Resolução da ficha n.º 2 - Complexos - 12.º ano

(1) $Re(z) \geq -1 \wedge Im\, z \geq 0 \wedge |z-i| \geq |z+1|$ (C)

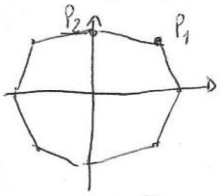
$x \geq -1$ $\quad y \geq 0 \quad z-(0+i) \quad z-(-1+0i)$

$(0,1) \quad (-1,0)$

(2) Rotação de $\frac{2\pi}{4}$ com o mesmo ρ (B)

(3) $z_1 = 2e^{i\frac{\pi}{4}}$

$z_2 = 2i$



$n=8$ (C)

(4) $e^{i\frac{\pi}{6}}$ e $e^{i\frac{5\pi}{6}}$ (A)

$\left(e^{i\frac{\pi}{6}}\right)^3 = e^{i\left(\frac{3\pi}{6}\right)} = e^{i\frac{\pi}{2}}$ ✓

$\left(e^{i\frac{5\pi}{6}}\right)^3 = e^{i\frac{15\pi}{6}} = e^{i\frac{5\pi}{2}} = e^{i\frac{\pi}{2}}$ ✓

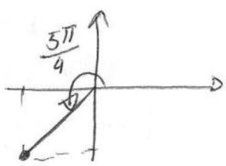
$\frac{5\pi}{6} - \frac{\pi}{6} = \frac{4\pi}{6} = \frac{2\pi}{3}$ ✓

(5) $z_1 = -6+3i \qquad z_2 = 1-2i$

$$\frac{z_1 + i^{23}}{z_2} = \frac{-6+3i+i^3}{1-2i} = \frac{-6+3i-i}{1-2i} = \frac{-6+2i}{1-2i} = \frac{(-6+2i)(1+2i)}{1-4i^2} =$$

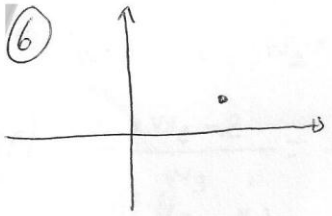
$$= \frac{-6-12i+2i-4}{1+4} = \frac{-10-10i}{5} = -2-2i = 2\sqrt{2}e^{i\frac{5\pi}{4}}$$

$\rho = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$



$$\begin{cases} \cos\alpha = \frac{-2}{2\sqrt{2}} = -\frac{\sqrt{2}}{2} \\ \sin\alpha = \frac{-2}{2\sqrt{2}} = -\frac{\sqrt{2}}{2} \end{cases}$$

(6)



$z = \rho e^{i\theta}$

$z^3 = \rho^3 e^{i(3\theta)}$

$0 < \theta < \frac{\pi}{2}$

$0 < 3\theta < \frac{3\pi}{2}$

$\theta \notin 4.^{\circ} Q$

(7) $z = 4 - 3i$

a) $2i + \frac{z^2}{i} = 2i + \frac{(4-3i)^2}{i} = 2i + \frac{16 - 24i - 9}{i} =$

$= 2i + \frac{7 - 24i}{i} = 2i + \frac{(7-24i)(-i)}{1} = 2i - 7i + 24i^2 = -24 - 5i$

b) $z = 4 - 3i$

$\rho = \sqrt{16+9} = 5$

$$\left.\begin{array}{l} z = 5e^{i\alpha} \\ \bar{z} = 5e^{i(-\alpha)} \\ i = e^{i\frac{\pi}{2}} \end{array}\right\} \quad i \times \bar{z} = e^{i\frac{\pi}{2}} \times 5e^{i(-\alpha)} = 5e^{i\left(\frac{\pi}{2} - \alpha\right)}$$

(8) $N = \frac{2+i}{1-i} - i = \frac{(2+i)(1+i)}{1-i^2} - i = \frac{2+2i+i+i^2}{2} - i =$

$= \frac{1+3i}{2} - i = \frac{1+3i-2i}{2} = \frac{1+i}{2} = \frac{1}{2} + \frac{1}{2}i = \frac{\sqrt{2}}{2} e^{i\frac{\pi}{4}}$

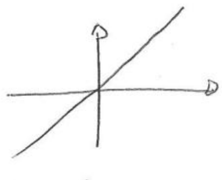
$\rho = \sqrt{\frac{1}{4} + \frac{1}{4}} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

(9) $z_1 = e^{i\alpha}$ $\quad z_2 = e^{i\left(\frac{\pi}{2} - \alpha\right)} = \cos\left(\frac{\pi}{2} - \alpha\right) + i \sin\left(\frac{\pi}{2} - \alpha\right) =$

$= \sin\alpha + i\cos\alpha$

$z_1 + z_2 = \cos\alpha + i\sin\alpha + \sin\alpha + i\cos\alpha =$

$= (\cos\alpha + \sin\alpha) + i(\cos\alpha + \sin\alpha)$



Parte real = Parte imaginária

⇓

imagem geométrica é bissectriz dos quadrantes ímpares.

(10) $w_1 = 1+i \qquad w_2 = \sqrt{2}\, e^{i\frac{\pi}{12}} \qquad w_3 = \sqrt{3}\, e^{i(-\frac{\pi}{2})}$

a) $\dfrac{w_1 \times w_2 - 2}{w_3} = \dfrac{(1+i)\times \sqrt{2}\, e^{i\frac{\pi}{12}} - 2}{\sqrt{3}\, e^{i(-\frac{\pi}{2})}} = \dfrac{\sqrt{2}\, e^{i\frac{\pi}{4}} \times \sqrt{2}\, e^{i\frac{\pi}{12}} - 2}{\sqrt{3}\, e^{i(-\frac{\pi}{2})}} =$

$= \dfrac{2e^{i(\frac{\pi}{4}+\frac{\pi}{12})} - 2}{\sqrt{3}\, e^{i(-\frac{\pi}{2})}} = \dfrac{2e^{i\frac{\pi}{3}} - 2}{\sqrt{3}\, e^{i(-\frac{\pi}{2})}} = \dfrac{2\left(\cos\frac{\pi}{3} + i \sin\frac{\pi}{3}\right) - 2}{\sqrt{3}\, e^{i(-\frac{\pi}{2})}} =$

$= \dfrac{2\left(\frac{1}{2} + \frac{\sqrt{3}}{2} i\right) - 2}{\sqrt{3}\, e^{i(-\frac{\pi}{2})}} = \dfrac{1 + \sqrt{3}\, i - 2}{\sqrt{3}\, e^{i(-\frac{\pi}{2})}} = \dfrac{-1 + \sqrt{3}\, i}{-\sqrt{3}\, i} =$

$= \dfrac{(-1+\sqrt{3}\, i)(\sqrt{3}\, i)}{-3i^2} = \dfrac{-\sqrt{3}\, i + 3i^2}{+3} = -\left(1 + \dfrac{\sqrt{3}}{3} i\right) = -1 - \dfrac{\sqrt{3}}{3} i$

b) $Re(z) \geqslant Re(w_1) \;\wedge\; |z - w_3| \leqslant \sqrt{3}$

$\Downarrow \qquad\qquad\qquad \Downarrow$

$x \geqslant 1 \qquad\qquad |z - (0 - \sqrt{3}\, i)| \leqslant \sqrt{3}$