Resolução da ficha n.º 3 - Trigonometria - 12.º ano

① Igual ao ex.º ~~1~~ da ficha 5 de funções

② $a = A\sqrt{\cos\theta}$ e $a = \pi r^2$ e $A = \pi R^2$ $\quad R = \sqrt[4]{2}\, r$

Substituindo a e A, temos $\quad \swarrow r^2 = (\sqrt[4]{2}\, r)^2 \sqrt{\cos\theta}$

$\cancel{\pi} r^2 = \cancel{\pi} R^2 \sqrt{\cos\theta} \Leftrightarrow r^2 = R^2 \sqrt{\cos\theta} \Leftrightarrow r^2 = (\sqrt[4]{2}\, r)^2\sqrt{\cos\theta}$

$\Leftrightarrow r^2 = (2^{\frac{1}{4}} \times r)^2 \sqrt{\cos\theta} \Leftrightarrow r^2 = (2^{\frac{1}{4}})^2 r^2 \sqrt{\cos\theta} \Leftrightarrow$

$\Leftrightarrow r^2 = 2^{\frac{1}{2}} r^2 \sqrt{\cos\theta} \Leftrightarrow \cancel{r^2} = \sqrt{2}\, \cancel{r^2} \sqrt{\cos\theta} \Leftrightarrow 1 = \sqrt{2}\sqrt{\cos\theta} \Leftrightarrow$

$\Leftrightarrow \sqrt{\cos\theta} = \frac{1}{\sqrt{2}} \Leftrightarrow \cos\theta = \left(\frac{1}{\sqrt{2}}\right)^2 \Leftrightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$

$\theta \in \left]0; \frac{\pi}{2}\right[$

③ $f(x) = e^{x-1} \quad g(x) = \sin x \quad f'(x) = e^{x-1} \quad g'(x) = \cos x$

$h(x) = f'(x) - g'(x) = e^{x-1} - \cos x$

a) h é cont. em IR (pois f' e g' são cont. em IR) logo é t.s. cont. em $\left[0; \frac{\pi}{2}\right]$

$h(0) = e^{0-1} - \cos 0 \simeq -0{,}63 < 0$

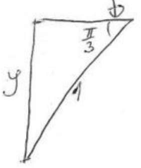
$h\left(\frac{\pi}{2}\right) = e^{\frac{\pi}{2}-1} - \cos\left(\frac{\pi}{2}\right) \simeq 1{,}77 > 0$

Como $h(0) \times h\left(\frac{\pi}{2}\right) < 0$, pelo Corolário do Teorema de Bolzano, concluímos que h tem pelo menos um zero em $\left]0; \frac{\pi}{2}\right[$.

b) Pela alínea a) existe $a \in \left]0; \frac{\pi}{2}\right[$ tal que $h(a) = 0$, ou seja, $f'(a) - g'(a) = 0 \Leftrightarrow f'(a) = g'(a)$. Como as derivadas são iguais em a (os declives das ~~as~~ retas tangentes t.s. o são). Assim podemos concluir que as retas são paralelas.

④ $\alpha(0) = \frac{\pi}{2} - \frac{\pi}{6}\cos(0) = \frac{\pi}{2} - \frac{\pi}{6} = \frac{2\pi}{6} = \frac{\pi}{3}$

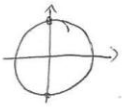
a)



$\text{sen}\left(\frac{\pi}{3}\right) = \frac{y}{1} \Leftrightarrow \frac{\sqrt{3}}{2} = \frac{y}{1} \Leftrightarrow y = \frac{\sqrt{3}}{2}$

b) $\alpha(t) = \frac{\pi}{2} \Leftrightarrow \cancel{\frac{\pi}{2}} - \frac{\pi}{6}\cos(\sqrt{9{,}8}\,t) = \cancel{\frac{\pi}{2}} \Leftrightarrow \cos(\sqrt{9{,}8}\,t) = 0 \Leftrightarrow$

$\Leftrightarrow \sqrt{9{,}8}\,t = \frac{\pi}{2} \Leftrightarrow t = \frac{\frac{\pi}{2}}{\sqrt{9{,}8}} \Leftrightarrow t \simeq 0{,}5$



⑤

a) distância mínima $= d(0) = 149{,}6(1 - 0{,}0167\cos 0) \simeq 147{,}1$

" máxima $= d(\pi) = 149{,}6(1 - 0{,}0167\cos\pi) \simeq 152{,}1$

5.1) $x = \pi$ $\frac{2\pi t}{T} = \pi - 0{,}0167\sin\pi \Leftrightarrow \frac{2\pi t}{T} = \pi \Leftrightarrow 2\cancel{\pi}t = \cancel{\pi}T \Leftrightarrow$

$\Leftrightarrow t = \frac{T}{2}$ c.q.d.

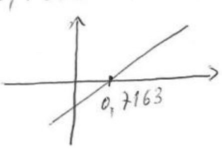
Interpretação: o tempo que decorre entre a passagem da Terra pelo Periélio até ao ponto da órbita da Terra mais afastado do Sol é metade do tempo que a Terra demora a descrever uma órbita completa.

5.2) Entre 4 Jan e 14 Fev. decorrem 41 dias

$t = 41$ e $T = 365{,}24$

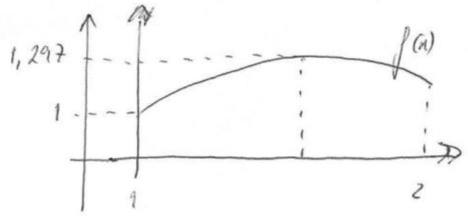
$\frac{2\pi \times 41}{365{,}24} = x - 0{,}0167\sin x \Leftrightarrow 0{,}7053 = x - 0{,}0167\sin x \Leftrightarrow$

$\Leftrightarrow x - 0{,}0167\sin x - 0{,}7053 = 0$



$d(0{,}7163) =$
$= 149{,}6(1 - 0{,}0167\cos 0{,}7163) =$
$= 147{,}7$ milhões

(6)



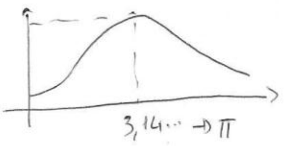
$$\left.\begin{array}{l} D'_f = [1 ; 1{,}297] \longrightarrow \text{amplitude} = 0{,}297 \\ D'_g = [4 ; 5] \longrightarrow \text{amplitude} = 1 \end{array}\right\} \ a = \frac{1}{0{,}297} = 3{,}37$$

Se $a = 3{,}367$ $\quad a \times f(x) = 3{,}367 \times f(x)$ tem por contradomínio

$$[1 \times 3{,}367 ; 1{,}297 \times 3{,}367] = [3{,}367 ; 4{,}367] \xrightarrow{+0{,}633} [4,5]$$

Para que o contradomínio seja $[4,5]$, então $b \simeq 0{,}63$.

(7)



(C)

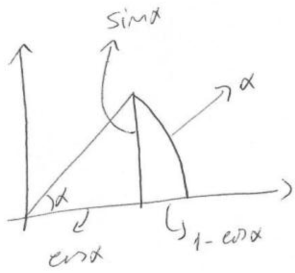
ou $f'(x) = 2 \sin x$

$f'(x) = 0 \Leftrightarrow$
$\Leftrightarrow 2 \sin x = 0 \Leftrightarrow$
$\Leftrightarrow \sin x = 0$
$\Leftrightarrow x = \pi + k\pi , k \in \mathbb{Z}$

	0		π		2π
f'		+	0	−	
f		↗	M	↘	

(8)

$P_{\triangle} = \alpha r = \alpha \times 1 = \alpha$



Perímetro pretendido =
$= 1 + \alpha + \sin\alpha - \cos\alpha$

(D)

(9) $g(x) = 2 + \sin(4x)$

a) $g'(0) = \lim\limits_{x \to 0} \dfrac{g(x) - g(0)}{x - 0} = \lim\limits_{x \to 0} \dfrac{\cancel{2} + \sin(4x) - \cancel{2} - \sin 0}{x} =$

$= \lim\limits_{x \to 0} \dfrac{\sin(4x)}{x} = \lim\limits_{x \to 0} \left(\underbrace{\dfrac{\sin(4x)}{4x}}_{1} \times 4 \right) = 1 \times 4 = 4$

b) $g'(x) = 4\cos(4x)$

Zeros de g'

$4\cos(4x) = 0 \Leftrightarrow$

$\Leftrightarrow \cos(4x) = 0$

$\Leftrightarrow 4x = \frac{\pi}{2} + k\pi, k \in \mathbb{Z}$

$\Leftrightarrow x = \frac{\pi}{8} + \frac{k\pi}{4}; k \in \mathbb{Z}$

$k = 0 \to x = \frac{\pi}{8}$

$k = 1 \to x = \frac{3\pi}{8}$

$k = 2 \to \cancel{x = \frac{\pi}{8} + \frac{\pi}{2}}$

x	0		$\frac{\pi}{8}$		$\frac{3\pi}{8}$		$\frac{\pi}{2}$
g'		$+$	0	$-$	0	$+$	
g		$\nearrow$	M	$\searrow$	m	$\nearrow$	

$\nearrow$ em $]0; \frac{\pi}{8}[$ e em $]\frac{3\pi}{8}; \frac{\pi}{8}[$

$\searrow$ em $]\frac{\pi}{8}; \frac{3\pi}{8}[$

máx $= g\left(\frac{\pi}{8}\right) = 2 + \sin\left(\frac{4\pi}{8}\right) = 2 + \sin\left(\frac{\pi}{2}\right) = 2 + 1 = 3$

mín $= g\left(\frac{3\pi}{8}\right) = 2 + \sin\left(\frac{12\pi}{8}\right) = 2 + \sin\left(\frac{3\pi}{2}\right) = 2 - 1 = 1$

(10) A. H:

$\lim\limits_{x \to +\infty} f(x) = \lim\limits_{x \to +\infty} e^{-4x+1} = e^{-\infty+1} = e^{-\infty} = 0 \Rightarrow y = 0$ A. H.

A. V.

Se existir, será apenas para $x = 0$

$\lim\limits_{x \to 0^+} f(x) = \lim\limits_{x \to 0^+} e^{-4x+1} = e^{1} = e$

$\lim\limits_{x \to 0^-} f(x) = \lim\limits_{x \to 0^-} \dfrac{3\sin x}{x^2} = \lim\limits_{x \to 0^-} \left(3 \underbrace{\dfrac{\sin x}{x}}_{1} \times \dfrac{1}{x} \right) = 3 \times 1 \times \dfrac{1}{0^-} = -\infty$

logo $x = 0$ A. V.

(11) Resumidamente:

- Exclui o gráfico 1 porque x varia de 0 a π
- " " 4 " " " de $]0; \pi[$
- " " 3 " e área é constante (base e altura
permanecem constantes).

OBS: A composição deve ser mais detalhada e rigorosa.

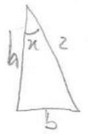
(12) $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \log_{\sqrt{2}}(k+x) = \log_{\sqrt{2}}(k)$

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \frac{\sin(2x)}{x} = \lim_{x\to 0^-} \left(\frac{\sin(2x)}{2x}\right)\times 2 = 2$, com $\frac{\sin(2x)}{2x} \to 1$

$\log_{\sqrt{2}} k = 2 \Leftrightarrow k = \sqrt{2}^2 \Leftrightarrow k = 4$

(13) $\sin x = \frac{b}{2} \Leftrightarrow b = 2\sin x$

$\cos x = \frac{h}{2} \Rightarrow h = 2\cos x$



$A_\Delta = \frac{b\times h}{2} = \frac{\cancel{2}\sin x \times 2\cos x}{\cancel{2}}$

$= 2\sin x\cos x = \sin(2x)$

$A_{\text{pedida}} = A_{\bigcirc} - A_\Delta = \pi - \sin(2x)$

(14) $f(x) = \sin(2x)\cos x$

a) $f'(x) = (\sin(2x))'\cos x + (\sin(2x))(\cos x)'$

$= 2\cos(2x)\cos x + \sin(2x)(-\sin x)$

$= 2\cos(2x)\cos x - \sin(2x)\sin x$

$m = f'(0) = 2\cos 0\cos 0 - \sin 0\sin 0 = 2$

$f(0) = \sin 0\cos 0 = 0$

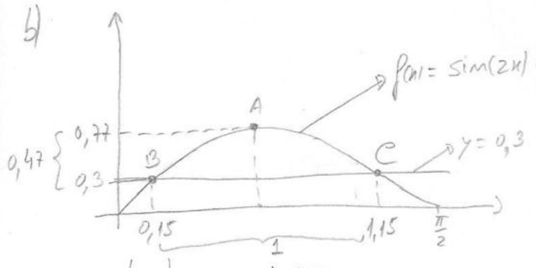
$y = 2x + b$

$(0,0) \rightsquigarrow 0 = 2\times 0 + b$

$0 = b$

$\therefore y = 2x$

b)



$A = \frac{b\times h}{2} = \frac{1\times 0{,}47}{2} \simeq 0{,}2$

(15)

a) $\lim_{x\to -\infty} \left(f(x) - (ax+b)\right) =$

$= \lim_{x\to -\infty} \left(\cancel{ax} + \cancel{b} + e^x - \cancel{ax} - \cancel{b}\right) =$

$= \lim_{x\to -\infty} (e^x) = e^{-\infty} = 0$

Como $\lim_{x\to -\infty} \left(f(x) - (ax+b)\right) = 0$

então $y = ax + b$ é A. O.

b) $\lim_{x\to 0^-} (ax + b + e^x) = 0 + b + e^0 = b + 1$

$\lim_{x\to 0^+} \frac{x - \sin(2x)}{x} = \lim_{x\to 0^+} \left(\frac{x}{x} - \frac{\sin(2x)}{2x}\times 2\right)$, com $\frac{\sin(2x)}{2x} \to 1$

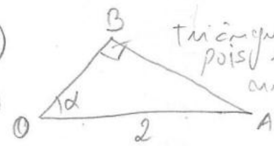
$= 1 - 1\times 2 = -1$

$f(0) = b + 1$

Então $b + 1 = -1 \Leftrightarrow b = -2$

(16) Triângulo retângulo, pois está inscrito numa circunf.

a)



$\sin\alpha = \frac{\overline{AB}}{2} \Leftrightarrow \overline{AB} = 2\sin\alpha$

$\cos\alpha = \frac{\overline{OB}}{2} \Leftrightarrow \overline{OB} = 2\cos\alpha$

$f(\alpha) = 2 + 2\sin\alpha + 2\cos\alpha =$

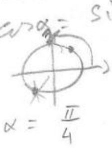
$2(1 + \cos\alpha + \sin\alpha)$ c.q.d.

b) $f'(\alpha) = 2\cos\alpha - 2\sin\alpha$

Raízes

$2(\cos\alpha - \sin\alpha) = 0$

$\cos\alpha = \sin\alpha$



$\alpha = \frac{\pi}{4}$

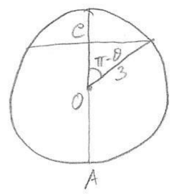
	0		$\frac{\pi}{4}$		$\frac{\pi}{2}$
f'	ND	+	0	−	ND
f	ND	↗	Max	↘	ND

R: $\alpha = \frac{\pi}{4}$

17

a)

Se $\theta \in \left]\frac{\pi}{2}, \pi\right[$



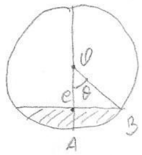
$\cos(\pi-\theta) = \frac{\overline{OC}}{3}$

$\overline{OC} = 3\cos(\pi-\theta)$

$\overline{OC} = -3\cos\theta$

$\overline{AC} = \overline{AO} + \overline{OC} = 3 - 3\cos\theta$

Se $\theta \in \left]0, \frac{\pi}{2}\right]$



$\cos\theta = \frac{\overline{OC}}{3}$

$\overline{OC} = 3\cos\theta$

$\overline{AC} = 3 - \overline{OC} =$

$= 3 - 3\cos\theta$

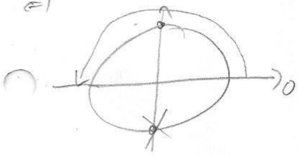
b) $h(\theta) = 3 \Leftrightarrow$

$\Leftrightarrow \cancel{3} - 3\cos(\theta) = \cancel{3} \Leftrightarrow$

$\Leftrightarrow -3\cos(\theta) = 0$

$\Leftrightarrow \cos(\theta) = 0$

$\Leftrightarrow$



$\theta = \frac{\pi}{2}$

Para que a altura do combustível seja 3 cm, o depósito tem que estar meio e o ângulo θ formado é $\frac{\pi}{2}$ rad.