Resolução de ficha nº4 - Funções - 12.º ano

(1) $\log_2\left(\frac{a^5}{8}\right) = \log_2 a^5 - \log_2 8 = 5 \times \log_2 a - 3 = 5 \times \frac{1}{5} - 3 = -2$ (B)

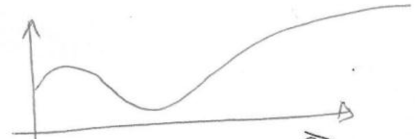
(2) $\log_p 16 = 4 \Leftrightarrow p^4 = 16 \Leftrightarrow p = 2$ (C)

(3) $P(0) = 400$

(A) $P(0) = \frac{1000}{1+e^0} = \frac{1000}{2} = 500$ X

(C) $\lim_{t \to +\infty} P(t) = \lim_{t \to +\infty} \frac{1200}{1+2\underbrace{e^{-t}}_{\to 0}} = \frac{1200}{1+2\times 0} = 1200$ X

(D)

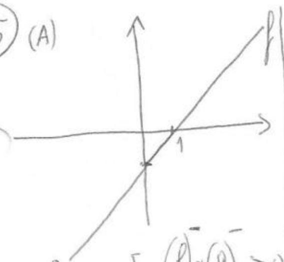


X

Resposta correcta: (B)

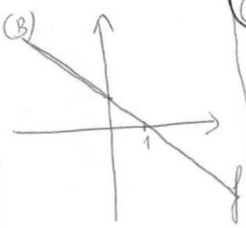
(4) $\frac{2\times 5}{0{,}7x} = \frac{10}{0{,}7x}$ (B)

(5) (A)



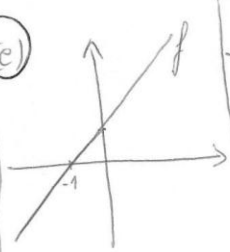
em $]-1;0[$ $\overset{-}{f} \times \overset{-}{g} > 0$ X
$h < 0$

(B)

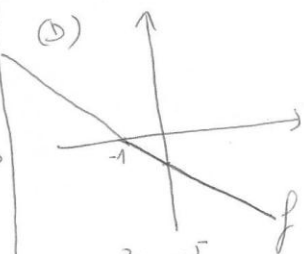


em $]-\infty;-2[$ $\overset{+}{f} \times \overset{+}{g} > 0$ X
$h < 0$

(C)



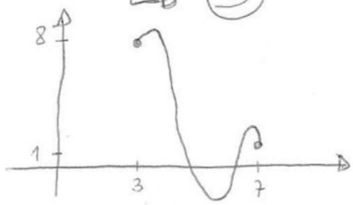
(D)



em $]0;2[$
$\overset{-}{f} \times \overset{-}{g} > 0$ X
$h < 0$

(6) Deslocamento de 1 unidade para a esquerda e 1 para cima.
$\to$ (D)

(7)



$\leftarrow$ gráfico possível

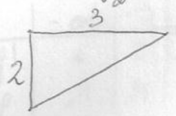
(D)

$g(x) = \sqrt{f(x)}$

$f(x) \geq 0 \Leftrightarrow x \in [-3; +\infty[$ (D)

(9) $f(x) = \log_2 (x+1) \rightsquigarrow f(3) = \log_2 4 = 2$

(C)




$\text{Área} = \frac{3 \times 2}{2} = 3$

(10) a) $\lim_{t \to +\infty} P(t) = \lim_{t \to +\infty} 5,2 \times 10^7 \times e^{(N-M)t} = 0$ (−) $\to 0$

No contexto do problema, significa que passado muito tempo a população de aves será extinta (vai diminuindo sempre)

b) $2000 - 1970 = 30$

$P(30) = \frac{1}{2} P(0)$ e $N = 7,56$ $M = ?$

$\cancel{5,2} \times \cancel{10^7} \times e^{(7,56-M)\times 30} = \frac{1}{2} \times \cancel{5,2} \times \cancel{10^7} \times e^{(7,56-M)\times 0} \Leftrightarrow$

$\Leftrightarrow e^{226,8 - 30M} = \frac{1}{2} \Leftrightarrow 226,8 - 30M = \ln\left(\frac{1}{2}\right) \Leftrightarrow$

$\Leftrightarrow M = \frac{226,8 - \ln(\frac{1}{2})}{30} \Leftrightarrow M \simeq 7,58$

(11) $\lim_{x \to 0^-} g(x) = \lim_{x \to 0^-} (K + \cos x) = K + \cos 0 = K + 1$

$\lim_{x \to 0^+} g(x) = \lim_{x \to 0^+} \frac{\ln(1+x)}{x} \underset{(*)}{=} 1$. Logo $K + \cancel{1} = \cancel{1} \Leftrightarrow K = 0$ (B)

$g(0) = K + \cos 0 = K + 1$

$(*) \lim_{x \to 0} \frac{\ln(1+x)}{x} = \lim_{y \to 0} \frac{y}{e^y - 1} = \lim_{y \to 0} \frac{1}{\frac{e^y - 1}{y}} = \frac{1}{1} = 1$ L.N. $\to 1$

$\boxed{\begin{array}{l} \ln(1+x) = y \to 0 \\ 1 + x = e^y \\ x = e^y - 1 \end{array}}$

(12) $\lim_{x \to -\infty} \frac{1}{f(x)} = \frac{1}{\lim_{x \to -\infty} f(x)} = \frac{1}{0^+} = +\infty$ (C)

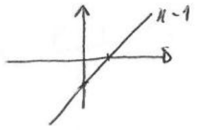
(13) $\lim_{x \to 5} f(x) = -3$ $\lim_{x \to +\infty} f(x) = 2$ $\lim_{x \to -\infty} [f(x) - x] = 0$

$\Downarrow$ $y = 2$ A.H. $\Downarrow$ $y = x$ A.O.

(A)

14) Para $x=1$ temos o único P.I, logo $h''$ tem apenas 1 zero

| | | 1 | |
|---|---|---|---|
| $h(x)$ | $\cap$ | P.I | $\cup$ |
| $x-1 = h''(x)$ | $-$ | 0 | $+$ |



(e)

15)

| | $-\infty$ | $-3$ | | $3$ | $+\infty$ |
|---|---|---|---|---|---|
| $f'$ | $+$ | 0 | $-$ | $-$ | $-$ |
| $f$ | $\nearrow$ | M | $\searrow$ | $\searrow$ | $\searrow$ |

Se $f(0)=2$, como $f$ é $\searrow$ a partir de $x=-3$, então $f(3)$ tem de ser menor que 2. (A) (A)

16) $f(x) = 1+3x^2e^{-x}$

a) $f'(x) = 3\times(2xe^{-x} - e^{-x}x^2)$

$f'(x)=0 \Leftrightarrow 2xe^{-x} - x^2e^{-x} = 0 \Leftrightarrow xe^{-x}(2-x)=0 \Leftrightarrow$

$\Leftrightarrow x=0 \vee x=2$

| | $-\infty$ | 0 | | 2 | $+\infty$ |
|---|---|---|---|---|---|
| $f'$ | $-$ | 0 | $+$ | 0 | $-$ |
| $f$ | $\searrow$ | m | $\nearrow$ | M | $\searrow$ |

$m = f(0) = 1+3\times0^2e^{-0} = 1$

b) $f$ é cont. em IR, em particular em $[-1;0]$

$f(0) = 1$

$f(-1) = 1+3e^1 = 1+3e \simeq 9,15$

Pelo Teor. de Bolzano-Cauchy, existe pelo menos um objecto em $]-1;0[$ cuja imagem é 4.

17) $f(x) = \dfrac{e^x-1}{x}$

a.1) $f'(x) = \dfrac{(e^x-1)'x - (e^x-1)x'}{x^2} = \dfrac{e^x x - (e^x-1)\times1}{x^2} = \dfrac{xe^x - e^x+1}{x^2}$

$m = f'(1) = \dfrac{1e^1 - e^1+1}{1^2} = \dfrac{\cancel{e}-\cancel{e}+1}{1} = 1$

$f(1) = \dfrac{e^1-1}{1} = e-1$

$y = mx+b$

$y = x+b$

$(1;\ e-1) \rightarrow e-1 = 1+b \Leftrightarrow b = e-2$ Logo $y = x+e-2$

2) $D_f = \mathbb{R} \setminus \{0\}$

$\lim\limits_{x\to 0} f(x) = \lim\limits_{x\to 0} \frac{e^x-1}{x} = 1$ $\rightarrow$ Não tem A.V.

$\lim\limits_{x\to -\infty} f(x) = \lim\limits_{x\to -\infty} \frac{e^x-1}{x} = \frac{0-1}{-\infty} = 0 \rightarrow \boxed{y = 0 \text{ A.H.}}$

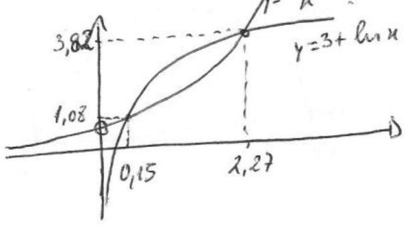
$\lim\limits_{x\to +\infty} f(x) = \lim\limits_{x\to +\infty} \frac{e^x-1}{x} \underset{(*)}{=} +\infty$ | (*) $\lim \frac{e^x-1}{x} = \lim\limits_{x\to+\infty}\left(\underbrace{\frac{e^x}{x}}_{\text{lim. not. } +\infty} - \underbrace{\frac{1}{x}}_{0}\right) = +\infty - 0 = +\infty$

b) $f(x) \leq 3 + \ln x \Leftrightarrow \frac{e^x-1}{x} \leq 3 + \ln x$

$y_1 = (e^x - 1) : x$

$y_2 = 3 + \ln x$

Janela: $[-2, 5] \times [-2, 4]$




R: $a = 0{,}15$

$b = 2{,}27$

18) $f(x) = x^{\alpha}$

$f'(x) = \alpha x^{\alpha - 1}$

$f''(x) = \underbrace{\underset{+}{\alpha}\,\underset{-}{(\alpha-1)}}_{-}\,\underset{+}{x^{\alpha-2}}$

| | 0 | | 1 |
|---|---|---|---|
| $f''$ | | $-$ | |
| $f$ | | $\cap$ | |

19) $f'(x) = 2 + x \ln x$

a) $f'(1) = 2 + \ln 1 = 2 + 0 = 2 \rightarrow m = 2$

$y = mx + b$

$\Downarrow$

$y = 2x + b$

$(1;3) \rightarrow 3 = 2 + b \Leftrightarrow b = 1$

$y = 2x + 1$

intersecção de $y = 2x+1$ com $OX$: $y = 0 \Leftrightarrow 2x + 1 = 0 \Leftrightarrow$

$\Leftrightarrow 2x = -1 \Leftrightarrow x = -\frac{1}{2}$

b) $f'(x) = 2 + x \ln x$

$f''(x) = x' \ln x + x (\ln x)' = \ln x + x\left(\frac{1}{x}\right) = 1 + \ln x$

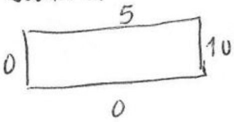
$f''(x) = 0 \Leftrightarrow 1 + \ln x = 0 \Leftrightarrow \ln x = -1 \Leftrightarrow x = e^{-1}$

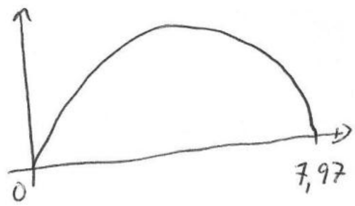
| | 0 | | $e^{-1}$ | |
|---|---|---|---|---|
| $f''$ | | $-$ | 0 | $+$ |
| $f$ | | $\cap$ | P.I | $\cup$ |

(20) $h(x) = 2x + 10 \ln(1 - 0{,}1x)$

a)

Janela:




$a = 7{,}97$

b) $h'(x) = 2 + 10\left(\frac{-0{,}1}{1 - 0{,}1x}\right) = 2 - \frac{1}{1 - 0{,}1x}$

$h'(x) = 0 \Leftrightarrow 2 = \frac{1}{1 - 0{,}1x} \Leftrightarrow 2 - 0{,}2x = 1 \Leftrightarrow 0{,}2x = 1 \Leftrightarrow$

$\Leftrightarrow x = \frac{1}{0{,}2} \Leftrightarrow x = 5$

| | 0 | | 5 | |
|---|---|---|---|---|
| $h'$ | | $+$ | 0 | $-$ |
| $h$ | 0 | $\nearrow$ | M | $\searrow$ |

$\nearrow$ em $[0; 5[$

$\searrow$ em $]5; 7{,}97]$

Máx $= h(5) = 10 + 10 \ln 0{,}5 \approx 3{,}07$

c)

$\frac{h(3) - h(1)}{3 - 1} = \frac{6 + 10 \ln 0{,}7 - 2 - 10 \ln 0{,}9}{2}$

C.A:

$h(3) = 6 + 10 \ln 0{,}7$

$h(1) = 2 + 10 \ln 0{,}9$

$= \frac{4 + 10(\ln 0{,}7 - \ln 0{,}9)}{2} =$

$= 2 + 5 \ln\left(\frac{0{,}7}{0{,}9}\right) = 2 + 5 \ln\left(\frac{7}{9}\right) = 2 + \ln\left(\frac{7}{9}\right)^5 = \ln e^2 + \ln\left(\frac{7}{9}\right)^5 = \ln\left[e^2\left(\frac{7}{9}\right)^5\right]$