: **(B)**

4.



$F\hat{A}O=F\hat{O}A=\frac{\pi}{3}$; $O\hat{F}E=E\hat{O}F=\frac{1}{2}\times\frac{\pi}{2}=\frac{\pi}{4}$

$B\hat{A}O=\frac{\pi}{2}-\frac{\pi}{3}=\frac{\pi}{6}$; $A\hat{O}C=\pi-\frac{\pi}{6}=\frac{5\pi}{6}$

$C\hat{O}E=2\pi-\frac{\pi}{4}-\frac{\pi}{3}-\frac{5\pi}{6}=\frac{7\pi}{12}$; $D\hat{C}O=\pi-\frac{7\pi}{12}=\frac{5\pi}{12}$

Resposta: **(A)**

5. O ponto T tem coordenadas $(1, \tan\theta)$ sendo θ o ângulo cujo lado origem é o semieixo positivo Ox e o lado extremidade é a semirreta $\dot{O}T$.

Logo, $\tan\theta = -\frac{\sqrt{3}}{3}$, pelo que $\theta = -\frac{\pi}{6}$ e o ponto P tem coordenadas $\left(\cos\left(\pi - \frac{\pi}{6}\right), \sin\left(\pi - \frac{\pi}{6}\right)\right)$.

$\cos\left(\pi - \frac{\pi}{6}\right) = -\cos\frac{\pi}{6} = -\frac{\sqrt{3}}{2}$

$\sin\left(\pi - \frac{\pi}{6}\right) = \sin\frac{\pi}{6} = \frac{1}{2}$

Logo, $P\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$.

Resposta: **(D)**

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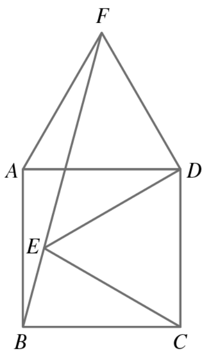
6.



6.1. $A\hat{D}E = A\hat{D}C - E\hat{D}C = \frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{6}$

6.2. $F\hat{D}E = F\hat{D}A + A\hat{D}E = \frac{\pi}{3} + \frac{\pi}{6} = \frac{\pi}{2}$

Logo, o triângulo $[DFE]$ é retângulo em D.
Como $\overline{FD} = \overline{AD}$ e $\overline{ED} = \overline{DC} = \overline{AD}$ vem $\overline{FD} = \overline{AD}$. Assim, o triângulo $[DFE]$ é isósceles.

6.3. $D\hat{E}F = \frac{\pi}{4}$ (o triângulo $[DEF]$ é retângulo em D e isósceles)

$C\hat{E}D = \frac{\pi}{3}$ (o triângulo $[CDE]$ é equilátero)

$E\hat{C}B = D\hat{C}B - D\hat{C}E = \frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{6}$

$C\hat{B}E = B\hat{E}C$ porque o triângulo $[BCE]$ é isósceles, sendo $\overline{EC} = \overline{BC}$.

Logo, $B\hat{E}C = \frac{1}{2}\left(\pi - E\hat{C}B\right) = \frac{1}{2}\left(\pi - \frac{\pi}{6}\right) = \frac{5\pi}{12}$

$B\hat{E}F = D\hat{E}F + C\hat{E}D + B\hat{E}C = \frac{\pi}{4} + \frac{\pi}{3} + \frac{5\pi}{12} = \pi$

Como o ângulo BEF é raso, o ponto E pertence à reta BF.

7. $360° : 20 = 18°$

7.1. Para obter 90 pontos a roda terá de rodar oito setores, para além das voltas inteiras.

$8 \times 18° = 144°$

$144° + 360° = 504°$

$144 + 2 \times 360° = 864°$

A amplitude do arco descrito por P_0 pode ser 504° ou 864°, por exemplo.

7.2.
- $810° = 90° + 2 \times 360$ $\quad \begin{array}{r|l} 810 & 360 \\ \hline 90 & 2 \end{array}$

  $90° : 18° = 5$

  O 1.º jogador obteve 60 pontos (5 setores)
- $\frac{16\pi}{5}$ rad $= \frac{16}{5} \times 180° = 576°$

  $576° = 216° + 360°$

  $216° : 18 = 12$

  O 2.º jogador obteve 25 pontos (12 setores)
- 7π rad $= 7 \times 180° =$

  $= 180° + 3 \times 360°$

  $180 : 18 = 10$

  O 3.º jogador obteve 5 pontos (10 setores)

A melhor pontuação (60 pontos) foi obtida pelo 1.º jogador.

8. $360° : 12 = 30°$

Em cada hora, o ponteiro das horas descreve um arco de 30° de amplitude.

60 min	——	30°
20 min	——	x

$x = \frac{20 \times 30}{60} = 10$

$30° - 10° = 20°$

$20° + 4 \times 30° = 140°$

Graus		Rad
180	——	π
140	——	x

$x = \frac{140\pi}{180} = \frac{7\pi}{9}$

Às 11 h 20 min, os ponteiros formam um ângulo de amplitude $\frac{7\pi}{9}$ rad.

9.1.
$$\sin\left(-\frac{2\pi}{3}\right) - \cos\frac{17\pi}{6} - \tan\left(-\frac{5\pi}{4}\right) =$$
$$= -\sin\frac{2\pi}{3} - \cos\left(\frac{12\pi}{6} + \frac{5\pi}{6}\right) + \tan\left(\frac{5\pi}{4}\right) =$$
$$= -\sin\left(\frac{3\pi}{3} - \frac{\pi}{3}\right) - \cos\left(2\pi + \frac{6\pi}{6} - \frac{\pi}{6}\right) + \tan\left(\frac{4\pi}{4} + \frac{\pi}{4}\right) =$$
$$= -\sin\left(\pi - \frac{\pi}{3}\right) - \cos\left(\pi - \frac{\pi}{6}\right) + \tan\left(\pi + \frac{\pi}{4}\right) =$$
$$= -\sin\frac{\pi}{3} + \cos\frac{\pi}{6} + \tan\frac{\pi}{4} = -\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} + 1 = 1$$

9.2.
$$\sin^2\left(\frac{\pi}{8}\right) + \sin^2\left(\frac{3\pi}{8}\right) - \tan^2\left(\frac{7\pi}{4}\right) =$$
$$= \sin^2\left(\frac{\pi}{8}\right) + \left[\sin\left(\frac{4\pi}{8} - \frac{\pi}{8}\right)\right]^2 - \left[\tan\left(\frac{8\pi}{4} - \frac{\pi}{4}\right)\right]^2 =$$
$$= \sin^2\left(\frac{\pi}{8}\right) + \left(\sin\left(\frac{\pi}{2} - \frac{\pi}{8}\right)\right)^2 - \left[\tan\left(2\pi - \frac{\pi}{4}\right)\right]^2 =$$
$$= \sin^2\left(\frac{\pi}{8}\right) + \cos^2\left(\frac{\pi}{8}\right) - \left[\tan\left(-\frac{\pi}{4}\right)\right]^2 = 1 - (-1)^2 = 0$$

10. $1+\cos(\pi\times x)\times\sin\left(x-\frac{3\pi}{2}\right)=$

$=1-\cos x\times\left[-\sin\left(\frac{3\pi}{2}-x\right)\right]=$

$=1+\cos x\times\sin\left[\pi+\left(\frac{\pi}{2}-x\right)\right]=$

$=1-\cos x\times\sin\left(\frac{\pi}{2}-x\right)=$

$=1-\cos x\times\cos x=$

$=1-\cos^2 x=$

$=\sin^2 x$

11. $\tan^2 x-\sin^2 x=\frac{\sin^2 x}{\cos^2 x}-\sin^2 x=$

$=\sin^2 x\times\left(\frac{1}{\cos^2 x}-1\right)=$

$=\sin^2 x\times\frac{1-\cos^2 x}{\cos^2 x}=$

$=\sin^2 x\times\frac{\sin^2 x}{\cos^2 x}=$

$=\sin^2 x\times\tan^2 x=$

$=\tan^2 x\times\sin^2 x$

12.1. Base do triângulo [*PQR*] =

$=\overline{RQ}=\overline{OA}=1$

Altura do triângulo [*PQR*] =

= ordenada de *Q* – ordenada de *P* =

= tan *x* – sin *x*

Logo, $A_{[PQR]}=\frac{\tan x-\sin x}{2}$.

12.2. Abcissa do ponto *P* = cos *x*

$\cos x=\frac{3}{5}$

$\sin^2 x+\cos^2 x=1$

$\sin^2 x+\left(\frac{3}{5}\right)^2=1\Leftrightarrow$

$\Leftrightarrow\sin^2 x=1-\frac{9}{25}\Leftrightarrow$

$\Leftrightarrow\sin^2 x=\frac{16}{25}$

Como $x\in\left]0\,,\ \frac{\pi}{2}\right[$, $\sin x=\frac{4}{5}$.

$\tan x=\frac{\sin x}{\cos x}=\frac{\frac{4}{5}}{\frac{3}{5}}=\frac{4}{3}$

$A_{[PQR]}=\frac{\frac{4}{3}-\frac{4}{5}}{2}=\frac{\frac{8}{15}}{2}=\frac{4}{15}$

Atividade inicial 3

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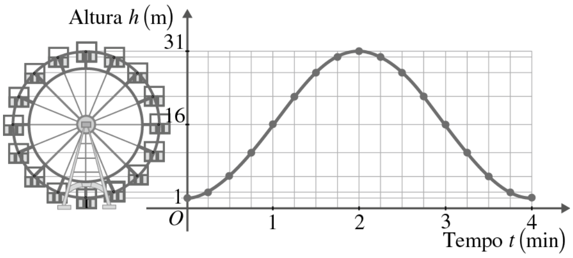
3.1. e **3.2**.



3.3. A altura máxima foi de 31 metros e em cada volta ocorre uma vez.

3.4. A cabine esteve a 16 metros de altura após 1 minuto e após 3 minutos do instante inicial. Em cada volta esta altura ocorre duas vezes.

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1. $f(x) = 3 - 2\sin\left(\frac{x}{2}\right)$; $D_f = \mathbb{R}$

1.1. $x \in \mathbb{R} \Leftrightarrow \frac{x}{2} \in \mathbb{R} \Leftrightarrow$

$\Leftrightarrow -1 \leq \sin\left(\frac{x}{2}\right) \leq 1 \Leftrightarrow$

$\Leftrightarrow 2 \geq -2\sin\left(\frac{x}{2}\right) \geq -2 \Leftrightarrow$

$\Leftrightarrow -2 \leq -2\sin\left(\frac{x}{2}\right) \leq 2 \Leftrightarrow$

$\Leftrightarrow 3-2 \leq 3 - 2\sin\left(\frac{x}{2}\right) \leq 3+2 \Leftrightarrow$

$\Leftrightarrow 1 \leq f(x) \leq 5$, logo $D'_f = [1\ ,\ 5]$.

1.2. $0 \notin D'_f$. Logo, a função *f* não tem zeros.

1.3. Se $x \in D_f$, $x + 4\pi \in D_f$ porque $D_f = \mathbb{R}$

$f(x+4\pi) = 3 - 2\sin\left(\frac{x+4\pi}{2}\right) = 3 - 2\sin\left(\frac{x}{2} + \frac{4\pi}{2}\right) =$

$= 3 - 2\sin\left(\frac{x}{2} + 2\pi\right) = 3 - 2\sin\frac{x}{2} =$

$= f(x)$

$\forall x \in D_f,\ f(x+4\pi) = f(x)$

Portanto, a função *f* é periódica de período 4π.

1.4. $f(a) + f(-a) = 3 - 2\sin\frac{a}{2} + 3 - 2\sin\left(\frac{-a}{2}\right) =$

$= 6 - 2\sin\frac{a}{2} - 2\sin\left(-\frac{a}{2}\right) =$

$= 6 - \sin\frac{a}{2} + 2\sin\frac{a}{2} =$ $\quad (\sin(-x) = -\sin x,\ \forall x \in \mathbb{R})$

$= 6$

2. $g(x) = 1 + 2\sin\left(x - \frac{\pi}{2}\right)$

O gráfico da função g pode ser obtido do gráfico de $y = \sin x$ por uma dilatação vertical de coeficiente 2 seguida de uma translação de vetor $\left(\frac{\pi}{2}\ ,\ 1\right)$.

3.1. $\arcsin\frac{\sqrt{2}}{2} = x \Leftrightarrow \frac{\sqrt{2}}{2} = \sin x \wedge x \in \left[-\frac{\pi}{2}\ ,\ \frac{\pi}{2}\right] \Leftrightarrow$

$\Leftrightarrow x = \frac{\pi}{4}$

$\arcsin\left(-\frac{\sqrt{3}}{2}\right) = y \Leftrightarrow -\frac{\sqrt{3}}{2} = \sin y \wedge y \in \left[-\frac{\pi}{2}\ ,\ \frac{\pi}{2}\right] \Leftrightarrow$

$\Leftrightarrow y = -\frac{\pi}{3}$

$\arcsin\left(\frac{\sqrt{2}}{2}\right) - \arcsin\left(-\frac{\sqrt{3}}{2}\right) = \frac{\pi}{\underset{(3)}{4}} - \left(-\frac{\pi}{\underset{(4)}{3}}\right) = \frac{3\pi + 4\pi}{12} =$

$= \frac{7\pi}{12}$

3.2. $\arcsin\left(\frac{1}{2} + \sin\frac{\pi}{6}\right) = \arcsin\left(\frac{1}{2} + \frac{1}{2}\right) = \arcsin(1) = \frac{\pi}{2}$

3.3. $2\arcsin\left(\sin\frac{\pi}{12}\right) - \frac{1}{2}\arcsin(1) = 2 \times \frac{\pi}{12} - \frac{1}{2} \times \frac{\pi}{2} =$

$= \frac{\pi}{\underset{(2)}{6}} - \frac{\pi}{\underset{(3)}{4}} = \frac{2\pi - 3\pi}{12} = -\frac{\pi}{12}$

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4. $h(x) = 1 + 2\cos\left(\frac{x}{3}\right)$, $D_h = \mathbb{R}$

4.1. $x \in \mathbb{R} \Leftrightarrow \frac{x}{3} \in \mathbb{R} \Leftrightarrow -1 \leq \cos\frac{x}{3} \leq 1 \Leftrightarrow$

$\Leftrightarrow -2 \leq 2\cos\frac{x}{3} \leq 2 \Leftrightarrow$

$\Leftrightarrow 1 - 2 \leq 1 + 2\cos\frac{x}{3} \leq 1 + 2 \Leftrightarrow$

$\Leftrightarrow -1 \leq h(x) \leq 3$

$D'_h = [-1\ ,\ 3]$

4.2. $D_h = \mathbb{R}$

Se $x \in D_h$, então $-x \in D_h$, pois $D_h = \mathbb{R}$.

$h(-x) = 1 + 2\cos\left(\frac{-x}{3}\right) = 1 + 2\cos\left(-\frac{x}{3}\right) =$

$= 1 + 2\cos\left(\frac{x}{3}\right) = h(x)$

$h(-x) = h(x),\ \forall x \in \mathbb{R}$. Logo, h é uma função par.

4.3. Seja P o período positivo mínimo da função h.
Se $x \in D_f$, então $x + P \in D_f$ porque $D_f = \mathbb{R}$.

$\forall x \in \mathbb{R},\ f(x+P) = f(x) \Leftrightarrow$

$\Leftrightarrow \forall x \in \mathbb{R},\ 1 + 2\cos\left(\frac{x+P}{3}\right) = 1 + 2\cos\frac{x}{3} \Leftrightarrow$

$\Leftrightarrow \forall x \in \mathbb{R},\ 1 + 2\cos\left(\frac{x}{3} + \frac{P}{3}\right) = 1 + 2\cos\frac{x}{3} \Leftrightarrow$

$\Leftrightarrow \forall x \in \mathbb{R},\ \cos\left(\frac{x}{3} + \frac{P}{3}\right) = \cos\frac{x}{3} \Leftrightarrow$

$\Leftrightarrow \frac{P}{3} = 2\pi$ porque 2π é o período positivo mínimo da função cosseno. Logo, $P = 6\pi$.

Logo, a função h é uma função periódica de período fundamental igual a 6π.

4.4. O gráfico da função h pode ser obtido do gráfico da função $y=\cos x$ por uma dilatação horizontal de coeficiente 3 seguida de uma dilatação vertical de coeficiente 2 e de uma translação de vetor (0 , 1).

5.1. $\arccos\left(-\frac{1}{2}\right)=x \Leftrightarrow$

$\Leftrightarrow -\frac{1}{2}=\cos x \wedge x\in[0\ ,\ \pi] \Leftrightarrow$

$\Leftrightarrow x=\pi-\frac{\pi}{3} \Leftrightarrow x=\frac{2\pi}{3}$

$\pi-\arccos\left(-\frac{1}{2}\right)=\pi-\frac{2\pi}{3}=\frac{\pi}{3}$

5.2. $\arccos\left(\frac{1}{2}-\sin\frac{\pi}{2}\right)+\arccos\left(-\frac{1}{2}\right)=$

$=\arccos\left(\frac{1}{2}-1\right)+\frac{2\pi}{3}=$

$=\arccos\left(-\frac{1}{2}\right)+\frac{2\pi}{3}=$

$=\frac{2\pi}{3}+\frac{2\pi}{3}=\frac{4\pi}{3}$

5.3. $\arccos\left(\cos\frac{\pi}{12}\right)+\arccos\left(\sin\left(-\frac{\pi}{4}\right)\right)=$

$=\frac{\pi}{12}+\arccos\left(-\sin\frac{\pi}{4}\right)=$

$=\frac{\pi}{12}+\arccos\left(-\frac{\sqrt{2}}{2}\right)=$

$=\frac{\pi}{12}+\frac{3\pi}{4}=\frac{5\pi}{6}$

Cálculo auxiliar:

$\arccos\left(-\frac{\sqrt{2}}{2}\right)=x \Leftrightarrow -\frac{\sqrt{2}}{2}=\cos x \wedge x\in[0\ ,\ \pi] \Leftrightarrow$

$\Leftrightarrow x=\pi-\frac{\pi}{4} \Leftrightarrow x=\frac{3\pi}{4}$

5.4. $\sin\left(\frac{\pi}{2}-\arccos\frac{\sqrt{3}}{2}\right)=\sin\left(\frac{\pi}{2}-\frac{\pi}{6}\right)=\sin\frac{\pi}{3}=\frac{\sqrt{3}}{2}$

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6. $f(x)=a\tan\left(2x-\frac{\pi}{3}\right),\ a\in\mathbb{R}\setminus\{0\}$

6.1. $D_f=\mathbb{R}\setminus\left\{x:\ 2x-\frac{\pi}{3}=\frac{\pi}{2}+k\pi,\ k\in\mathbb{Z}\right\}=$

$=\mathbb{R}\setminus\left\{x:\ x=\frac{5\pi}{12}+\frac{k\pi}{2},\ k\in\mathbb{Z}\right\}$

Cálculo auxiliar:

$2x-\frac{\pi}{3}=\frac{\pi}{2}+k\pi,\ k\in\mathbb{Z} \Leftrightarrow$

$\Leftrightarrow 2x=\frac{\pi}{2}+\frac{\pi}{3}+k\pi,\ k\in\mathbb{Z} \Leftrightarrow$

$\Leftrightarrow 2x=\frac{5\pi}{6}+k\pi,\ k\in\mathbb{Z} \Leftrightarrow$

$\Leftrightarrow x=\frac{5\pi}{12}+\frac{k\pi}{2},\ k\in\mathbb{Z}$

6.2. $f(x)=0 \Leftrightarrow a\tan\left(2x-\frac{\pi}{3}\right)=0 \Leftrightarrow$

$\Leftrightarrow \tan\left(2x-\frac{\pi}{3}\right)=0 \Leftrightarrow$

$\Leftrightarrow 2x-\frac{\pi}{3}=k\pi,\ k\in\mathbb{Z} \Leftrightarrow$

$\Leftrightarrow 2x=\frac{\pi}{3}+k\pi,\ k\in\mathbb{Z} \Leftrightarrow$

$\Leftrightarrow x=\frac{\pi}{6}+\frac{k\pi}{2},\ k\in\mathbb{Z}$

6.3. A função f não é ímpar.

Por exemplo, $f\left(-\frac{\pi}{6}\right)\neq -f\left(\frac{\pi}{6}\right)$ pois $f\left(\frac{\pi}{6}\right)=0$ e

$f\left(-\frac{\pi}{6}\right)=a\tan\left(-\frac{2\pi}{6}-\frac{\pi}{3}\right)=$

$=a\tan\left(-\frac{2\pi}{3}\right)=-a\tan\frac{2\pi}{3}=-a\tan\left(\pi-\frac{\pi}{3}\right)=$

$=-a\tan\left(-\frac{\pi}{3}\right)=a\tan\frac{\pi}{3}=a\sqrt{3}\neq 0$

6.4. Se $x\in D_f$, então $x+\frac{\pi}{2}\in D_f$.

$\forall x\in D_f,\ f\left(x+\frac{\pi}{2}\right)=a\tan\left(2\left(x+\frac{\pi}{2}\right)-\frac{\pi}{3}\right)=$

$=a\tan\left(2x+\pi-\frac{\pi}{3}\right)=a\tan\left[\pi+\left(2x-\frac{\pi}{3}\right)\right]=$

$=a\tan\left(2x-\frac{\pi}{3}\right)=f(x)$

6.5. $f\left(\frac{\pi}{3}\right)-3f\left(\frac{\pi}{12}\right)=6 \Leftrightarrow$

$\Leftrightarrow a\tan\left(\frac{2\pi}{3}-\frac{\pi}{3}\right)-3a\tan\left(2\times\frac{\pi}{12}-\frac{\pi}{3}\right)=6 \Leftrightarrow$

$\Leftrightarrow a\tan\frac{\pi}{3}-3a\tan\left(\frac{\pi}{6}-\frac{\pi}{3}\right)=6 \Leftrightarrow$

$\Leftrightarrow a\times\sqrt{3}-3a\tan\left(-\frac{\pi}{6}\right)=6 \Leftrightarrow$

$\Leftrightarrow \sqrt{3}a+3a\tan\frac{\pi}{6}=6 \Leftrightarrow$

$\Leftrightarrow \sqrt{3}a+3a\times\frac{\sqrt{3}}{3}=6 \Leftrightarrow$

$\Leftrightarrow \sqrt{3}a+\sqrt{3}a=6 \Leftrightarrow$

$\Leftrightarrow 2\sqrt{3}a=6 \Leftrightarrow$

$\Leftrightarrow a=\frac{6}{2\sqrt{3}} \Leftrightarrow$

$\Leftrightarrow a=\frac{6\sqrt{3}}{2\times\sqrt{3}\times\sqrt{3}} \Leftrightarrow$

$\Leftrightarrow a=\sqrt{3}$

7.1. $\cos(\arctan(-1))=\cos\left(-\frac{\pi}{4}\right)=\cos\frac{\pi}{4}=\frac{\sqrt{2}}{2}$

Cálculo auxiliar:

$\arctan(-1)=x \Leftrightarrow -1=\tan x \wedge x\in\left]-\frac{\pi}{2}\ ,\ \frac{\pi}{2}\right[\Leftrightarrow x=-\frac{\pi}{4}$

7.2. $\arctan\left(\sin\frac{\pi}{2}\right)=\arctan(1)=\frac{\pi}{4}$

7.3. $\arctan\left(\cos\frac{\pi}{6}-\frac{5\sqrt{3}}{6}\right)=\arctan\left(\frac{\sqrt{3}}{2}-\frac{5\sqrt{3}}{6}\right)=$

$=\arctan\left(\frac{3\sqrt{3}-5\sqrt{3}}{6}\right)=\arctan\left(-\frac{2\sqrt{3}}{6}\right)=$

$=\arctan\left(-\frac{\sqrt{3}}{3}\right)=-\frac{\pi}{6}$

Cálculo auxiliar:

$\arctan\left(-\frac{\sqrt{3}}{3}\right)=x\Leftrightarrow -\frac{\sqrt{3}}{3}=\tan x\wedge x\in\left]-\frac{\pi}{2},\frac{\pi}{2}\right[\;\Leftrightarrow x=-\frac{\pi}{6}$

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8.1. $2\sin x-1=0\Leftrightarrow \sin x=\frac{1}{2}\Leftrightarrow \sin x=\sin\frac{\pi}{6}\Leftrightarrow$

$\Leftrightarrow x=\frac{\pi}{6}+2k\pi\vee x=\pi-\frac{\pi}{6}+2k\pi,\ k\in\mathbb{Z}\Leftrightarrow$

$\Leftrightarrow x=\frac{\pi}{6}+2k\pi\vee x=\frac{5\pi}{6}+2k\pi,\ k\in\mathbb{Z}$

8.2. $2\sin\left(x-\frac{\pi}{3}\right)-\sqrt{3}=0\Leftrightarrow\sin\left(x-\frac{\pi}{3}\right)=\frac{\sqrt{3}}{2}\Leftrightarrow$

$\Leftrightarrow\sin\left(x-\frac{\pi}{3}\right)=\sin\frac{\pi}{3}\Leftrightarrow$

$\Leftrightarrow x-\frac{\pi}{3}=\frac{\pi}{3}+2k\pi\vee x-\frac{\pi}{3}=\pi-\frac{\pi}{3}+2k\pi,\ k\in\mathbb{Z}\Leftrightarrow$

$\Leftrightarrow x=\frac{2\pi}{3}+2k\pi\vee x=\pi+2k\pi,\ k\in\mathbb{Z}$

8.3. $4\sin\left(2x+\frac{\pi}{4}\right)+\sqrt{8}=0\Leftrightarrow\sin\left(2x+\frac{\pi}{4}\right)=-\frac{2\sqrt{2}}{4}\Leftrightarrow$

$\Leftrightarrow\sin\left(2x+\frac{\pi}{4}\right)=-\frac{\sqrt{2}}{2}\Leftrightarrow\sin\left(2x+\frac{\pi}{4}\right)=\sin\left(-\frac{\pi}{4}\right)\Leftrightarrow$

$\Leftrightarrow 2x+\frac{\pi}{4}=-\frac{\pi}{4}+2k\pi\vee$

$\vee 2x+\frac{\pi}{4}==\pi-\left(-\frac{\pi}{4}\right)+2k\pi,\ k\in\mathbb{Z}\Leftrightarrow$

$\Leftrightarrow 2x=-\frac{\pi}{2}+2k\pi\vee 2x=\pi+2k\pi,\ k\in\mathbb{Z}\Leftrightarrow$

$\Leftrightarrow x=-\frac{\pi}{4}+k\pi\vee x=\frac{\pi}{2}+k\pi,\ k\in\mathbb{Z}$

8.4. $3\sin x+2\sqrt{3}\sin^2 x=0\Leftrightarrow\sin x\left(3+2\sqrt{3}\sin x\right)=0\Leftrightarrow$

$\Leftrightarrow\sin x=0\vee 3+2\sqrt{3}\sin x=0\Leftrightarrow$

$\Leftrightarrow\sin x=0\vee\sin x=-\frac{3}{2\sqrt{3}}\Leftrightarrow$

$\Leftrightarrow\sin x=0\vee\sin x=-\frac{\sqrt{3}}{2}\Leftrightarrow$

$\Leftrightarrow\sin x=0\vee\sin x=\sin\left(-\frac{\pi}{3}\right)\Leftrightarrow$

$\Leftrightarrow x=k\pi\vee x=-\frac{\pi}{3}+2k\pi\vee$

$\vee x=\pi-\left(-\frac{\pi}{3}\right)+2k\pi,\ k\in\mathbb{Z}\Leftrightarrow$

$\Leftrightarrow x=k\pi\vee x=-\frac{\pi}{3}+2k\pi\vee x=\frac{4\pi}{3}+2k\pi,\ k\in\mathbb{Z}$

8.5. $\sin^2\left(x-\frac{\pi}{4}\right)=1\Leftrightarrow\sin\left(x-\frac{\pi}{4}\right)=1\vee\sin\left(x-\frac{\pi}{4}\right)=-1\Leftrightarrow$

$\Leftrightarrow x-\frac{\pi}{4}=\frac{\pi}{2}+2k\pi\vee x-\frac{\pi}{4}=\frac{3\pi}{2}+2k\pi,\ k\in\mathbb{Z}\Leftrightarrow$

$\Leftrightarrow x=\frac{3\pi}{4}+2k\pi\vee x=\frac{7\pi}{4}+2k\pi,\ k\in\mathbb{Z}$

8.6. $2\sin^2 x-1=0\Leftrightarrow\sin^2 x=\frac{1}{2}\Leftrightarrow$

$\Leftrightarrow\sin x=\sqrt{\frac{1}{2}}\vee\sin x=-\sqrt{\frac{1}{2}}\Leftrightarrow$

$\Leftrightarrow\sin x=\frac{\sqrt{2}}{2}\vee\sin x=-\frac{\sqrt{2}}{2}\Leftrightarrow$

$\Leftrightarrow\sin x=\sin\frac{\pi}{4}\vee\sin x=\sin\left(-\frac{\pi}{4}\right)\Leftrightarrow$

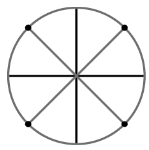
$\Leftrightarrow x=\frac{\pi}{4}+2k\pi\vee x=\pi-\frac{\pi}{4}+2k\pi\vee$

$\vee x=-\frac{\pi}{4}+2k\pi\vee x=\pi+\frac{\pi}{4}+2k\pi\Leftrightarrow$

$\Leftrightarrow x=\frac{\pi}{4}+2k\pi\vee x=\frac{3\pi}{4}+2k\pi\vee x=-\frac{\pi}{4}+2k\pi\vee$

$\vee x=\frac{5\pi}{4}+2k\pi,\ k\in\mathbb{Z}\Leftrightarrow$

$\Leftrightarrow x=\frac{\pi}{4}+\frac{k\pi}{2},\ k\in\mathbb{Z}$



8.7. $2\sin^2 x-\sin x-1=0\Leftrightarrow$

$y=\sin x$

$\Leftrightarrow 2y^2-y-1=0\Leftrightarrow$

$\Leftrightarrow y^2=\frac{1\pm\sqrt{1+8}}{4}\Leftrightarrow\ y=1\vee y=-\frac{1}{2}\Leftrightarrow$

$\Leftrightarrow\sin x=1\vee\sin x=-\frac{1}{2}\Leftrightarrow$

$\Leftrightarrow\sin x=1\vee\sin x=\sin\left(-\frac{\pi}{6}\right)\Leftrightarrow$

$\Leftrightarrow x=\frac{\pi}{2}+2k\pi\vee x=-\frac{\pi}{6}+2k\pi\vee$

$\vee x=\pi-\left(-\frac{\pi}{6}\right)+2k\pi,\ k\in\mathbb{Z}\Leftrightarrow$

$\Leftrightarrow x=\frac{\pi}{2}+2k\pi\vee x=-\frac{\pi}{6}+2k\pi\vee x=\frac{7\pi}{6}+2k\pi,\ k\in\mathbb{Z}$

8.8. $4\sin^2(3x)=8\sin(3x)-3\Leftrightarrow$

$\Leftrightarrow 4\sin^2(3x)-8\sin(3x)+3=0\Leftrightarrow$

$y=\sin(3x)$

$\Leftrightarrow 4y^2-8y+3=0\Leftrightarrow$

$\Leftrightarrow y=\frac{8\pm\sqrt{64-4\times4\times3}}{8}\Leftrightarrow$

$\Leftrightarrow y=\frac{1}{2}\vee y=\frac{3}{2}\Leftrightarrow$

$\Leftrightarrow\sin(3x)=\frac{1}{2}\vee\sin(3x)=\frac{3}{2}\Leftrightarrow$

$\Leftrightarrow\sin(3x)=\sin\frac{\pi}{6}\vee$ condição impossível $\Leftrightarrow$

$\Leftrightarrow 3x=\frac{\pi}{6}+2k\pi\vee 3x=\pi-\frac{\pi}{6}+2k\pi,\ k\in\mathbb{Z}\Leftrightarrow$

$\Leftrightarrow x=\frac{\pi}{18}+\frac{2k\pi}{3}\vee x=\frac{5\pi}{18}+\frac{2k\pi}{3},\ k\in\mathbb{Z}$

8.9. $\sin^3 x - 2\sin^2 x = 0 \Leftrightarrow$
$\Leftrightarrow \sin^2 x(\sin x - 2) = 0 \Leftrightarrow$
$\Leftrightarrow \sin^2 x = 0 \vee \sin x - 2 = 0 \Leftrightarrow$
$\Leftrightarrow \sin x = 0 \vee \sin x = 2 \Leftrightarrow$
$\Leftrightarrow x = k\pi,\ k \in \mathbb{Z} \vee$ condição impossível $\Leftrightarrow$
$\Leftrightarrow x = k\pi,\ k \in \mathbb{Z}$

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9.1. $\sqrt{2}\sin x - 1 = 0 \wedge x \in [0\ ,\ 2\pi[$

$\sqrt{2}\sin x - 1 = 0 \Leftrightarrow$
$\Leftrightarrow \sin x = \frac{1}{\sqrt{2}} \Leftrightarrow \sin x = \frac{\sqrt{2}}{2} \Leftrightarrow$
$\Leftrightarrow \sin x = \sin\frac{\pi}{4} \Leftrightarrow$
$\Leftrightarrow x = \frac{\pi}{4} + 2k\pi \vee x = \pi - \frac{\pi}{4} + 2k\pi,\ k \in \mathbb{Z} \Leftrightarrow$
$\Leftrightarrow x = \frac{\pi}{4} + 2k\pi \vee x = \frac{3\pi}{4} + 2k\pi,\ k \in \mathbb{Z}$

Atribuindo os valores a k, obtemos as soluções do intervalo $[0\ ,\ 2\pi[$:

$k = 0 \Rightarrow x = \frac{\pi}{4} \vee x = \frac{3\pi}{4}$

$S = \left\{\frac{\pi}{4}\ ,\ \frac{3\pi}{4}\right\}$

9.2. $2\sqrt{3}\sin\left(x + \frac{\pi}{4}\right) - 3 = 0 \wedge x \in [-\pi\ ,\ \pi[$

$2\sqrt{3}\sin\left(x + \frac{\pi}{4}\right) - 3 = 0 \Leftrightarrow$
$\Leftrightarrow 2\sqrt{3}\sin\left(x + \frac{\pi}{4}\right) = 3 \Leftrightarrow$
$\Leftrightarrow \sin\left(x + \frac{\pi}{4}\right) = \frac{3}{2\sqrt{3}} \Leftrightarrow$
$\Leftrightarrow \sin\left(x + \frac{\pi}{4}\right) = \frac{\sqrt{3}}{2} \Leftrightarrow$
$\Leftrightarrow \sin\left(x + \frac{\pi}{4}\right) = \sin\frac{\pi}{3} \Leftrightarrow$
$\Leftrightarrow x + \frac{\pi}{4} = \frac{\pi}{3} + 2k\pi \vee x + \frac{\pi}{4} = \pi - \frac{\pi}{3} + 2k\pi,\ k \in \mathbb{Z} \Leftrightarrow$
$\Leftrightarrow x = \frac{\pi}{3} - \frac{\pi}{4} + 2k\pi \vee x = \frac{2\pi}{3} - \frac{\pi}{4} + 2k\pi,\ k \in \mathbb{Z} \Leftrightarrow$
$\Leftrightarrow x = \frac{\pi}{12} + 2k\pi \vee x = \frac{5\pi}{12} + 2k\pi,\ k \in \mathbb{Z}$

Soluções no intervalo $[-\pi\ ,\ \pi[$:

$x = \frac{\pi}{12} + 2k\pi$

$k = -1 \Rightarrow x = \frac{\pi}{12} - 2\pi \Leftrightarrow x = -\frac{23\pi}{12}$ ✗

$k = 0 \Rightarrow x = \frac{\pi}{12}$ ✔

$k = 1 \Rightarrow x = \frac{\pi}{12} + 2\pi$ ✗

$x = \frac{5\pi}{12} + 2k\pi$

$k = -1 \Rightarrow x = \frac{5\pi}{12} - 2\pi \Leftrightarrow x = -\frac{19\pi}{12}$ ✗

$k = 0 \Rightarrow x = \frac{5\pi}{12}$ ✔

$k = 1 \Rightarrow x = \frac{5\pi}{12} + 2\pi$ ✗

$S = \left\{\frac{\pi}{12}\ ,\ \frac{5\pi}{12}\right\}$

9.3. $\sin\left(x + \frac{\pi}{3}\right) - \cos x = 0 \wedge x \in [-\pi\ ,\ \pi[$

$\sin\left(x + \frac{\pi}{3}\right) - \cos x = 0 \Leftrightarrow$
$\Leftrightarrow \sin\left(x + \frac{\pi}{3}\right) = \cos x \Leftrightarrow$
$\Leftrightarrow \sin\left(x + \frac{\pi}{3}\right) = \sin\left(\frac{\pi}{2} - x\right) \Leftrightarrow$
$\Leftrightarrow x + \frac{\pi}{3} = \frac{\pi}{2} - x + 2k\pi \vee$
$\vee x + \frac{\pi}{3} = \pi - \left(\frac{\pi}{2} - x\right) + 2k\pi,\ k \in \mathbb{Z} \Leftrightarrow$
$\Leftrightarrow 2x = \frac{\pi}{6} + 2k\pi \vee x - x = \frac{\pi}{2} - \frac{\pi}{3} + 2k\pi,\ k \in \mathbb{Z} \Leftrightarrow$
$\Leftrightarrow x = \frac{\pi}{12} + k\pi,\ k \in \mathbb{Z} \vee$ condição impossível $\Leftrightarrow$
$\Leftrightarrow x = \frac{\pi}{12} + k\pi,\ k \in \mathbb{Z}$

Soluções no intervalo $[-\pi\ ,\ \pi[$:

$k = -1 \Rightarrow x = \frac{\pi}{12} - \pi \Leftrightarrow x = -\frac{11\pi}{12}$ ✔

$k = -2 \Rightarrow x = \frac{\pi}{12} - 2\pi \Leftrightarrow x = -\frac{23\pi}{12}$ ✗

$k = 0 \Rightarrow x = \frac{\pi}{12}$ ✔

$k = 1 \Rightarrow x = \frac{\pi}{12} + \pi$ ✗

$S = \left\{-\frac{11\pi}{12}\ ,\ \frac{\pi}{12}\right\}$

9.4. $\sin^2(\pi x) - 1 = 0 \wedge x \in [0\ ,\ 2]$

$\sin^2(\pi x) - 1 = 0 \Leftrightarrow \sin^2(\pi x) = 1 \Leftrightarrow$
$\Leftrightarrow \sin(\pi x) = 1 \vee \sin(\pi x) = -1 \Leftrightarrow$
$\Leftrightarrow \pi x = \frac{\pi}{2} + 2k\pi \vee \pi x = \frac{3\pi}{2} + 2k\pi,\ k \in \mathbb{Z} \Leftrightarrow$
$\Leftrightarrow x = \frac{1}{2} + 2k \vee x = \frac{3}{2} + 2k,\ k \in \mathbb{Z}$

As soluções que pertencem ao intervalo [0 , 2] são as que se obtém para $k = 0$, ou seja, $S = \left\{\frac{1}{2}\ ,\ \frac{3}{2}\right\}$.

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10.1. $2\cos x - 1 = 0 \Leftrightarrow \cos x = \frac{1}{2} \Leftrightarrow \cos x = \cos\frac{\pi}{3} \Leftrightarrow$
$\Leftrightarrow x = \frac{\pi}{3} + 2k\pi \vee x = -\frac{\pi}{3} + 2k\pi,\ k \in \mathbb{Z}$

10.2. $2\cos x + 1 = 0 \Leftrightarrow \cos x = -\frac{1}{2} \Leftrightarrow \cos x = -\cos\frac{\pi}{3} \Leftrightarrow$

$\Leftrightarrow \cos x = \cos\left(\pi - \frac{\pi}{3}\right) \Leftrightarrow \cos x = \cos\frac{2\pi}{3} \Leftrightarrow$

$\Leftrightarrow x = \frac{2\pi}{3} + 2k\pi \vee x = -\frac{2\pi}{3} + 2k\pi,\ k \in \mathbb{Z}$

10.3. $2\cos\left(x - \frac{\pi}{3}\right) = \sqrt{2} \Leftrightarrow \cos\left(x - \frac{\pi}{3}\right) = \frac{\sqrt{2}}{2} \Leftrightarrow$

$\Leftrightarrow \cos\left(x - \frac{\pi}{3}\right) = \cos\frac{\pi}{4} \Leftrightarrow$

$\Leftrightarrow x - \frac{\pi}{3} = \frac{\pi}{4} + 2k\pi \vee x - \frac{\pi}{3} = -\frac{\pi}{4} + 2k\pi,\ k \in \mathbb{Z} \Leftrightarrow$

$\Leftrightarrow x = \frac{\pi}{4} + \frac{\pi}{3} + 2k\pi \vee x = -\frac{\pi}{4} + \frac{\pi}{3} + 2k\pi,\ k \in \mathbb{Z} \Leftrightarrow$

$\Leftrightarrow x = \frac{7\pi}{12} + 2k\pi \vee x = \frac{\pi}{12} + 2k\pi,\ k \in \mathbb{Z}$

10.4. $\sqrt{8}\cos\left(2x + \frac{\pi}{4}\right) + \sqrt{6} = 0 \Leftrightarrow \cos\left(2x + \frac{\pi}{4}\right) = -\frac{\sqrt{6}}{\sqrt{8}} \Leftrightarrow$

$\Leftrightarrow \cos\left(2x + \frac{\pi}{4}\right) = -\sqrt{\frac{3}{4}} \Leftrightarrow$

$\Leftrightarrow \cos\left(2x + \frac{\pi}{4}\right) = -\frac{\sqrt{3}}{2} \Leftrightarrow$

$\Leftrightarrow \cos\left(2x + \frac{\pi}{4}\right) = \cos\left(\pi - \frac{\pi}{6}\right) \Leftrightarrow$

$\Leftrightarrow \cos\left(2x + \frac{\pi}{4}\right) = \cos\frac{5\pi}{6} \Leftrightarrow$

$\Leftrightarrow 2x + \frac{\pi}{4} = \frac{5\pi}{6} + 2k\pi \vee 2x + \frac{\pi}{4} = -\frac{5\pi}{6} + 2k\pi,\ k \in \mathbb{Z}$

$\Leftrightarrow 2x = \frac{5\pi}{6} - \frac{\pi}{4} + 2k\pi \vee 2x = -\frac{5\pi}{6} - \frac{\pi}{4} + 2k\pi,\ k \in \mathbb{Z}$

$\Leftrightarrow 2x = \frac{7\pi}{12} + 2k\pi \vee 2x = -\frac{13\pi}{12} + 2k\pi,\ k \in \mathbb{Z}$

$\Leftrightarrow x = \frac{7\pi}{24} + k\pi \vee x = -\frac{13\pi}{24} + k\pi,\ k \in \mathbb{Z}$

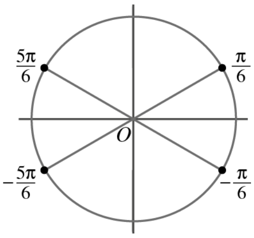
10.5. $\cos^2 x = \frac{3}{4} \Leftrightarrow$

$\Leftrightarrow \cos x = \frac{\sqrt{3}}{2} \vee \cos x = -\frac{\sqrt{3}}{2} \Leftrightarrow$

$\Leftrightarrow \cos x = \cos\frac{\pi}{6} \vee \cos x = \cos\left(\pi - \frac{\pi}{6}\right) \Leftrightarrow$

$\Leftrightarrow x = \frac{\pi}{6} + 2k\pi \vee x = -\frac{\pi}{6} + 2k\pi \vee x = \frac{5\pi}{6} + 2k\pi \vee$

$\vee x = -\frac{5\pi}{6} + 2k\pi \Leftrightarrow$



$\Leftrightarrow x = \frac{\pi}{6} + k\pi \vee x = -\frac{\pi}{6} + k\pi,\ k \in \mathbb{Z}$

10.6. $2\cos^2 x + \cos x - 1 = 0 \Leftrightarrow$ $y = \cos x$

$\Leftrightarrow 2y^2 + y - 1 = 0 \Leftrightarrow$

$\Leftrightarrow y = \frac{-1 \pm \sqrt{1+8}}{4} \Leftrightarrow$

$\Leftrightarrow y = -1 \vee y = \frac{1}{2} \Leftrightarrow$

$\Leftrightarrow \cos x = -1 \vee \cos x = \frac{1}{2} \Leftrightarrow$

$\Leftrightarrow \cos x = -1 \vee \cos x = \cos\frac{\pi}{3} \Leftrightarrow$

$\Leftrightarrow x = \pi + 2k\pi \vee x = \frac{\pi}{3} + 2k\pi \vee x = -\frac{\pi}{3} + 2k\pi,\ k \in \mathbb{Z}$

10.7. $\cos^2\left(x - \frac{\pi}{8}\right) - 1 = 0 \Leftrightarrow$

$\Leftrightarrow \cos^2\left(x - \frac{\pi}{8}\right) = 1 \Leftrightarrow$

$\Leftrightarrow \cos\left(x - \frac{\pi}{8}\right) = -1 \vee \cos\left(x - \frac{\pi}{8}\right) = 1 \Leftrightarrow$

$\Leftrightarrow x - \frac{\pi}{8} = \pi + 2k\pi \vee x - \frac{\pi}{8} = 2k\pi,\ k \in \mathbb{Z} \Leftrightarrow$

$\Leftrightarrow x = \frac{9\pi}{8} + 2k\pi \vee x = \frac{\pi}{8} + 2k\pi,\ k \in \mathbb{Z}$

10.8. $\left(\sqrt{2}\cos x + 1\right)\left(\frac{\sin x}{2} - 1\right) = 0 \Leftrightarrow$

$\Leftrightarrow \sqrt{2}\cos x + 1 = 0 \vee \frac{\sin x}{2} - 1 = 0 \Leftrightarrow$

$\Leftrightarrow \cos x = -\frac{1}{\sqrt{2}} \vee \sin x - 2 = 0 \Leftrightarrow$

$\Leftrightarrow \cos x = -\frac{\sqrt{2}}{2} \vee \sin x = 2 \Leftrightarrow$

$\Leftrightarrow \cos x = \cos\left(\pi - \frac{\pi}{4}\right) \vee$ condição impossível $\Leftrightarrow$

$\Leftrightarrow x = \frac{3\pi}{4} + 2k\pi \vee x = -\frac{3\pi}{4} + 2k\pi,\ k \in \mathbb{Z}$

10.9. $4\sin^2 x + 8\cos x - 7 = 0 \Leftrightarrow$

$\Leftrightarrow 4\left(1 - \cos^2 x\right) + 8\cos x - 7 = 0 \Leftrightarrow$

$\Leftrightarrow 4 - 4\cos^2 x + 8\cos x - 7 = 0 \Leftrightarrow$

$\Leftrightarrow 4\cos^2 x - 8\cos x + 3 = 0 \Leftrightarrow$

$\Leftrightarrow 4y^2 - 8y + 3 = 0 \Leftrightarrow$

$\Leftrightarrow y = \frac{8 \pm \sqrt{64 - 4 \times 4 \times 3}}{8} \Leftrightarrow$

$\Leftrightarrow y = \frac{1}{2} \vee y = \frac{3}{2} \Leftrightarrow$

$\Leftrightarrow \cos x = \frac{1}{2} \vee \cos x = \frac{3}{2}$

$\Leftrightarrow \cos x = \cos\frac{\pi}{3} \vee$ condição impossível $\Leftrightarrow$

$\Leftrightarrow x = \frac{\pi}{3} + 2k\pi \vee x = -\frac{\pi}{3} + 2k\pi,\ k \in \mathbb{Z}$

10.10. $\cos^2(2x) = \sin^2(2x) \Leftrightarrow$

$\Leftrightarrow \cos^2(2x) = 1 - \cos^2(2x) \Leftrightarrow$

$\Leftrightarrow 2\cos^2(2x) = 1 \Leftrightarrow$

$\Leftrightarrow \cos^2(2x) = \frac{1}{2} \Leftrightarrow$

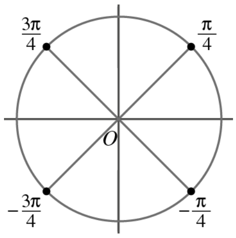
$\Leftrightarrow \cos(2x) = -\sqrt{\frac{1}{2}} \vee \cos(2x) = \sqrt{\frac{1}{2}} \Leftrightarrow$

$\Leftrightarrow \cos(2x) = -\frac{\sqrt{2}}{2} \vee \cos(2x) = \frac{\sqrt{2}}{2} \Leftrightarrow$

$\Leftrightarrow \cos(2x) = \cos\left(\pi - \frac{\pi}{4}\right) \vee \cos(2x) = \cos\frac{\pi}{4} \Leftrightarrow$

$\Leftrightarrow 2x = -\frac{3\pi}{4} + 2k\pi \vee 2x = \frac{3\pi}{4} + 2k\pi,\ k \in \mathbb{Z} \Leftrightarrow$

$\Leftrightarrow 2x = \frac{\pi}{4} + \frac{k\pi}{2},\ k \in \mathbb{Z} \Leftrightarrow$



$\Leftrightarrow x = \frac{\pi}{8} + \frac{k\pi}{4},\ k \in \mathbb{Z}$

11.1. $2\cos x + \sqrt{3} = 0 \wedge x \in \left]-\pi\ ,\ \pi\right]$

$2\cos x + \sqrt{3} = 0 \Leftrightarrow$

$\Leftrightarrow \cos x = -\frac{\sqrt{3}}{2} \Leftrightarrow$

$\Leftrightarrow \cos = \cos\left(\pi - \frac{\pi}{6}\right) \Leftrightarrow$

$\Leftrightarrow \cos x = \cos\frac{5\pi}{6} \Leftrightarrow$

$\Leftrightarrow x = \frac{5\pi}{6} + 2k\pi \vee x = -\frac{5\pi}{6} + 2k\pi,\ k \in \mathbb{Z}$

As únicas soluções que pertencem ao intervalo $\left]-\pi\ ,\ \pi\right]$ são as que se obtêm para $k = 0$, ou seja, $S = \left\{-\frac{5\pi}{6}\ ,\ \frac{5\pi}{6}\right\}$.

11.2. $\cos x - \sqrt{0{,}5} = 0 \wedge x \in \left[0\ ,\ 2\pi\right[$

$\cos x - \sqrt{0{,}5} = 0 \Leftrightarrow \cos x = \sqrt{\frac{1}{2}} \Leftrightarrow$

$\Leftrightarrow \cos x = \frac{\sqrt{2}}{2} \Leftrightarrow \cos x = \cos\frac{\pi}{4} \Leftrightarrow$

$\Leftrightarrow x = -\frac{\pi}{4} + 2k\pi \vee x = \frac{\pi}{4} + 2k\pi,\ k \in \mathbb{Z}$

Atribuindo valores a k, obtemos:

$x = -\frac{\pi}{4} + 2k\pi$

$k = 0 \Rightarrow x = -\frac{\pi}{4}$ ✗

$k = 1 \Rightarrow x = \frac{7\pi}{4}$ ✓

$k = 2 \Rightarrow x = -\frac{\pi}{4} + 4\pi$ ✗

$x = \frac{\pi}{4} + 2k\pi$

$k = 0 \Rightarrow x = \frac{\pi}{4}$ ✓

$k = 1 \Rightarrow x = \frac{\pi}{4} + 2\pi$ ✗

$S = \left\{\frac{\pi}{4}\ ,\ \frac{7\pi}{4}\right\}$

11.3. $8\cos\left(\frac{\pi x}{2}\right) + \sqrt{32} = 0 \wedge x \in [-2\ ,\ 2]$

$8\cos\left(\frac{\pi x}{2}\right) + \sqrt{32} = 0 \Leftrightarrow \cos\left(\frac{\pi x}{2}\right) = -\frac{\sqrt{32}}{8} \Leftrightarrow$

$\Leftrightarrow \cos\left(\frac{\pi x}{2}\right) = -\sqrt{\frac{32}{64}} \Leftrightarrow$

$\Leftrightarrow \cos\left(\frac{\pi x}{2}\right) = -\sqrt{\frac{1}{2}} \Leftrightarrow$

$\Leftrightarrow \cos\left(\frac{\pi x}{2}\right) = -\frac{\sqrt{2}}{2} \Leftrightarrow$

$\Leftrightarrow \cos\left(\frac{\pi x}{2}\right) = \cos\left(\pi - \frac{\pi}{4}\right) \Leftrightarrow$

$\Leftrightarrow \underset{(2)}{\frac{\pi x}{2}} = \frac{3\pi}{4} + \underset{(4)}{2k\pi} \vee \underset{(2)}{\frac{\pi x}{2}} = -\frac{3\pi}{4} + \underset{(4)}{2k\pi},\ k \in \mathbb{Z} \Leftrightarrow$

$\Leftrightarrow 2\pi x = 3\pi + 8k\pi \vee 2\pi x - 3\pi + 8k\pi,\ k \in \mathbb{Z} \Leftrightarrow$

$\Leftrightarrow 2x = 3 + 8k \vee 2x = -3 + 8k,\ k \in \mathbb{Z} \Leftrightarrow$

$\Leftrightarrow x = \frac{3}{2} + 4k \vee x = -\frac{3}{2} + 4k,\ k \in \mathbb{Z}$

Atribuindo valores a k obtemos as soluções que pertencem ao intervalo $[-2\ ,\ 2]$.

$k = 0 \Rightarrow x = \frac{3}{2} \vee x = -\frac{3}{2}$ ✓

$k = -1 \Rightarrow x = \frac{3}{2} - 4 \vee x = -\frac{3}{2} - 4$ ✗

$k = 1 \Rightarrow x = \frac{3}{2} + 4 \vee x = -\frac{3}{2} + 4$ ✗

$S = \left\{-\frac{3}{2}\ ,\ \frac{3}{2}\right\}$

11.4. $\cos x + \sin x = 0 \wedge x \in \left]-\pi\ ,\ \pi\right[$

$\cos x + \sin x = 0 \Leftrightarrow$

$\Leftrightarrow \cos x = -\sin x \Leftrightarrow$

$\Leftrightarrow \cos x = \sin(-x) \Leftrightarrow$

$\Leftrightarrow \cos x = \cos\left(\frac{\pi}{2} - (-x)\right) \Leftrightarrow$

$\Leftrightarrow \cos x = \cos\left(\frac{\pi}{2} + x\right) \Leftrightarrow$

$\Leftrightarrow x = \frac{\pi}{2} + x + 2k\pi \vee x = -\frac{\pi}{2} - x + 2k\pi,\ k \in \mathbb{Z} \Leftrightarrow$

$\Leftrightarrow$ Equação impossível $\vee 2x = -\frac{\pi}{2} + 2k\pi,\ k \in \mathbb{Z} \Leftrightarrow$

$\Leftrightarrow x = -\frac{\pi}{4} + k\pi,\ k \in \mathbb{Z}$

$k = -1 \Rightarrow x = -\frac{\pi}{4} - \pi$ ✗

$k = 0 \Rightarrow x = -\frac{\pi}{4}$ ✓

$k = 1 \Rightarrow x = -\dfrac{\pi}{4} + \pi = \dfrac{3\pi}{4}$ ✔

$k = 2 \Rightarrow x = -\dfrac{\pi}{4} + 2\pi = \dfrac{7\pi}{4}$ ✕

$S = \left\{ -\dfrac{\pi}{4} \, , \, \dfrac{3\pi}{4} \right\}$

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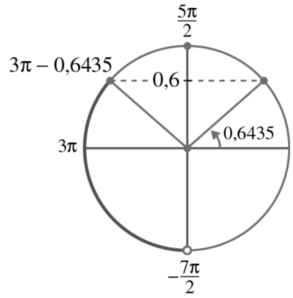
12.1. $\sin x = 0{,}6 \wedge x \in \left] \dfrac{5\pi}{2} \, , \, \dfrac{7\pi}{2} \right[$

Recorrendo à calculadora, obtemos, $\arcsin(0{,}6) \approx 0{,}6435$.



$\sin x = 0{,}6 \wedge x \in \left] \dfrac{5\pi}{2} \, , \, \dfrac{7\pi}{2} \right[\Rightarrow$

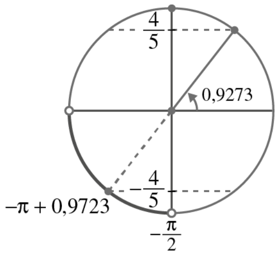
$\Rightarrow x \approx (3\pi - 0{,}6435) \text{ rad}$

$\Rightarrow x \approx 8{,}78 \text{ rad}$

12.2. $5\sin(3x) + 4 = 0 \wedge x \in \left] -\dfrac{\pi}{3} \, , \, -\dfrac{\pi}{6} \right[\Leftrightarrow$

$\Leftrightarrow \sin(3x) = -\dfrac{4}{5} \wedge 3x \in \left] -\pi \, , \, -\dfrac{\pi}{2} \right[$

Recorrendo à calculadora, obtemos $\arcsin\left(\dfrac{4}{5}\right) \approx 0{,}9273$.



$\sin(3x) = -\dfrac{4}{5} \wedge 3x \in \left] -\pi \, , \, -\dfrac{\pi}{2} \right[\Rightarrow$

$\Rightarrow 3x \approx (-\pi + 0{,}9273) \text{ rad} \Rightarrow$

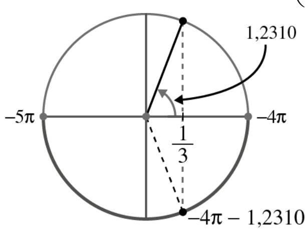
$\Rightarrow 3x \approx -2{,}2143 \text{ rad} \Rightarrow$

$\Rightarrow x \approx -0{,}74 \text{ rad}$

12.3. $3\cos(2x) = 1 \wedge x \in \left[-\dfrac{5\pi}{2} \, , \, -2\pi \right] \Leftrightarrow$

$\Leftrightarrow \cos(2x) = \dfrac{1}{3} \wedge 2x \in [-5\pi \, , \, -4\pi]$

Recorrendo à calculadora obtemos $\arccos\left(\dfrac{1}{3}\right) \approx 1{,}2310$.



$\cos(2x) = \dfrac{1}{3} \wedge 2x \in [-5\pi \, , \, -4\pi] \Rightarrow$

$\Rightarrow 2x \approx (-4\pi - 1{,}2310) \text{ rad} \Rightarrow$

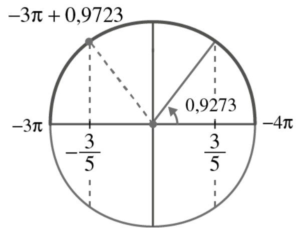
$\Rightarrow 2x \approx -13{,}7974 \text{ rad} \Rightarrow x \approx -6{,}90 \text{ rad}$

12.4. $2\cos\dfrac{x}{2} + \dfrac{6}{5} = 0 \wedge x \in [-8\pi \, , \, -6\pi] \Leftrightarrow$

$\Leftrightarrow 2\cos\dfrac{x}{2} = -\dfrac{6}{5} \wedge \dfrac{x}{2} \in [-4\pi \, , \, -3\pi] \Leftrightarrow$

$\Leftrightarrow \cos\dfrac{x}{2} = -\dfrac{3}{5} \wedge \dfrac{x}{2} \in [-4\pi \, , \, -3\pi]$

Na calculadora obtemos $\arccos\left(\dfrac{3}{5}\right) \approx 0{,}9273$.



$\cos\dfrac{x}{2} = -\dfrac{3}{5} \wedge \dfrac{x}{2} \in [-4\pi \, , \, -3\pi] \Rightarrow$

$\Rightarrow \dfrac{x}{2} \approx (-3\pi - 0{,}9273) \text{ rad} \Rightarrow$

$\Rightarrow \dfrac{x}{2} \approx -10{,}3521 \text{ rad} \Rightarrow x \approx -20{,}70 \text{ rad}$

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13.1. $\tan(\pi x) + 1 = 0 \wedge x \in [0 \, , \, 1]$

$\tan(\pi x) + 1 = 0 \Leftrightarrow \tan(\pi x) = -1 \Leftrightarrow$

$\Leftrightarrow \tan(\pi x) = \tan\left(-\dfrac{\pi}{4}\right) \Leftrightarrow \pi x = -\dfrac{\pi}{4} + k\pi, \; k \in \mathbb{Z} \Leftrightarrow$

$\Leftrightarrow x = -\dfrac{1}{4} + k, \; k \in \mathbb{Z}$

Para $k = 1$ obtemos $x = \dfrac{3}{4}$ que é a única solução do intervalo [0 , 1], ou seja, $S = \left\{ \dfrac{3}{4} \right\}$.

13.2. $6\tan\left(2x - \dfrac{\pi}{6}\right) - \sqrt{12} = 0 \wedge x \in \left] -\dfrac{\pi}{3} \, , \, \dfrac{2\pi}{3} \right[$

$6\tan\left(2x - \dfrac{\pi}{6}\right) - \sqrt{12} = 0 \Leftrightarrow$

$\Leftrightarrow \tan\left(2x - \dfrac{\pi}{6}\right) = \dfrac{\sqrt{12}}{6} \Leftrightarrow$

$\dfrac{\sqrt{12}}{6} = \dfrac{2\sqrt{3}}{6} = \dfrac{\sqrt{3}}{3}$

$\Leftrightarrow \tan\left(2x - \dfrac{\pi}{6}\right) = \dfrac{\sqrt{3}}{3} \Leftrightarrow$

$\Leftrightarrow \tan\left(2x - \dfrac{\pi}{6}\right) = \tan\dfrac{\pi}{6} \Leftrightarrow 2x - \dfrac{\pi}{6} = \dfrac{\pi}{6} + k\pi, \; k \in \mathbb{Z} \Leftrightarrow$

$\Leftrightarrow 2x = 2 \times \dfrac{\pi}{6} + k\pi, \; k \in \mathbb{Z} \Leftrightarrow x = \dfrac{\pi}{6} + \dfrac{k\pi}{2}, \; k \in \mathbb{Z}$

$k = -1 \Rightarrow x = \dfrac{\pi}{6} - \dfrac{\pi}{2} = -\dfrac{\pi}{3}$ ✕

$k = 0 \Rightarrow x = \dfrac{\pi}{6}$ ✔

$k = 1 \Rightarrow x = \dfrac{\pi}{6} + \dfrac{\pi}{2} = \dfrac{2\pi}{3}$ ✕

$S = \left\{ \dfrac{\pi}{6} \right\}$

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14.1. $3\tan\left(2x-\frac{\pi}{6}\right)=0 \Leftrightarrow \tan\left(2x-\frac{\pi}{6}\right)=0 \Leftrightarrow$

$\Leftrightarrow 2x-\frac{\pi}{6}=k\pi,\ k\in\mathbb{Z} \Leftrightarrow$

$\Leftrightarrow 2x=\frac{\pi}{6}+k\pi,\ k\in\mathbb{Z} \Leftrightarrow$

$\Leftrightarrow x=\frac{\pi}{12}+\frac{k\pi}{2},\ k\in\mathbb{Z}$

14.2. $3\tan^2\left(\frac{\pi x}{2}\right)=1 \Leftrightarrow \tan^2\left(\frac{\pi x}{2}\right)=\frac{1}{3} \Leftrightarrow$

$\Leftrightarrow \tan\left(\frac{\pi x}{2}\right)=\pm\sqrt{\frac{1}{3}} \Leftrightarrow$

$\Leftrightarrow \tan\left(\frac{\pi x}{2}\right)=\frac{\sqrt{3}}{3} \vee \tan\left(\frac{\pi x}{2}\right)=-\frac{\sqrt{3}}{3} \Leftrightarrow$

$\Leftrightarrow \tan\left(\frac{\pi x}{2}\right)=\tan\frac{\pi}{6} \vee \tan\left(\frac{\pi x}{2}\right)=\tan\left(-\frac{\pi}{6}\right) \Leftrightarrow$

$\Leftrightarrow \frac{\pi x}{2}=\frac{\pi}{6}+k\pi \vee \frac{\pi x}{2}=-\frac{\pi}{6}+k\pi,\ k\in\mathbb{Z} \Leftrightarrow$

$\Leftrightarrow 3\pi x=\pi+6k\pi \vee 3\pi x=-\pi+6k,\ k\in\mathbb{Z} \Leftrightarrow$

$\Leftrightarrow x=\frac{1}{3}+2k \vee x=-\frac{1}{3}+2k,\ k\in\mathbb{Z}$

14.3. $\tan(2x)=\tan\left(\frac{\pi}{6}-x\right) \Leftrightarrow 2x=\frac{\pi}{6}-x+k\pi,\ k\in\mathbb{Z} \Leftrightarrow$

$\Leftrightarrow 3x=\frac{\pi}{6}+k\pi,\ k\in\mathbb{Z} \Leftrightarrow x=\frac{\pi}{18}+\frac{k\pi}{3},\ k\in\mathbb{Z}$

14.4. $\tan x+\sin x=0 \Leftrightarrow \frac{\sin x}{\cos x}+\sin x=0 \Leftrightarrow$

$\Leftrightarrow \sin x\left(\frac{1}{\cos x}+1\right)=0 \Leftrightarrow$

$\Leftrightarrow \sin x=0 \vee \frac{1}{\cos x}+1=0 \Leftrightarrow$

$\Leftrightarrow \sin x=0 \vee \frac{1}{\cos x}=-1 \Leftrightarrow$

$\Leftrightarrow (\sin x=0 \vee \cos x=-1) \wedge \cos x\neq 0 \Leftrightarrow$

$\Leftrightarrow (x=k\pi \vee x=\pi+2k\pi) \wedge x\neq\frac{\pi}{2}+k\pi,\ k\in\mathbb{Z} \Leftrightarrow$

$\Leftrightarrow x=k\pi,\ k\in\mathbb{Z}$

14.5. $\sin x=\tan^2 x\sin x=0 \Leftrightarrow$

$\Leftrightarrow \sin x-\tan^2 x\sin x=0 \Leftrightarrow$

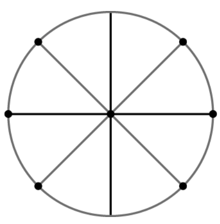
$\Leftrightarrow \sin x\left(1-\tan^2 x\right)=0 \Leftrightarrow$

$\Leftrightarrow \sin x=0 \vee \tan^2 x=1 \Leftrightarrow$

$\Leftrightarrow \sin x=0 \vee \tan x=1 \vee \tan x=-1 \Leftrightarrow$

$\Leftrightarrow \left(x=k\pi \vee x=\frac{\pi}{4}+k\pi \vee x=-\frac{\pi}{4}+k\pi\right) \wedge$

$\wedge x\neq\frac{\pi}{2}+k\pi,\ k\in\mathbb{Z} \Leftrightarrow$



$\Leftrightarrow x=k\pi \vee x=\frac{\pi}{4}+\frac{k\pi}{2},\ k\in\mathbb{Z}$

14.6. $3\tan^2 x+2\sqrt{3}\tan x=3$

$\Leftrightarrow 3y^2+2\sqrt{3}y-3=0 \Leftrightarrow$ $\quad y=\tan x$

$\Leftrightarrow y=\frac{-2\sqrt{3}\pm\sqrt{12+36}}{6} \Leftrightarrow y=\frac{-2\sqrt{3}\pm\sqrt{48}}{6} \Leftrightarrow$

$\Leftrightarrow y=\frac{-2\sqrt{3}\pm 4\sqrt{3}}{6} \Leftrightarrow y=\frac{-6\sqrt{3}}{6} \vee y=\frac{\sqrt{3}}{3} \Leftrightarrow$

$\Leftrightarrow y=-\sqrt{3} \vee y=\frac{\sqrt{3}}{3} \Leftrightarrow$

$\Leftrightarrow \tan x=-\sqrt{3} \vee \tan x=\frac{\sqrt{3}}{3} \Leftrightarrow$

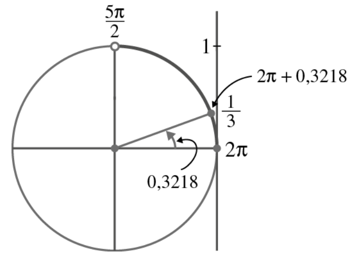
$\Leftrightarrow \tan x=\tan\left(-\frac{\pi}{3}\right) \vee \tan x=\tan\frac{\pi}{6} \Leftrightarrow$

$\Leftrightarrow x=-\frac{\pi}{3}+k\pi \vee x=\frac{\pi}{6}+k\pi,\ k\in\mathbb{Z}$

15.1. $3\tan(2x)=1 \wedge x\in\left[\pi\ ,\ \frac{5\pi}{4}\right[\Leftrightarrow$

$\Leftrightarrow \tan(2x)=\frac{1}{3} \wedge 2x\in\left[2\pi\ ,\ \frac{5\pi}{2}\right[$

Na calculadora obtém-se $\arctan\left(\frac{1}{3}\right)\approx 0{,}3218$.



$\tan(2x)=\frac{1}{3} \wedge 2x\in\left[2\pi\ ,\ \frac{5\pi}{2}\right[\Rightarrow$

$\Rightarrow 2x\approx 2\pi+0{,}3218\ \text{rad} \Rightarrow$

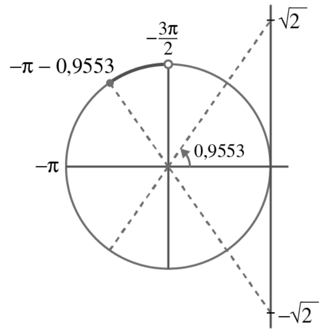
$\Rightarrow 2x\approx 6{,}6050\ \text{rad} \Rightarrow$

$\Rightarrow x\approx 3{,}30\ \text{rad}$

15.2. $\tan\frac{x}{2}+\sqrt{2}=0 \wedge x\in\left]-3\pi\ ,\ -2\pi\right] \Leftrightarrow$

$\Leftrightarrow \tan\frac{x}{2}=-\sqrt{2} \wedge \frac{x}{2}\in\left]-\frac{3\pi}{2}\ ,\ -\pi\right]$

Na calculadora obtemos $\arctan\left(\sqrt{2}\right)\approx 0{,}9553$.



$\tan\frac{x}{2}=-\sqrt{2} \wedge \frac{x}{2}\in\left]-\frac{3\pi}{2}\ ,\ -\pi\right[\Rightarrow$

$\Rightarrow \frac{x}{2}\approx(-\pi-0{,}9553)\ \text{rad} \Rightarrow \frac{x}{2}\approx -4{,}0969\ \text{rad} \Rightarrow$

$\Rightarrow x\approx -8{,}19\ \text{rad}$

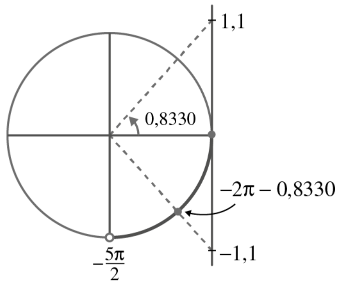
15.3. $\tan^2 x = 1{,}21 \wedge x \in \left]-\frac{5\pi}{2}\ ,\ -2\pi\right] \Leftrightarrow$

$x \in 4.^{\text{o}} Q$
$\tan x < 0$

$\Leftrightarrow \tan x = -\sqrt{1{,}21} \wedge x \in \left]-\frac{5\pi}{2}\ ,\ -2\pi\right] \Leftrightarrow$

$\Leftrightarrow \tan x = -1{,}1 \wedge x \in \left]-\frac{5\pi}{2}\ ,\ -2\pi\right]$

Na calculadora obtém-se $\arctan(1{,}1) \approx 0{,}8330$.



$\tan x = -1{,}1 \wedge x \in \left]-\frac{5\pi}{2}\ ,\ -2\pi\right] \Rightarrow$

$\Rightarrow x \approx (-2\pi - 0{,}8330) \text{ rad } \Rightarrow x \approx -7{,}1 \text{ rad}$

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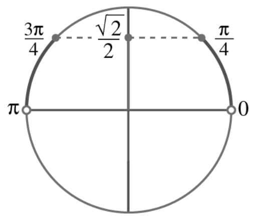
16.1. $2\sin x - \sqrt{2} \le 0 \wedge x \in [0\ ,\ \pi] \Leftrightarrow$

$\Leftrightarrow \sin x \le \frac{\sqrt{2}}{2} \wedge x \in [0\ ,\ \pi] \Leftrightarrow$

$\sin x = \frac{\sqrt{2}}{2} \wedge x \in [0\ ,\ \pi] \Leftrightarrow$

$\Leftrightarrow x = \frac{\pi}{4} \vee x = \pi - \frac{\pi}{4} \Leftrightarrow$

$\Leftrightarrow x = \frac{\pi}{4} \vee x = \frac{3\pi}{4}$



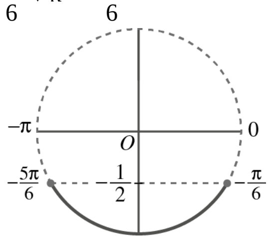
$\sin x \le \frac{\sqrt{2}}{2} \wedge x \in [0\ ,\ \pi] \Leftrightarrow x \in \left[0\ ,\ \frac{\pi}{4}\right] \cup \left[\frac{3\pi}{4}\ ,\ \pi\right]$

16.2. $2\sin x + 1 \le 0 \wedge x \in [-\pi\ ,\ \pi] \Leftrightarrow$

$\Leftrightarrow \sin x \le -\frac{1}{2} \wedge x \in [-\pi\ ,\ \pi]$

$\sin x = -\frac{1}{2} \wedge x \in [-\pi\ ,\ \pi\ ] \Leftrightarrow x = -\pi + \frac{\pi}{6} \vee x = -\frac{\pi}{6} \Leftrightarrow$

$\Leftrightarrow x = -\frac{5\pi}{6} \vee x = -\frac{\pi}{6}$



$\sin x \le -\frac{1}{2} \wedge x \in [-\pi\ ,\ \pi] \Leftrightarrow x \in \left[-\frac{5\pi}{6}\ ,\ -\frac{\pi}{6}\right]$

16.3. $4\sin x + \sqrt{12} > 0 \wedge x \in [0\ ,\ 2\pi[\Leftrightarrow$

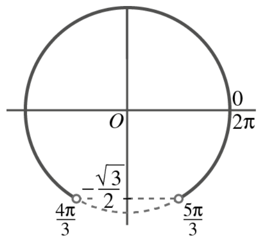
$\Leftrightarrow \sin x > -\frac{\sqrt{12}}{4} \wedge x \in [0\ ,\ 2\pi[\Leftrightarrow$

$\sqrt{12} = 2\sqrt{3}$

$\Leftrightarrow \sin x > -\frac{\sqrt{3}}{2} \wedge x \in [0\ ,\ 2\pi[$

$\sin x = -\frac{\sqrt{3}}{2} \wedge x \in [0\ ,\ 2\pi[\Leftrightarrow x = \pi + \frac{\pi}{3} \vee x = 2\pi - \frac{\pi}{3} \Leftrightarrow$

$\Leftrightarrow x = \frac{4\pi}{3} \vee x = \frac{5\pi}{3}$



$\sin x > -\frac{\sqrt{3}}{2} \wedge x \in [0\ ,\ 2\pi[\Leftrightarrow$

$\Leftrightarrow x \in \left[0\ ,\ \frac{4\pi}{3}\right[\cup \left]\frac{5\pi}{3}\ ,\ 2\pi\right[$

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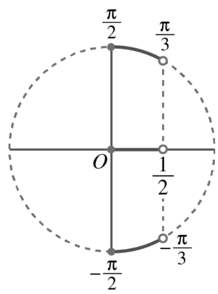
17.1. $2\cos x < 1 \wedge x \in \left[-\frac{\pi}{2}\ ,\ \frac{\pi}{2}\right] \Leftrightarrow \cos x < \frac{1}{2} \wedge x \in \left[-\frac{\pi}{2}\ ,\ \frac{\pi}{2}\right]$

$\cos x = \frac{1}{2} \wedge x \in \left[-\frac{\pi}{2}\ ,\ \frac{\pi}{2}\right] \Leftrightarrow x = -\frac{\pi}{3} \vee x = \frac{\pi}{3}$



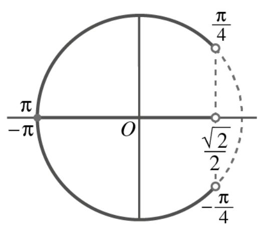
$\cos x < \frac{1}{2} \wedge x \in \left[-\frac{\pi}{2}\ ,\ \frac{\pi}{2}\right] \Leftrightarrow x \in \left[-\frac{\pi}{2}\ ,\ -\frac{\pi}{3}\right[\cup \left]\frac{\pi}{3}\ ,\ \frac{\pi}{2}\right]$

17.2. $\sqrt{2} - 2\cos x > 0 \wedge x \in [-\pi\ ,\ \pi] \Leftrightarrow$

$\Leftrightarrow 2\cos x < \sqrt{2} \wedge x \in [-\pi\ ,\ \pi] \Leftrightarrow$

$\Leftrightarrow \cos x < \frac{\sqrt{2}}{2} \wedge x \in [-\pi\ ,\ \pi]$

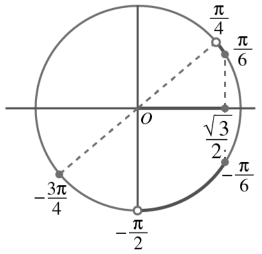
$\cos x = \frac{\sqrt{2}}{2} \wedge x \in [-\pi\ ,\ \pi] \Leftrightarrow x = -\frac{\pi}{4} \vee x = \frac{\pi}{4}$



$\cos x < \frac{\sqrt{2}}{2} \wedge x \in [-\pi\ ,\ \pi] \Leftrightarrow x \in \left[-\pi\ ,\ -\frac{\pi}{4}\right[\cup \left]\frac{\pi}{4}\ ,\ \pi\right]$

17.3. $0 \le \cos x \le \frac{\sqrt{3}}{2} \wedge x \in \left[-\frac{3\pi}{4}, \frac{\pi}{4}\right[$

$\cos x = \frac{\sqrt{3}}{2} \wedge x \in \left[-\frac{3\pi}{4}, \frac{\pi}{4}\right[\Leftrightarrow x = -\frac{\pi}{6} \vee x = \frac{\pi}{6}$

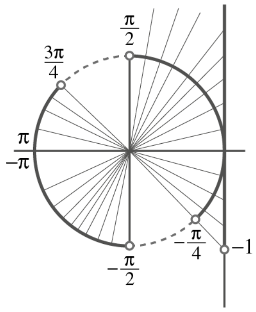


$0 \le \cos x \le \frac{\sqrt{3}}{2} \wedge x \in \left[-\frac{3\pi}{4}, \frac{\pi}{4}\right[\Leftrightarrow$

$\Leftrightarrow x \in \left[-\frac{\pi}{2}, -\frac{\pi}{6}\right] \cup \left[\frac{\pi}{6}, \frac{\pi}{4}\right[$

17.4. $\tan x > -1 \wedge x \in \left]-\pi, \pi\right]$

$\tan x = -1 \wedge x \in \left]-\pi, \pi\right] \Leftrightarrow x = -\frac{\pi}{4} \vee x = \frac{3\pi}{4}$



$\tan x > -1 \wedge x \in \left]-\pi, \pi\right] \Leftrightarrow$

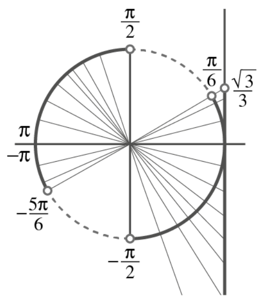
$\Leftrightarrow x \in \left]-\pi, -\frac{\pi}{2}\right[\cup \left]-\frac{\pi}{4}, \frac{\pi}{2}\right[\cup \left]\frac{3\pi}{4}, \pi\right]$

17.5. $3\tan(2x) < \sqrt{3} \wedge x \in \left]-\frac{\pi}{2}, \frac{\pi}{2}\right] \Leftrightarrow$

$\Leftrightarrow \tan(2x) < \frac{\sqrt{3}}{3} \wedge 2x \in \left]-\pi, \pi\right]$

$\tan(2x) = \frac{\sqrt{3}}{3} \wedge 2x \in \left]-\pi, \pi\right] \Leftrightarrow$

$\Leftrightarrow 2x = \frac{\pi}{6} \vee 2x = -\pi + \frac{\pi}{6} \Leftrightarrow 2x = \frac{\pi}{6} \vee 2x = -\frac{5\pi}{6}$



$\tan(2x) < \frac{\sqrt{3}}{3} \wedge 2x \in \left]-\pi, \pi\right] \Leftrightarrow$

$\Leftrightarrow 2x \in \left]-\pi, -\frac{5\pi}{6}\right[\cup \left]-\frac{\pi}{2}, \frac{\pi}{6}\right[\cup \left]\frac{\pi}{2}, \pi\right] \Leftrightarrow$

$\Leftrightarrow x \in \left]-\frac{\pi}{2}, -\frac{5\pi}{12}\right[\cup \left]-\frac{\pi}{4}, \frac{\pi}{12}\right[\cup \left]\frac{\pi}{4}, \frac{\pi}{2}\right]$

17.6. $\tan^2 x < 3 \wedge x \in \left[0, 2\pi\right[$

$y^2 < 3 \Leftrightarrow y^2 - 3 < 0 \Leftrightarrow -\sqrt{3} < y < \sqrt{3}$ $\quad \tan x = y$

Logo, $\tan^2 x < 3 \wedge x \in \left[0, 2\pi\right[\Leftrightarrow$

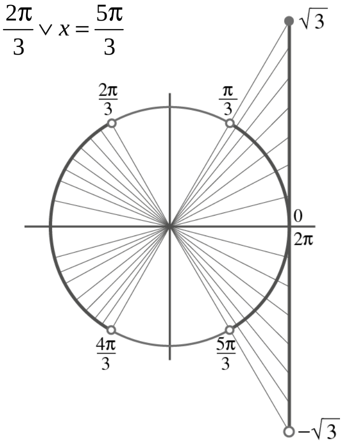
$\Leftrightarrow -\sqrt{3} < \tan x < \sqrt{3} \wedge x \in \left[0, 2\pi\right[$

No intervalo $\left[0, 2\pi\right[$:

$\tan x = \sqrt{3} \Leftrightarrow x = \frac{\pi}{3} \vee x = \pi + \frac{\pi}{3} \Leftrightarrow x = \frac{\pi}{3} \vee x = \frac{4\pi}{3}$

$\tan x = -\sqrt{3} \Leftrightarrow x = \pi - \frac{\pi}{3} \vee x = 2\pi - \frac{\pi}{3} \Leftrightarrow$

$\Leftrightarrow x = \frac{2\pi}{3} \vee x = \frac{5\pi}{3}$



$-\sqrt{3} < \tan x < \sqrt{3} \wedge x \in \left[0, 2\pi\right[\Leftrightarrow$

$\Leftrightarrow x \in \left[0, \frac{\pi}{3}\right[\cup \left]\frac{2\pi}{3}, \frac{4\pi}{3}\right[\cup \left]\frac{5\pi}{3}, 2\pi\right[$

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18. $f(x) = 1 - \sin\frac{x}{2}$; $g(x) = 3 - 2\cos^2\frac{x}{2}$; $D_f = D_g = \left[0, 4\pi\right]$

18.1. $f(x) = g(x) \Leftrightarrow 1 - \sin\left(\frac{x}{2}\right) = 3 - 2\cos^2\left(\frac{x}{2}\right) \Leftrightarrow$

$\Leftrightarrow 2\cos^2\frac{x}{2} - \sin\frac{x}{2} - 2 = 0 \Leftrightarrow$

$\Leftrightarrow 2\left(1 - \sin^2\frac{x}{2}\right) - \sin\frac{x}{2} - 2 = 0 \Leftrightarrow$

$\Leftrightarrow 2 - 2\sin^2\frac{x}{2} - \sin\frac{x}{2} - 2 = 0 \Leftrightarrow$

$\Leftrightarrow 2\sin^2\frac{x}{2} + \sin\frac{x}{2} = 0 \Leftrightarrow \sin\frac{x}{2}\left(2\sin\frac{x}{2} + 1\right) = 0 \Leftrightarrow$

$\Leftrightarrow \sin\frac{x}{2} = 0 \vee 2\sin\frac{x}{2} + 1 = 0 \Leftrightarrow$

$\Leftrightarrow \sin\frac{x}{2} = 0 \vee \sin\frac{x}{2} = -\frac{1}{2} \Leftrightarrow$

$\Leftrightarrow \sin\frac{x}{2} = 0 \vee \sin\frac{x}{2} = \sin\left(-\frac{\pi}{6}\right) \Leftrightarrow$

$\Leftrightarrow \frac{x}{2} = k\pi \vee \frac{x}{2} = -\frac{\pi}{6} + 2k\pi \vee \frac{x}{2} = \frac{7\pi}{6} + 2k\pi \Leftrightarrow$

$\Leftrightarrow x = 2k\pi \vee x = -\frac{\pi}{3} + 4k\pi \vee x = \frac{7\pi}{3} + 4k\pi,\ k \in \mathbb{Z}$

Atribuindo valores a k obtemos as soluções do intervalo $\left[0, 4\pi\right]$:

$$x=0 \vee x=2\pi \vee x=4\pi \vee x=-\frac{\pi}{3}+4\pi \vee x=\frac{7\pi}{3} \Leftrightarrow$$

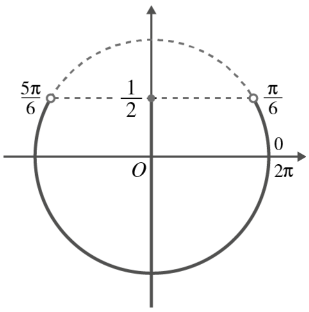
$$\Leftrightarrow x=0 \vee x=2\pi \vee x=\frac{7\pi}{3} \vee x=\frac{11\pi}{3} \vee x=4\pi$$

18.2. $f(x)>\frac{1}{2} \Leftrightarrow 1-\sin\frac{x}{2}>\frac{1}{2} \wedge x\in[0\ ,\ 4\pi] \Leftrightarrow$

$$\Leftrightarrow \sin\frac{x}{2}<1-\frac{1}{2} \wedge \frac{x}{2}\in[0\ ,\ 2\pi] \Leftrightarrow$$

$$\Leftrightarrow \sin\frac{x}{2}<\frac{1}{2} \wedge \frac{x}{2}\in[0\ ,\ 2\pi]$$

$$\sin\frac{x}{2}=\frac{1}{2} \wedge x\in[0\ ,\ 2\pi] \Leftrightarrow \frac{x}{2}=\frac{\pi}{6} \vee \frac{x}{2}=\frac{5\pi}{6}$$



$$\sin\frac{x}{2}<\frac{1}{2} \wedge \frac{x}{2}\in[0\ ,\ 2\pi] \Leftrightarrow \frac{x}{2}\in\left[0\ ,\ \frac{\pi}{6}\right[\cup \left]\frac{5\pi}{6}\ ,\ 2\pi\right]$$

$$\Leftrightarrow x\in\left[0\ ,\ \frac{\pi}{3}\right[\cup \left]\frac{5\pi}{3}\ ,\ 4\pi\right]$$

18.3. $|g(x)-f(x)|=1 \Leftrightarrow \left|3-2\cos^2\frac{x}{2}-1+\sin\frac{x}{2}\right|=1 \Leftrightarrow$

$$\Leftrightarrow \left|2-2\left(1-\sin^2\frac{x}{2}\right)+\sin\frac{x}{2}\right|=1 \Leftrightarrow$$

$$\Leftrightarrow \left|\cancel{2}-\cancel{2}+2\sin^2\frac{x}{2}+\sin\frac{x}{2}\right|=1 \Leftrightarrow$$

$$\Leftrightarrow 2\sin^2\frac{x}{2}+\sin\frac{x}{2}=1 \vee 2\sin^2\frac{x}{2}+\sin\frac{x}{2}=-1 \Leftrightarrow$$

$$\Leftrightarrow 2\sin^2\frac{x}{2}+\sin\frac{x}{2}-1=0 \vee 2\sin^2\frac{x}{2}+\sin\frac{x}{2}+1=0 \Leftrightarrow$$

$$\Leftrightarrow 2y^2+y-1=0 \vee 2y^2+y+1=0 \Leftrightarrow \qquad \left(y=\sin\frac{x}{2}\right)$$

$$\Leftrightarrow y=\frac{-1\pm\sqrt{1+8}}{4} \vee y=\frac{-1\pm\sqrt{1-8}}{4} \Leftrightarrow$$

$$\Leftrightarrow y=-1 \vee y=\frac{1}{2} \vee \text{ condição impossível} \Leftrightarrow$$

$$\Leftrightarrow \sin\frac{x}{2}=-1 \vee \sin\frac{x}{2}=\frac{1}{2} \Leftrightarrow \sin\frac{x}{2}=-1 \vee \sin\frac{x}{2}=\sin\left(\frac{\pi}{6}\right)$$

$$\Leftrightarrow \frac{x}{2}=\frac{3\pi}{2}+2k\pi \vee \frac{x}{2}=\frac{\pi}{6}+2k\pi \vee$$

$$\vee \frac{x}{2}=\pi-\frac{\pi}{6}+2k\pi,\ k\in\mathbb{Z} \Leftrightarrow$$

$$\Leftrightarrow x=3\pi+4k\pi \vee x=\frac{\pi}{3}+4k\pi \vee x=\frac{5\pi}{3}+4k\pi,\ k\in\mathbb{Z}$$

No intervalo $[0\ ,\ 4\pi]$, temos:

$$x=3\pi \vee x=\frac{\pi}{3} \vee x=\frac{5\pi}{3} \Leftrightarrow x=\frac{\pi}{3} \vee x=\frac{5\pi}{3} \vee x=3\pi$$

$$\begin{cases} f\left(\frac{\pi}{3}\right)=1-\sin\frac{\pi}{6}=1-\frac{1}{2}=\frac{1}{2} & ;\ A\left(\frac{\pi}{3}\ ,\ \frac{1}{2}\right) \\ g\left(\frac{\pi}{3}\right)=3-2\cos^2\frac{\pi}{6}=3-2\times\left(\frac{\sqrt{3}}{2}\right)^2=\frac{3}{2} & ;\ B\left(\frac{\pi}{3}\ ,\ \frac{3}{2}\right) \end{cases}$$

$$\begin{cases} f\left(\frac{5\pi}{3}\right)=1-\sin\frac{5\pi}{6}=1-\frac{1}{2}=\frac{1}{2} & ;\ A\left(\frac{5\pi}{3}\ ,\ \frac{1}{2}\right) \\ g\left(\frac{5\pi}{3}\right)=3-2\cos^2\frac{5\pi}{6}=3-2\times\left(-\frac{\sqrt{3}}{2}\right)^2=\frac{3}{2}; & B\left(\frac{5\pi}{3}\ ,\ \frac{3}{2}\right) \end{cases}$$

$$\begin{cases} f(3\pi)=1-\sin\frac{3\pi}{2}=1+1=2 & ;\ A(3\pi\ ,\ 2) \\ g(3\pi)=3-2\cos^2\frac{3\pi}{2}=3-0=3 & ;\ B(3\pi\ ,\ 3) \end{cases}$$

$A\left(\frac{\pi}{3}\ ,\ \frac{1}{2}\right)$ e $B\left(\frac{\pi}{3}\ ,\ \frac{3}{2}\right)$; $A\left(\frac{5\pi}{3}\ ,\ \frac{1}{2}\right)$ e $B\left(\frac{5\pi}{3}\ ,\ \frac{3}{2}\right)$ ou $A(3\pi\ ,\ 2)$ e $B(3\pi\ ,\ 3)$

Atividades complementares

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19. $f(x)=\frac{1-\sin(4x)}{2}$, $D_f=\mathbb{R}$

19.1. $x\in\mathbb{R} \Leftrightarrow 4x\in\mathbb{R} \Leftrightarrow -1\leq\sin(4x)\leq 1 \Leftrightarrow$

$$\Leftrightarrow 1-1\leq 1-\sin(4x)\leq 1+1 \Leftrightarrow$$

$$\Leftrightarrow \frac{0}{2}\leq\frac{1-\sin(4x)}{2}\leq\frac{2}{2} \Leftrightarrow 0\leq f(x)\leq 1$$

$D'_f=[0\ ,\ 1]$

19.2. Se $x\in D_f$ então $x+\frac{\pi}{2}\in D_f$ porque $D_f=\mathbb{R}$

$$f\left(x+\frac{\pi}{2}\right)=\frac{1-\sin\left[4\left(x+\frac{\pi}{2}\right)\right]}{2}=$$

$$=\frac{1-\sin(4x+2\pi)}{2}=$$

$$=\frac{1-\sin(4x)}{2}=$$

$$=f(x)$$

$$f\left(x+\frac{\pi}{2}\right)=f(x),\ \forall x\in\mathbb{R}$$

19.3. $f(\pi+\alpha)\times f(\pi-\alpha)=\frac{4}{25} \Leftrightarrow$

$$\Leftrightarrow \frac{1-\sin[4(\pi+\alpha)]}{2}\times\frac{1-\sin[4(\pi-\alpha)]}{2}=\frac{4}{25} \Leftrightarrow$$

$$\Leftrightarrow \frac{1-\sin(4\pi+4\alpha)}{2}\times\frac{1-\sin(4\pi-4\alpha)}{2}=\frac{4}{25} \Leftrightarrow$$

$$\Leftrightarrow \frac{1-\sin 4\alpha}{2}\times\frac{1-\sin(-4\alpha)}{2}=\frac{4}{25} \Leftrightarrow$$

$$\Leftrightarrow \frac{(1-\sin(4\alpha))+(1+\sin(4\alpha))}{4}=\frac{4}{25} \Leftrightarrow$$

$$\Leftrightarrow 1-\sin^2(4\alpha)=\frac{16}{25} \Leftrightarrow \sin^2(4\alpha)=1-\frac{16}{25} \Leftrightarrow$$

$$\Leftrightarrow \sin(4\alpha)=\pm\sqrt{\frac{9}{25}}$$

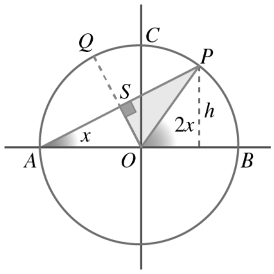
Se $\alpha \in \left]\frac{\pi}{4}, \frac{\pi}{2}\right[$, $4\alpha \in \left]\pi, 2\pi\right[$.

Logo, $\sin(4\alpha) = -\frac{3}{5}$.

$$f(\alpha) = \frac{1-\sin(4\alpha)}{2} = \frac{1-\left(-\frac{3}{5}\right)}{2} = \frac{1+\frac{3}{5}}{2} = \frac{\frac{8}{5}}{2} = \frac{4}{5}$$

20.1. Se $B\hat{A}P = x$, então $B\hat{O}P = 2x$.

Tomando $[OA]$ para base do triângulo $[AOP]$ a altura h é dada por $\sin(2x)$.



A área do triângulo $[AOP]$ é dada por:

$$\frac{\overline{AO}\times h}{2} = \frac{1\times\sin(2x)}{2} = \frac{\sin(2x)}{2}$$

Como $\overline{OA} = \overline{OP}$, $[OS]$ é a mediana do triângulo $[AOP]$ relativa ao vértice O.

Então, $A_{[OPS]} = \frac{1}{2}A_{[AOP]} = \frac{1}{2}\times\frac{\sin(2x)}{2} = \frac{1}{4}\sin(2x)$.

Portanto, $g(x) = \frac{1}{4}\sin(2x)$.

20.2. Se $x \in \left]0, \frac{\pi}{2}\right[$, então $2x \in \left]0, \pi\right[$. Logo, $0 < \sin(2x) \leq 1$ e $0 < \frac{\sin(2x)}{4} \leq \frac{1}{4}$. Assim, $D'_g = \left]0, \frac{1}{4}\right]$ e a área máxima do triângulo $[OPS]$ é $\frac{1}{4}$.

20.3. $P(\cos(2x), \sin(2x))$

$$\cos(2x) = \frac{5}{13}$$

$$\cos^2(2x) + \sin^2(2x) = 1$$

$$\left(\frac{5}{13}\right)^2 + \sin^2(2x) = 1 \Leftrightarrow \sin^2(2x) = 1 - \frac{25}{169} \Leftrightarrow$$

$$\Leftrightarrow \sin^2(2x) = \frac{144}{169}$$

Como $2x \in \left]0, \pi\right[$, temos $\sin(2x) = \frac{12}{13}$.

Se $\sin(2x) = \frac{12}{13}$, $g(x) = \frac{1}{4}\times\frac{12}{13} = \frac{3}{13}$.

21.1. $\arcsin\left(-\frac{\sqrt{3}}{2}\right) = x \Leftrightarrow -\frac{\sqrt{3}}{2} = \sin x \wedge x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \Leftrightarrow$

$$\Leftrightarrow x = -\frac{\pi}{3}$$

$\arcsin\frac{1}{2} = x \Leftrightarrow \frac{1}{2} = \sin x \wedge x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \Leftrightarrow$

$$\Leftrightarrow x = \frac{\pi}{6}$$

$$\arcsin\left(-\frac{\sqrt{3}}{2}\right) + 2\arcsin\frac{1}{2} = -\frac{\pi}{3} + 2\times\frac{\pi}{6} = 0$$

21.2. $\sin\left(\frac{\pi}{2} + \arcsin\frac{\sqrt{2}}{2}\right) = \sin\left(\frac{\pi}{2} + \frac{\pi}{4}\right) =$

$$= \sin\left(\frac{3\pi}{4}\right) = \frac{\sqrt{2}}{2}$$

22. $\cos\left(\arcsin\frac{3}{5}\right) = \cos x$

$$\arcsin\frac{3}{5} = x \Leftrightarrow \frac{3}{5} = \sin x \wedge x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$\sin^2 x + \cos^2 x = 1$$

$$\left(\frac{3}{5}\right)^2 + \cos^2 x = 1 \Leftrightarrow \cos^2 x = 1 - \frac{9}{25} \Leftrightarrow \cos^2 x = \frac{16}{25}$$

Como $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, $\cos x > 0$. Logo, $\cos x = \frac{4}{5}$.

$$\cos\left(\arcsin\frac{3}{5}\right) = \cos x = \frac{4}{5}$$

23. $f(x) = 3\cos\left(\frac{x}{2}\right)$; $g(x) = \frac{1}{3}\sin(3x)$; $D_f = D_g = \mathbb{R}$

23.1. Se $x \in D_f$, então $x + 4\pi \in D_f$ porque $D_f = \mathbb{R}$.

$$f(x+4\pi) = 3\cos\left(\frac{x+4\pi}{2}\right) = 3\cos\left(\frac{x}{2} + \frac{4\pi}{2}\right) =$$

$$= 3\cos\left(\frac{x}{2} + 2\pi\right) = 3\cos\frac{x}{2} = f(x)$$

$$f(x+4\pi) = f(x),\ \forall x \in \mathbb{R}$$

Logo, f é uma função periódica de período 4π.

23.2. Seja P o período positivo mínimo da função g.
Se $x \in D_g$, então $x + P \in D_g$ porque $D_g = \mathbb{R}$.

$$\forall x \in \mathbb{R},\ g(x+P) = g(x) \Leftrightarrow$$

$$\Leftrightarrow \forall x \in \mathbb{R},\ \frac{1}{3}\sin\left[3(x+P)\right] = \frac{1}{3}\sin(3x) \Leftrightarrow$$

$$\Leftrightarrow \forall x \in \mathbb{R},\ \sin(3x+3P) = \sin(3x)$$

Como 2π é o período positivo mínimo da função seno e P é o menor valor positivo para o qual a proposição é verdadeira, terá de ser $3P = 2\pi \Leftrightarrow P = \frac{2\pi}{3}$. Logo, a função g é periódica de período positivo mínimo $P_0 = \frac{2\pi}{3}$.

23.3. $f(\pi - 2\alpha) = 1 \Leftrightarrow 3\cos\left(\frac{\pi-2\alpha}{2}\right) = 1 \Leftrightarrow$

$$\Leftrightarrow 3\cos\left(\frac{\pi}{2} - \alpha\right) = 1 \Leftrightarrow 3\sin\alpha = 1 \Leftrightarrow \sin\alpha = \frac{1}{3}$$

$$g\left(\frac{\pi}{2} + \frac{\alpha}{3}\right) = \frac{1}{3}\sin\left[3\left(\frac{\pi}{2} + \frac{\alpha}{3}\right)\right] =$$

$$= \frac{1}{3}\sin\left(\frac{3\pi}{2} + \alpha\right) = \frac{1}{3}\sin\left(\pi + \frac{\pi}{2} + \alpha\right) =$$

$$= -\frac{1}{3}\sin\left(\frac{\pi}{2} + \alpha\right) = -\frac{1}{3}\cos\alpha = -\frac{1}{3}\times\left(-\frac{2\sqrt{2}}{3}\right) = \frac{2\sqrt{2}}{9}$$

Cálculo auxiliar:

$\frac{1}{9} + \cos^2\alpha = 1 \Leftrightarrow \cos^2\alpha = 1 - \frac{1}{9} \Leftrightarrow \cos^2\alpha = \frac{8}{9}$

Como $\alpha \in \left]\frac{\pi}{2}, \frac{3\pi}{2}\right[$, $\cos\alpha < 0$, pelo que $\cos\alpha = -\frac{\sqrt{8}}{3} = -\frac{2\sqrt{2}}{3}$.

23.4. $D_h = D_f \cap D_g = \mathbb{R}$

Se $x \in D_h$, então $-x \in D_h$.

$h(-x) = (f \times g)(-x) = f(-x) \times g(-x) =$

$= 3\cos\left(\frac{-x}{2}\right) \times \frac{1}{3}\sin(3\times(-x)) =$

$= 3\cos\left(-\frac{x}{2}\right) \times \frac{1}{3}\sin(-3x) =$

$= 3\cos\frac{x}{2} \times \frac{1}{3}\left[-\sin(3x)\right] =$

$= -3\cos\frac{x}{2} \times \frac{1}{3}\sin(3x) =$

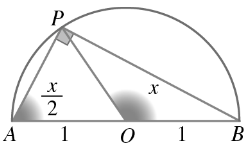
$= -(f \times g)(x) = -h(x)$

$\forall x \in \mathbb{R},\ h(-x) = -h(x)$

Logo, *h* é uma função ímpar.

24. $f(x) = \overline{BP}$; $g(x) = \overline{AP}$

24.1. O triângulo [*ABP*] é retângulo em *P* porque o ângulo *APB* está inscrito numa semicircunferência.



24.2. Atendendo a que o triângulo [*ABP*] é retângulo em *P* e a que $B\hat{A}P = \frac{1}{2}B\hat{O}P = \frac{x}{2}$, temos:

$\frac{\overline{BP}}{\overline{AB}} = \sin\left(\frac{x}{2}\right) \Leftrightarrow \frac{\overline{BP}}{2} = \sin\left(\frac{x}{2}\right) \Leftrightarrow \overline{BP} = 2\sin\left(\frac{x}{2}\right)$

$\frac{\overline{AP}}{\overline{AB}} = \cos\left(\frac{x}{2}\right) \Leftrightarrow \frac{\overline{AP}}{2} = \cos\left(\frac{x}{2}\right) \Leftrightarrow \overline{AP} = 2\cos\left(\frac{x}{2}\right)$

Portanto, $f(x) = 2\sin\left(\frac{x}{2}\right)$ e $g(x) = 2\cos\left(\frac{x}{2}\right)$.

24.3. $f(\pi - 2\alpha) = 1 \Leftrightarrow 2\sin\left(\frac{\pi - 2\alpha}{2}\right) = 1 \Leftrightarrow$

$\Leftrightarrow 2\sin\left(\frac{\pi}{2} - \alpha\right) = 1 \Leftrightarrow 2\cos\alpha = 1 \Leftrightarrow$

$\Leftrightarrow \cos\alpha = \frac{1}{2} \Leftrightarrow$

$\alpha \in \left]0\ ,\ \pi\right[$

$\Leftrightarrow \alpha = \frac{\pi}{3}$

$\overline{AP} = g(\alpha) = g\left(\frac{\pi}{3}\right) = 2\cos\left(\frac{\frac{\pi}{3}}{2}\right) = 2\cos\left(\frac{\pi}{6}\right) = \sqrt{3}$

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25.1. $\arccos\left(-\frac{\sqrt{3}}{2}\right) = x \Leftrightarrow -\frac{\sqrt{3}}{2} = \cos x \wedge x \in [0\ ,\ \pi]$

$\Leftrightarrow x = \pi - \frac{\pi}{6} \Leftrightarrow x = \frac{5\pi}{6}$

$\arccos\left(\frac{1}{2}\right) = \frac{\pi}{3}$

$\arccos\left(-\frac{\sqrt{3}}{2}\right) - \arccos\left(\frac{1}{2}\right) = \frac{5\pi}{2} - \frac{\pi}{3} = \frac{\pi}{2}$

25.2. $\cos\left(2\arccos\frac{\sqrt{2}}{2} - \arccos\frac{\sqrt{3}}{2}\right) =$

$= \cos\left(2\times\frac{\pi}{4} - \frac{\pi}{6}\right) = \cos\left(\frac{\pi}{2} - \frac{\pi}{6}\right)$

$= \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

26. $\sin\left(\arccos\frac{12}{13}\right) = \sin x$

$x = \arccos\frac{12}{13} \wedge x \in [0\ ,\ \pi] \Leftrightarrow$

$\Leftrightarrow \cos x = \frac{12}{13} \wedge x \in [0\ ,\ \pi]$

$\sin^2 x + \cos^2 x = 1$

$\sin^2 x + \left(\frac{12}{13}\right)^2 = 1 \Leftrightarrow \sin^2 x = 1 - \frac{144}{169} \Leftrightarrow \sin^2 x = \frac{25}{169}$

Como $x \in [0\ ,\ \pi]$, $\sin x > 0$. Logo, $\sin x = \frac{5}{13}$.

$\sin\left(\arccos\frac{12}{13}\right) = \sin x = \frac{5}{13}$

27. $h(x) = 2\tan\left(\frac{x}{2} + \frac{\pi}{2}\right)$

27.1. $D_h = \mathbb{R} \setminus \left\{x :\ \frac{x}{2} + \frac{\pi}{2} = \frac{\pi}{2} + k\pi,\ k \in \mathbb{Z}\right\}$

$= \mathbb{R} \setminus \{x :\ x = 2k\pi,\ k \in \mathbb{Z}\}$

27.2. $h(\pi - x) \times h(x) = 2\tan\left(\frac{\pi - x}{2} + \frac{\pi}{2}\right) \times 2\tan\left(\frac{x}{2} + \frac{\pi}{2}\right) =$

$= 4\tan\left(\frac{\pi}{2} - \frac{x}{2} + \frac{\pi}{2}\right) \times \frac{\sin\left(\frac{\pi}{2} + \frac{x}{2}\right)}{\cos\left(\frac{\pi}{2} + \frac{x}{2}\right)} =$

$= 4\tan\left(\pi - \frac{x}{2}\right) \times \frac{\cos\left(\frac{x}{2}\right)}{-\sin\left(\frac{x}{2}\right)} = -4\tan\left(\frac{x}{2}\right) \times \frac{\cos\left(\frac{x}{2}\right)}{-\sin\left(\frac{x}{2}\right)} =$

$= 4\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)} \times \frac{\cos\left(\frac{x}{2}\right)}{\sin\left(\frac{x}{2}\right)} = 4$ se $\cos\left(\frac{x}{2}\right) \neq 0 \wedge \sin\left(\frac{x}{2}\right) \neq 0$

$\cos\left(\frac{x}{2}\right) \neq 0 \wedge \sin\left(\frac{x}{2}\right) \neq 0 \Leftrightarrow \frac{x}{2} \neq \frac{k\pi}{2},\ k \in \mathbb{Z} \Leftrightarrow$

$\Leftrightarrow x \neq k\pi,\ k \in \mathbb{Z}$

27.3. Se $x \in D_h$, então $x + 2\pi \in D_h$.

$h(x + 2\pi) = 2\tan\left(\frac{x + 2\pi}{2} + \frac{\pi}{2}\right) = 2\tan\left(\frac{x}{2} + \pi + \frac{\pi}{2}\right) =$

$= 2\tan\left[\left(\frac{x}{2} + \frac{\pi}{2}\right) + \pi\right] =$

$= 2\tan\left(\frac{x}{2} \times \frac{\pi}{2}\right) = h(x)$

$\forall x \in D_h,\ h(x + 2\pi) = h(x)$

Logo, a função *h* é periódica de período 2π .

28.1. $\arctan\left(-\sqrt{3}\right)=x \Leftrightarrow -\sqrt{3}=\tan x \wedge x\in \left]-\frac{\pi}{2}\ ,\ \frac{\pi}{2}\right[\Leftrightarrow$

$\Leftrightarrow x=-\frac{\pi}{3}$

$3\arctan\left(\frac{\sqrt{3}}{3}\right)+\arctan\left(-\sqrt{3}\right)=3\times\frac{\pi}{6}-\frac{\pi}{3}=\frac{\pi}{6}$

28.2. $\sin\left[\arctan(-1)\right]=\sin\left(-\frac{\pi}{4}\right)=-\sin\frac{\pi}{4}=-\frac{\sqrt{2}}{2}$

Cálculo auxiliar:

$\arctan(-1)=x \Leftrightarrow -1=\tan x \wedge x\in\left]-\frac{\pi}{2}\ ,\ \frac{\pi}{2}\right[\Leftrightarrow x=-\frac{\pi}{4}$

29. $\tan\left(\arccos\frac{4}{5}\right)=\tan x$ com $\arccos\frac{4}{5}=x$

$\arccos\frac{4}{5}=x \Leftrightarrow \frac{4}{5}=\cos x \wedge x\in\left[0\ ,\ \pi\right]$

$1+\tan^2 x=\frac{1}{\cos^2 x}$

$1+\tan^2 x=\left(\frac{5}{4}\right)^2 \Leftrightarrow \tan^2 x=\frac{25}{16}-1 \Leftrightarrow$

$\Leftrightarrow \tan^2 x=\frac{9}{16}$

Como $x\in\left[0\ ,\ \pi\right]\wedge\cos x>0,\ \tan x>0$.

Logo, $\tan x=\sqrt{\frac{9}{16}}=\frac{3}{4}$.

Então, $\tan\left(\arccos\frac{4}{5}\right)=\tan x=\frac{3}{4}$.

30.1. $\sin\left(2x-\frac{\pi}{3}\right)=\frac{\sqrt{3}}{2} \Leftrightarrow \sin\left(2x-\frac{\pi}{3}\right)=\sin\frac{\pi}{3} \Leftrightarrow$

$\Leftrightarrow 2x-\frac{\pi}{3}=\frac{\pi}{3}+2k\pi \vee 2x-\frac{\pi}{3}=\pi-\frac{\pi}{3}+2k\pi,\ k\in\mathbb{Z} \Leftrightarrow$

$\Leftrightarrow 2x=\frac{2\pi}{3}+2k\pi \vee 2x=\pi+2k\pi,\ k\in\mathbb{Z} \Leftrightarrow$

$\Leftrightarrow x=\frac{\pi}{3}+k\pi \vee x=\frac{\pi}{2}+k\pi,\ k\in\mathbb{Z}$

30.2. $\sin^3 x=\sin x \Leftrightarrow \sin^3 x-\sin x=0 \Leftrightarrow$

$\Leftrightarrow \sin x\left(\sin^2 x-1\right)=0 \Leftrightarrow$

$\Leftrightarrow \sin x=0 \vee \sin^2 x-1=0 \Leftrightarrow$

$\Leftrightarrow \sin x=0 \vee -\cos^2 x=0 \Leftrightarrow$

$\Leftrightarrow \sin x=0 \vee \cos x=0 \Leftrightarrow$

$\Leftrightarrow x=k\pi \vee x=\frac{\pi}{2}+k\pi,\ k\in\mathbb{Z} \Leftrightarrow x=k\frac{\pi}{2},\ k\in\mathbb{Z}$

30.3. $\sqrt{3}-\sin x=\sin x \Leftrightarrow$

$\Leftrightarrow \sin x+\sin x=\sqrt{3} \Leftrightarrow$

$\Leftrightarrow 2\sin x=\sqrt{3} \Leftrightarrow$

$\Leftrightarrow \sin x=\frac{\sqrt{3}}{2} \Leftrightarrow$

$\Leftrightarrow \sin x=\sin\frac{\pi}{3}$

$\Leftrightarrow x=\frac{\pi}{3}+2k\pi \vee x=\pi-\frac{\pi}{3}+2k\pi,\ k\in\mathbb{Z}$

$\Leftrightarrow x=\frac{\pi}{3}+2k\pi \vee x=\frac{2\pi}{3}+2k\pi,\ k\in\mathbb{Z}$

30.4. $1-\sin x=\cos^2 x \Leftrightarrow$

$\Leftrightarrow 1-\sin x=1-\sin^2 x \Leftrightarrow$

$\Leftrightarrow \sin^2 x-\sin x=0 \Leftrightarrow$

$\Leftrightarrow \sin x\left(\sin x-1\right)=0 \Leftrightarrow$

$\Leftrightarrow \sin x=0 \vee \sin x-1=0 \Leftrightarrow$

$\Leftrightarrow \sin x=0 \vee \sin x=1 \Leftrightarrow$

$\Leftrightarrow x=k\pi \vee x=\frac{\pi}{2}+2k\pi,\ k\in\mathbb{Z}$

30.5. $\sin x-1=\cos x-1 \Leftrightarrow$

$\Leftrightarrow \sin x=\cos x \Leftrightarrow$

$\Leftrightarrow \cos\left(\frac{\pi}{2}-x\right)=\cos x \Leftrightarrow$

$\Leftrightarrow \frac{\pi}{2}-x=x+2k\pi \vee \frac{\pi}{2}-x=-x+2k\pi,\ k\in\mathbb{Z} \Leftrightarrow$

$\Leftrightarrow 2x=\frac{\pi}{2}+2k\pi,\ k\in\mathbb{Z} \Leftrightarrow$

$\Leftrightarrow x=\frac{\pi}{4}+k\pi,\ k\in\mathbb{Z}$

30.6. $2\cos^2 x+3\sin x-3=0 \Leftrightarrow$

$\Leftrightarrow 2\left(1-\sin^2 x\right)+3\sin x-3=0 \Leftrightarrow$

$\Leftrightarrow 2-2\sin^2 x+3\sin x-3=0 \Leftrightarrow$

$\Leftrightarrow 2\sin^2 x-3\sin x+1=0 \Leftrightarrow$

$y=\sin x$

$\Leftrightarrow 2y^2-3y+1=0 \Leftrightarrow$

$\Leftrightarrow y=\frac{3\pm\sqrt{9-8}}{4} \Leftrightarrow y=\frac{1}{2} \vee y=1 \Leftrightarrow$

$\Leftrightarrow \sin x=\frac{1}{2} \vee \sin x=1 \Leftrightarrow \sin x=\sin\frac{\pi}{6} \vee \sin x=1 \Leftrightarrow$

$\Leftrightarrow x=\frac{\pi}{6}+2k\pi \vee x=\frac{5\pi}{6}+2k\pi \vee x=\frac{\pi}{2}+2k\pi,\ k\in\mathbb{Z}$

31.1. $\left(\sin x-1\right)\left(2\sin x-1\right)=0 \wedge x\in\left]0\ ,\ \pi\right[\Leftrightarrow$

$\Leftrightarrow \left(\sin x=1 \vee 2\sin x-1=0\right) \wedge x\in\left]0\ ,\ \pi\right[\Leftrightarrow$

$\Leftrightarrow \left(\sin x=1 \vee \sin x=\frac{1}{2}\right) \wedge x\in\left]0\ ,\ \pi\right[\Leftrightarrow$

$\Leftrightarrow x=\frac{\pi}{2} \vee x=\frac{\pi}{6} \vee x=\frac{5\pi}{6}$

$S=\left\{\frac{\pi}{6}\ ,\ \frac{\pi}{2}\ ,\ \frac{5\pi}{6}\right\}$

31.2. $2\sin^2(2x)-\sin(2x)-1=0 \wedge x\in\left[\frac{\pi}{4}\ ,\ \frac{3\pi}{4}\right] \Leftrightarrow$

$y=\sin x$

$\Leftrightarrow 2y^2-y-1=0 \wedge x\in\left[\frac{\pi}{4}\ ,\ \frac{3\pi}{4}\right] \Leftrightarrow$

$\Leftrightarrow y=\frac{1\pm\sqrt{1+8}}{4} \wedge x\in\left[\frac{\pi}{4}\ ,\ \frac{3\pi}{4}\right] \Leftrightarrow$

$\Leftrightarrow y=1 \vee y=-\frac{1}{2} \wedge x\in\left[\frac{\pi}{4}\ ,\ \frac{3\pi}{4}\right] \Leftrightarrow$

$\Leftrightarrow \left(\sin(2x)=1 \vee \sin(2x)=-\frac{1}{2}\right) \wedge 2x\in\left[\frac{\pi}{2}\ ,\ \frac{3\pi}{2}\right] \Leftrightarrow$

$\Leftrightarrow 2x=\frac{\pi}{2} \vee 2x=\pi+\frac{\pi}{6} \Leftrightarrow x=\frac{\pi}{4} \vee x=\frac{7\pi}{12}$

$S=\left\{\frac{\pi}{4}\ ,\ \frac{7\pi}{12}\right\}$

31.3. $\sin(2x) - \cos x = 0 \wedge x \in \left]-\pi\,,\ \pi\right[$

$\sin(2x) - \cos x = 0 \Leftrightarrow \sin(2x) = \cos x \Leftrightarrow$

$\Leftrightarrow \sin(2x) = \sin\left(\dfrac{\pi}{2} - x\right) \Leftrightarrow$

$\Leftrightarrow 2x = \dfrac{\pi}{2} - x + 2k\pi \vee 2x = \pi - \left(\dfrac{\pi}{2} - x\right) + 2k\pi,\ k \in \mathbb{Z}$

$\Leftrightarrow 3x = \dfrac{\pi}{2} + 2k\pi \vee 2x = \pi - \dfrac{\pi}{2} + x + 2k\pi,\ k \in \mathbb{Z} \Leftrightarrow$

$\Leftrightarrow x = \dfrac{\pi}{6} + \dfrac{2k\pi}{3} \vee x = \dfrac{\pi}{2} + 2k\pi,\ k \in \mathbb{Z}$

Soluções no intervalo $\left]-\pi\,,\ \pi\right[$:

- $x = \dfrac{\pi}{6} + \dfrac{2k\pi}{3}$

  $k = 0 \Rightarrow x = \dfrac{\pi}{6}$ ✔

  $k = -1 \Rightarrow x = \dfrac{\pi}{6} - \dfrac{2\pi}{3} = -\dfrac{\pi}{2}$ ✔

  $k = -2 \Rightarrow x = \dfrac{\pi}{6} - \dfrac{4\pi}{3} = -\dfrac{7\pi}{6}$ ✗

  $k = 1 \Rightarrow x = \dfrac{\pi}{6} + \dfrac{2\pi}{3} = \dfrac{5\pi}{6}$ ✔

  $k = 2 \Rightarrow x = \dfrac{\pi}{6} + \dfrac{4\pi}{3} = \dfrac{3\pi}{2}$ ✗

- $x = \dfrac{\pi}{2} + k\pi$

  $k = 0 \Rightarrow x = \dfrac{\pi}{2}$ ✔

  $k = -1 \Rightarrow x = \dfrac{\pi}{2} - 2\pi = -\dfrac{3\pi}{2}$ ✗

  $k = 1 \Rightarrow x = \dfrac{\pi}{2} + 2\pi = \dfrac{5\pi}{2}$ ✗

$S = \left\{-\dfrac{\pi}{2}\,,\ \dfrac{\pi}{6}\,,\ \dfrac{\pi}{2}\,,\ \dfrac{5\pi}{6}\right\}$

32.1. $\cos^2 x + \sin x(\sin x - 2\cos x) = 1 \Leftrightarrow$

$\Leftrightarrow \cos^2 x + \sin^2 x - 2\sin x\cos x = 1 \Leftrightarrow$

$\Leftrightarrow 1 - 2\sin x\cos x = 1 \Leftrightarrow 2\sin x\cos x = 0 \Leftrightarrow$

$\Leftrightarrow \sin x = 0 \vee \cos x = 0 \Leftrightarrow$

$\Leftrightarrow x = k\pi \vee x = \dfrac{\pi}{2} + k\pi,\ k \in \mathbb{Z} \Leftrightarrow x = \dfrac{k\pi}{2},\ k \in \mathbb{Z}$

32.2. $\cos(2x) - \sin x = 0 \Leftrightarrow \cos(2x) = \sin x \Leftrightarrow$

$\Leftrightarrow \cos(2x) = \cos\left(\dfrac{\pi}{2} - x\right) \Leftrightarrow$

$\Leftrightarrow 2x = \dfrac{\pi}{2} - x + 2k\pi \vee 2x = -\dfrac{\pi}{2} + x + 2k\pi,\ k \in \mathbb{Z} \Leftrightarrow$

$\Leftrightarrow 3x = \dfrac{\pi}{2} + 2k\pi \vee x = -\dfrac{\pi}{2} + 2k\pi,\ k \in \mathbb{Z} \Leftrightarrow$

$\Leftrightarrow x = \dfrac{\pi}{6} + \dfrac{2k\pi}{3} \vee x = -\dfrac{\pi}{2} + 2k\pi,\ k \in \mathbb{Z}$

32.3. $\sqrt{3}\sin^2 x = \sqrt{3} + 2\cos x \Leftrightarrow$

$\Leftrightarrow \sqrt{3}\left(1 - \cos^2 x\right) = \sqrt{3} + 2\cos x \Leftrightarrow$

$\Leftrightarrow \sqrt{3} - \sqrt{3}\cos^2 x - \sqrt{3} - 2\cos x = 0 \Leftrightarrow$

$\Leftrightarrow \sqrt{3}\cos^2 x + 2\cos x = 0 \Leftrightarrow \cos x\left(\sqrt{3}\cos x + 2\right) = 0 \Leftrightarrow$

$\Leftrightarrow \cos x = 0 \vee \cos x = -\dfrac{2}{\sqrt{3}} \Leftrightarrow$

$\Leftrightarrow x = \dfrac{\pi}{2} + k\pi,\ k \in \mathbb{Z} \vee$ condição impossível

$\Leftrightarrow x = \dfrac{\pi}{2} + k\pi,\ k \in \mathbb{Z}$

32.4. $3\sin^2 x = \cos^2 x \Leftrightarrow 3\left(1 - \cos^2 x\right) - \cos^2 x = 0 \Leftrightarrow$

$\Leftrightarrow 3 - 3\cos^2 x - \cos^2 x = 0 \Leftrightarrow$

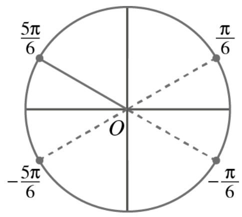
$\Leftrightarrow 3 - 4\cos^2 x = 0 \Leftrightarrow \cos^2 x = \dfrac{3}{4} \Leftrightarrow$

$\Leftrightarrow \cos x = \dfrac{\sqrt{3}}{2} \vee \cos x = -\dfrac{\sqrt{3}}{2} \Leftrightarrow$

$\Leftrightarrow \cos x = \cos\dfrac{\pi}{6} \vee \cos x = \cos\left(\pi - \dfrac{\pi}{6}\right) \Leftrightarrow$

$\Leftrightarrow x = \dfrac{\pi}{6} + 2k\pi \vee x = -\dfrac{\pi}{6} + 2k\pi \vee x = \dfrac{5\pi}{6} + 2k\pi \vee$

$\vee x = -\dfrac{5\pi}{6} + 2k\pi,\ k \in \mathbb{Z} \Leftrightarrow$



$\Leftrightarrow x = \dfrac{\pi}{6} + k\pi \vee x = -\dfrac{\pi}{6} + k\pi,\ k \in \mathbb{Z}$

32.5. $\cos^3 x = \cos x \Leftrightarrow \cos^3 x - \cos x = 0 \Leftrightarrow$

$\Leftrightarrow \cos x\left(\cos^2 x - 1\right) = 0 \Leftrightarrow \cos x = 0 \vee \cos^2 x = 1 \Leftrightarrow$

$\Leftrightarrow \cos x = 0 \vee \cos x = 1 \vee \cos x = -1 \Leftrightarrow$

$\Leftrightarrow x = \dfrac{\pi}{2} + k\pi \vee x = 2k\pi \vee x = \pi + 2k\pi,\ k \in \mathbb{Z} \Leftrightarrow$

$\Leftrightarrow x = \dfrac{k\pi}{2},\ k \in \mathbb{Z}$

32.6. $\cos^2 x + \sin x + 1 = 0 \Leftrightarrow 1 - \sin^2 x + \sin x + 1 = 0 \Leftrightarrow$

$\Leftrightarrow \sin^2 x - \sin x - 2 = 0 \Leftrightarrow$

$\Leftrightarrow y^2 - y - 2 = 0 \Leftrightarrow$ $\quad y = \sin x$

$\Leftrightarrow y = \dfrac{1 \pm \sqrt{1+8}}{2} \Leftrightarrow y = -1 \vee y = 2 \Leftrightarrow$

$\Leftrightarrow \sin x = -1 \vee \sin x = 2 \Leftrightarrow x = \dfrac{3\pi}{2} + 2k\pi,\ k \in \mathbb{Z}$

33.1. $(1 - \cos x)(1 + 2\cos x) = 0 \wedge x \in \left]-\pi\,,\ \pi\right[\Leftrightarrow$

$\Leftrightarrow (1 - \cos x = 0 \vee 1 + 2\cos x = 0) \wedge x \in \left]-\pi\,,\ \pi\right[\Leftrightarrow$

$\Leftrightarrow \left(\cos x = 1 \vee \cos x = -\dfrac{1}{2}\right) \wedge x \in \left]-\pi\,,\ \pi\right[\Leftrightarrow$

$\Leftrightarrow x = 0 \vee x = -\pi + \dfrac{\pi}{3} \vee x = \pi - \dfrac{\pi}{3} \Leftrightarrow$

$\Leftrightarrow x = -\dfrac{2\pi}{3} \vee x = 0 \vee x = \dfrac{2\pi}{3}$

$S = \left\{-\dfrac{2\pi}{3}\,,\ 0\,,\ \dfrac{2\pi}{3}\right\}$

33.2. $\sqrt{3}\sin x + 2\sin x\cos x = 0 \wedge x \in \left]-\frac{\pi}{2}, \frac{3\pi}{2}\right[\Leftrightarrow$

$\Leftrightarrow \sin x\left(\sqrt{3}+2\cos x\right) = 0 \wedge x \in \left]-\frac{\pi}{2}, \frac{3\pi}{2}\right[\Leftrightarrow$

$\Leftrightarrow \sin x = 0 \vee \cos x = -\frac{\sqrt{3}}{2} \wedge x \in \left]-\frac{\pi}{2}, \frac{3\pi}{2}\right[\Leftrightarrow$

$\Leftrightarrow \sin x = 0 \vee \cos x = \cos\left(\pi - \frac{\pi}{6}\right) \wedge x \in \left]-\frac{\pi}{2}, \frac{3\pi}{2}\right[\Leftrightarrow$

$\Leftrightarrow \left(x = k\pi \vee x = \frac{5\pi}{6} + 2k\pi \vee x = -\frac{5\pi}{6} + 2k\pi\right) \wedge$

$\wedge x \in \left]-\frac{\pi}{2}, \frac{3\pi}{2}\right[\Leftrightarrow$

$\Leftrightarrow x = 0 \vee x = \frac{5\pi}{6} \vee x = \pi \vee x = \frac{7\pi}{6}$

Cálculo auxiliar:

$x = k\pi$

$k=-1 \Rightarrow x = -\pi$ ✗

$k=0 \Rightarrow x = 0$ ✔

$k=1 \Rightarrow x = \pi$ ✔

$k=2 \Rightarrow x = 2\pi$ ✗

$x = -\frac{5\pi}{6} + 2k\pi$	$x = \frac{5\pi}{6} + 2k\pi$
$k=0 \Rightarrow x = -\frac{5\pi}{6}$ ✗	$k=0 \Rightarrow x = \frac{5\pi}{6}$ ✔
$k=1 \Rightarrow x = \frac{7\pi}{6}$ ✔	$k=-1 \Rightarrow x = -\frac{7\pi}{6}$ ✗
$k=2 \Rightarrow x = \frac{19\pi}{6}$ ✗	$k=1 \Rightarrow x = \frac{17\pi}{6}$ ✗

$S = \left\{0, \frac{5\pi}{6}, \pi, \frac{7\pi}{6}\right\}$

33.3. $\sqrt{8}\cos\left(\frac{x}{2}\right) - 4\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right) = 0 \wedge x \in \left]0, 2\pi\right[\Leftrightarrow$

$\Leftrightarrow \cos\left(\frac{x}{2}\right)\left(\sqrt{8} - 4\sin\frac{x}{2}\right) = 0 \wedge \frac{x}{2} \in \left]0, \pi\right[\Leftrightarrow$

$\Leftrightarrow \left(\cos\left(\frac{x}{2}\right) = 0 \vee \sqrt{8} - 4\sin\left(\frac{x}{2}\right) = 0\right) \wedge \frac{x}{2} \in \left]0, \pi\right[\Leftrightarrow$

$\Leftrightarrow \left(\cos\left(\frac{x}{2}\right) = 0 \vee \sin\left(\frac{x}{2}\right) = \frac{\sqrt{8}}{4}\right) \wedge \frac{x}{2} \in \left]0, \pi\right[\Leftrightarrow$

$\Leftrightarrow \left(\cos\left(\frac{x}{2}\right) = 0 \vee \sin\left(\frac{x}{2}\right) = \frac{\sqrt{2}}{2}\right) \wedge \frac{x}{2} \in \left]0, \pi\right[\Leftrightarrow$

$\Leftrightarrow \frac{x}{2} = \frac{\pi}{2} \vee \frac{x}{2} = \frac{\pi}{4} \vee \frac{x}{2} = \pi - \frac{\pi}{4} \Leftrightarrow$

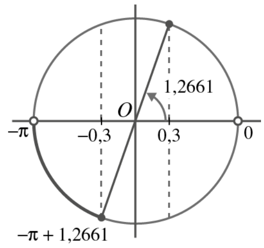
$\Leftrightarrow x = \pi \vee x = \frac{\pi}{2} \vee x = \frac{3\pi}{2}$

$S = \left\{\frac{\pi}{2}, \pi, \frac{3\pi}{2}\right\}$

34.1. $10\cos(2x) + 3 = 0 \wedge x \in \left]-\frac{\pi}{2}, 0\right[\Leftrightarrow$

$\Leftrightarrow \cos(2x) = -\frac{3}{10} \wedge 2x \in \left]-\pi, 0\right[$

Utilizando a calculadora, obtemos $\arccos\left(\frac{3}{10}\right) \approx 1{,}2661$.



$\cos(2x) = -0{,}3 \wedge 2x \in \left]-\pi, 0\right[\Rightarrow$

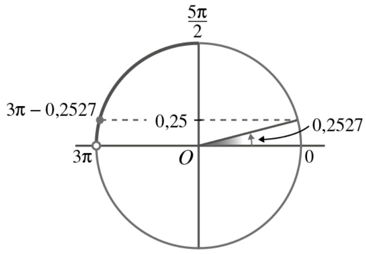
$\Rightarrow 2x = (-\pi + 1{,}2661)$ rad $\Rightarrow$

$\Rightarrow 2x \approx -1{,}8755$ rad $\Rightarrow x \approx -0{,}94$ rad

34.2. $2\sin x = 0{,}5 \wedge x \in \left]\frac{5\pi}{2}, 3\pi\right[\Leftrightarrow$

$\Leftrightarrow \sin x = 0{,}25 \wedge x \in \left]\frac{5\pi}{2}, 3\pi\right[$

Com a calculadora obtemos: $\arcsin(0{,}25) \approx 0{,}2527$.



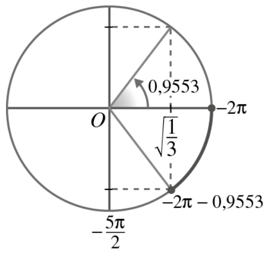
$\sin x = 0{,}25 \wedge x \in \left]\frac{5\pi}{2}, 3\pi\right[\Rightarrow$

$\Rightarrow x \approx (3\pi - 0{,}2527)$ rad $\Rightarrow x \approx 9{,}17$ rad

34.3. $\cos\frac{x}{2} = \sqrt{\frac{1}{3}} \wedge x \in \left]-5\pi, -4\pi\right[\Leftrightarrow$

$\Leftrightarrow \cos\frac{x}{2} = \sqrt{\frac{1}{3}} \wedge \frac{x}{2} \in \left]-\frac{5\pi}{2}, -2\pi\right[$

Na calculadora obtemos $\arccos\left(\sqrt{\frac{1}{3}}\right) \approx 0{,}9553$.



$\cos\left(\frac{x}{2}\right) = \sqrt{\frac{1}{3}} \wedge \frac{x}{2} \in \left]-\frac{5\pi}{2}, -2\pi\right[\Rightarrow$

$\Rightarrow \frac{x}{2} \approx (-2\pi - 0{,}9553)$ rad $\Rightarrow$

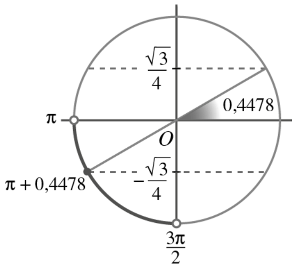
$\Rightarrow \frac{x}{2} \approx -7{,}2385$ rad $\Rightarrow$

$\Rightarrow x \approx -14{,}48$ rad

34.4. $4\sin(4x) + \sqrt{3} = 0 \wedge x \in \left]\frac{\pi}{4}, \frac{3\pi}{8}\right[\Leftrightarrow$

$\Leftrightarrow \sin(4x) = -\frac{\sqrt{3}}{4} \wedge 4x \in \left]\pi, \frac{3\pi}{2}\right[$

Utilizando uma calculadora, $\arcsin\left(\frac{\sqrt{3}}{4}\right) \approx 0{,}4478$.



$$\sin(4x) = -\frac{\sqrt{3}}{4} \wedge 4x \in \left]\pi\ ,\ \frac{3\pi}{2}\right[\Rightarrow$$
$$\Rightarrow 4x \approx (\pi + 0{,}4478) \text{ rad} \Rightarrow$$
$$\Rightarrow 4x \approx 3{,}5894 \text{ rad} \Rightarrow x \approx 0{,}90 \text{ rad}$$

35.1. $(\tan x + 1)\tan x = 0 \wedge x \in [0\ ,\ \pi] \Leftrightarrow$
$$\Leftrightarrow (\tan x + 1 = 0 \vee \tan x = 0) \wedge x \in [0\ ,\ \pi] \Leftrightarrow$$
$$\Leftrightarrow \tan x = -1 \vee \tan x = 0 \wedge x \in [0\ ,\ \pi] \Leftrightarrow$$
$$\Leftrightarrow x = \pi - \frac{\pi}{4} \vee x = 0 \vee x = \pi \Leftrightarrow$$
$$\Leftrightarrow x = 0 \vee x = \frac{3\pi}{4} \vee x = \pi$$
$$S = \left\{0\ ,\ \frac{3\pi}{4}\ ,\ \pi\right\}$$

35.2. $3\tan\left(2x + \frac{\pi}{6}\right) - \sqrt{3} = 0 \wedge x \in \left]-\pi\ ,\ 0\right[$
$$3\tan\left(2x + \frac{\pi}{6}\right) - \sqrt{3} = 0 \Leftrightarrow \tan\left(2x + \frac{\pi}{6}\right) = \frac{\sqrt{3}}{3} \Leftrightarrow$$
$$\Leftrightarrow 2x + \frac{\pi}{6} = \frac{\pi}{6} + k\pi,\ k \in \mathbb{Z} \Leftrightarrow x = \frac{k\pi}{2},\ k \in \mathbb{Z}$$
Para $k = -1$, $x = -\frac{\pi}{2}$ única solução do intervalo $\left]-\pi\ ,\ 0\right[$.
$$S = \left\{-\frac{\pi}{2}\right\}$$

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36.1. $\tan\left(\frac{3x}{2}\right) + 1 = 0 \Leftrightarrow \tan\left(\frac{3x}{2}\right) = -1 \Leftrightarrow$
$$\Leftrightarrow \tan\left(\frac{3x}{3}\right) = \tan\left(-\frac{\pi}{4}\right) \Leftrightarrow$$
$$\Leftrightarrow \frac{3x}{2} = -\frac{\pi}{4} + k\pi,\ k \in \mathbb{Z} \Leftrightarrow$$
$$\Leftrightarrow 3x = -\frac{\pi}{2} + 2k\pi,\ k \in \mathbb{Z} \Leftrightarrow$$
$$\Leftrightarrow x = -\frac{\pi}{6} + \frac{2k\pi}{3},\ k \in \mathbb{Z}$$

36.2. $3 - \tan^2(2x) = 0 \Leftrightarrow$
$$\Leftrightarrow \tan^2(2x) = 3 \Leftrightarrow$$
$$\Leftrightarrow \tan(2x) = \sqrt{3} \vee \tan(2x) = -\sqrt{3} \Leftrightarrow$$
$$\Leftrightarrow \tan(2x) = \tan\left(\frac{\pi}{3}\right) \vee \tan(2x) = \tan\left(-\frac{\pi}{3}\right) \Leftrightarrow$$
$$\Leftrightarrow 2x = \frac{\pi}{3} + k\pi \vee 2x = -\frac{\pi}{3} + k\pi,\ k \in \mathbb{Z} \Leftrightarrow$$
$$\Leftrightarrow x = \frac{\pi}{6} + \frac{k\pi}{2} \vee x = -\frac{\pi}{6} + \frac{k\pi}{2},\ k \in \mathbb{Z}$$

36.3. $\tan^3 x = \tan x \Leftrightarrow$
$$\Leftrightarrow \tan^3 x - \tan x = 0 \Leftrightarrow$$
$$\Leftrightarrow \tan x\left(\tan^2 x - 1\right) = 0 \Leftrightarrow$$
$$\Leftrightarrow \tan x = 0 \vee \tan^2 x - 1 = 0 \Leftrightarrow$$
$$\Leftrightarrow \tan x = 0 \vee \tan x = 1 \vee \tan x = -1 \Leftrightarrow$$
$$\Leftrightarrow \tan x = 0 \vee \tan x = \tan\frac{\pi}{4} \vee \tan x = \tan\left(-\frac{\pi}{4}\right) \Leftrightarrow$$
$$\Leftrightarrow x = k\pi \vee x = \frac{\pi}{4} + k\pi \vee x = -\frac{\pi}{4} + k\pi,\ k \in \mathbb{Z}$$

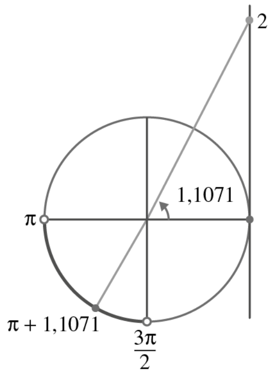
36.4. $2\tan x - \tan x \sin^2 x = 0 \Leftrightarrow$
$$\Leftrightarrow \tan x\left(2 - \sin^2 x\right) = 0 \Leftrightarrow$$
$$\Leftrightarrow \tan x = 0 \vee \sin^2 x = 2 \Leftrightarrow$$
$$\Leftrightarrow x = k\pi \vee \text{condição impossível},\ k \in \mathbb{Z} \Leftrightarrow$$
$$\Leftrightarrow x = k\pi,\ k \in \mathbb{Z}$$

36.5. $\cos x + \sin x \tan x = 2 \Leftrightarrow$
$$\Leftrightarrow \cos x + \sin x \times \frac{\sin x}{\cos x} - 2 = 0 \Leftrightarrow$$
$$\Leftrightarrow \cos^2 x + \sin^2 x - 2\cos x = 0 \wedge \cos x \neq 0 \Leftrightarrow$$
$$\Leftrightarrow 1 - 2\cos x = 0 \wedge \cos x \neq 0 \Leftrightarrow$$
$$\Leftrightarrow \cos x = \frac{1}{2} \Leftrightarrow \cos x = \cos\frac{\pi}{3} \Leftrightarrow$$
$$\Leftrightarrow x = \frac{\pi}{3} + 2k\pi \vee x = -\frac{\pi}{3} + 2k\pi,\ k \in \mathbb{Z}$$

36.6. $\tan^2 x + 1 = 2\tan x \Leftrightarrow$
$$\Leftrightarrow \tan^2 x - 2\tan x + 1 = 0 \Leftrightarrow$$
$$\Leftrightarrow y^2 - 2y + 1 = 0 \Leftrightarrow \qquad y = \tan x$$
$$\Leftrightarrow y = \frac{2 \pm \sqrt{4-4}}{2} \Leftrightarrow y = 1 \Leftrightarrow$$
$$\Leftrightarrow \tan x = 1 \Leftrightarrow \tan x = \tan\frac{\pi}{4} \Leftrightarrow$$
$$\Leftrightarrow x = \frac{\pi}{4} + k\pi,\ k \in \mathbb{Z}$$

36.7. $\tan^2 x + 1 = \frac{2}{\cos x} \Leftrightarrow$
$$\Leftrightarrow \frac{\sin^2 x}{\cos^2 x} - \frac{2}{\cos x} + 1 = 0 \Leftrightarrow$$
$$\Leftrightarrow \sin^2 x - 2\cos x + \cos^2 x = 0 \wedge \cos x \neq 0 \Leftrightarrow$$
$$\Leftrightarrow 1 - 2\cos x = 0 \wedge \cos \neq 0 \Leftrightarrow$$
$$\Leftrightarrow \cos x = \frac{1}{2} \wedge \cos x \neq 0 \Leftrightarrow \cos x = \cos\frac{\pi}{3} \Leftrightarrow$$
$$\Leftrightarrow x = \frac{\pi}{3} + 2k\pi \vee x = -\frac{\pi}{3} + 2k\pi,\ k \in \mathbb{Z}$$

37. $2\cos^2 x = \tan x - 2\sin^2 x \wedge x \in \left]\pi\ ,\ \frac{3\pi}{2}\right[\Leftrightarrow$
$$\Leftrightarrow 2\left(\cos^2 x + \sin^2 x\right) = \tan x \wedge x \in \left]\pi\ ,\ \frac{3\pi}{2}\right[\Leftrightarrow$$
$$\Leftrightarrow \tan x = 2 \wedge x \in \left]\pi\ ,\ \frac{3\pi}{2}\right[$$
Recorrendo à calculadora, $\arctan(2) = 1{,}1071$.



$$\tan x = 2 \wedge x \in \left]\pi \, , \, \frac{3\pi}{2}\right[\Rightarrow x \approx (\pi + 1{,}1071) \text{ rad} \Rightarrow$$

$$\Rightarrow x \approx 4{,}25 \text{ rad}$$

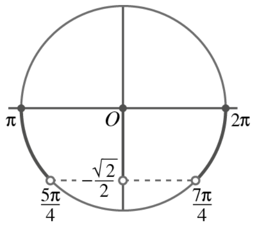
38.1. $6\sin x + \sqrt{18} > 0 \wedge x \in [\pi \, , \, 2\pi] \Leftrightarrow$

$$\Leftrightarrow \sin x > -\frac{3\sqrt{2}}{6} \wedge x \in [\pi \, , \, 2\pi] \Leftrightarrow$$

$$\Leftrightarrow \sin x > -\frac{\sqrt{2}}{2} \wedge x \in [\pi \, , \, 2\pi]$$

- $\sin x = -\dfrac{\sqrt{2}}{2} \wedge x \in [\pi \, , \, 2\pi] \Leftrightarrow$

$$\Leftrightarrow x = \pi + \frac{\pi}{4} \vee x = 2\pi - \frac{\pi}{4} \Leftrightarrow x = \frac{5\pi}{4} \vee x = \frac{7\pi}{4}$$



- $\sin x > -\dfrac{\sqrt{2}}{2} \wedge x \in [\pi \, , \, 2\pi] \Leftrightarrow$

$$\Leftrightarrow x \in \left[\pi \, , \, \frac{5\pi}{4}\right[\cup \left]\frac{7\pi}{4} \, , \, 2\pi\right]$$

38.2. $\sqrt{12}\cos x + \sqrt{3} > 0 \wedge x \in \left[\dfrac{\pi}{4} \, , \, \dfrac{7\pi}{4}\right] \Leftrightarrow$

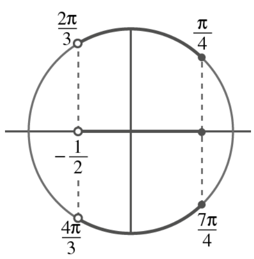
$$\Leftrightarrow \cos x > \frac{-\sqrt{3}}{\sqrt{12}} \wedge x \in \left[\frac{\pi}{4} \, , \, \frac{7\pi}{4}\right] \Leftrightarrow$$

$$\Leftrightarrow \cos x > -\sqrt{\frac{1}{4}} \wedge x \in \left[\frac{\pi}{4} \, , \, \frac{7\pi}{4}\right] \Leftrightarrow$$

$$\Leftrightarrow \cos x > -\frac{1}{2} \wedge x \in \left[\frac{\pi}{4} \, , \, \frac{7\pi}{4}\right]$$

- $\cos x = -\dfrac{1}{2} \wedge x \in \left[\dfrac{\pi}{4} \, , \, \dfrac{7\pi}{4}\right] \Leftrightarrow$

$$\Leftrightarrow x = \pi - \frac{\pi}{3} \vee x = \pi + \frac{\pi}{3} \Leftrightarrow x = \frac{2\pi}{3} \vee x = \frac{4\pi}{3}$$



$$\cos x > -\frac{1}{2} \wedge x \in \left[\frac{\pi}{4} \, , \, \frac{7\pi}{4}\right] \Leftrightarrow$$

$$\Leftrightarrow x \in \left[\frac{\pi}{4} \, , \, \frac{2\pi}{3}\right[\cup \left]\frac{4\pi}{3} \, , \, \frac{7\pi}{4}\right]$$

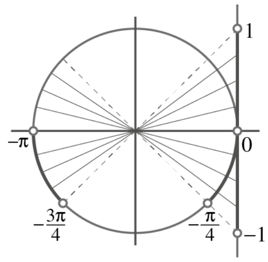
38.3. $|\tan x| < 1 \wedge x \in \left]-\pi \, , \, 0\right[\Leftrightarrow$

$$\Leftrightarrow \tan x > -1 \wedge \tan x < 1 \wedge x \in \left]-\pi \, , \, 0\right[\Leftrightarrow$$

$$\Leftrightarrow -1 < \tan x < 1 \wedge x \in \left]-\pi \, , \, 0\right[$$

$$(\tan x = -1 \vee \tan x = 1) \wedge x \in \left]-\pi \, , \, 0\right[\Leftrightarrow$$

$$\Leftrightarrow x = -\pi + \frac{\pi}{4} \vee x = -\frac{\pi}{4} \Leftrightarrow x = -\frac{3\pi}{4} \vee x = -\frac{\pi}{4}$$



$$-1 < \tan x < 1 \wedge x \in \left]-\pi \, , \, 0\right[\Leftrightarrow$$

$$\Leftrightarrow x \in \left]-\pi \, , \, -\frac{3\pi}{4}\right[\cup \left]-\frac{\pi}{4} \, , \, 0\right[$$

39. $f(x) = 1 - 2\cos(2\pi x)$

$g(x) = \sin(\pi x + a), \; a \in \mathbb{R}$

$D_f = D_g = [0 \, , \, 2]$

39.1. $x \in [0 \, , \, 2] \Leftrightarrow 2\pi x \in [0 \, , \, 4\pi]$

$-1 \leq \cos(2\pi x) \leq 1$

$-2 \leq -2\cos(2\pi x) \leq 2$

$1 - 2 \leq 1 - 2\cos(2\pi x) \leq 1 + 2$

$-1 \leq f(x) \leq 3$

$D'_f = [-1 \, , \, 3]$

39.2. $f(1) = g(1) \Leftrightarrow$

$$\Leftrightarrow 1 - 2\cos(2\pi) = \sin(\pi + a) \Leftrightarrow$$

$$\Leftrightarrow 1 - 2 = -\sin a \Leftrightarrow \sin a = 1 \Leftrightarrow$$

$$\Leftrightarrow a = \frac{\pi}{2} + k\pi, \; k \in \mathbb{Z}$$

O menor valor positivo de a é $\dfrac{\pi}{2}$.

39.3. Seja A o ponto de menor abcissa tal que $\overline{AB} = \dfrac{3}{4}$.

Então, B tem abcissa $x + \dfrac{3}{4}$ e $f(x) = f\left(x + \dfrac{3}{4}\right)$.

$$f(x) = f\left(x + \frac{3}{4}\right) \Leftrightarrow$$

$$\Leftrightarrow 1 - 2\cos(2\pi x) = 1 - 2\cos\left[2\pi\left(x + \frac{3}{4}\right)\right] \Leftrightarrow$$

$$\Leftrightarrow \cos(2\pi x) = \cos\left(2\pi x + \frac{3\pi}{2}\right) \Leftrightarrow$$

$$\Leftrightarrow \cancel{2\pi x} = \cancel{2\pi x} + \frac{3\pi}{2} + 2k\pi \vee 2\pi x =$$

$$= -2\pi x - \frac{3\pi}{2} + 2k\pi, \; k \in \mathbb{Z} \Leftrightarrow$$

$$\Leftrightarrow \text{ condição impossível} \vee\ 4\pi x = -\frac{3\pi}{2} + 2k\pi,\ k \in \mathbb{Z}$$

$$\Leftrightarrow 4x = -\frac{3}{2} + 2k,\ k \in \mathbb{Z} \Leftrightarrow x = -\frac{3}{8} + \frac{k}{2},\ k \in \mathbb{Z}$$

Como $x \in [0\ ,\ 2]$ e $x + \frac{3}{4} \in [0\ ,\ 2]$, temos:

$$k = 0 \Rightarrow x = -\frac{3}{8} \notin D_f$$

$$k = 1 \Rightarrow x = \frac{1}{8} \text{ e } x + \frac{3}{4} = \frac{1}{8} + \frac{3}{4} = \frac{7}{8}$$

$$k = 2 \Rightarrow x = \frac{5}{8} \text{ e } x + \frac{3}{4} = \frac{5}{8} + \frac{3}{4} = \frac{11}{8}$$

$$k = 3 \Rightarrow x = \frac{9}{8} \text{ e } x + \frac{3}{4} = \frac{9}{8} + \frac{3}{4} = \frac{15}{8}$$

$$k = 4 \Rightarrow x = \frac{13}{8} \text{ e } x + \frac{3}{4} = \frac{13}{8} + \frac{3}{4} = \frac{19}{8} \notin D_f$$

$$f\left(\frac{1}{8}\right) = 1 - 2\cos\frac{\pi}{4} = 1 - \sqrt{2} = f\left(\frac{7}{8}\right)$$

$$f\left(\frac{5}{8}\right) = 1 - 2\cos\frac{5\pi}{4} = 1 + \sqrt{2} = f\left(\frac{11}{8}\right)$$

$$f\left(\frac{9}{8}\right) = 1 - 2\cos\frac{9\pi}{4} = 1 - \sqrt{2} = f\left(\frac{15}{8}\right)$$

Os pares de pontos $(A\ ,\ B)$ que verificam a condição são

$A\left(\frac{1}{8}\ ,\ 1-\sqrt{2}\right)$ e $B\left(\frac{7}{8}\ ,\ 1-\sqrt{2}\right)$; $A\left(\frac{5}{8}\ ,\ 1+\sqrt{2}\right)$ e $B\left(\frac{11}{8}\ ,\ 1+\sqrt{2}\right)$ ou $A\left(\frac{9}{8}\ ,\ 1-\sqrt{2}\right)$ e $B\left(\frac{15}{8}\ ,\ 1-\sqrt{2}\right)$

40.1. $[OP] \perp [OA]$

$$\frac{\overline{OP}}{\overline{OA}} = \cos\alpha \Leftrightarrow \frac{\overline{OP}}{r} = \cos\alpha \Leftrightarrow \overline{OP} = r\cos\alpha$$

$$\frac{\overline{PA}}{\overline{OA}} = \sin\alpha \Leftrightarrow \frac{\overline{PA}}{r} = \sin\alpha \Leftrightarrow \overline{PA} = r\sin\alpha$$

$$A_{[OAP]} = \frac{\overline{PA} \times \overline{OP}}{2} = \frac{r\sin\alpha \times r\cos\alpha}{2} =$$

$$= \frac{r^2}{2}\sin\alpha\cos\alpha$$

40.2. $3\overline{AP} = \sqrt{3}\,\overline{OP} \Leftrightarrow \overline{AP} = \frac{\sqrt{3}}{3}\overline{OP} \Leftrightarrow$

$$\Leftrightarrow \frac{\overline{AP}}{\overline{OP}} = \frac{\sqrt{3}}{3} \Leftrightarrow \tan\alpha = \frac{\sqrt{3}}{3}$$

Se $\tan\alpha = \frac{\sqrt{3}}{3} \wedge \alpha \in \left]0\ ,\ \frac{\pi}{2}\right[$, então $\alpha = \frac{\pi}{6}$ e $\cos\alpha = \frac{\sqrt{3}}{2}$.

$$\overline{OP} = r\cos\alpha = \frac{\sqrt{3}}{2}r$$

40.3. $\overline{OP} = \frac{r}{2} \Leftrightarrow r\cos\alpha = \frac{r}{2} \overset{r \neq 0}{\Leftrightarrow} \cos\alpha = \frac{1}{2}$

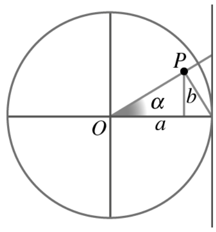
Como $\alpha \in \left]0\ ,\ \frac{\pi}{2}\right[$, temos que $\alpha = \frac{\pi}{3}$.

$$A_{[OAP]} = \frac{r^2}{2}\sin\alpha\cos\alpha =$$

$$= \frac{r^2}{2} \times \sin\frac{\pi}{3} \times \cos\frac{\pi}{3} =$$

$$= \frac{r^2}{2} \times \frac{\sqrt{3}}{2} \times \frac{1}{2} = \frac{\sqrt{3}}{8}r^2$$

40.4. $r = 1$; $P(a\ ,\ b)$ e $a = \frac{3}{4}$



$$\frac{a}{\overline{OP}} = \cos\alpha$$

$$a = \overline{OP} \times \cos\alpha$$

$$\frac{3}{4} = r\cos\alpha \times \cos\alpha$$

$r = 1$

$$\frac{3}{4} = \cos^2\alpha$$

$$\cos\alpha = \pm\frac{\sqrt{3}}{2}$$

Como $\alpha \in$ 1.º Q, $\cos\alpha = \frac{\sqrt{3}}{2}$ e $\alpha = \frac{\pi}{6}$.

$$\frac{b}{\overline{OP}} = \sin\alpha \Leftrightarrow b = \overline{OP} \times \sin\alpha \Leftrightarrow$$

$$\Leftrightarrow b = r\cos\alpha \times \sin\alpha$$

Para $r = 1$ e $\alpha = \frac{\pi}{6}$, vem:

$$b = 1 \times \sin\frac{\pi}{6} \times \cos\frac{\pi}{6} = \frac{1}{2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}$$

40.5. $A_{[OAQ]} = \frac{\overline{OA} \times \overline{AQ}}{2}$

$$\frac{\overline{AQ}}{\overline{OA}} = \tan\alpha \Leftrightarrow \frac{\overline{AQ}}{r} = \tan\alpha \Leftrightarrow \overline{AQ} = r\tan\alpha$$

$$A_{[OAQ]} = \frac{r \times r\tan\alpha}{2} = \frac{r^2}{2}\tan\alpha$$

$$A_{[AQP]} = A_{[OAQ]} - A_{[OAP]} = \frac{r^2}{2}\tan\alpha - \frac{r^2}{2}\sin\alpha\cos\alpha =$$

$$= \frac{r^2}{2}\left(\tan\alpha - \sin\alpha\cos\alpha\right) =$$

$$= \frac{r^2}{2}\left(\frac{\sin\alpha}{\cos\alpha} - \sin\alpha\cos\alpha\right) =$$

$$= \frac{r^2}{2}\,\frac{\sin\alpha - \sin\alpha\cos^2\alpha}{\cos\alpha} = \frac{r^2\sin\alpha\left(1 - \cos^2\alpha\right)}{2\cos\alpha} =$$

$$= \frac{r^2\sin\alpha \times \sin^2\alpha}{2\cos\alpha} = \frac{r^2\sin^3\alpha}{2\cos\alpha}$$

40.6. $r = 1$; $\overline{AQ} = \sqrt{8}$

$$r\tan\alpha = \sqrt{8}$$

$r = 1$

$$\tan\alpha = \sqrt{8}$$

$$1 + \tan^2\alpha = \frac{1}{\cos^2\alpha}$$

$$1 + \left(\sqrt{8}\right)^2 = \frac{1}{\cos^2\alpha} \Leftrightarrow 1 + 8 = \frac{1}{\cos^2\alpha} \Leftrightarrow \cos^2\alpha = \frac{1}{9}$$

Como $\alpha \in$ 1.º Q, vem $\cos\alpha = \frac{1}{3}$.

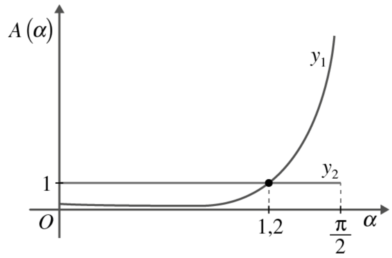
$$\sin^2\alpha + \frac{1}{9} = 1 \Leftrightarrow \sin^2\alpha = \frac{8}{9}$$

Como $\alpha \in$ 1.º Q, $\sin\alpha = \frac{\sqrt{8}}{3}$.

$$A = \frac{1 \times \left(\frac{\sqrt{8}}{3}\right)^3}{2 \times \frac{1}{3}} = \frac{\frac{8 \times \sqrt{8}}{27}}{\frac{2}{3}} = \frac{8 \times 2\sqrt{2} \times 3}{27 \times 2} = \frac{8\sqrt{2}}{9}$$

40.7. Se $r = 1$, $A(\alpha) = \dfrac{\sin^3 \alpha}{2\cos\alpha}$.

Fazendo $y_1 = \dfrac{\sin^3 x}{2\cos x}$ e $y_2 = 1$, determinou-se, no intervalo $\left]0\, ,\, \dfrac{\pi}{2}\right[$ a abcissa do ponto de interseção dos dois gráficos:



Logo, $\alpha \approx 1,2$.

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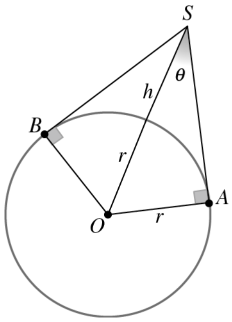
41.



41.1. $\dfrac{r}{h+r} = \sin\theta \Leftrightarrow r = h\sin\theta + r\sin\theta \Leftrightarrow$

$\Leftrightarrow h\sin\theta = r - r\sin\theta \Leftrightarrow h = \dfrac{r(1-\sin\theta)}{\sin\theta}$

Como r = 6370 km: $h = \dfrac{6370(1-\sin\theta)}{\sin\theta}$

41.2. d = comprimento do arco AB

$2\pi R$	——	2π
d	——	$A\hat{O}B$

$d = r \times A\hat{O}B$

$A\hat{O}B = 2 \times A\hat{O}S = 2\left(\dfrac{\pi}{2} - \theta\right) = \pi - 2\theta$

$d = r(\pi - 2\theta)$

$d = 6370(\pi - 2\theta)$

41.3. h = 150 km

$150 = \dfrac{6370(1-\sin\theta)}{\sin\theta} \Leftrightarrow$

$\Leftrightarrow 150\sin\theta = 6370 - 6370\sin\theta \Leftrightarrow$

$\Leftrightarrow 150\sin\theta + 6370\sin\theta = 6370 \Leftrightarrow$

$\Leftrightarrow 6520\sin\theta = 6370 \Leftrightarrow$

$\Leftrightarrow \sin\theta = \dfrac{6370}{6520} \Leftrightarrow$

$\Leftrightarrow \theta = \arcsin\left(\dfrac{637}{652}\right)$

$d = 6370(\pi - 2\theta) =$

$= 6370\left(\pi - 2\arcsin\dfrac{637}{652}\right) \approx 2738$ km

41.4. d = 1450 km

$d = 6370(\pi - 2\theta)$

$1450 = 6370\pi - 12\,740\theta$

$\theta = \dfrac{6370\pi - 1450}{12740}$

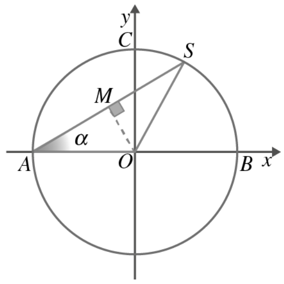
$\theta \approx 1,45698$

$h = \dfrac{6370(1-\sin\theta)}{\sin\theta}$

$h \approx \dfrac{6370(1-\sin(1,45698))}{\sin(1,45698)}$

$h \approx 41,5$ km

42.



42.1. $\overline{AO} = \overline{OS} = r$

Seja M o ponto médio de [AS]

O triângulo [AOM] é retângulo em M.

$\dfrac{\overline{AM}}{\overline{AO}} = \cos\alpha \Leftrightarrow \dfrac{\overline{AM}}{r} = \cos\alpha \Leftrightarrow \overline{AM} = r\cos\alpha$

$\overline{AS} = 2\overline{AM} = 2r\cos\alpha$

$P_{[AOS]} = \overline{AO} + \overline{OS} + \overline{AS} = r + r + 2r\cos\alpha =$

$= 2r + 2r\cos\alpha$

$P_{[AOS]} = 2r(1+\cos\alpha)$

42.2. $r = 1$

$\overset{\frown}{BS} = 2\overset{\frown}{SA} \Leftrightarrow \overset{\frown}{SA} = \dfrac{\overset{\frown}{BS}}{2}$

$\overset{\frown}{BS} + \overset{\frown}{SA} = \pi \Leftrightarrow \overset{\frown}{BS} + \dfrac{\overset{\frown}{BS}}{2} = \pi \Leftrightarrow$

$\Leftrightarrow 2\overset{\frown}{BS} + \overset{\frown}{BS} = 2\pi \Leftrightarrow \overset{\frown}{BS} = \dfrac{2\pi}{3}$

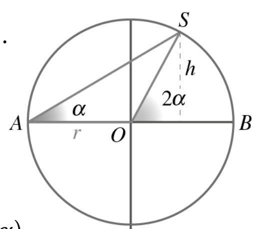
$\alpha = \dfrac{1}{2}\overset{\frown}{BS} = \dfrac{1}{2} \times \dfrac{2\pi}{3} = \dfrac{\pi}{3}$

$P_{[AOS]} = 2 \times 1\left(1 + \cos\dfrac{\pi}{3}\right) = 2\left(1 + \dfrac{1}{2}\right) = 3$

42.3. $P = 3r \Leftrightarrow$

$2r(1+\cos\alpha) = 3r \Leftrightarrow 1 + \cos\alpha = \dfrac{3}{2} \Leftrightarrow \cos\alpha = \dfrac{1}{2}$

Como $\alpha \in \left]\alpha\, ,\, \dfrac{\pi}{2}\right[$, temos $\alpha = \dfrac{\pi}{3}$.



Tomando [AO] para base do triângulo [AOS], a sua altura, h, é:

$\dfrac{h}{\overline{OS}} = \sin(2\alpha) \Leftrightarrow h = r\sin(2\alpha)$

$A_{[AOS]} = \dfrac{\overline{AO} \times r\sin(2\alpha)}{2} = \dfrac{r^2\sin(2\alpha)}{2}$

Para $\alpha = \dfrac{\pi}{3}$:

$$A_{[AOS]} = \frac{r^2 \times \sin\left(\frac{2\pi}{3}\right)}{2} = \frac{r^2 \times \sin\left(\pi + \frac{\pi}{3}\right)}{2}$$

$$= \frac{1}{2} \times r^2 \times \sin\frac{\pi}{3} = \frac{r^2}{2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4} r^2$$

42.4. $r = 1$; $P = 2 + \sqrt{3}$

$P = 2r(1 + \cos\alpha)$

$2 + \sqrt{3} = 2(1 + \cos\alpha) \Leftrightarrow 2 + \sqrt{3} = 2 + 2\cos\alpha \Leftrightarrow$

$\Leftrightarrow 2\cos\alpha = \sqrt{3} \Leftrightarrow \alpha = \dfrac{\sqrt{3}}{2}$

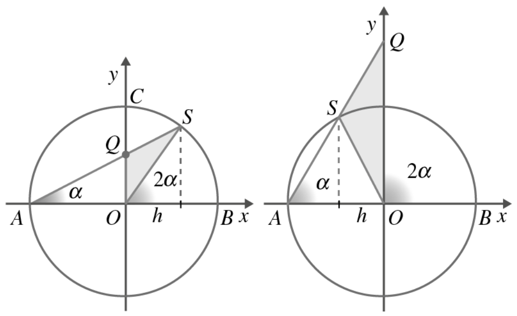
Como $\alpha \in \left]0\,,\, \dfrac{\pi}{2}\right[$, vem $\alpha = \dfrac{\pi}{6}$.

Seja C o comprimento do arco BS.

O ângulo ao centro correspondente ao arco BS tem amplitude $2\alpha = 2 \times \dfrac{\pi}{6} = \dfrac{\pi}{3}$.

$C = r \times \dfrac{\pi}{3} = 1 \times \dfrac{\pi}{3} = \dfrac{\pi}{3}$

42.5.



a) Tomando [OQ] para base do triângulo [OSQ], a sua altura h é dada por:

$\dfrac{h}{r} = \cos 2\alpha \Leftrightarrow h = r\cos 2\alpha$, se $2\alpha \le \dfrac{\pi}{2}$

$\dfrac{h}{r} = \cos(\pi - 2\alpha) \Leftrightarrow h = -r\cos(2\alpha)$, se $2\alpha > \dfrac{\pi}{2}$

Portanto, $h = r\left|\cos(2\alpha)\right|$.

$\dfrac{\overline{OQ}}{\overline{AO}} = \tan\alpha \Leftrightarrow \overline{OQ} = r\tan\alpha$

$A_{[OSQ]} = \dfrac{\overline{OQ} \times h}{2} = \dfrac{1}{2} \times r\tan\alpha \times r\left|\cos(2\alpha)\right|$

$A(\alpha) = \dfrac{1}{2} r^2 \tan\alpha \left|\cos(2\alpha)\right|$

b) $A(\alpha) = 0 \Leftrightarrow \dfrac{1}{2} r^2 \tan\alpha\left|\cos(2\alpha)\right| = 0 \Leftrightarrow$

$\overset{r \neq 0}{\Leftrightarrow} \tan\alpha = 0 \vee \left|\cos(2\alpha)\right| = 0 \Leftrightarrow$

$\Leftrightarrow \cos(2\alpha) = 0$ pois $\tan\alpha \neq 0,\ \forall \alpha \in \left]0\,,\, \dfrac{\pi}{2}\right[$

$\Leftrightarrow 2\alpha = \dfrac{\pi}{2} \Leftrightarrow$

$\Leftrightarrow \alpha = \dfrac{\pi}{4}$

Se $\alpha = \dfrac{\pi}{4}$, [OSQ] reduz-se ao segmento de reta [OC].

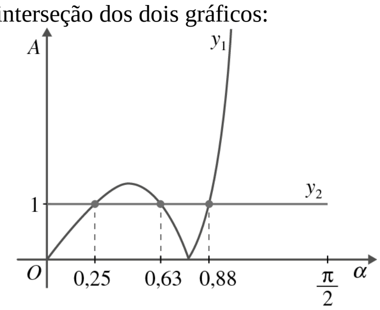
c) $r = 3$

$A(\alpha) = \dfrac{9}{2}\tan\alpha\left|\cos(2\alpha)\right|$

Considerando na calculadora gráfica:

$$y_1 = \frac{9}{2}\tan x\left|\cos(2x)\right| \quad \text{e} \quad y_2 = 1$$

com $x \in \left]0\,,\, \dfrac{\pi}{2}\right[$, determinam-se as abcissas dos pontos de interseção dos dois gráficos:



A área do triângulo [OSQ] é igual a 1 para,
$\alpha \approx 0{,}25 \vee \alpha = 0{,}63 \vee \alpha \approx 0{,}88$

Avaliação 3

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1. Como $P_0 < 5$, da análise do gráfico, podemos concluir que $P_0 = 3$.

$f(x + kP) = f(x),\ \forall x \in D_f$

- $f(2) = f(2 + 6 \times 3) = f(20)$ ((**A**) é verdadeira)
- $f(13) = f(13 + 4 \times 3) = f(25)$
  $f(25) = f(13) \Leftrightarrow f(25) - f(13) = 0$ ((**B**) é verdadeira)
- $f(12) = f(0 + 4 \times 3) = f(0) \neq f(4)$ ((**C**) é falsa)
- $f(8) = f(2 + 2 \times 3) = f(2)$
  $f(15) = f(6 \times 3 \times 3) = f(6)$
  Logo, $f(8) + f(6) = f(15) + f(2)$ ((**D**) é verdadeira)

Resposta: **(C)**

2. $f(x) = \cos(\pi x)$; $\overline{AB} = \dfrac{1}{2}$

Se x é abcissa de A, $x + \dfrac{1}{2}$ é a abcissa de B.

$f(x) = f\left(x + \dfrac{1}{2}\right)$

$\cos(\pi x) = \cos\left(\pi\left(x + \dfrac{1}{2}\right)\right) \Leftrightarrow \cos(\pi x) = \cos\left(\pi x + \dfrac{\pi}{2}\right) \Leftrightarrow$

$\Leftrightarrow \pi x = \pi x + \dfrac{\pi}{2} + 2k\pi \vee \pi x = -\pi x - \dfrac{\pi}{2} + 2k\pi,\ k \in \mathbb{Z}$

$\Leftrightarrow$ condição impossível $\vee\ 2x = -\dfrac{1}{2} + 2k,\ k \in \mathbb{Z} \Leftrightarrow$

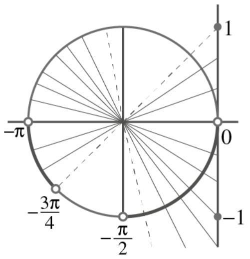
$\Leftrightarrow x = -\dfrac{1}{4} + k,\ k \in \mathbb{Z}$

Para $k = 1$, temos: $x = \dfrac{3}{4}$ e $x + \dfrac{1}{2} = \dfrac{5}{4}$

Resposta: **(C)**

3. $\tan x \le 1 \wedge x \in \left]-\pi\,,\ 0\right[$

No intervalo $\left]-\pi\,,\ 0\right[$, $\tan x = 1 \Leftrightarrow x = -\dfrac{3\pi}{4}$.



$\tan x \le 1 \wedge x \in \left]-\pi\,,\ 0\right[\Leftrightarrow$

$$\Leftrightarrow x \in \left]-\pi\,,\ -\frac{3\pi}{4}\right] \cup \left]-\frac{\pi}{2}\,,\ 0\right[$$

Resposta: **(A)**

4. $f(x) = \sin x + \cos x$

4.1. $f(-x) = \sin(-x) + \cos(-x) =$

$= -\sin x + \cos x$

A função *f* não é par nem ímpar.

$$\forall x \in \mathbb{R},\ f\left(\frac{\pi}{2} + x\right) = \sin\left(\frac{\pi}{2} + x\right) + \cos\left(\frac{\pi}{2} + x\right) =$$

$$= \cos x - \sin x =$$

$$= \cos(-x) + \sin(-x) =$$

$$= f(-x)$$

Resposta: **(D)**

4.2. $g(x) = f(-x) + f(x) =$

$$= \left[\sin(-x) + \cos(-x)\right] \times (\sin x + \cos x) =$$

$$= (-\sin x + \cos x) \times (\sin x + \cos x) =$$

$$= (\cos x - \sin x) \times (\cos x + \sin x) =$$

$$= \cos^2 x - \sin^2 x = 1 - \sin^2 x - \sin^2 x =$$

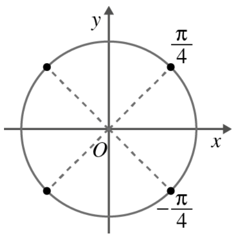
$$= 1 - 2\sin^2 x$$

Resposta: **(D)**

5. $|\tan x| - 1 = 0 \Leftrightarrow |\tan x| = 1 \Leftrightarrow \tan x = 1 \vee \tan x = -1 \Leftrightarrow$

$$\Leftrightarrow \tan x = \tan\frac{\pi}{4} \vee \tan x = \tan\left(-\frac{\pi}{4}\right) \Leftrightarrow$$

$$\Leftrightarrow x = \frac{\pi}{4} + k\pi \vee x = -\frac{\pi}{4} + k\pi,\ k \in \mathbb{Z} \Leftrightarrow$$



$$\Leftrightarrow x = \frac{\pi}{4} + \frac{k\pi}{2},\ k \in \mathbb{Z}$$

$$\Leftrightarrow x = \frac{\pi}{4} - \frac{k\pi}{2},\ k \in \mathbb{Z}$$

Resposta: **(A)**

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6. $h(x) = 2\sin(4\pi x + a),\ a \in \mathbb{R}$; $D_h = \mathbb{R}$

6.1. Se $x \in D_h$, então $x + \dfrac{1}{2} \in D_h$ porque $D_h = \mathbb{R}$.

$$h\left(x + \frac{1}{2}\right) = 2\sin\left[4\pi\left(x + \frac{1}{2}\right) + a\right] =$$

$$= 2\sin(4\pi x + 2\pi + a) =$$

$$= 2\sin\left[(4\pi x + a) + 2\pi\right] =$$

$$= 2\sin(4\pi x + a) = h(x)$$

$$\forall x \in D_h,\ h\left(x + \frac{1}{2}\right) = h(x)$$

Logo, *h* é uma função periódica de período fundamental $P_0 = \dfrac{1}{2}$.

6.2. $h\left(\dfrac{1}{3}\right) = 0 \Leftrightarrow 2\sin\left(4\pi \times \dfrac{1}{3} + a\right) = 0 \Leftrightarrow$

$$\Leftrightarrow \sin\left(\frac{4\pi}{3} + a\right) = 0 \Leftrightarrow 4\frac{\pi}{3} + a = k\pi,\ k \in \mathbb{Z} \Leftrightarrow$$

$$\Leftrightarrow a = -\frac{4\pi}{3} + k\pi,\ k \in \mathbb{Z}$$

$$k = 0 \Rightarrow a = -\frac{4\pi}{3}$$

$$k = 1 \Rightarrow a = -\frac{\pi}{3}$$

$$k = 2 \Rightarrow a = \frac{2\pi}{3}$$

Portanto, $a = \dfrac{2\pi}{3}$.

6.3. Se $x \in D_h$, então $-x \in D_h$, pois $D_h = \mathbb{R}$.

$$\forall x \in \mathbb{R},\ h(-x) = h(x) \Leftrightarrow$$

$$\Leftrightarrow \forall x \in \mathbb{R},\ 2\sin\left[4\pi(-x) + a\right] = 2\sin(4\pi x + a) \Leftrightarrow$$

$$\Leftrightarrow \forall x \in \mathbb{R},\ \sin(-4\pi x + a) = \sin(4\pi x + a) \Leftrightarrow$$

$$\Leftrightarrow \forall x \in \mathbb{R},\ -4\pi x + \cancel{a} = 4\pi x + \cancel{a} + 2k\pi \vee$$

$$\vee -4\pi x + a = \pi - (4\pi x + a) + 2k\pi,\ k \in \mathbb{Z} \Leftrightarrow$$

$$\Leftrightarrow \forall x \in \mathbb{R},\ -\cancel{4\pi x} + a = \pi - \cancel{4\pi x} - a + 2k\pi,\ k \in \mathbb{Z}$$

$$\Leftrightarrow 2a = \pi + 2k\pi,\ k \in \mathbb{Z} \Leftrightarrow$$

$$\Leftrightarrow a = \frac{\pi}{2} + k\pi,\ k \in \mathbb{Z}$$

7. $D_f = \{x \in \mathbb{R} :\ \cos x \ne 0\} = \mathbb{R} \setminus \left\{x :\ x = \dfrac{\pi}{2} + k\pi,\ k \in \mathbb{Z}\right\}$

$$D_g = \{x \in \mathbb{R} :\ \sin x \ne 1\} = \mathbb{R} \setminus \left\{x :\ \frac{\pi}{2} + 2k\pi,\ k \in \mathbb{Z}\right\}$$

$$D_f \cap D_g = D_f = D = \{x \in \mathbb{R} :\ \cos x \ne 0\}$$

$$f(x) = \frac{1 + \sin x}{\cos x} = \frac{(1 + \sin x)(1 - \sin x)}{\cos x(1 - \sin x)}$$

$$= \frac{1 - \sin^2 x}{\cos x(1 - \sin x)} = \frac{\cos^2 x}{\cos x(1 - \sin x)} = \frac{\cos x}{1 - \sin x} = g(x)$$

Logo, em $D = \{x :\ \cos x \ne 0\}$ as funções *f* e *g* coincidem.

8. $f(x) = -1 + \sin x$ e $g(x) = 2\sin^2 x - 1$

$D_f = D_g = [0\ ,\ 2\pi]$

8.1. $g(x) = 0 \Leftrightarrow 2\sin^2 x - 1 = 0 \wedge x \in [0\ ,\ 2\pi] \Leftrightarrow$

$$\Leftrightarrow \sin^2 x = \frac{1}{2} \wedge x \in [0\ ,\ 2\pi] \Leftrightarrow$$

$$\Leftrightarrow \sin x = \pm\sqrt{\frac{1}{2}} \wedge x \in [0\ ,\ 2\pi] \Leftrightarrow$$

$$\Leftrightarrow \left(\sin x = \frac{\sqrt{2}}{2} \vee \sin x = -\frac{\sqrt{2}}{2}\right) \wedge x \in [0\ ,\ 2\pi] \Leftrightarrow$$

$$\Leftrightarrow \left(\sin x = \sin\frac{\pi}{4} \vee \sin x = \sin\left(-\frac{\pi}{4}\right)\right) \wedge x \in [0\ ,\ 2\pi] \Leftrightarrow$$

$$\Leftrightarrow x = \frac{\pi}{4} \vee x = \frac{3\pi}{4} \vee x = \frac{5\pi}{4} \vee x = \frac{7\pi}{4}$$

$$S = \left\{\frac{\pi}{4}\ ,\ \frac{3\pi}{4}\ ,\ \frac{5\pi}{4}\ ,\ \frac{7\pi}{4}\right\}$$

8.2. $f(x) = g(x) \Leftrightarrow$

$$\Leftrightarrow -1 + \sin x = 2\sin^2 x - 1 \wedge x \in [0\ ,\ 2\pi] \Leftrightarrow$$

$$\Leftrightarrow 2\sin^2 x - \sin x = 0 \wedge x \in [0\ ,\ 2\pi] \Leftrightarrow$$

$$\Leftrightarrow \sin x(2\sin x - 1) = 0 \wedge x \in [0\ ,\ 2\pi] \Leftrightarrow$$

$$\Leftrightarrow \left(\sin x = 0 \vee \sin x = \frac{1}{2}\right) \wedge x \in [0\ ,\ 2\pi] \Leftrightarrow$$

$$\Leftrightarrow x = 0 \vee x = \pi \vee x = 2\pi \vee x = \frac{\pi}{6} \vee x = \pi - \frac{\pi}{6} \Leftrightarrow$$

$$\Leftrightarrow x = 0 \vee x = \frac{\pi}{6} \vee x = \frac{5\pi}{6} \vee x = \pi \vee x = 2\pi$$

$$f(0) = -1 + \sin x = -1 = f(\pi) = f(2\pi)$$

$$f\left(\frac{\pi}{6}\right) = -1 + \sin\frac{\pi}{6} = -\frac{1}{2} = f\left(\frac{5\pi}{6}\right)$$

Logo, $(0\ ,\ -1)$, $\left(\frac{\pi}{6}\ ,\ -\frac{1}{2}\right)$, $\left(\frac{5\pi}{6}\ ,\ -\frac{1}{2}\right)$, $(\pi\ ,\ -1)$ e $(2\pi\ ,\ -1)$ são os pontos de interseção dos dois gráficos.

8.3. São os pontos de abcissa *x* tais que $|f(x) - g(x)| = 1$.

$$|f(x) - g(x)| = 1 \Leftrightarrow$$

$$\Leftrightarrow \left|-1 + \sin x - \left(2\sin^2 x - 1\right)\right| = 1 \Leftrightarrow$$

$$\Leftrightarrow \left|2\sin^2 x - \sin x\right| = 1 \Leftrightarrow$$

$$\Leftrightarrow 2\sin^2 x - \sin x = 1 \vee 2\sin^2 x - \sin x = -1 \Leftrightarrow$$

$$\Leftrightarrow 2\sin^2 x - \sin x - 1 = 0 \vee 2\sin^2 x - \sin x + 1 = 0$$

Fazendo $y = \sin x$, temos:

$$2y^2 - y - 1 = 0 \vee 2y^2 - y + 1 = 0 \Leftrightarrow$$

$$\Leftrightarrow y = \frac{1 \pm \sqrt{1+8}}{4} \vee y = \frac{1 \pm \sqrt{1-8}}{4} \Leftrightarrow$$

$$\Leftrightarrow y = -\frac{1}{2} \vee y = 1 \Leftrightarrow$$

$$\Leftrightarrow \sin x = -\frac{1}{2} \vee \sin x = 1$$

No intervalo $[0\ ,\ 2\pi]$, temos:

$$x = \frac{\pi}{2} \vee x = \pi + \frac{\pi}{6} \vee x = 2\pi - \frac{\pi}{6} \Leftrightarrow$$

$$\Leftrightarrow x = \frac{\pi}{2} \vee x = \frac{7\pi}{6} \vee x = \frac{11\pi}{6}$$

$$f\left(\frac{\pi}{2}\right) = -1 + \sin\frac{\pi}{2} = 0 \quad ; \quad g\left(\frac{\pi}{2}\right) = 2\sin^2\frac{\pi}{2} - 1 = 1$$

$$f\left(\frac{7\pi}{6}\right) = -1 + \sin\frac{7\pi}{6} = -\frac{3}{2} \quad ; \quad g\left(\frac{7\pi}{6}\right) = 2\sin^2\frac{7\pi}{6} - 1 = -\frac{1}{2}$$

$$f\left(\frac{11\pi}{6}\right) = -1 + \sin\frac{11\pi}{6} = -\frac{3}{2}$$

$$g\left(\frac{11\pi}{6}\right) = 2\sin^2\frac{11\pi}{6} - 1 = -\frac{1}{2}$$

Portanto:

$A\left(\frac{\pi}{2}\ ,\ 0\right)$ e $B\left(\frac{\pi}{2}\ ,\ 1\right)$; $A\left(\frac{7\pi}{6}\ ,\ -\frac{3}{2}\right)$ e $B\left(\frac{7\pi}{6}\ ,\ -\frac{1}{2}\right)$

ou $A\left(\frac{11\pi}{6}\ ,\ -\frac{3}{2}\right)$ e $B\left(\frac{11\pi}{6}\ ,\ -\frac{1}{2}\right)$

9. $2\cos(2x) + 3\tan(2x) = 0 \wedge x \in \left]-\frac{\pi}{4}\ ,\ \frac{\pi}{4}\right[\Leftrightarrow$

$$\Leftrightarrow 2\cos(2x) + 3\frac{\sin(2x)}{\cos(2x)} = 0 \wedge 2x \in \left]-\frac{\pi}{2}\ ,\ \frac{\pi}{2}\right[\Leftrightarrow$$

$$\Leftrightarrow 2\cos^2(2x) + 3\sin(2x) = 0 \wedge \cos(2x) \neq 0 \wedge$$

$$\wedge 2x \in \left]-\frac{\pi}{2}\ ,\ \frac{\pi}{2}\right[\Leftrightarrow$$

$$\Leftrightarrow 2\left(1 - \sin^2(2x)\right) + 3\sin(2x) = 0 \wedge 2x \in \left]-\frac{\pi}{2}\ ,\ \frac{\pi}{2}\right[,$$

dado que se $2x \in \left]-\frac{\pi}{2}\ ,\ \frac{\pi}{2}\right[$, $\cos(2x) \neq 0 \Leftrightarrow$

$$\Leftrightarrow 2 - 2\sin^2(2x) + 3\sin(2x) = 0 \wedge 2x \in \left]-\frac{\pi}{2}\ ,\ \frac{\pi}{2}\right[\Leftrightarrow$$

$$\Leftrightarrow 2\sin^2(2x) - 3\sin(2x) - 2 = 0 \wedge 2x \in \left]-\frac{\pi}{2}\ ,\ \frac{\pi}{2}\right[\Leftrightarrow$$

$$\Leftrightarrow \sin(2x) = \frac{3 \pm \sqrt{9+16}}{4} \wedge 2x \in \left]-\frac{\pi}{2}\ ,\ \frac{\pi}{2}\right[\Leftrightarrow$$

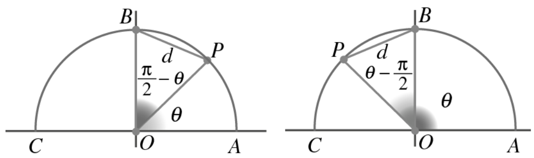
$$\Leftrightarrow \left(\sin(2x) - \frac{1}{2} \vee \sin(2x) = 2\right) \wedge 2x \in \left]-\frac{\pi}{2}\ ,\ \frac{\pi}{2}\right[\Leftrightarrow$$

$$\Leftrightarrow 2x = -\frac{\pi}{6} \Leftrightarrow$$

$$\Leftrightarrow x = -\frac{\pi}{12}$$

$$S = \left\{-\frac{\pi}{12}\right\}$$

10.1.



Se $0 < \theta \leq \frac{\pi}{2}$, $P\hat{O}B = \frac{\pi}{2} - \theta$.

Se $\frac{\pi}{2} < \theta < \pi$, $P\hat{O}B = \theta - \frac{\pi}{2}$.

$$\cos\left(\theta - \frac{\pi}{2}\right) = \cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$$

$$\cos\left(P\hat{O}B\right) = \sin\theta$$

Pelo Teorema de Carnot:

$$d^2 = \overline{OP}^2 + \overline{OB}^2 - 2\times\overline{OP}\times\overline{OB}\times\cos\left(P\hat{O}B\right)$$

$$d^2 = r^2 + r^2 - 2r\times r\times\sin\theta$$

$$d^2 = 2r^2 - 2r^2\sin\theta$$

$$d^2 = 2r^2\left(1-\sin\theta\right)$$

10.2. $d = r$

$$r^2 = 2r^2\left(1-\sin\theta\right) \Leftrightarrow$$

$$\Leftrightarrow 1-\sin\theta = \frac{r^2}{2r^2}$$

$$\Leftrightarrow 1-\sin\theta = \frac{1}{2}$$

$$\Leftrightarrow \sin\theta = \frac{1}{2}$$

Como $\theta\in\left]0\ ,\ \pi\right[,\ \sin\theta = \frac{1}{2} \Leftrightarrow$

$$\Leftrightarrow \theta = \frac{\pi}{6} \vee \theta = \pi - \frac{\pi}{6} \Leftrightarrow$$

$$\Leftrightarrow \theta = \frac{\pi}{6} \vee \theta = \frac{5\pi}{6}$$

Se $d = r$ o triângulo [OPB] é equilátero pelo que $P\hat{O}B = \frac{\pi}{3}$.

Então, $\frac{\pi}{2} - \theta = \frac{\pi}{3}$ ou $\theta - \frac{\pi}{2} = \frac{\pi}{3}$. Logo, $\theta = \frac{\pi}{6}$ ou $\theta = \frac{5\pi}{6}$.

10.3. Seja y a ordenada de P.

Então, $\frac{y}{r} = \sin\theta \vee \frac{y}{r} = \sin\left(\pi-\theta\right) \Leftrightarrow$

$$\Leftrightarrow y = r\sin\theta \vee y = r\sin\theta \Leftrightarrow$$

$$\Leftrightarrow y = r\sin\theta$$

Se $y = \frac{r}{9}$, temos: $r\sin\theta = \frac{r}{9} \Leftrightarrow \sin\theta = \frac{1}{9}$

Se $\sin\theta = \frac{1}{9}$: $d^2 = 2r^2\left(1-\frac{1}{9}\right) \Leftrightarrow d^2 = \frac{16}{9}r^2 \Leftrightarrow d = \frac{4}{3}r$

10.4. Base do triângulo [OPB] : [OB]

Altura h do triângulo [OBP]:

Se $0 < \theta \le \frac{\pi}{2}$, $\frac{h}{r} = \cos\theta \Leftrightarrow h = r\cos\theta$

Se $\frac{\pi}{2} < \theta < \pi$, $\frac{h}{r} = \cos\left(\pi-\theta\right) \Leftrightarrow h = r\left(-\cos\theta\right)$

Logo, $h = r\left|\cos\theta\right|$.

$$A_{[OBP]} = \frac{\overline{OB}\times h}{2} = \frac{r\times r\left|\cos\theta\right|}{2} = \frac{r^2}{2}\left|\cos\theta\right|$$

10.5. $r = 2$; $A_{[OBP]} = \frac{\sqrt{15}}{4}$

$$\frac{2^2}{2}\left|\cos\theta\right| = \frac{\sqrt{15}}{4} \Leftrightarrow 2\left|\cos\theta\right| = \frac{\sqrt{15}}{4} \Leftrightarrow \left|\cos\theta\right| = \frac{\sqrt{15}}{8}$$

$$\sin^2\theta + \left(\frac{\sqrt{15}}{8}\right)^2 = 1 \Leftrightarrow \sin^2\theta = 1 - \frac{15}{64}$$

$$\Leftrightarrow \sin^2\theta = \frac{49}{64} \Leftrightarrow$$

$\sin\theta > 0$

$$\Leftrightarrow \sin\theta = \frac{7}{8}$$

$$d^2 = 2r^2\left(1-\sin\theta\right)$$

$$d^2 = 2\times 4\left(1-\frac{7}{8}\right) \Leftrightarrow d^2 = 8-7 \Leftrightarrow d = 1$$

Comprimento do arco PB =

$= r\times P\hat{O}B = 2P\hat{O}B$

Se $0 < \theta \le \frac{\pi}{2}$, $\cos\theta = \frac{\sqrt{15}}{8} \Leftrightarrow \theta = \arccos\frac{\sqrt{15}}{8}$

$$P\hat{O}B = \frac{\pi}{2} - \theta = \frac{\pi}{2} - \arccos\frac{\sqrt{15}}{8}$$

$$2P\hat{O}B = 2\left(\frac{\pi}{2} - \arccos\frac{\sqrt{15}}{8}\right) \approx 1{,}01$$

Se $\frac{\pi}{2} < 0 \le \pi$, $\cos\theta = -\frac{\sqrt{15}}{8} \Leftrightarrow \theta = \arccos\left(-\frac{\sqrt{15}}{8}\right)$

$$P\hat{O}B = \theta - \frac{\pi}{2} = \arccos\left(-\frac{\sqrt{15}}{8}\right) - \frac{\pi}{2}$$

$$2P\hat{O}B = 2\left[\arccos\left(-\frac{\sqrt{15}}{8}\right) - \frac{\pi}{2}\right] \approx 1{,}01$$

Portanto, o comprimento do arco é aproximadamente 1,01.

Avaliação global

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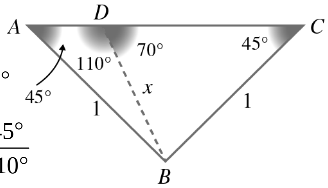
1. $C\hat{B}A = 90°$



Como $\overline{AB} = \overline{BC}$, então:

$$B\hat{C}A = B\hat{A}C = \frac{180° - 90°}{2} = 45°$$

$$\frac{\sin 45°}{x} = \frac{\sin 110°}{1} \Leftrightarrow x = \frac{\sin 45°}{\sin 110°}$$

$$B\hat{D}C = 180° - 110° = 70°$$

$$\frac{\sin 45°}{x} = \frac{\sin 70°}{1} \Leftrightarrow x = \frac{\sin 45°}{\sin 70°}$$

Resposta: **(B)**

2. $A\left(1\ ,\ \tan\alpha\right)$; $\tan\alpha = \frac{3}{4}$; $A\hat{O}B = \frac{\pi}{2}$

$$B\left(\cos\left(\frac{\pi}{2}+\alpha\right)\ ,\ \sin\left(\frac{\pi}{2}+\alpha\right)\right)$$

$$\cos\left(\frac{\pi}{2}+\alpha\right) = -\sin\alpha$$

$$1 + \tan^2\alpha = \frac{1}{\cos^2\alpha}$$

$$1 + \left(\frac{3}{4}\right)^2 = \frac{1}{\cos^2\alpha} \Leftrightarrow \frac{1}{\cos^2\alpha} = \frac{25}{16} \Leftrightarrow \cos^2\alpha = \frac{16}{25}$$

$$\sin^2\alpha + \cos^2\alpha = 1$$

$$\sin^2\alpha + \frac{16}{25} = 1 \Leftrightarrow \sin^2\alpha = 1 - \frac{16}{25} \Leftrightarrow \sin^2\alpha = \frac{9}{25}$$

Como $\alpha\in$ 1.º Q, $\sin\alpha = \frac{3}{5}$, logo $-\sin\alpha = -\frac{3}{5}$.

Resposta: **(C)**

3. $\cos^2 x = 1 \Leftrightarrow \cos x = -1 \vee \cos x = 1$

No intervalo $\left[0\ ,\ 2\pi\right[$, a equação $\cos^2 x = 1$ tem duas soluções: $x = 0 \vee x = \pi$

Como a função cosseno é periódica de período fundamental 2π, em cada um dos 500 intervalos $\left[0\ ,\ 2\pi\right[$,

$[2\pi\ ,\ 4\pi[\ ,\ \ldots\ ,\ [998\pi\ ,\ 1000\pi[$ a equação tem duas soluções.

Dado que $\cos^2(1000\pi) = \cos^2 0 = 1$, a equação tem, no intervalo $[0\ ,\ 1000\pi]$, $500 \times 2 + 1 = 1001$ soluções.

Resposta: **(D)**

4. $f(-x) = -f(x),\ \forall x \in \mathbb{R}$

$f(-0) = -f(0) \Leftrightarrow f(0) + f(0) = 0 \Leftrightarrow$

$\Leftrightarrow 2f(0) = 0 \Leftrightarrow f(0) = 0$

Resposta: **(A)**

5. α é solução da equação $\cos x = a$. Então, $\cos\alpha = a$.

A: $\cos(\pi + \alpha) = -a$?

$\cos(\pi + \alpha) = -\cos\alpha = -a$

Resposta: **(A)**

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6.1. $d = \overline{AB}$

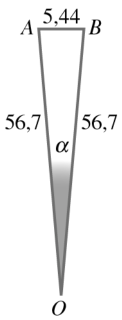
$d^2 = \overline{OA}^2 + \overline{OB}^2 - 2\overline{OA} \times \overline{OB} \times \cos\left(B\hat{O}A\right)$

$d^2 = 56{,}7^2 + 56{,}7^2 - 2 \times 56{,}7 \times 56{,}7 \times \cos(3{,}99°)$

$d^2 \approx 15{,}5844$

$d \approx \sqrt{15{,}5844}$

$\overline{AB} \approx 3{,}95$ m



6.2. $\cos\alpha = \dfrac{\overline{OA}^2 + \overline{OB}^2 - \overline{AB}^2}{2 \times \overline{OA} \times \overline{OB}}$

$\cos\alpha = \dfrac{56{,}7^2 + 56{,}7^2 - 5{,}44^2}{2 \times 56{,}7 \times 56{,}7}$

$\cos\alpha \approx 0{,}9954$

$\alpha \approx \cos^{-1}(0{,}9954) \approx 5{,}5°$

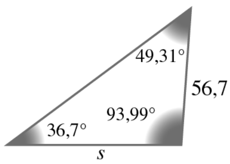
6.3. $90° + 3{,}99° = 93{,}99°$

$180° - 93{,}99° - 36{,}7° = 49{,}31°$

$\dfrac{\sin 49{,}31°}{s} = \dfrac{\sin 36{,}7°}{56{,}7}$

$s = \dfrac{56{,}7 \times \sin 49{,}31°}{\sin 36{,}7°}$

$s \approx 71{,}9$ m

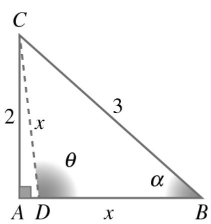


7. $\overline{AC} = 2$

$B\hat{A}C = 90°$

$\overline{DC} = \overline{DB}$

$\cos\alpha = \dfrac{\sqrt{5}}{3}$



7.1. Seja $\overline{CD} = x$. Então, $\overline{DB} = x$.

$\cos^2\alpha + \sin^2\alpha = 1$

$\left(\dfrac{\sqrt{5}}{3}\right)^2 + \sin^2\alpha = 1 \Leftrightarrow \sin^2\alpha = 1 - \dfrac{5}{9} \Leftrightarrow \sin^2\alpha = \dfrac{4}{9}$

Como α é um ângulo agudo, $\sin\alpha = \dfrac{2}{3}$.

$\dfrac{\overline{AC}}{\overline{BC}} = \sin\alpha$

$\dfrac{2}{\overline{BC}} = \dfrac{2}{3} \Leftrightarrow 6 = 2BC \Leftrightarrow \overline{BC} = 3$

Pelo Teorema de Carnot:

$\overline{DC}^2 = \overline{BC}^2 + \overline{BD}^2 - 2\overline{BC} \times \overline{BD} \times \cos\alpha$

$x^2 = 3^2 + x^2 - 2 \times 3 \times x \times \dfrac{\sqrt{5}}{3} \Leftrightarrow 2\sqrt{5}x = 9 \Leftrightarrow$

$\Leftrightarrow x = \dfrac{9}{2\sqrt{5}} \Leftrightarrow x = \dfrac{9 \times \sqrt{5}}{2 \times \sqrt{5} \times \sqrt{5}} \Leftrightarrow x = \dfrac{9\sqrt{5}}{10}$

Logo, $\overline{CD} = \dfrac{9\sqrt{5}}{10}$.

7.2. $\dfrac{\sin\theta}{\overline{BC}} = \dfrac{\sin\alpha}{x}$ (Lei dos senos)

$\dfrac{\sin\theta}{3} = \dfrac{\frac{2}{3}}{\frac{9\sqrt{5}}{10}} \Leftrightarrow \sin\theta = 3 \times \dfrac{2 \times 10}{3 \times 9\sqrt{5}} \Leftrightarrow$

$\Leftrightarrow \sin\theta = \dfrac{20\sqrt{5}}{9 \times \sqrt{5} \times \sqrt{5}} \Leftrightarrow \sin\theta = \dfrac{4\sqrt{5}}{9}$

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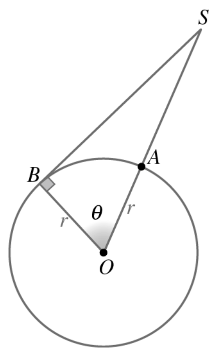
8. $26°\ 42' = \left(26 + \dfrac{42}{60}\right)^{\circ} = 26{,}7°$

$13°\ 18' = \left(13 + \dfrac{18}{60}\right)^{\circ} = 13{,}3°$

$\theta = 26{,}7° - 13{,}3° = 13{,}4°$



$\dfrac{\overline{SB}}{\overline{OB}} = \tan\theta \Leftrightarrow \overline{SB} = \overline{OB}\tan\theta$

$\overline{SB} = 6370 \times \tan(13{,}4°) \Leftrightarrow$

$\Leftrightarrow \overline{SB} \approx 1518$ km

$\dfrac{\overline{OB}}{\overline{OS}} = \cos\theta$

$\dfrac{6370}{6370 + \overline{AS}} = \cos 13{,}4°$

$6370 = 6370 \times \cos 13{,}4° + \overline{AS} \times \cos 13{,}4°$

$\overline{AS} = \dfrac{6370 - 637 \times \cos 13{,}4°}{\cos 13{,}4°}$

$\overline{AS} \approx 178$ km

9. $r = 6370$ km

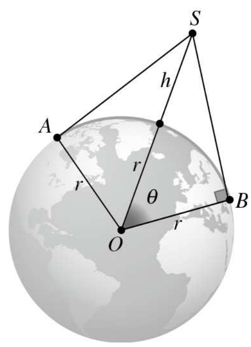
$h = 20\ 200$ km

$\dfrac{r}{h + r} = \cos\theta$

$\dfrac{6370}{20\ 200 + 6370} = \cos\theta$

$\cos\theta = \dfrac{6370}{26\ 570}$

$2\theta = 2\arccos\left(\dfrac{637}{2657}\right)$



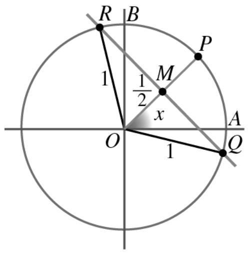
Comprimento do arco AB =

$= 2\theta \times r$ (θ em radianos)

$= 2 \times \arccos\left(\dfrac{637}{2657}\right) \times 6370$

$\approx 16\ 927$ km

10.



10.1. $\overline{OA} = \overline{OB} = \overline{OR} = \overline{OQ} = 1$

$\overline{OM} = \dfrac{1}{2}$

Os triângulos [OQM] e [OMR] são iguais e retângulos em M.

$\cos x$ = abcissa de P

$\cos x = \dfrac{\sqrt{2}}{2}$. Logo, $A\hat{O}P = x = \dfrac{\pi}{4}$.

$\dfrac{\overline{OM}}{\overline{OQ}} = \cos\left(Q\hat{O}M\right)$

$\dfrac{1}{2} = \cos\left(Q\hat{O}M\right)$

Logo, $Q\hat{O}M = \dfrac{\pi}{3}$.

$Q\hat{O}A = Q\hat{O}M - A\hat{O}M = \dfrac{\pi}{3} - \dfrac{\pi}{4} = \dfrac{\pi}{12}$

Uma amplitude do ângulo orientado de lado origem $\dot{O}A$ e lado extremidade $\dot{O}Q$ é, por exemplo, $-\dfrac{\pi}{12}$ rad.

10.2. $A\hat{O}R = \dfrac{\pi}{2} + \dfrac{\pi}{12} = \dfrac{7\pi}{2}$

Uma amplitude do ângulo orientado AOR é, por exemplo, $\dfrac{7\pi}{12}$.

11.1. $\tan^2\alpha - \sin^2\alpha = \dfrac{\sin^2\alpha}{\cos^2\alpha} - \sin^2\alpha =$

$= \dfrac{\sin^2\alpha - \sin^2\alpha\cos^2\alpha}{\cos^2\alpha} =$

$= \dfrac{\sin^2\alpha\left(1-\cos^2\alpha\right)}{\cos^2\alpha} =$

$= \dfrac{\sin^2\alpha}{\cos^2\alpha} \times \sin^2\alpha =$

$= \tan^2\alpha \times \sin^2\alpha$

11.2. $\dfrac{\sin\alpha}{1-\cos\alpha} - \dfrac{1}{\tan\alpha} = \dfrac{\sin\alpha}{1-\cos\alpha} - \dfrac{\cos\alpha}{\sin\alpha} =$

$= \dfrac{\sin^2\alpha - \cos\alpha(1-\cos\alpha)}{(1-\cos\alpha)\sin\alpha} =$

$= \dfrac{\sin^2\alpha - \cos\alpha + \cos^2\alpha}{(1-\cos\alpha)\sin\alpha} =$

$= \dfrac{\left(\sin^2\alpha + \cos^2\alpha\right) - \cos\alpha}{(1-\cos\alpha)\sin\alpha} =$

$= \dfrac{1-\cos\alpha}{(1-\cos\alpha)\sin\alpha} =$

$= \dfrac{1}{\sin\alpha}$

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12.1. $1 - \dfrac{\cos x}{\sin x} = \dfrac{1}{\sin^2 x} \Leftrightarrow \dfrac{\sin^2 x - \sin x\cos x - 1}{\sin^2 x} = 0 \Leftrightarrow$

$\Leftrightarrow -\left(1-\sin^2 x\right) - \sin x\cos x = o \wedge \sin^2 x \neq 0 \Leftrightarrow$

$\Leftrightarrow -\cos^2 x - \sin x\cos x = 0 \wedge \sin x \neq 0 \Leftrightarrow$

$\Leftrightarrow \cos x\left(-\cos x - \sin x\right) = 0 \wedge \sin x \neq 0 \Leftrightarrow$

$\Leftrightarrow \left(\cos x = 0 \vee \cos x + \sin x = 0\right) \wedge \sin x \neq 0 \Leftrightarrow$

$\Leftrightarrow \cos x = 0 \vee \cos x = -\sin x \Leftrightarrow$

$\Leftrightarrow \cos x = 0 \vee \cos x = \sin(-x) \Leftrightarrow$

$\Leftrightarrow \cos x = 0 \vee \cos x = \cos\left(\dfrac{\pi}{2} - (-x)\right) \Leftrightarrow$

$\Leftrightarrow x = \dfrac{\pi}{2} + k\pi \vee x = \dfrac{\pi}{2} + x + 2k\pi \vee$

$\vee x = -\dfrac{\pi}{2} - x + 2k\pi,\ k \in \mathbb{Z} \Leftrightarrow$

$\Leftrightarrow x = \dfrac{\pi}{2} + k\pi \vee$ condição impossível $\vee$

$\vee 2x = -\dfrac{\pi}{2} + 2k\pi,\ k \in \mathbb{Z}$

$\Leftrightarrow x = \dfrac{\pi}{2} + k\pi \vee x = -\dfrac{\pi}{4} + k\pi,\ k \in \mathbb{Z}$

12.2. $\sin x + \cos\left(x - \dfrac{\pi}{4}\right) = 0 \wedge x \in \left]-\pi, \pi\right[$

$\sin x + \cos\left(x - \dfrac{\pi}{4}\right) = 0 \Leftrightarrow \cos\left(x - \dfrac{\pi}{4}\right) = -\sin x \Leftrightarrow$

$\Leftrightarrow \cos\left(x - \dfrac{\pi}{4}\right) = \cos\left(\dfrac{\pi}{2} + x\right) \Leftrightarrow$

$\Leftrightarrow \cos\left(x - \dfrac{\pi}{4}\right) = \cos\left(\dfrac{\pi}{2} + x\right) \Leftrightarrow$

$\Leftrightarrow \cancel{x} - \dfrac{\pi}{4} = \dfrac{\pi}{2} + \cancel{x} + 2k\pi \vee x - \dfrac{\pi}{4} = -\dfrac{\pi}{2} - x + 2k\pi,\ k \in \mathbb{Z}$

$\Leftrightarrow$ condição impossível $\vee\ 2x = -\dfrac{\pi}{4} + 2k\pi,\ k \in \mathbb{Z} \Leftrightarrow$

$\Leftrightarrow x = -\dfrac{\pi}{8} + k\pi,\ k \in \mathbb{Z}$

$k = -1 \Rightarrow x = -\dfrac{\pi}{8} - \pi$ ✗ $\quad k = 0 \Rightarrow x = -\dfrac{\pi}{8}$ ✓

$k = 1 \Rightarrow x = -\dfrac{\pi}{8} + \pi = \dfrac{7\pi}{8}$ ✓ $\quad k = 2 \Rightarrow x = -\dfrac{\pi}{8} + 2\pi$ ✗

$S = \left\{-\dfrac{\pi}{8}, \dfrac{7\pi}{8}\right\}$

13. Se $x \in \left]-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right[$, $\cos x \neq 0$.

$\sin x + 2\cos x = 0 \wedge x \in \left]-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right[\Leftrightarrow$

$\Leftrightarrow \dfrac{\sin x}{\cos x} + \dfrac{2\cos x}{\cos x} = \dfrac{0}{\cos x} \wedge x \in \left]-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right[\Leftrightarrow$

$\Leftrightarrow \tan x + 2 = 0 \wedge x \in \left]-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right[\Leftrightarrow$

$\Leftrightarrow \tan x = -2 \wedge x \in \left]-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right[\Leftrightarrow$

$\Leftrightarrow x = \arctan(-2) \Rightarrow$

$\Rightarrow x \approx -1{,}11$ rad

14.1. $\overline{AQ} = \overline{OQ} - \overline{OA} = \overline{OQ} - r$

$$\frac{r}{\overline{OQ}} = \cos\theta \Leftrightarrow \overline{OQ} = \frac{r}{\cos\theta}$$

$$\overline{AQ} = \frac{r}{\cos\theta} - r = \frac{r - r\cos\theta}{\cos\theta}$$

$$d = \frac{r(1-\cos\theta)}{\cos\theta}$$

14.2. Se $d = r$:

$$r = \frac{r(1-\cos\theta)}{\cos\theta} \Leftrightarrow 1 = \frac{1-\cos\theta}{\cos\theta} \Leftrightarrow$$

$$\Leftrightarrow \cos\theta = 1 - \cos\theta \Leftrightarrow \quad \text{pois } \cos\theta \neq 0$$

$$\Leftrightarrow 2\cos\theta = 1 \Leftrightarrow \cos\theta = \frac{1}{2}$$

Como $\theta \in \left]0\,,\,\frac{\pi}{2}\right[$, $\theta = \frac{\pi}{3}$.

14.3. $r = 1$ e $d = 2$

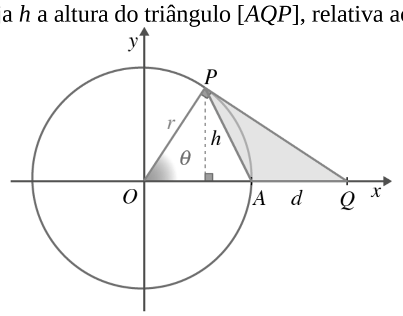
$$2 = \frac{1(1-\cos\theta)}{\cos\theta} \Leftrightarrow 2\cos\theta = 1 - \cos\theta \Leftrightarrow \cos\theta = \frac{1}{3}$$

$$\left(\frac{1}{3}\right)^2 + \sin^2\theta = 1 \Leftrightarrow \sin^2\theta = 1 - \frac{1}{9} \Leftrightarrow \sin^2\theta = \frac{8}{9}$$

Como $\theta \in \left]0\,,\,\frac{\pi}{2}\right[$, $\sin\theta = \frac{\sqrt{8}}{3} = \frac{2\sqrt{2}}{3}$.

Logo, $P\left(\frac{1}{3}\,,\,\frac{2\sqrt{2}}{3}\right)$.

14.4. a) Seja *h* a altura do triângulo [*AQP*], relativa ao vértice *P*



$$\frac{h}{r} = \sin\theta \Leftrightarrow h = r\sin\theta$$

$$A_{[AQP]} = \frac{d \times h}{2} = \frac{1}{2} \times d \times h =$$

$$= \frac{1}{2} \times \frac{r(1-\cos\theta)}{\cos\theta} \times r\sin\theta =$$

$$= \frac{1}{2} \times r^2 \times (1-\cos\theta) \times \frac{\sin\theta}{\cos\theta}$$

$$A = \frac{r^2}{2}(1-\cos\theta)\tan\theta$$

b) $\tan\alpha + \frac{1}{\tan\alpha} = 2 \wedge \alpha \in \left]0\,,\,\frac{\pi}{2}\right[$

Como $\tan\alpha \neq 0$, vem:

$$\tan^2\alpha + 1 = 2\tan\alpha \wedge \alpha \in \left]0\,,\,\frac{\pi}{2}\right[\Leftrightarrow$$

$$\Leftrightarrow \tan^2\alpha - 2\tan\alpha + 1 = 0 \wedge \alpha \in \left]0\,,\,\frac{\pi}{2}\right[\Leftrightarrow$$

$$\Leftrightarrow \tan\alpha = \frac{2 \pm \sqrt{4-4}}{2} \wedge \alpha \in \left]0\,,\,\frac{\pi}{2}\right[\Leftrightarrow$$

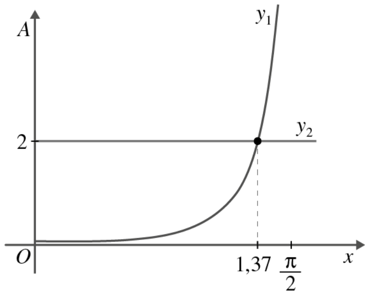
$$\Leftrightarrow \tan\alpha = 1 \wedge \alpha \in \left]0\,,\,\frac{\pi}{2}\right[\Leftrightarrow \alpha = \frac{\pi}{4}$$

$$A\left(\frac{\pi}{4}\right) = \frac{2^2}{2} \times \left(1 - \cos\frac{\pi}{4}\right) \times \tan\frac{\pi}{4} =$$

$$= 2 \times \left(1 - \frac{\sqrt{2}}{2}\right) \times 1 =$$

$$= 2 - \sqrt{2}$$

c) Fazendo, na calculadora gráfica, $y_1 = \frac{1}{2}(1-\cos x)\tan x$ e $y_2 = 2$, determinou-se a abcissa do ponto de interseção dos respetivos gráficos, no intervalo $\left]0\,,\,\frac{\pi}{2}\right[$:



Assim, $\theta \approx 1{,}37$.

15. Se P é o período de f, então:

(1) • Se $x \in D_f$, $x + P \in D_f$

(2) • $f(x+P) = f(x)$, $\forall x \in D_f$

Por (1):

Se $x \in D_f$, $x + P \in D_f$.

Logo:

Se $(x+P) \in D_f$, então $(x+P) + P \in D_f$.

Se $x \in D_f$, então $x + 2P \in D_f$.

Por (2):

$$f(x+2P) = f((x+P)+P)$$

$$= f(x+P)$$

$$= f(x),\ \forall x \in D_f$$

Logo, se P é período de uma função *f*, então $2P$ também é período da função *f*.