Resolução da ficha nº 2 - Trigonometria - 12º ANO

(1) $f(x) = x + \operatorname{sen} x \qquad \left[-\frac{\pi}{2} ; \frac{3\pi}{2}\right]$

a) $f'(0) = \lim\limits_{x \to 0} \frac{f(x) - f(0)}{x - 0} = \lim\limits_{x \to 0} \frac{x + \operatorname{sen} x - 0}{x} =$

$= \lim\limits_{x \to 0} \frac{x + \operatorname{sen} x}{x} = \lim\limits_{x \to 0} \left(\frac{x}{x} + \underbrace{\frac{\operatorname{sen} x}{x}}_{1} \right) = 1 + 1 = 2$

b) $f'(x) = 1 + \cos x$

$f''(x) = -\sin x$

$-\sin x = 0 \Leftrightarrow$

$\Leftrightarrow \sin x = 0 \Leftrightarrow$

$\Leftrightarrow x = K\pi$

$K = 0 \to x = 0$ ✓

$K = 1 \to x = \pi$ ✓

$K = 2 \to x = 2\pi$ ✗

$K = -1 \to x = -\pi$ ✗

	$-\frac{\pi}{2}$		0		π		$\frac{3\pi}{2}$
f''	+	+	0	−	0	+	+
f		∪	P.I	∩	P.I	∪	

$f(0) = 0 + \operatorname{sen} 0 = 0 \to (0,0)$ P.I.

$f(\pi) = \pi + \sin \pi = \pi \to (\pi, \pi)$ P.I

- Conc. voltada para baixo em $]0, \pi[$
- Conc. voltada para cima em $[-\frac{\pi}{2}, 0[$ e em $]\pi, \frac{3\pi}{2}]$

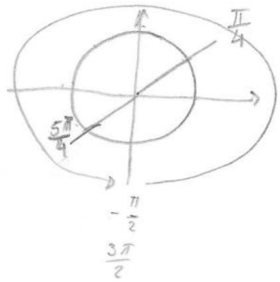
c) $f(x) = x + \cos x \Leftrightarrow \cancel{x} + \sin x = \cancel{x} + \cos x \Leftrightarrow$

$\Leftrightarrow \sin x = \cos x \Leftrightarrow \sin x = \sin\left(\frac{\pi}{2} - x\right) \Leftrightarrow$

$\Leftrightarrow x = \frac{\pi}{2} - x + 2K\pi \ \vee\ \cancel{x} = \pi - \frac{\pi}{2} + \cancel{x} + 2K\pi \ ; K \in \mathbb{Z} \Leftrightarrow$

$\Leftrightarrow 2x = \frac{\pi}{2} + 2K\pi \ \vee\ \text{imp} \Leftrightarrow$

$\Leftrightarrow x = \frac{\pi}{4} + K\pi \ ; \ K \in \mathbb{Z}$



$K = 0 \to x = \frac{\pi}{4}$ ✓

$K = 1 \to x = \frac{5\pi}{4}$ ✓

$K = 2 \to x = \frac{9\pi}{4}$ ✗

$K = -1 \to x = -\frac{3\pi}{4}$ ✗

$\text{C.S} = \left\{ \frac{\pi}{4} ; \frac{5\pi}{4} \right\}$

② $f(x) = a + b\sin^2 x$

a) $\left.\begin{matrix} a = 2 \\ b = -5 \end{matrix}\right\}$ $f(x) = 2 - 5\sin^2 x$

$\text{tg}\,\theta = \frac{1}{2}$ $\quad 1 + \text{tg}^2\theta = \frac{1}{\cos^2\theta} \Leftrightarrow 1 + \frac{1}{4} = \frac{1}{\cos^2\theta} \Leftrightarrow$

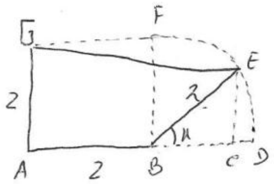
$\Leftrightarrow \frac{5}{4} = \frac{1}{\cos^2\theta} \Leftrightarrow \boxed{\cos^2\theta = \frac{4}{5}}$

$\sin^2\theta + \cos^2\theta = 1 \Leftrightarrow \sin^2\theta = 1 - \frac{4}{5} \Leftrightarrow \boxed{\sin^2\theta = \frac{1}{5}}$

$f(\theta) = 2 - 5\sin^2\theta = 2 - \cancel{5} \times \frac{1}{\cancel{5}} = 2 - 1 = 1$

b) $\left\{\begin{matrix} f(0) = 1 \\ f(\frac{\pi}{2}) = -3 \end{matrix}\right.$ $\quad \begin{matrix} a + b \times 0 = 1 \\ a + b = -3 \end{matrix}$ $\quad \left\{\begin{matrix} \boxed{a = 1} \\ \boxed{b = -4} \end{matrix}\right.$

③



$\cos x = \frac{\overline{BC}}{2} \Rightarrow \overline{BC} = 2\cos x$

$\sin x = \frac{\overline{EC}}{2} \Rightarrow \overline{EC} = 2\sin x$

a) $A_{trapézio} = \frac{B+b}{2} \times h = \frac{2 + \overline{EC}}{2} \times (2 + \overline{BC}) =$

$= \frac{2 + 2\sin x}{2} \times (2 + 2\cos x) = (1 + \sin x)(2 + 2\cos x)$

$A_{triângulo} = \frac{\overline{BC} \times \overline{EC}}{2} = \frac{\cancel{2}\cos x\, 2\sin x}{\cancel{2}} = 2\sin x\cos x$

$A_{sombreada} = A_{trapézio} - A_{triângulo} = (1 + \sin x)(2 + 2\cos x) - 2\sin x\cos x =$

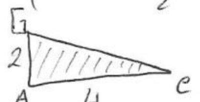
$= 2 + 2\cos x + 2\sin x + \cancel{2\sin x\cos x} - \cancel{2\sin x\cos x} =$

$= 2(1 + \cos x + \sin x)$ c.q.d.

b) $A(0) = 2(1 + \cos 0 + \sin 0) = 2(1 + 1 + 0) = 4$

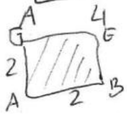
$A(\frac{\pi}{2}) = 2(1 + \cos\frac{\pi}{2} + \sin\frac{\pi}{2}) = 2(1 + 0 + 1) = 4$

$A(0) \rightarrow$



$A = \frac{2 \times 4}{2} = \frac{8}{2} = 4\,u$ (triângulo)

$A(\frac{\pi}{2}) \rightarrow$

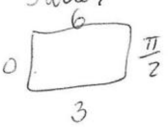


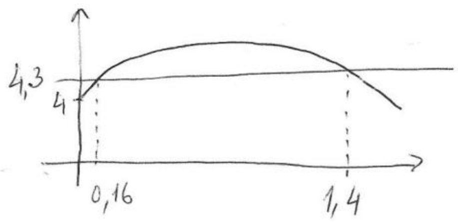
$A = 2 \times 2 = 4\,u$ (quadrado)

c) $A(x) = 2(1 + \sin x + \cos x)$

$A(x) = 4,3$?

Janela





R: A área é 4,3 para $x \simeq 0,2$ ou $x \simeq 1,4$.

(4) $f(x) = \frac{1}{3} + 2e^{1-x}$ $\quad g(x) = 2\sin x - \cos x$

4.1.a) $f(x) = \frac{1}{3} + 2e^{1-x}$

↳ F.ão contínua em IR, logo não tem A.V.

$$\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty}\left(\frac{1}{3} + 2\underbrace{e^{1-x}}_{0}\right) = \frac{1}{3} + 2\times 0 = \frac{1}{3} \Rightarrow \boxed{y = \frac{1}{3}} \text{ A.H.}$$

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty}\left(\frac{1}{3} + 2\underbrace{e^{1-x}}_{+\infty}\right) = +\infty$$

4.1.b) $f(x) = g(\pi) \Leftrightarrow \frac{1}{3} + 2e^{1-x} = 1 \Leftrightarrow$

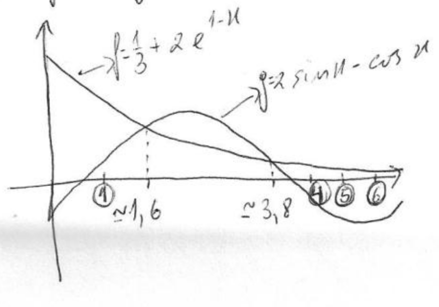
$\Leftrightarrow 2e^{1-x} = 1 - \frac{1}{3} \Leftrightarrow 2e^{1-x} = \frac{2}{3} \Leftrightarrow$

$\Leftrightarrow e^{1-x} = \frac{1}{3} \Leftrightarrow 1 - x = \ln\left(\frac{1}{3}\right) \Leftrightarrow x = 1 - \ln\left(\frac{1}{3}\right)$

$\Leftrightarrow x = \ln e - \ln\left(\frac{1}{3}\right) \Leftrightarrow x = \ln\left(\frac{e}{\frac{1}{3}}\right) \Leftrightarrow x = \ln(3e)$ //

$g(\pi) =$
$= 2\sin\pi - \cos\pi =$
$= 1$

4.2) $f(x) > g(x)$ em $[0; 2\pi]$



$C.S. = \{1; 4; 5; 6\}$

(5) $f'(x) = x + 2\cos x \qquad [-\pi ; \pi]$

5.1.a) $\lim\limits_{x \to 0} \frac{f(x) - f(0)}{x} = f'(0) = 0 + 2\cos 0 = 2$

5.1.b) $f''(x) = 1 - 2\sin x$

$1 - 2\sin x = 0 \Leftrightarrow$

$\Leftrightarrow 2\sin x = 1 \Leftrightarrow$

$\Leftrightarrow \sin x = \frac{1}{2} \Leftrightarrow$

$\Leftrightarrow \sin x = \sin \frac{\pi}{6} \Leftrightarrow$

$\Leftrightarrow x = \frac{\pi}{6} + 2k\pi \vee x = \pi - \frac{\pi}{6} + 2k\pi ; k \in \mathbb{Z}$

$\Leftrightarrow x = \frac{\pi}{6} + 2k\pi \vee x = \frac{5\pi}{6} + 2k\pi ; k \in \mathbb{Z}$

$k = 0 \rightarrow x = \frac{\pi}{6} \vee x = \frac{5\pi}{6}$

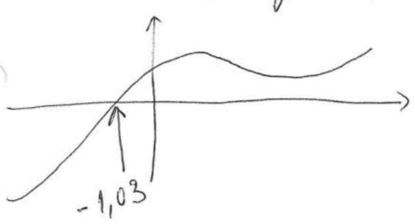
$k = 1 \rightarrow x = \cancel{\frac{13\pi}{6}} \vee x = \cancel{\frac{17\pi}{6}}$

$k = -1 \rightarrow x = -\cancel{\frac{11\pi}{6}} \vee x = -\cancel{\frac{7\pi}{6}}$

	$-\pi$		$\frac{\pi}{6}$		$\frac{5\pi}{6}$		π
f''	+	+	0	−	0	+	+
f		∪	P.I	∩	P.I	∪	

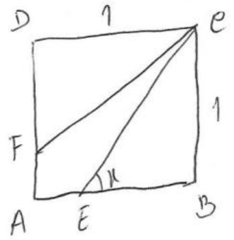
- Conc. voltada para baixo em $\left]\frac{\pi}{6}, \frac{5\pi}{6}\right[$
- Conc. voltada para cima em $\left[-\pi, \frac{\pi}{6}\right[$ e em $\left]\frac{5\pi}{6}, \pi\right]$

(5.2) Se a reta tg é horizontal, trata-se de um extremo, logo $f' = 0$.



6)

a)



$\sin x = \frac{1}{\overline{EC}} \Rightarrow \boxed{\overline{EC} = \frac{1}{\sin x}}$

logo $\boxed{\overline{FC} = \frac{1}{\sin x}}$

$tg\, x = \frac{1}{\overline{EB}} \Rightarrow \overline{EB} = \frac{1}{tg\, x} \Rightarrow \boxed{\overline{AE} = 1 - \frac{1}{tg\, x}}$ e $\boxed{\overline{AF} = 1 - \frac{1}{tg\, x}}$

Perímetro $= 2 \times \frac{1}{\sin x} + 2\left(1 - \frac{1}{tg\, x}\right) = \frac{2}{\sin x} + 2 - \frac{2}{tg\, x}$ c.q.d.

b) $\lim\limits_{x \to \frac{\pi}{2}^-} f(x) = \lim\limits_{x \to \frac{\pi}{2}^-} \left(\frac{2}{\sin x} + 2 - \frac{2}{tg\, x}\right) = \left(\frac{2}{1} + 2 - \frac{2}{\infty}\right) =$

$= 4$

Se $x \to \frac{\pi}{2}^-$ temos Perímetro do quadrado 1 1 1 1.

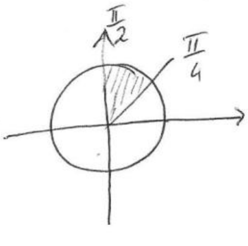
c) $f(x) = 2 - \frac{2}{tg\, x} + \frac{2}{\sin x}$

$$f'(x) = -\frac{2'\, tg\, x - 2(tg\, x)'}{tg^2 x} + \frac{2' \sin x - 2(\sin x)'}{\sin^2 x} =$$

$$= -\frac{0 - 2\frac{1}{\cos^2 x}}{tg^2 x} + \frac{0 - 2\cos x}{\sin^2 x} =$$

$$= \frac{2}{\cos^2 x\, tg^2 x} - \frac{2\cos x}{\sin^2 x} = \frac{2}{\cos^2 x \frac{\sin^2 x}{\cos^2 x}} - \frac{2\cos x}{\sin^2 x} =$$

$$= \frac{2}{\sin^2 x} - \frac{2\cos x}{\sin^2 x} = \frac{2 - 2\cos x}{\sin^2 x}$$ c.q.d.



$2 - 2\cos x = 0$

$2\cos x = 2$

$\cancel{\cos x = 1}$ impossível em $]\frac{\pi}{4}; \frac{\pi}{2}[$

$\sin^2 x = 0$

$\sin x = 0$ impossível em $]\frac{\pi}{4}; \frac{\pi}{2}[$

	$\frac{\pi}{4}$		$\frac{\pi}{2}$
$2 - 2\cos x$	X	+	X
$\sin^2 x$	X	+	X
f'	X	+	X
f	X	↗	X

f é ↗ em $]\frac{\pi}{4}; \frac{\pi}{2}[$

⑦ $f(x) = \dfrac{\cos x}{1+\cos x}$ em $]-\pi, \pi[$

a) $D =]-\pi, \pi[$

Como a função está definida em $]-\pi, \pi[$, não podem existir assimptotas não verticais. (Domínio limitado)

	-2π		$-\pi$		π		2π
$1+\cos x$		+	0	+	0	+	

Verticais:

$\lim\limits_{x \to -\pi^+} f(x) = \lim\limits_{x \to -\pi^+} \dfrac{\cos x}{1+\cos x} = \dfrac{-1}{0^+} = -\infty \rightarrow \boxed{x = -\pi}$ A.V. unilateral

$\lim\limits_{x \to \pi^-} f(x) = \lim\limits_{x \to \pi^-} \dfrac{\cos x}{1+\cos x} = \dfrac{-1}{0^+} = -\infty \rightarrow \boxed{x = \pi}$ A.V. unilateral

b) $f'(x) = \dfrac{(\cos x)'(1+\cos x) - \cos x(1+\cos x)'}{(1+\cos x)^2} = \dfrac{-\sin x(1+\cos x) - \cos x(-\sin x)}{(1+\cos x)^2} =$

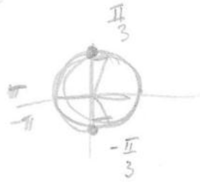
$= \dfrac{-\sin x - \cancel{\sin x \cos x + \cos x \sin x}}{(1+\cos x)^2} = \dfrac{-\sin x}{(1+\cos x)^2}$

O sinal de $f'(x)$ apenas depende de $-\sin x$ visto que $(1+\cos x)^2 > 0$, $\forall x \in]-\pi, \pi[$.

$f'(x) = 0 \Leftrightarrow -\sin x = 0 \Leftrightarrow \sin x = 0 \Leftrightarrow x = 0$ por $x \in]-\pi, \pi[$

	$-\pi$		0		π
$f'(x)$	X	+	0	−	X
$f(x)$	X	↗	M	↘	X

Máx $= f(0) = \dfrac{\cos 0}{1+\cos 0} = \dfrac{1}{1+1} = \dfrac{1}{2}$ ✓



c) $A_{\text{trapézio}} = \dfrac{B+b}{2} \times h$

$h = \dfrac{1}{3}$

$B = ?$ $f(x) = 0 \Leftrightarrow \cos x = 0 \Leftrightarrow x = \dfrac{\pi}{2}$ em $]-\pi, \pi[$, logo $B = \dfrac{\pi}{2}$

$b = ?$ $f(x) = \dfrac{1}{3} \Leftrightarrow \dfrac{\cos x}{1+\cos x} = \dfrac{1}{3} \Leftrightarrow 3\cos x = 1 + \cos x \Leftrightarrow 2\cos x = 1 \Leftrightarrow \cos x = \dfrac{1}{2} \Leftrightarrow$

$\Leftrightarrow x = \dfrac{\pi}{3}$ em $]-\pi, \pi[$, logo $b = \dfrac{\pi}{3}$

$\therefore A = \dfrac{\frac{\pi}{2} + \frac{\pi}{3}}{2} \times \dfrac{1}{3} = \dfrac{5\pi}{12} \times \dfrac{1}{3} = \dfrac{5\pi}{36}$

⑧ $f(x) = x + 2\cos x$ em $[0, 2\pi]$

a) $f'(x) = 1 - 2\sin x$

$f'(x) = 0 \Leftrightarrow 1 - 2\sin x = 0 \Leftrightarrow \sin x = \frac{1}{2} \Leftrightarrow x = \frac{\pi}{6} \vee x = \frac{5\pi}{6}$ em $[0, 2\pi]$

Como A e B são pontos do gráfico com tangente horizontal, então as abcissas de A e B são, respetivamente, $\frac{\pi}{6}$ e $\frac{5\pi}{6}$.

$f\left(\frac{\pi}{6}\right) = \frac{\pi}{6} + 2\cos\left(\frac{\pi}{6}\right) = \frac{\pi}{6} + \cancel{2} \times \frac{\sqrt{3}}{\cancel{2}} = \frac{\pi + 6\sqrt{3}}{6}$

$f\left(\frac{5\pi}{6}\right) = \frac{5\pi}{6} + 2\cos\left(\frac{5\pi}{6}\right) = \frac{5\pi}{6} + \cancel{2} \times \left(-\frac{\sqrt{3}}{\cancel{2}}\right) = \frac{5\pi - 6\sqrt{3}}{6}$.

$A\left(\frac{\pi}{6} ; \frac{\pi + 6\sqrt{3}}{6}\right) \quad B\left(\frac{5\pi}{6} , \frac{5\pi - 6\sqrt{3}}{6}\right)$

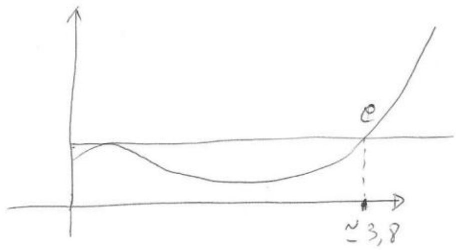
b) Em $[0, 2\pi]$, e pela observação do gráfico, o mínimo é $f\left(\frac{5\pi}{6}\right)$ e o máximo é $f(2\pi)$.

Como $f(2\pi) = 2\pi + 2\cos(2\pi) = 2\pi + 2$, logo

$D' = \left[\frac{5\pi - 6\sqrt{3}}{6} ; 2 + 2\pi\right]$

c) $y_1 = x + 2\cos x$

$y_2 = \frac{\pi + 6\sqrt{3}}{6}$



$[0, 2\pi] \times [0, 2 + 2\pi]$

$\therefore x \simeq 3,8$

⑨

$\cos x = \frac{1}{h} \Leftrightarrow h = \frac{1}{\cos x}$

$A_{\text{quinta}} = 4 + 4 \times \frac{\cancel{2} \times \frac{1}{\cos x}}{\cancel{2}} = 4 + \frac{4}{\cos x} = \frac{4\cos x + 4}{\cos x}$

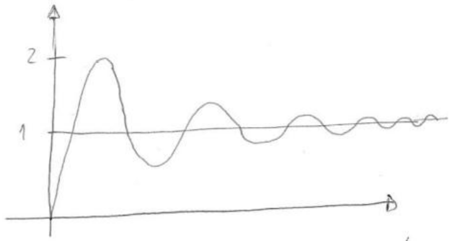
com $x \in \left]0, \frac{\pi}{2}\right[$

b) $\lim_{x \to \frac{\pi}{2}^-} A(x) = \lim_{x \to \frac{\pi}{2}^-} \frac{4 + 4\cos x}{\cos x} = \frac{4}{0^+} = +\infty$

Quando x aumenta, as arestas laterais tornam-se no limite, retas perpendiculares à base; a área aumenta infinitamente.
prisma quadrangular com base superior

(10) $f(x) = \dfrac{x + 3\sin\frac{x}{2}}{\ln(e^x + x)}$

a)



$[0, 150] \times [0, 2]$

$\lim_{x \to +\infty} f(x) = 1$.

b) A observação do comportamento de uma função não pode ser conclusiva, pois é sempre uma observação limitada, já que também a máquina é limitada.

(11) $f(x) = 2x - \cos x$

a) $f(x)$ é contínua em $\mathbb{R}$ (diferença de 2 f.ões contínuas), logo é contínua em $[0, \pi]$.

$f(0) = -1 < 0$
$f(\pi) = 2\pi + 1 > 0$

Pelo Corolário do Teorema de Bolzano, existe, pelo menos, um zero em $]0, \pi[$.

b) $f'(x) = 2 + \sin x$

$-1 < \sin x < 1 \Leftrightarrow -1 + 2 < 2 + \sin x < 1 + 2 \Leftrightarrow 1 < 2 + \sin x < 3$

Então $f'(x) > 0, \forall x \in \mathbb{R}$, logo f é estrita/ crescente. Conclusão: f terá no máximo um zero. Pela alínea anterior, prova-se que existe. Então é único.

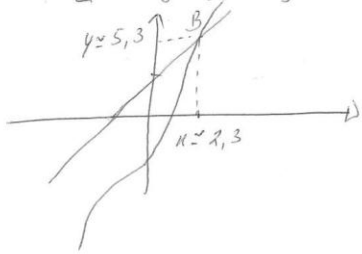
c) Coordenadas de B ?

e) Calculadora

$y = x + 3 \quad (tg\,45° = 1)$

$y = 2x - \cos x$

$[-10,10] \times [-10,10]$



$A_{\Delta} = \frac{b \times h}{2} = \frac{3 \times 5,3}{2} \simeq 8.$

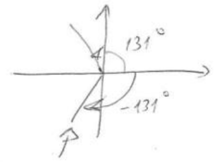
(12) $d = \frac{7820}{1 + 0,07 \cos x}$

a) $x = 180° \rightarrow d = \frac{7820}{1 + 0,07(-1)} \simeq 8409$ Km

Altitude $= 8409 - 6378 = 2031$ Km

b) $d(x) = 8200 \Leftrightarrow \frac{782\cancel{0}}{1 + 0,07 \cos x} = 820\cancel{0} \Leftrightarrow 1 + 0,07 \cos x = \frac{782}{820} \Leftrightarrow$

$\Leftrightarrow 0,07 \cos x = \frac{782}{820} - 1 \Leftrightarrow \ldots \; x \simeq 131° \vee x \simeq -131°$



Como $x \in 3.^{\circ}Q$ então $x \simeq 180° + (180° - 131°) = 229°$

(13) $f(n) = 12,2 + 2,64 \sin \frac{\pi(n-81)}{183}$

a)

24 Março $\rightarrow 31 + 29 + 24 = 84$

$f(84) \simeq 12,336 \rightarrow$ 12h 20 min

$$\begin{array}{r} 6\text{h } 30 \text{ min} \\ +\,12\text{h } 20 \text{ min} \\ \hline 18\text{h } 50 \text{ min} \end{array}$$

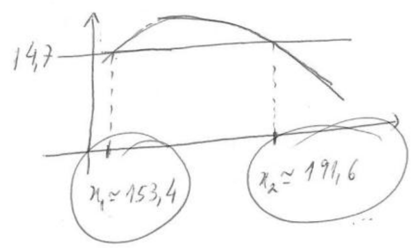
O pôr-do-sol ocorreu às 18h50min.

b) $f(n) > 14,7 \Leftrightarrow n \in ?$

$[150, 250] \times [12, 15]$

$y_1 = 12,2 + 2,64 \sin \frac{\pi(n-81)}{183}$

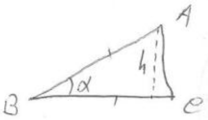
$y_2 = 14,7$



$191,6 - 153,4 \simeq 38$ dias

14

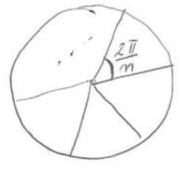
a) $A_\Delta = \frac{b\times h}{2} = \frac{\overline{BC}\times\overline{BC}\sin\alpha}{2} = \frac{\overline{BC}}{2}\sin\alpha$



$(\alpha \in]0,\pi[$

$\sin\alpha = \frac{h}{\overline{AB}} = \frac{h}{\overline{BC}}$

b)



$A_\Delta = \frac{r}{2}\sin\left(\frac{2\pi}{n}\right) = \frac{1}{2}\sin\left(\frac{2\pi}{n}\right) \Leftrightarrow h = \overline{BC}\sin\alpha$

Para n Δs, temos

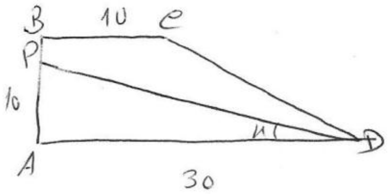
$A_{polígono} = n\times\frac{1}{2}\sin\left(\frac{2\pi}{n}\right) = \frac{n}{2}\sin\left(\frac{2\pi}{n}\right) = A_n$

c) $\lim\limits_{n\to+\infty} A_n = \lim\limits_{n\to+\infty}\frac{n}{2}\sin\left(\frac{2\pi}{n}\right) = \lim\limits_{n\to+\infty}\frac{\sin\left(\frac{2\pi}{n}\right)}{\frac{2}{n}} =$

$= \lim\limits_{\frac{2\pi}{n}\to 0}\underbrace{\frac{\sin\left(\frac{2\pi}{n}\right)}{\frac{2\pi}{n}}}_{1}\times\pi = \pi$

Quando o número de lados aumenta, o polígono tende para o círculo.
Quando $n\to+\infty$, a área do polígono tende para a área do círculo, que, neste caso, tem $r=1$ e portanto, área $=\pi$ ($\pi r^2 = A$).

15



$A = \frac{B+b}{2} \times h = \frac{30+10}{2} \times 10 =$

$= 200$

Metade $= 100$

$tg\, x = \frac{\overline{AP}}{30} \Rightarrow \overline{AP} = 30\, tg\, x$, logo $A = \frac{b \times h}{2} =$

$= \frac{30 \times 30\, tg\, x}{2} = \frac{30^2\, tg\, x}{2}$

Então $\frac{30^2\, tg\, x}{2} = 100$. Resposta (B)

16 $\begin{cases} d(0) = 2 \\ d\left(\frac{\pi}{2}\right) = 1 \\ d(\pi) = 0 \end{cases}$

$\boxed{(A)\ d(\alpha) = 1 + \cos \alpha}$

(B) $d(\alpha) = 2 + \sin \alpha$

$d\left(\frac{\pi}{2}\right) = 2 + 1 = 3$ X

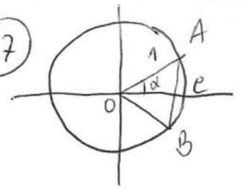
(C) $d(\alpha) = 1 - \cos \alpha$

$d(0) = 1 - 1 = 0$ X

(D) $d(\alpha) = 2 - \sin \alpha$

$d(\pi) = 2 - 0 = 2$ X

(17)



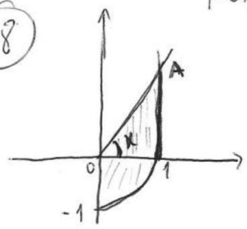
$\sin\alpha = \frac{\overline{Ae}}{1} \Rightarrow \boxed{\overline{Ae} = \sin\alpha}$

$\boxed{\cos\alpha = \overline{Oe}}$

$A_{\triangledown} = \frac{b \times h}{2} = \frac{2\sin\alpha\cos\alpha}{\cancel{2}} = \sin\alpha\cos\alpha$

Resposta (A)

(18)



$A_{total} = A_{\text{sector}} + A_{\triangle}$

$\operatorname{tg} x = \frac{h}{1} \Rightarrow h = \operatorname{tg} x$

$A_{\text{sector}} = \frac{1}{4}\pi r^2 = \frac{1}{4}\pi$

$A_{\triangle} = \frac{b \times h}{2} = \frac{1 \times \operatorname{tg} x}{2}$

$A_{total} = \frac{\pi}{4} + \frac{\operatorname{tg} x}{2}$ Resposta (C)

(19) $\lim_{x \to 0^+} \frac{\ln x}{\sin x} = \frac{-\infty}{0^+} = -\infty$ Resposta (A)

(20) $h(x) = \sin x$

$h'(x) = \cos x \rightarrow$ "Declive"

(A) $y = \sqrt{2}\,x - 9$ imp
>1

$\boxed{(B)\ y = x}$

$h'(0) = \cos 0 = 1$

(C) $y = 2x + \pi$ imp
>1

$h(0) = \sin 0 = 0$

(D) $y = -2$ imp

$y = x + 0 \Leftrightarrow y = x$

(21) (D) Não existe $\lim_{x \to +\infty} \sin x$.