Resolução da 2.ª ficha Trigonometria - 12.º ano.

(1) $V(x) = 80(x - \operatorname{sen} x)$

$[0; 2\pi]$

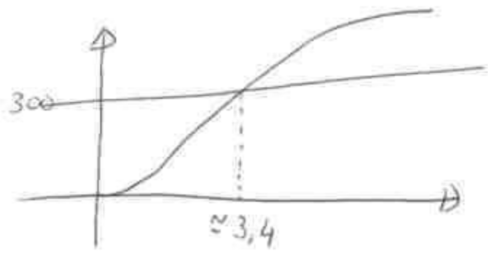
a) $V(2\pi) = 80(2\pi - \operatorname{sen} 2\pi) = 80 \times 2\pi = 160\pi \simeq 503\ m^3$

b) $V(x) = 300$

$y_1 = 80(x - \operatorname{sen} x)$

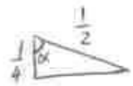
$y_2 = 300$

Janela:



R: 3,4 rad

c)



$\cos\alpha = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{2}{4} = \frac{1}{2}$

$\Downarrow$

$\alpha = \frac{\pi}{3} \rightarrow 2\alpha = \frac{2\pi}{3}$

$V\left(\frac{2\pi}{3}\right) = 80\left(\frac{2\pi}{3} - \operatorname{sen}\frac{2\pi}{3}\right) \simeq 98\ m^3$

d) D X → a altura não cresce linearmente

C X → " " " decresce nunca

A X → a " cresce + rápido/ no início e no fim e não o contrário.

Resposta: (B)

(2) A área é máx. para $\frac{\pi}{2}$ e $\frac{3\pi}{2}$ logo (A)

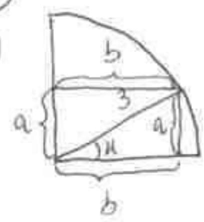
(3) $P = \frac{14\pi}{9} - \frac{2\pi}{9} = \frac{12\pi}{9} = \frac{4\pi}{3}$ (D)

(4) $f(x) = \cos x$

$f'(\pi) = \lim_{x \to \pi} \frac{\cos x - \cos\pi}{x - \pi} = \lim_{x \to \pi} \frac{\cos x + 1}{x - \pi}$ (A)

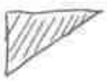
(5)

a)



$\sin x = \frac{a}{3} \Leftrightarrow a = 3\sin x$

$\cos x = \frac{b}{3} \Leftrightarrow b = 3\cos x$



$A = \frac{3\sin x \; 3\cos x}{2} = \frac{9 \sin x \cos x}{2}$

$2\pi \;—\; \pi r^2$

$x \;-\; A$

$A = \frac{\cancel{\pi} r^2 x}{2\cancel{\pi}} = \frac{9x}{2}$

Área Total $= 4 \times \frac{9\sin x \cos x}{2} + 4 \times \frac{9x}{2} = \frac{36 \sin x \cos x}{2} + \frac{36x}{2} =$

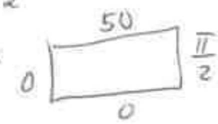
$= 18 \sin x \cos x + 18x = 18(x + \sin x \cos x)$

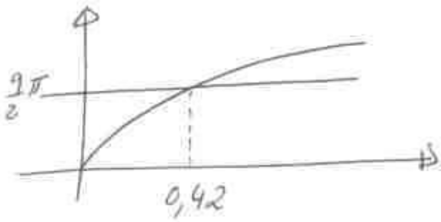
b) $A(x) = \frac{1}{2} A_{\circ} \Leftrightarrow A(x) = \frac{1}{2}\pi 9 \Leftrightarrow A(x) = \frac{9}{2}\pi$

$y_1 = 18(x + \sin x \times \cos x)$

$y_2 = 9\pi : 2$

Janela:





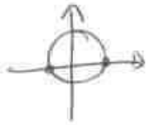
R: $x = 0{,}42$ rad

(6) a) $b(t) = p(t) \Leftrightarrow \cancel{10} + e^{-0,1t}\operatorname{sen}(\pi t) = \cancel{10} - 1{,}37 e^{-0,1t}\operatorname{sen}(\pi t) \Leftrightarrow$

$\Leftrightarrow e^{-0,1t}\operatorname{sen}(\pi t) + 1{,}37\, e^{-0,1t}\operatorname{sen}(\pi t) = 0 \Leftrightarrow$

$\Leftrightarrow e^{-0,1t}\left(\operatorname{sen}(\pi t) + 1{,}37 \operatorname{sen}(\pi t)\right) = 0 \Leftrightarrow$

$\Leftrightarrow \cancel{e^{-0,1t} = 0} \lor 2{,}37 \operatorname{sen}(\pi t) = 0 \Leftrightarrow$



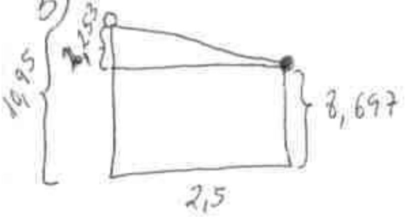
$\Leftrightarrow \operatorname{sen}(\pi t) = 0 \Leftrightarrow$

$\Leftrightarrow \cancel{\pi} t = K\cancel{\pi} \Leftrightarrow$

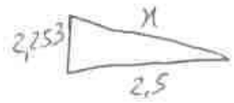
$\Leftrightarrow t = K$

R: Entre os instantes $t = 0$ e $t = 5$ temos:
$t = 0$; $t = 1$; $t = 2$; $t = 3$; $t = 4$ e $t = 5$ (6 vezes)

b)



$b(0{,}5) = 10{,}95$
$p(0{,}5) = 8{,}697$



$x^2 = 2{,}253^2 + 2{,}5^2 \Leftrightarrow x \simeq 3{,}4$ cm

⑦ $[0; 2\pi]$ $f(x) = \operatorname{sen} x$

a) declive de a $= f'(a)$
declive de b $= f'(b)$

$a + b = 2\pi \Rightarrow b = 2\pi - a$. Então $\cos b = \cos(2\pi - a)$, ou seja, $\cos b = \cos a$ isto é $f'(b) = f'(a)$ c.q.d.

b) $]0; 2\pi[\setminus \{\pi\}$ $g(x) = \dfrac{x}{\operatorname{sen} x}$

Verticais:

$\lim\limits_{x\to 0^+} g(x) = \lim\limits_{x\to 0^+} \dfrac{x}{\sin x} = \lim\limits_{x\to 0^+} \dfrac{1}{\frac{\sin x}{x}} = \dfrac{1}{1} = 1$

$\lim\limits_{x\to 2\pi^-} g(x) = \lim\limits_{x\to 2\pi^-} \dfrac{x}{\sin x} = \dfrac{2\pi}{0^-} = -\infty \rightarrow \boxed{x = 2\pi \text{ A.V.}}$

$\lim\limits_{x\to \pi} g(x) = \lim\limits_{x\to \pi} \dfrac{x}{\sin x} = \dfrac{\pi}{0} = \infty \rightarrow \boxed{x = \pi \text{ A.V.}}$

Atendendo ao domínio, não pode haver assínt. não verticais.