

## Atividade de diagnóstico

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$$1.1. \overline{AB} = \sqrt{(3+1)^2 + (2-2)^2} = \sqrt{4^2 + 0^2} = 4$$

$$1.2. \overline{AB} = \sqrt{(2+1)^2 + (-2-3)^2} = \sqrt{3^2 + (-5)^2} = \sqrt{9+25} = \sqrt{34}$$

$$2.1. A(-1, 2); B(2, -1); P(x, y)$$

$$\begin{aligned} \overline{AP} = \overline{BP} &\Leftrightarrow \sqrt{(x+1)^2 + (y-2)^2} = \sqrt{(x-2)^2 + (y+1)^2} \Leftrightarrow \\ &\Leftrightarrow x^2 + 2x + 1 + y^2 - 4y + 4 = \\ &= x^2 - 4x + 4 + y^2 + 2x + 1 \Leftrightarrow \\ &\Leftrightarrow -6y = -6x \Leftrightarrow y = x \end{aligned}$$

A bissetriz dos quadrantes ímpares é a mediatriz de  $[AB]$ .

$$2.2. B(2, -1); C(3, 3); P(x, y)$$

$$\begin{aligned} \overline{BP} = \overline{CP} &\Leftrightarrow \\ &\Leftrightarrow \sqrt{(x-2)^2 + (y+1)^2} = \sqrt{(x-3)^2 + (y-3)^2} \Leftrightarrow \\ &\Leftrightarrow x^2 - 4x + 4 + y^2 + 2y + 1 = \\ &= x^2 - 6x + 9 + y^2 - 6y + 9 \Leftrightarrow \\ &\Leftrightarrow 8y = -2x + 13 \Leftrightarrow y = -\frac{2}{8}x + \frac{13}{8} \Leftrightarrow y = -\frac{1}{4}x + \frac{13}{8} \end{aligned}$$

A equação reduzida da mediatriz de  $[BC]$  é  $y = -\frac{1}{4}x + \frac{13}{8}$ .

$$3.1. (x+3)^2 + (y-2)^2 = 9$$

$$3.2. \text{Centro: } (1, 0); \text{raio: } r = \sqrt{3}$$

Por exemplo, o ponto de coordenadas  $(1, \sqrt{3})$  pertence à circunferência  $C_1$ .

$$4. 1 \leq x^2 + y^2 \leq 4$$

$$5.1. \text{Reta vertical: } x = 0$$

Retas horizontais:  $y = 0; y = 3$

Retas oblíquas:  $y = mx + b$

$$\left. \begin{aligned} (3, 0); (5, 3) \\ m = \frac{3-0}{5-3} = \frac{3}{2} \\ 0 = \frac{3}{2} \times 3 + b \Leftrightarrow -\frac{9}{2} = b \\ b = 2 \end{aligned} \right\} \Rightarrow r: y = \frac{3}{2}x - \frac{9}{2}$$

$$\left. \begin{aligned} r // s \Rightarrow m = \frac{3}{2} \end{aligned} \right\} \Rightarrow r: y = \frac{3}{2}x + 2$$

$$\text{Condição: } 0 \leq y \leq 3 \wedge x \geq 0 \wedge \frac{3}{2}x - \frac{9}{2} \leq y \leq \frac{3}{2}x + 2$$

$$5.2. \text{Circunferência: } (x-x_1)^2 + (y-y_1)^2 = r^2$$

$$\text{Centro: } (1, 1) \Rightarrow (x-1)^2 + (y-1)^2 = 1$$

Retas verticais:  $x = 0; x = 2$

Retas horizontais:  $y = 0; y = 3$

$$\text{Condição: } (0 \leq x \leq 2 \wedge 0 \leq y \leq 1 \wedge (x-1)^2 + (y-1)^2 \geq 1) \vee$$

$$\vee (y \geq 1 \wedge (x-1)^2 + (y-1)^2 \leq 1)$$

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$$5.3. \text{Circunferência: } (x-x_1)^2 + (y-y_1)^2 = r^2$$

$$\left. \begin{aligned} \text{Centro: } (1, 2) \\ \text{raio: } r = 2 \end{aligned} \right\} \Rightarrow (x-1)^2 + (y-2)^2 = 4$$

$$\left. \begin{aligned} \text{Reta: } y = mx \\ (0, 0); (3, 3) \end{aligned} \right\} \Rightarrow y = x$$

$$\text{Condição: } (x-1)^2 + (y-2)^2 < 4 \wedge y \geq x$$

$$5.4. \text{Circunferência: } (x-x_1)^2 + (y-y_1)^2 = r^2$$

$$\left. \begin{aligned} \text{Centro: } (1, 1) \\ \text{Raio: } r = 1 \end{aligned} \right\} \Rightarrow (x-1)^2 + (y-1)^2 = 1$$

Retas:  $y = mx + b$

A reta  $r$  passa nos pontos de coordenadas  $(0, 1)$  e  $(2, 2)$ .

$$\left. \begin{aligned} b = 1 \\ m = \frac{2-1}{2-0} = \frac{1}{2} \end{aligned} \right\} \Rightarrow r: y = \frac{1}{2}x + 1$$

A reta  $s$  passa nos pontos de coordenadas  $(1, 0)$  e  $(2, 2)$ .

$$\left. \begin{aligned} m = \frac{2-0}{2-1} = \frac{2}{1} = 2 \\ 0 = 2 \times 1 + b \Leftrightarrow b = -2 \end{aligned} \right\} \Rightarrow s: y = 2x - 1$$

Condição:

$$\left[ (x-1)^2 + (y-1)^2 \leq 1 \wedge \left( y \geq \frac{1}{2}x + 1 \vee y \leq 2x - 1 \right) \right] \vee \vee \left( (x-1)^2 + (y-1)^2 \geq 1 \wedge 0 \leq x \leq 1 \wedge 0 \leq y \leq 1 \right)$$

$$6.1. \text{a) } \overline{AB} = B - A = (0, 2) - \left( \frac{1}{2}, -1 \right) = \left( -\frac{1}{2}, 3 \right)$$

$$\text{b) } \overline{BC} = C - B = \left( -\frac{3}{2}, 2 \right) - (0, 2) = \left( -\frac{3}{2}, 0 \right)$$

$$\text{c) } \overline{AB} + \overline{BC} = \left( -\frac{1}{2}, 3 \right) + \left( -\frac{3}{2}, 0 \right) = (-2, 3)$$

$$\begin{aligned} \text{d) } 2\overline{AB} - 3\overline{CB} &= 2\overline{AB} + 3\overline{BC} = 2\left( -\frac{1}{2}, 3 \right) + 3\left( -\frac{3}{2}, 0 \right) = \\ &= (-1, 6) + \left( -\frac{9}{2}, 0 \right) = \left( -\frac{11}{2}, 6 \right) \end{aligned}$$

6.2. Os vetores  $\overline{AB}$  e  $\overline{BC}$  não são colineares, porque a ordenada de  $\overline{BC}$  é nula enquanto que a ordenada de  $\overline{AB}$  é não nula.

$$7.1. \overline{OA}(-1, 3) \quad 7.2. \overline{OB}(0, 2)$$

$$7.3. C = A + \vec{u}(-2, -3) = (-1, 3) + (-2, -3) = (-3, 0)$$

Logo,  $\overline{OC}(-3, 0)$ .

$$8.1. A(1, -1); C(-2, 3)$$

$$\overline{AC} = C - A = (-2, 3) - (1, -1) = (-3, 4)$$

$$\|\overline{AC}\| = \sqrt{(-3)^2 + 4^2} = \sqrt{9+16} = \sqrt{25} = 5$$

$$8.2. B(x, 3)$$

$$\overline{BC} = C - B = (-2, 3) - (x, 3) = (-2-x, 0)$$

$$\|\overline{BC}\| = 5 \Leftrightarrow \sqrt{(-2-x)^2 + 0^2} = 5 \Leftrightarrow (-2-x)^2 = 25 \Leftrightarrow$$

$$\Leftrightarrow -2-x = 5 \vee -2-x = -5 \Leftrightarrow x = 3 \vee x = -7$$

Como o ponto  $B$  pertence ao 1.º quadrante, a sua abcissa é 3.

## Atividade inicial

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$$1. \quad 4^3 - 15 \times 4 - 4 = 64 - 60 - 4 = 0$$

Logo, 4 é solução da equação  $x^3 - 15x - 4 = 0$ .

$$2.1. \quad p = -15; q = -4$$

$$D = \frac{(-4)^2}{4} + \frac{(-15)^3}{27} = \frac{4^2}{4} - \frac{3^3 \times 5^3}{3^3} = 4 - 5^3 =$$

$$= 4 - 125 = -121 = -11^2$$

$$D = \frac{q^2}{4} + \frac{p^3}{27} = -11^2$$

$$2.2. \quad x = \sqrt[3]{-\frac{q}{2} + \sqrt{D}} + \sqrt[3]{-\frac{q}{2} - \sqrt{D}} =$$

$$= \sqrt[3]{-\frac{-4}{2} + \sqrt{-11^2}} + \sqrt[3]{-\frac{-4}{2} - \sqrt{-11^2}} =$$

$$= \sqrt[3]{2 + \sqrt{-1 \times 11^2}} + \sqrt[3]{2 - \sqrt{-1 \times 11^2}} =$$

$$= \sqrt[3]{2 + 11\sqrt{-1}} + \sqrt[3]{2 - 11\sqrt{-1}}$$

$$2.3. \quad (2 + \sqrt{-1})^3 = {}^3C_0 \times 2^3 + {}^3C_1 \times 2^2 \times \sqrt{-1} +$$

$$+ {}^3C_2 \times 2(\sqrt{-1})^2 + {}^3C_3(\sqrt{-1})^3 =$$

$$= 8 + 3 \times 4\sqrt{-1} + 3 \times 2 \times (-1) + (\sqrt{-1})^2 \times \sqrt{-1} =$$

$$= 8 + 12\sqrt{-1} - 6 - 1 \times \sqrt{-1} = 2 + 11\sqrt{-1}$$

$$(2 - \sqrt{-1})^3 = {}^3C_0 \times 2^3 + {}^3C_1 \times 2^2 \times (-\sqrt{-1}) +$$

$$+ {}^3C_2 \times 2 \times (-\sqrt{-1})^2 + {}^3C_3(-\sqrt{-1})^3 =$$

$$= 8 + 3 \times 4 \times (-\sqrt{-1}) + 3 \times 2 \times (-1) - (\sqrt{-1})^2 \times \sqrt{-1} =$$

$$= 8 - 12\sqrt{-1} - 6 + \sqrt{-1} = 2 - 11\sqrt{-1}$$

Logo,  $(2 \pm \sqrt{-1})^3 = 2 \pm 11\sqrt{-1}$ .

$$2.4. \quad \sqrt[3]{2 + 11\sqrt{-1}} + \sqrt[3]{2 - 11\sqrt{-1}} = (2 + 11\sqrt{-1}) + (2 - 11\sqrt{-1}) =$$

$$= 2 + 2 + 11\sqrt{-1} - 11\sqrt{-1} = 4$$

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$$1.1. \quad z_1 + z_2 + z_3 = (2 - i) + (-1 + i) + 2i = 1 + 2i$$

$$1.2. \quad z_1 - z_2 - z_3 = (2 - i) - (-1 + i) - 2i = 2 - i + 1 - i - 2i = 3 - 4i$$

$$1.3. \quad z_1 \times z_2 = (2 - i)(-1 + i) = -2 + 2i + i - i^2 = -2 + 1 + 3i =$$

$$= -1 + 3i$$

$$1.4. \quad (z_3)^2 + z_1 = (2i)^2 + 2 - i = 4i^2 + 2 - i = -4 + 2 - i = -2 - i$$

$$1.5. \quad (z_1 - z_2) \times z_3 = (2 - i - (-1 + i)) \times 2i = (2 - i + 1 - i) \times 2i =$$

$$= (3 - 2i) \times 2i = 6i - 4i^2 = 4 + 6i$$

$$1.6. \quad (z_1)^2 \times z_2 + z_3 = (2 - i)^2 \times (-1 + i) + 2i =$$

$$= (4 - 4i + i^2) \times (-1 + i) + 2i =$$

$$= (3 - 4i)(-1 + i) + 2i =$$

$$= -3 + 3i + 4i + 4 + 2i = 1 + 9i$$

$$1.7. \quad iz_1 + i^2 z_2 \times z_3 = i(2 - i) + i^2(-1 + i) \times 2i =$$

$$= 2i - i^2 - 1 \times (-2i + 2i^2) = 2i + 1 - (-2 - 2i) =$$

$$= 2i + 1 + 2 + 2i = 3 + 4i$$

$$1.8. \quad z_3 \times (z_1 - z_2)^2 = 2i(2 - i - (-1 + i))^2 = 2i(2 - i + 1 - i)^2 =$$

$$= 2i(3 - 2i)^2 = 2i(9 + 12i - 4) =$$

$$= 2i(5 - 12i) = 24 + 10i$$

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$$2.1. \quad i^{20} + i^{37} + i^{43} + i^{2021} =$$

20	14	37	4	43	4	2021	4
0	5	1	9	3	10	021	505

$$= i^0 + i^1 + i^3 + i^1 =$$

$$= 1 + i - i + i = 1 + i$$

$$2.2. \quad (2 - i^{110})(i^{11} - i^{166}) =$$

110	14	37	4	166	4
30	27	3	2	06	41
2				2	

$$= (2 - i^2)(i^3 - i^2) =$$

$$= (2 + 1)(-i + 1) = 3(1 - i) = 3 - 3i$$

$$2.3. \quad (2i^{19} - i^{10})^2 (3i^2 - 4i^3) = (2i^3 - i^2)^2 (-3 + 4i) =$$

19	4	10	4
3	4	2	2

$$= (-2i + 1)^2 (-3 + 4i) = (4i^2 - 4i + 1)(-3 + 4i) =$$

$$= (-4 + 1 - 4i)(-3 + 4i) = (-3 - 4i)(3 + 4i) =$$

$$= 9 - 16i^2 = 9 + 16 = 25$$

$$2.4. \quad S = \sum_{k=0}^{515} i^{3k} \text{ é a soma dos primeiros 516 termos de uma}$$

progressão geométrica de razão  $i^3 = -i$  e cujo primeiro termo é 1.

$$S = 1 \times \frac{1 - (-i)^{516}}{1 - (-i)} = \frac{1 - i^0}{1 + i} = \frac{1 - 1}{1 + i} = 0$$

516	4
11	129
36	
0	

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$$3.1. \quad z = (1 - 2i)(1 + i)^2 + 2(2 - i)^2 =$$

$$= (1 - 2i)(1 + 2i - 1) + 4 - 2i = (1 - 2i) \times 2i + 4 - 2i =$$

$$= 2i + 4 + 4 - 2i = 8$$

$$\operatorname{Re}(z) = 8; \operatorname{Im}(z) = 0$$

 $z$  é um número real.

$$3.2. \quad z = (\sqrt{2} - \sqrt{3}i) \times (\sqrt{3} - \sqrt{2}i)^2 =$$

$$= (2 - 2\sqrt{6}i + 3i^2)(3 - 2\sqrt{6}i + 2i^2) =$$

$$= (-1 - 2\sqrt{6}i)(1 - 2\sqrt{6}i) = (-2\sqrt{6}i - 1)(-2\sqrt{6}i + 1) =$$

$$= 4 \times 6i^2 - 1 = -24 - 1 = -25$$

$$\operatorname{Re}(z) = -25; \operatorname{Im}(z) = 0$$

 $z$  é um número real.

$$3.3. \quad z = (i^2)^6 - (i + 1)^2 = 1^{12} - (i^2 + 2i + 1) =$$

12	4	3
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$$= i^0 - (-1 + 2i + 1) = 1 - 2i$$

$$\operatorname{Re}(z) = 1; \operatorname{Im}(z) = -2$$

$$3.4. \quad z = (\sqrt{2} - 5i)(\sqrt{2} - i)(1 + \sqrt{2}i)^2 =$$

$$= \left(2 - \sqrt{2}i - 5\sqrt{2}i + \frac{5i^2}{-5}\right) \left(1 + 2\sqrt{2}i + \frac{2i^2}{-2}\right) =$$

$$= (-3 - 6\sqrt{2}i)(-1 + 2\sqrt{2}i) = 3 - 6\sqrt{2}i + 6\sqrt{2}i - 24i^2 =$$

$$= 27$$

$$\operatorname{Re}(z) = 27; \operatorname{Im}(z) = 0$$

 $z$  é um número real.

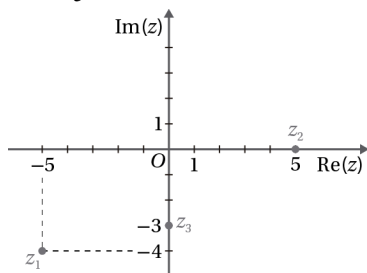
4.1.  $z = (x-i)^2 - (y-1)i = x^2 - 2xi + i^2 - (yi - i) =$   
 $= (x^2 - 1) + (-2x - y + 1)i$   
 $\begin{cases} \operatorname{Re}(z) = -1 \\ \operatorname{Im}(z) = 1 \end{cases} \Leftrightarrow \begin{cases} x^2 - 1 = -1 \\ -2x - y + 1 = 1 \end{cases} \Leftrightarrow \begin{cases} x^2 = 0 \\ -2x - y = 0 \end{cases} \Leftrightarrow \begin{cases} x = 0 \\ y = 0 \end{cases}$   
 Logo,  $x = 0$  e  $y = 0$ .

4.2.  $\begin{cases} \operatorname{Re}(z) = 0 \\ \operatorname{Im}(z) \neq 0 \end{cases} \Leftrightarrow \begin{cases} x^2 - 1 = 0 \\ -2x - y + 1 \neq 0 \end{cases} \Leftrightarrow \begin{cases} x^2 = 1 \\ \text{---} \end{cases} \Leftrightarrow$   
 $\Leftrightarrow \begin{cases} x = -1 \\ 2 - y + 1 \neq 0 \end{cases} \vee \begin{cases} x = 1 \\ -2 - y + 1 \neq 0 \end{cases} \Leftrightarrow$   
 $\Leftrightarrow \begin{cases} x = -1 \\ y \neq 3 \end{cases} \vee \begin{cases} x = 1 \\ y \neq -1 \end{cases}$   
 Logo,  $x = -1$  e  $y \neq 3$  ou  $x = 1$  e  $y \neq -1$ .

4.3.  $z = 3 \Leftrightarrow (x^2 - 1) + (-2x + y + 1)i = 3 + 0i \Leftrightarrow$   
 $\Leftrightarrow x^2 - 1 = 3 \wedge -2x - y + 1 = 0 \Leftrightarrow$   
 $\Leftrightarrow x^2 = 4 \wedge y = 1 - 2x \Leftrightarrow$   
 $\Leftrightarrow (x = -2 \wedge y = 5) \vee (x = 2 \wedge y = -3)$   
 Logo,  $x = -2$  e  $y = 5$  ou  $x = 2$  e  $y = -3$ .

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5.  $z_1 = (2-3i)^2 - (2i)^3 = 4 - 12i + 9i^2 - 8i^3 =$   
 $= 4 - 9 - 12i + 8i = -5 - 4i$   
 $z_2 = (5-i) - (1+i)^2 - i^2(i-2i^3) =$   
 $= 5 - i - (1 + 2i + i^2) + 1(i + 2i) =$   
 $= 5 - i - 1 - 2i + 1 + 3i = 5$   
 $z_3 = (1-2i)(1+2i) \times \frac{3i^3}{5} = (1-4i^2) \times \frac{-3i}{5} =$   
 $= (1+4) \times \frac{-3i}{5} = -3i$



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6.1.  $(1+i)^3 + {}^3C_0 1^3 + {}^3C_1 1^2 \times i + {}^3C_2 1 \times i^2 + {}^3C_3 i^3 =$   
 $= 1 + 3i - 3 - i = -2 + 2i$   
 $f(z) = z - (1+i)^3 = z - (-2 + 2i) = z + (2 - 2i)$   
 Sendo  $M$  o afixo de  $z$  em  $\mathbb{C}$ , então o afixo de  $f(z)$  é a imagem de  $M$  na translação de vetor  $(2, -2)$ .

6.2.  $A' = (0, 1) + (2, -2) = (2, -1)$   
 $B' = (0, 3) + (2, -2) = (2, 1)$   
 $C' = (-1, 2) + (2, -2) = (1, 0)$

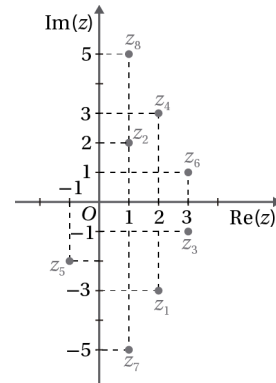
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7.1.  $f(z) = \bar{z} - i^3 = \bar{z} - (-i) = \bar{z} + i = \bar{z} + (0 + i)$   
 Sendo  $M$  o afixo de  $z$ , o afixo de  $f(z)$  é a imagem de  $M$  pela reflexão de eixo real seguida da translação de vetor  $(0, 1)$ .  
 $g(z) = 1 + 3i - \overline{z+1} = 1 + 3i - (\bar{z} + \bar{1}) = 1 + 3i - \bar{z} - 1 =$   
 $= -\bar{z} + 3i$

Sendo  $N$  o afixo de  $z$ , o afixo de  $g(z)$  é a imagem de  $N$  pela reflexão de eixo real, seguida da reflexão central de centro  $O$  e pela translação de vetor  $(0, 3)$ .

7.2. Transformações por  $f$ :  
 $A' = (0, -2) + (0, 1) = (0, -1)$   
 $B' = (-2, -1) + (0, 1) = (-2, 0)$   
 $C' = (-2, -3) + (0, 1) = (-2, -2)$   
 Transformações por  $g$ :  
 $A'' = -(0, -2) + (0, 3) = (0, 2) + (0, 3) = (0, 5)$   
 $B'' = -(-2, -1) + (0, 3) = (2, 1) + (0, 3) = (2, 4)$   
 $C'' = -(-2, -3) + (0, 3) = (2, 3) + (0, 3) = (2, 6)$

8.1.  $z_1 = 2 - 3i$   
 8.2.  $z_2 = 1 + 2i$   
 8.3.  $z_3 = z_1 + z_2 = 2 - 3i + 1 + 2i = 3 - i$   
 8.4.  $z_4 = \bar{z}_1 = 2 + 3i$   
 8.5.  $z_5 = -z_2 = -(1 + 2i) = -1 - 2i$   
 8.6.  $z_6 = \overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2 = 2 + 3i + 1 - 2i = 3 + i$   
 8.7.  $z_7 = z_1 - z_2 = 2 - 3i - 1 - 2i = 1 - 5i$   
 8.8.  $z_8 = \overline{z_1 - z_2} = \bar{z}_1 - \bar{z}_2 = 2 + 3i - (1 - 2i) = 1 + 5i$



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9.1.  $|z| = |3 + 3i| = \sqrt{3^2 + 3^2} = \sqrt{9 + 9} = \sqrt{18} = 3\sqrt{2}$   
 9.2.  $|z| = |-3 - 4i| = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$   
 9.3.  $|z| = |-2| = 2$   
 9.4.  $|z| = |-7i| = 7$   
 9.5.  $|z| = |0| = 0$   
 9.6.  $|z| = |15 - 8i| = \sqrt{15^2 + 8^2} = \sqrt{225 + 64} = \sqrt{289} = 17$   
 9.7.  $|z| = \left| -1 + \frac{24}{7}i \right| = \sqrt{1^2 \times \left(\frac{24}{7}\right)^2} = \sqrt{1 + \frac{576}{49}} = \sqrt{\frac{625}{49}} = \frac{25}{7}$   
 9.8.  $|z| = |2 - 2\sqrt{3}i| = \sqrt{2^2 + (2\sqrt{3})^2} = \sqrt{4 + 4 \times 3} = \sqrt{16} = 4$   
 9.9.  $|z| = |2,1 + 2i| = \sqrt{2,1^2 + 2^2} = \sqrt{4,41 + 4} = \sqrt{8,41} =$   
 $= \sqrt{\frac{841}{100}} = \frac{29}{10}$   
 9.10.  $|z| = |3 - 6i| = \sqrt{3^2 + 6^2} = \sqrt{9 + 36} = \sqrt{45} = 3\sqrt{5}$

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10.1.  $\overline{AB} = |z_B - z_A| = |(2 - i) - (-4 + 3i)| = |6 - 4i| =$   
 $= \sqrt{6^2 + 4^2} = \sqrt{36 + 16} = \sqrt{52}$   
 $\overline{BC} = |z_C - z_B| = |9i - (2 - i)| = |-2 + 10i| = \sqrt{2^2 + 10^2} =$   
 $= \sqrt{4 + 100} = \sqrt{104}$   
 $\overline{AC} = |z_C - z_A| = |9i - (-4 + 3i)| = |4 + 6i| = \sqrt{4^2 + 6^2} =$   
 $= \sqrt{16 + 36} = \sqrt{52}$

Como  $\overline{AB} = \overline{AC}$ , o triângulo  $[ABC]$  é isósceles.  

$$\left. \begin{aligned} (\sqrt{104})^2 &= 104 \\ (\sqrt{52})^2 + (\sqrt{52})^2 &= 52 + 52 = 104 \end{aligned} \right\} \Rightarrow (\sqrt{104})^2 = (\sqrt{52})^2 + (\sqrt{52})^2$$

Logo, o triângulo  $[ABC]$  é retângulo.

Portanto, o triângulo  $[ABC]$  é retângulo e isósceles.

10.2. O centro da circunferência é o ponto médio de  $[BC]$  e

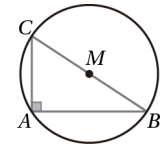
$$r = \frac{BC}{2} = \frac{\sqrt{104}}{2} = \frac{2\sqrt{26}}{2} = \sqrt{26}$$

$B(2, -1); C(0, 9)$

$$M\left(\frac{2+0}{2}, \frac{-1+9}{2}\right) = (1, 4)$$

A equação da circunferência pedida é:

$$(x-1)^2 + (y-4)^2 = 26$$



10.3.  $z_M = 1 + 4i$

$$\frac{z_B + z_C}{2} = \frac{2 - i - 9i}{2} = \frac{2 + 8i}{2} = \frac{2}{2} + \frac{8i}{2} = 1 + 4i = z_M$$

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11.1.  $u\bar{v} + \bar{u}v = u\bar{v} + \overline{\overline{u\bar{v}}} = u\bar{v} + \overline{u\bar{v}} = 2 \times \frac{u\bar{v} + \overline{u\bar{v}}}{2} = 2 \operatorname{Re}(u\bar{v})$

Logo,  $u\bar{v} + \bar{u}v$  é um número real.

$$\begin{aligned} 11.2. |1 - \bar{u}v|^2 - |u - v|^2 &= (1 - \bar{u}v)(1 - \overline{\bar{u}v}) - (u - v)(\overline{u - v}) = \\ &= (1 - \bar{u}v)(1 - u\bar{v}) - (u - v)(\bar{u} - \bar{v}) = \\ &= 1 - \bar{u}v - \bar{u}v + \bar{u}v \times u\bar{v} - u\bar{u} + u\bar{v} + \bar{u}v - v\bar{v} = \\ &= 1 + u\bar{u} \times v\bar{v} - |u|^2 - |v|^2 = 1 + |u|^2 |v|^2 - |u|^2 - |v|^2 = \\ &= (1 - |u|^2) - |v|^2 (1 - |u|^2) = (1 - |u|^2)(1 - |v|^2) \end{aligned}$$

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$$12.1. \frac{1}{1+i} = \frac{1(1-i)}{(1+i)(1-i)} = \frac{1-i}{1^2+1^2} = \frac{1-i}{2} = \frac{1}{2} - \frac{1}{2}i$$

$$\begin{aligned} 12.2. (1-3i)^{-2} &= \frac{1}{(1-3i)^2} = \frac{1}{1-6i-9} = \frac{1}{-8-6i} = \\ &= \frac{-8+6i}{(-8-6i)(-8+6i)} = \frac{-8+6i}{8^2+6^2} = \frac{-8+6i}{64+36} = \\ &= -\frac{8}{100} + \frac{6}{100}i = -\frac{2}{25} + \frac{3}{50}i \end{aligned}$$

$$\begin{aligned} 12.3. \frac{2-4i}{(1+i)^2} &= \frac{2-4i}{1+2i-1} = \frac{2-4i}{2i} = \frac{(2-4i)(-2i)}{2i \times (-2i)} = \\ &= \frac{-4i-8}{2^2} = \frac{-8-4i}{4} = -2-i \end{aligned}$$

$$12.4. \frac{1}{-i} \times (3-2i) = \frac{(3-2i) \times i}{-i \times i} = \frac{3i+2}{1^2} = 2+3i$$

$$\begin{aligned} 12.5. \frac{5i(1-2i)^2}{2-i} &= \frac{5i(1-4i-4)}{2-i} = \frac{5i(-3-4i)}{2-i} = \frac{-15i+20}{2-i} = \\ &= \frac{(20-15i)(2+i)}{(2-i)(2+i)} = \frac{40+20i-30i+15}{2^2+1^2} = \\ &= \frac{55-10i}{5} = 11-2i \end{aligned}$$

$$12.6. \frac{1}{1-i} + \frac{1}{1+i} = \frac{1+i+1-i}{(1-i)(1+i)} = \frac{2}{1^2+1^2} = \frac{2}{2} = 1$$

$$\begin{aligned} 12.7. \left(\frac{1-2i}{\sqrt{2}i}\right)^2 - \frac{3}{2} &= \frac{(1-2i)^2}{(\sqrt{2}i)^2} - \frac{3}{2} = \frac{1-4i-4}{-2} - \frac{3}{2} = \\ &= \frac{-3-4i}{-2} - \frac{3}{2} = \frac{3+4i-3}{2} = 2i \end{aligned}$$

$$\begin{aligned} 12.8. \frac{5(1+i)^2 - 10i^{42}}{i^{13} + (1-i)^3} &= \frac{5(1+2i-1) - 10i^2}{i-2-2i} \quad \begin{array}{l} \text{Cálculo auxiliar} \\ 42 = 4 \times 10 + 2 \\ 13 = 4 \times 3 + 1 \\ (1-i)^3 = (1-i)^2(1-i) = \\ = (1-2i-1)(1-i) = \\ = -2i(1-i) = -2-2i \end{array} \\ &= \frac{10i+10}{-2-i} = \frac{(10+10i)(-2+i)}{(-2-i)(-2+i)} = \\ &= \frac{-20+10i-20i-10}{2^2+1^2} = \frac{-30-10i}{5} = -6-2i \end{aligned}$$

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$$13.1. \left| \frac{3-4i}{(1+i)^2} \right| = \frac{|3-4i|}{|(1+i)^2|} = \frac{|3-4i|}{|1+i|^2} = \frac{\sqrt{3^2+4^2}}{1^2+1^2} = \frac{\sqrt{25}}{2} = \frac{5}{2}$$

$$13.2. \left| \frac{9-3i}{6+2i} \right| = \frac{|9-3i|}{|6+2i|} = \frac{\sqrt{9^2+(-3)^2}}{\sqrt{6^2+2^2}} = \frac{\sqrt{90}}{\sqrt{40}} = \sqrt{\frac{90}{40}} = \frac{3}{2}$$

$$\begin{aligned} 14. |z_1| = |z_2| &\Leftrightarrow \left| \frac{10+xi}{1+5i} \right| = \left| \frac{x-8i}{4+i} \right| \Leftrightarrow \frac{|10+xi|}{|1+5i|} = \frac{|x-8i|}{|4+i|} \Leftrightarrow \\ &\Leftrightarrow \frac{\sqrt{100+x^2}}{\sqrt{1+25}} = \frac{\sqrt{x^2+64}}{\sqrt{16+1}} \Leftrightarrow \\ &\Leftrightarrow \sqrt{17}\sqrt{100+x^2} = \sqrt{26}\sqrt{x^2+64} \Leftrightarrow \\ &\Leftrightarrow \sqrt{1700+17x^2} = \sqrt{26x^2+1664} \Leftrightarrow \\ &\Leftrightarrow 1700+17x^2 = 26x^2+1664 \Leftrightarrow \\ &\Leftrightarrow 9x^2 = 36 \Leftrightarrow x^2 = \frac{36}{9} \Leftrightarrow x^2 = 4 \Leftrightarrow x = \pm 2 \end{aligned}$$

Como  $-2$  e  $2$  são soluções de

$$\sqrt{1700+17x^2} = \sqrt{26x^2+1664}, \text{ então } x = -2 \vee x = 2.$$

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$$\begin{aligned} 15. \frac{\overline{z+i}}{w-3} = \frac{\bar{z}-i}{\bar{w}-3} &= \frac{8+2i-i}{3+4i-3} = \frac{8+i}{4i} = \frac{(8+i)(-4i)}{4i \times (-4i)} = \\ &= \frac{-32i+4}{16} = \frac{4}{16} - \frac{32}{16}i = \frac{1}{4} - 2i \end{aligned}$$

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$$\begin{aligned} 16. z' &= (z-2)(\bar{z}-i) = (x+yi-2)(x-yi-i) = \\ &= x^2 - \cancel{xy}i - xi + \cancel{xy}i + y^2 + y - 2x + 2yi + 2i = \\ &= x^2 + y^2 - 2x + y + (2y+2-x)i \end{aligned}$$

$$\begin{aligned} 16.1. z' \text{ é um número real} &\Leftrightarrow \operatorname{Im}(z') = 0 \Leftrightarrow \\ &\Leftrightarrow 2y+2-x=0 \Leftrightarrow y = \frac{1}{2}x-1 \end{aligned}$$

$E_1$  é a reta de equação  $y = \frac{1}{2}x - 1$ .

16.2. Como  $z'$  é um imaginário puro:

$$x^2 + y^2 - 2x + y = 0 \wedge 2y + 2 \neq 0$$

$$\bullet x^2 + y^2 - 2x + y = 0 \Leftrightarrow$$

$$\Leftrightarrow (x^2 - 2x + 1) - 1 + \left(y^2 + y + \frac{1}{4}\right) - \frac{1}{4} = 0 \Leftrightarrow$$

$$\Leftrightarrow (x-1)^2 + \left(y + \frac{1}{2}\right)^2 = \frac{5}{4}$$

•  $2y + 2 - x = 0 \Leftrightarrow y = \frac{1}{2}x - 1$

Interseção da circunferência com a reta:

$$(x-1)^2 + \left(\frac{1}{2}x - 1 + \frac{1}{2}\right)^2 = \frac{5}{4} \Leftrightarrow$$

$$\Leftrightarrow x^2 - 2x + 1 + \frac{1}{4}x^2 - \frac{1}{2}x + \frac{1}{4} = \frac{5}{4} \Leftrightarrow$$

$$\Leftrightarrow 4x^2 - 8x + 4 + x^2 - 2x + 1 = 5 \Leftrightarrow 5x^2 - 10x = 0 \Leftrightarrow$$

$$\Leftrightarrow 5x(x-2) = 0 \Leftrightarrow x = 0 \vee x = 2$$

Portanto,  $z'$  é um imaginário puro se os afixos de  $z$

pertencerem à circunferência de centro  $\left(1, -\frac{1}{2}\right)$  e raio

$\frac{\sqrt{5}}{2}$ , com exceção dos afixos de abscissas 0 e 2, ou seja, os pontos  $(0, -1)$  e  $(2, 0)$ .

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17.1.  $|z_1| = |1+i| = \sqrt{1+1} = \sqrt{2} \neq 1$

$z_1$  não é um número complexo unitário.

17.2.  $|z_2| = |i| = 1$

$z_2$  é um número complexo unitário.

17.3.  $|z_3| = \left|\frac{2}{3} + \frac{\sqrt{5}}{3}i\right| = \sqrt{\frac{4}{9} + \frac{5}{9}} = \sqrt{\frac{9}{9}} = \sqrt{1} = 1$

$z_3$  é um número complexo unitário.

17.4.  $z_4$  não é um número complexo unitário por não ser da forma  $z = \cos \theta + i \sin \theta$ .

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18.1.  $z_1 = \frac{1-\sqrt{3}i}{2} = \frac{1}{2} - \frac{\sqrt{3}}{2}i = \cos\left(\frac{\pi}{3}\right) - i \sin\left(\frac{\pi}{3}\right) =$

$$= \cos\left(-\frac{\pi}{3}\right) + i \sin\left(-\frac{\pi}{3}\right)$$

Logo,  $z_2$  é unitário por ser da forma  $z = \cos \theta + i \sin \theta$ ,

neste caso com  $\theta = -\frac{\pi}{3}$ .

18.2.  $z_1 = \cos\left(-\frac{\pi}{3}\right) + i \sin\left(-\frac{\pi}{3}\right)$ , logo  $\text{Arg}(z_1) = -\frac{\pi}{3}$

19.1.  $z_2 = 1 - 2 \sin \frac{\pi}{8} \left( \sin \frac{\pi}{8} - i \cos \frac{\pi}{8} \right) =$

$$= 1 - 2 \sin^2 \frac{\pi}{8} + 2i \sin \frac{\pi}{8} \cos \frac{\pi}{8} =$$

$$= 1 - \sin^2 \frac{\pi}{8} - \sin^2 \frac{\pi}{8} + i \sin \frac{\pi}{4} =$$

$$= \cos^2 \frac{\pi}{8} - \sin^2 \frac{\pi}{8} + i \sin \frac{\pi}{4} =$$

$$= \cos \frac{\pi}{4} + i \sin \frac{\pi}{4}$$

$$\left| \begin{array}{l} 2 \sin \frac{\pi}{8} \cos \frac{\pi}{8} = \\ = \sin \left( 2 \times \frac{\pi}{8} \right) = \sin \left( \frac{\pi}{4} \right) \end{array} \right.$$

$$\left| \begin{array}{l} \sin^2 \frac{\pi}{8} + \cos^2 \frac{\pi}{8} = 1 \end{array} \right.$$

$$\left| \begin{array}{l} \cos^2 \frac{\pi}{8} - \sin^2 \frac{\pi}{8} = \cos \left( 2 \times \frac{\pi}{8} \right) = \cos \left( \frac{\pi}{4} \right) \end{array} \right.$$

Logo,  $z_2$  é um número complexo da forma  $z = \cos \theta + i \sin \theta$ ,

com  $\theta = \frac{\pi}{4}$ , pelo que  $z_2$  é um número complexo unitário.

19.2.  $z_2 = \cos \frac{\pi}{4} + i \sin \frac{\pi}{4}$ , logo  $\text{Arg}(z_2) = \frac{\pi}{4}$ .

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20.1.  $|2 + 2\sqrt{3}i| = \sqrt{4+12} = 4$

Seja  $\theta$  um argumento de  $2 + 2\sqrt{3}i$ .

$$\left\{ \begin{array}{l} \tan \theta = \frac{2\sqrt{3}}{2} = \sqrt{3} \Rightarrow \frac{\pi}{3} \text{ é um argumento de } 2 + 2\sqrt{3}i \\ (2, 2\sqrt{3}) \in 1.^\circ \text{Q} \end{array} \right.$$

Logo,  $2 + 2\sqrt{3}i = 4e^{i\frac{\pi}{3}}$ .

20.2.  $|-3\sqrt{3} + 3i| = \sqrt{27+9} = 6$

Seja  $\theta$  um argumento de  $-3\sqrt{3} + 3i$ .

$$\left\{ \begin{array}{l} \tan \theta = \frac{3}{-3\sqrt{3}} = -\frac{\sqrt{3}}{3} \Rightarrow \pi - \frac{\pi}{6} = \frac{5\pi}{6} \text{ é um argumento de } -3\sqrt{3} + 3i \\ (-3\sqrt{3}, 3) \in 2.^\circ \text{Q} \end{array} \right.$$

Logo,  $-3\sqrt{3} + 3i = 6e^{i\frac{5\pi}{6}}$ .

20.3.  $|1-i| = \sqrt{1+1} = \sqrt{2}$ . Seja  $\theta$  um argumento  $1-i$ .

$$\left\{ \begin{array}{l} \tan(\theta) = \frac{-1}{1} = -1 \Rightarrow -\frac{\pi}{4} \text{ é um argumento de } 1-i. \\ (1, -1) \in 4.^\circ \text{Q} \end{array} \right.$$

Logo,  $1-i = \sqrt{2}e^{-i\frac{\pi}{4}}$ .

20.4.  $|2\sqrt{3} + 2i| = \sqrt{12+4} = 4$

Seja  $\theta$  um argumento de  $2\sqrt{3} + 2i$ .

$$\left\{ \begin{array}{l} \tan \theta = \frac{2}{2\sqrt{3}} = \frac{\sqrt{3}}{3} \Rightarrow \frac{\pi}{6} \text{ é um argumento de } 2\sqrt{3} + 2i \\ (2\sqrt{3}, 2) \in 1.^\circ \text{Q} \end{array} \right.$$

Logo,  $2\sqrt{3} + 2i = 4e^{i\frac{\pi}{6}}$ .

20.5.  $|-\sqrt{2} + \sqrt{2}i| = \sqrt{2+2} = 2$

Seja  $\theta$  um argumento de  $-\sqrt{2} + \sqrt{2}i$

$$\left\{ \begin{array}{l} \tan \theta = \frac{\sqrt{2}}{-\sqrt{2}} = -1 \Rightarrow \pi - \frac{\pi}{4} = \frac{3\pi}{4} \text{ é um argumento de } -\sqrt{2} + \sqrt{2}i \\ (-\sqrt{2}, \sqrt{2}) \in 2.^\circ \text{Q} \end{array} \right.$$

Logo,  $-\sqrt{2} + \sqrt{2}i = 2e^{i\frac{3\pi}{4}}$ .

20.6.  $\left|\frac{\sqrt{3}i-1}{4}\right| = \frac{\sqrt{3+1}}{4} = \frac{2}{4} = \frac{1}{2}$        $\frac{\sqrt{3}i-1}{4} = -\frac{1}{4} + \frac{\sqrt{3}}{4}i$

Seja  $\theta$  um argumento de  $-\frac{1}{4} + \frac{\sqrt{3}}{4}i$ .

$$\left\{ \begin{array}{l} \tan \theta = \frac{\sqrt{3}}{-1} = -\sqrt{3} \Rightarrow \pi - \frac{\pi}{3} = \frac{2\pi}{3} \text{ é um argumento de } -\frac{1}{4} + \frac{\sqrt{3}}{4}i \\ \left(-\frac{1}{4}, \frac{\sqrt{3}}{4}\right) \in 2.^\circ \text{Q} \end{array} \right.$$

Logo,  $\frac{\sqrt{3}i-1}{4} = \frac{1}{2}e^{i\frac{2\pi}{3}}$ .

20.7.  $|-\sqrt{2} - \sqrt{6}i| = \sqrt{2+6} = \sqrt{8} = 2\sqrt{2}$

Seja  $\theta$  um argumento de  $-\sqrt{2} - \sqrt{6}i$

$$\left\{ \begin{array}{l} \tan \theta = \frac{-\sqrt{6}}{-\sqrt{2}} = \sqrt{3} \Rightarrow \\ (-\sqrt{2}, -\sqrt{6}) \in 3.^\circ \text{Q} \end{array} \right.$$

$\Rightarrow -\pi + \frac{\pi}{3} = -\frac{2\pi}{3}$  é um argumento de  $-\sqrt{2} - \sqrt{6}i$ .

Logo,  $-\sqrt{2} - \sqrt{6}i = 2\sqrt{2}e^{-i\frac{2\pi}{3}}$ .

20.8.  $\left| \frac{-5\sqrt{3} - 5i}{2} \right| = \frac{|-5\sqrt{3} - 5i|}{|2|} = \frac{\sqrt{75+25}}{2} = \frac{10}{2} = 5$

Seja  $\theta$  um argumento de  $-\frac{5\sqrt{3}}{2} - \frac{5}{2}i$ .

$$\begin{cases} \tan \theta = \frac{-5}{-5\sqrt{3}} = \frac{\sqrt{3}}{3} \\ \left( \frac{-5\sqrt{3}}{2}, -\frac{5}{2} \right) \in 3.^\circ Q \end{cases} \Rightarrow$$

$\Rightarrow -\pi + \frac{\pi}{6} = -\frac{5\pi}{6}$  é um argumento de  $-\frac{5\sqrt{3}}{2} - \frac{5}{2}i$ .

Logo,  $\frac{-5\sqrt{3} - 5i}{2} = 5e^{-i\frac{5\pi}{6}}$ .

20.9.  $z = \frac{4+2i}{3-i} = \frac{(4+2i)(3+i)}{(3-i)(3+i)} = \frac{12+4i+6i-2}{9+1} = \frac{10+10i}{10} = 1+i$

$\left| \frac{4+2i}{3-i} \right| = |1+i| = \sqrt{1+1} = \sqrt{2}$

Seja  $\theta$  um argumento de  $z$ .

$$\begin{cases} \tan \theta = \frac{1}{1} = 1 \Rightarrow \frac{\pi}{4} \text{ é um argumento de } z \\ (1, 1) \in 1.^\circ Q \end{cases}$$

Logo,  $\frac{4+2i}{3-i} = \sqrt{2}e^{i\frac{\pi}{4}}$ .

20.10.  $2 + (1+i^{43})^2 = 2 + (1+i^3)^2 = 2 + (1-i)^2 = 2 + 1 - 2i - 1 = 2 - 2i$

$|2-2i| = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

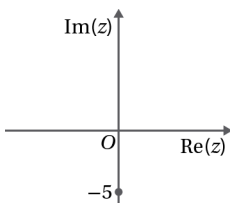
Seja  $\theta$  um argumento de  $2-2i$ .

$$\begin{cases} \tan \theta = \frac{-2}{2} = -1 \Rightarrow -\frac{\pi}{4} \text{ é um argumento de } 2-2i \\ (2, -2) \in 4.^\circ Q \end{cases}$$

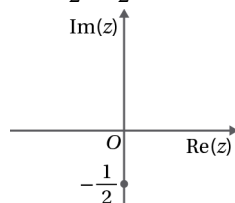
Logo,  $2 + (1+i^{43})^2 = 2\sqrt{2}e^{-i\frac{\pi}{4}}$

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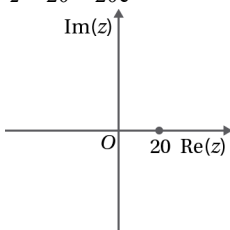
21.1.  $z = -5i = 5e^{-i\frac{\pi}{2}}$



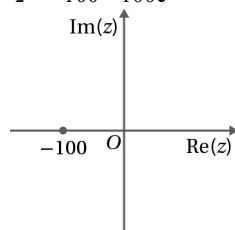
21.2.  $z = -\frac{1}{2}i = \frac{1}{2}e^{-i\frac{\pi}{2}}$



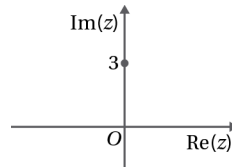
21.3.  $z = 20 = 20e^{i \times 0}$



21.4.  $z = -100 = 100e^{i\pi}$



21.5.  $z = 3i^5 = 3i = 3e^{i\frac{\pi}{2}}$



21.6.  $z = \frac{2}{i} = \frac{2 \times (-i)}{i \times (-i)} = \frac{-2i}{1} = -2i = 2e^{-i\frac{\pi}{2}}$

21.7.  $z = (1+i)^2 = 1+2i-1 = 2i = 2e^{i\frac{\pi}{2}}$

21.8.  $z = |1+i| = \sqrt{1+1} = \sqrt{2} = \sqrt{2}e^{i \times 0}$

21.9.  $z = |1+i| \times i = \sqrt{2}i = \sqrt{2}e^{i\frac{\pi}{2}}$

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22.1.  $z = 3 \left( \cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right) = 3 \left( \frac{\sqrt{3}}{2} + \frac{1}{2}i \right) = \frac{3\sqrt{3}}{2} + \frac{3}{2}i$

22.2.  $z = 5e^{-i\frac{\pi}{4}} = 5 \left( \cos \left( -\frac{\pi}{4} \right) + i \sin \left( -\frac{\pi}{4} \right) \right) = 5 \left( \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i \right) = \frac{5\sqrt{2}}{2} - \frac{5\sqrt{2}}{2}i$

22.3.  $z = 3\sqrt{2}e^{i\frac{3\pi}{4}} = 3\sqrt{2} \left( \cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} \right) = 3\sqrt{2} \left( -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i \right) = -3 + 3i$

22.4.  $z = \sqrt{3}e^{-i\frac{5\pi}{6}} = \sqrt{3} \left( \cos \left( -\frac{5\pi}{6} \right) + i \sin \left( -\frac{5\pi}{6} \right) \right) = \sqrt{3} \left( -\frac{\sqrt{3}}{2} - \frac{1}{2}i \right) = -\frac{3}{2} - \frac{\sqrt{3}}{2}i$

22.5.  $z = 2^{i\frac{5\pi}{3}} = 2 \left( \cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3} \right) = 2 \left( \frac{1}{2} - \frac{\sqrt{3}}{2}i \right) = 1 - \sqrt{3}i$

22.6.  $z = \sqrt{2}e^{i\frac{\pi}{2}} = \sqrt{2}i$

22.7.  $z = 2e^{i \times \pi} = -2$

22.8.  $z = 2e^{-i \times \pi} = -2$

22.9.  $z = 3e^{-i\frac{\pi}{2}} = -3i$

22.10.  $z = 2e^{-i\frac{19\pi}{3}} = 2e^{i\frac{5\pi}{3}} = 2e^{-i\frac{\pi}{3}} = 2 \left( \cos \left( -\frac{\pi}{3} \right) + i \sin \left( -\frac{\pi}{3} \right) \right) = 2 \left( \frac{1}{2} - \frac{\sqrt{3}}{2}i \right) = 1 - \sqrt{3}i$

22.11.  $z = 2\sqrt{3}e^{i\frac{13\pi}{6}} = 2\sqrt{3}e^{i\frac{\pi}{6}} = 2\sqrt{3} \left( \cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right) = 2\sqrt{3} \left( \frac{\sqrt{3}}{2} + \frac{1}{2}i \right) = 3 + \sqrt{3}i$

22.12.  $z = \sqrt{2}e^{-i\frac{21\pi}{4}} = \sqrt{2}e^{i\frac{3\pi}{4}} = \sqrt{2} \left( \cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} \right) = \sqrt{2} \left( -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i \right) = -1 + i$

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23.1.  $|z_1| = |z_2| = 2$

$\text{Arg}(z_1) = \frac{31\pi}{4} - 4 \times 2\pi = \frac{31\pi}{4} - \frac{32\pi}{4} = -\frac{\pi}{4}$

$\text{Arg}(z_2) = -\frac{\pi}{4}$

Como  $|z_1| = |z_2|$  e  $\text{Arg}(z_1) = \text{Arg}(z_2)$ , então  $z_1 = z_2$ .

$$23.2. |z_1| = |z_2| = 1$$

$$\text{Arg}(z_1) = \frac{12\pi}{5} - 2\pi = \frac{12\pi}{5} - \frac{10\pi}{5} = \frac{2\pi}{5}$$

$$\text{Arg}(z_2) = -\frac{8\pi}{5} + 2\pi = -\frac{8\pi}{5} + \frac{10\pi}{5} = \frac{2\pi}{5}$$

Como  $|z_1| = |z_2|$  e  $\text{Arg}(z_1) = \text{Arg}(z_2)$ , então  $z_1 = z_2$ .

$$23.3. |z_1| = \sqrt{8} = \sqrt{4 \times 2} = 2\sqrt{2}; |z_2| = 2\sqrt{2}$$

$$\text{Arg}(z_1) = -\frac{\pi}{7}; \text{Arg}(z_2) = \frac{13\pi}{7} - 2\pi = \frac{13\pi - 14\pi}{7} = -\frac{\pi}{7}$$

Como  $|z_1| = |z_2|$  e  $\text{Arg}(z_1) = \text{Arg}(z_2)$ , então  $z_1 = z_2$ .

$$23.4. |z_1| = |z_2| = 2$$

$$\text{Arg}(z_1) = -\pi + 2\pi = \pi; \text{Arg}(z_2) = \pi$$

Como  $|z_1| = |z_2|$  e  $\text{Arg}(z_1) = \text{Arg}(z_2)$ , então  $z_1 = z_2$ .

$$23.5. |z_1| = |z_2| = 2$$

$$\text{Arg}(z_1) = -\frac{5\pi}{2} + 2\pi = \frac{-5\pi + 4\pi}{2} = -\frac{\pi}{2}$$

$$\text{Arg}(z_2) = -\frac{13\pi}{2} + 6\pi = \frac{-13\pi + 12\pi}{2} = -\frac{\pi}{2}$$

Como  $|z_1| = |z_2|$  e  $\text{Arg}(z_1) = \text{Arg}(z_2)$ ,  $z_1 = z_2$

$$24.1. |z_1| = |z_2| = 3$$

$$\text{Arg}(z_1) = \frac{13\pi}{6} - 2\pi = \frac{13\pi - 12\pi}{6} = \frac{\pi}{6} \quad (1.^\circ \text{Q})$$

$$\text{Arg}(z_2) = -\frac{5\pi}{6} \quad (3.^\circ \text{Q})$$

Como  $\text{Arg}(z_1) \neq \text{Arg}(z_2)$ , então  $z_1 \neq z_2$ .

$$24.2. |z_1| = |z_2| = 1$$

$$\text{Arg}(z_1) = \frac{3\pi}{7}; \text{Arg}(z_2) = \frac{18\pi}{7} - 2\pi = \frac{18\pi - 14\pi}{7} = \frac{4\pi}{7}$$

Como  $\text{Arg}(z_1) \neq \text{Arg}(z_2)$ , então  $z_1 \neq z_2$ .

$$24.3. |z_1| = |z_2| = 4$$

$$\text{Arg}(z_1) = -\frac{\pi}{8}; \text{Arg}(z_2) = -\frac{17\pi}{8} + 2\pi = \frac{-17\pi + 16\pi}{8} = -\frac{\pi}{8}$$

Como  $|z_1| = |z_2|$  e  $\text{Arg}(z_1) = \text{Arg}(z_2)$ , então  $z_1 = z_2$ .

$$24.4. |z_1| = |z_2| = 5$$

$$\text{Arg}(z_1) = -\pi + 2\pi = \pi; \text{Arg}(z_2) = 4\pi - 4\pi = 0$$

Como  $\text{Arg}(z_1) \neq \text{Arg}(z_2)$ ,  $z_1 \neq z_2$

$$24.5. |z_1| = 5 \text{ e } |z_2| = 3. \text{ Como } |z_1| \neq |z_2|, \text{ então } z_1 \neq z_2.$$

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$$25.1. z_1 = e^{i\frac{\pi}{5}}; -z_1 = e^{i(\frac{\pi}{5}+\pi)} = e^{i\frac{6\pi}{5}} = e^{i(\frac{6\pi}{5}-2\pi)} = e^{-i\frac{4\pi}{5}};$$

$$\bar{z}_1 = e^{-i\frac{\pi}{5}}$$

$$25.2. z_2 = 2e^{-i\frac{2\pi}{3}}; -z_2 = 2e^{i(-\frac{2\pi}{3}+\pi)} = 2e^{i\frac{\pi}{3}}; \bar{z}_2 = 2e^{i\frac{2\pi}{3}}$$

$$25.3. z_3 = \sqrt{2}e^{i\frac{11\pi}{6}}$$

$$-z_3 = \sqrt{2}e^{i(\frac{11\pi}{6}+\pi)} = \sqrt{2}e^{i\frac{17\pi}{6}} = \sqrt{2}e^{i(\frac{17\pi}{6}-2\pi)} = \sqrt{2}e^{-i\frac{5\pi}{6}}$$

$$\bar{z}_3 = \sqrt{2}e^{-i\frac{11\pi}{6}} = \sqrt{2}e^{i(-\frac{11\pi}{6}+2\pi)} = \sqrt{2}e^{i\frac{\pi}{6}}$$

$$25.4. z_4 = 3e^{-i\frac{7\pi}{4}}; -z_4 = 3e^{i(-\frac{7\pi}{4}+\pi)} = 3e^{-i\frac{3\pi}{4}}$$

$$\bar{z}_4 = 3e^{i\frac{7\pi}{4}} = 3e^{i(\frac{7\pi}{4}-2\pi)} = 3e^{-i\frac{\pi}{4}}$$

$$26.1. z_1 = -2e^{i\frac{\pi}{6}} = 2e^{i(\frac{\pi}{6}+\pi)} = e^{i\frac{7\pi}{6}}$$

$$26.2. z_2 = 2e^{i\frac{\pi}{3}} = 2e^{-i\frac{2\pi}{3}}$$

$$26.3. z_3 = -3e^{-i\frac{4\pi}{3}} = 3e^{i(\frac{4\pi}{3}+\pi)} = 3e^{i\frac{7\pi}{3}} = 3e^{i\frac{\pi}{3}}$$

$$27.1. z_1 = 3e^{-i\frac{\pi}{6}}; \bar{z}_1 = 3e^{i\frac{\pi}{6}}; z_2 = 3e^{-i\frac{11\pi}{6}} = 3e^{i(-\frac{11\pi}{6}+2\pi)} = 3e^{i\frac{\pi}{6}}$$

Logo,  $\bar{z}_1 = z_2$ .

$$27.2. -z_1 = 3e^{i(\frac{\pi}{6}+\pi)} = 3e^{i\frac{7\pi}{6}}; z_3 = -3e^{-i\frac{11\pi}{6}} = 3e^{i(-\frac{11\pi}{6}+\pi)} = 3e^{-i\frac{5\pi}{6}}$$

Logo,  $-z_1 \neq z_3$ .

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$$28.1. \text{ Seja } z = \sqrt{3} - i. |z| = \sqrt{3+1} = 2$$

$$\begin{cases} \tan(\text{Arg}(z)) = \frac{-1}{\sqrt{3}} = -\frac{\sqrt{3}}{3} \Rightarrow -\frac{\pi}{6} \text{ é um argumento de } z. \\ (\sqrt{3}, -1) \in 2.^\circ \text{Q} \end{cases}$$

Portanto,  $z = 2e^{-i\frac{\pi}{6}}$ . Assim:

$$(\sqrt{3} - i)^6 = \left(2e^{-i\frac{\pi}{6}}\right)^6 = 2^6 e^{-i\frac{\pi}{6} \times 6} = 64e^{-i\pi} = 64e^{i\pi} = -64$$

$$28.2. \frac{(\sqrt{3} + 3i)^6}{2i^5} = \frac{(\sqrt{3} + 3i)^6}{2i}$$

$$\text{ Seja } z = \sqrt{3} + 3i. |z| = \sqrt{3+9} = \sqrt{12} = 2\sqrt{3}$$

$$\begin{cases} \tan(\text{Arg}(z)) = \frac{3}{\sqrt{3}} = \sqrt{3} \Rightarrow \frac{\pi}{3} \text{ é um argumento de } z. \\ (\sqrt{3}, 3) \in 1.^\circ \text{Q} \end{cases}$$

Logo,  $z = 2\sqrt{3}e^{i\frac{\pi}{3}}$ .

$$z^6 = \left(2\sqrt{3}e^{i\frac{\pi}{3}}\right)^6 = (2\sqrt{3})^6 e^{i\frac{\pi}{3} \times 6} = 64 \times 27e^{i2\pi} = 1728e^{i0} = 1728$$

$$\text{ Seja } w = (2i)^5 = 2^5 i^5 = 32i.$$

$$\frac{(\sqrt{3} + 3i)^6}{(2i)^5} = \frac{1728}{32i} = \frac{1728 \times i}{32i \times i} = -\frac{1728 \times i}{-32} = -54i$$

$$28.3. \text{ Seja } z_1 = 1 + i. |z_1| = \sqrt{1+1} = \sqrt{2}$$

$$\begin{cases} \tan(\text{Arg}(z_1)) = \frac{1}{1} = 1 \Rightarrow \frac{\pi}{4} \text{ é um argumento de } z_1 \\ (1, 1) \in 1.^\circ \text{Q} \end{cases}$$

Logo,  $z_1 = \sqrt{2}e^{i\frac{\pi}{4}}$ .

$$z_1^{10} = \left(\sqrt{2}e^{i\frac{\pi}{4}}\right)^{10} = (\sqrt{2})^{10} e^{i\frac{10\pi}{4}} = 2^5 e^{i\frac{5\pi}{2}} = 32e^{i\frac{\pi}{2}} \quad \left| \frac{5\pi}{2} - 2\pi = \frac{\pi}{2} \right|$$

$$\text{ Seja } z_2 = 1 - i. |z_2| = \sqrt{1+1} = \sqrt{2}$$

$$\begin{cases} \tan(\text{Arg}(z_2)) = \frac{-1}{1} = -1 \Rightarrow -\frac{\pi}{4} \text{ é um argumento de } z_2 \\ (1, -1) \in 4.^\circ \text{Q} \end{cases}$$

$$\text{ Logo, } z_2 = \sqrt{2}e^{-i\frac{\pi}{4}} \text{ e } z_2^{10} = \left(\sqrt{2}e^{-i\frac{\pi}{4}}\right)^{10} = \left| \frac{10\pi}{4} = \frac{5\pi}{2} \right|$$

$$= (\sqrt{2})^{10} e^{-i\frac{10\pi}{4}} = 32e^{-i\frac{\pi}{2}} = -32i \quad \left| \frac{5\pi}{2} + 2\pi = -\frac{\pi}{2} \right|$$

$$\text{ Portanto, } (1+i)^{10} - (1-i)^{10} = 32i - (-32i) = 64i.$$

29.1.  $z = 1 + i$ ;  $|z| = \sqrt{1+1} = \sqrt{2}$

$$\begin{cases} \tan(\text{Arg}(z)) = \frac{1}{1} \Rightarrow \frac{\pi}{4} \text{ é um argumento de } z. \\ (1, 1) \in 1.^\circ\text{Q} \end{cases}$$

Logo,  $z = \sqrt{2}e^{i\frac{\pi}{4}}$ .

$$(1+i)^{10} = \left(\sqrt{2}e^{i\frac{\pi}{4}}\right)^{10} = (\sqrt{2})^{10} e^{i\frac{10\pi}{4}} = 32e^{i\frac{5\pi}{2}} = 32e^{i\frac{\pi}{2}} = 32i$$

29.2. Seja  $z = \sqrt{3} - i$ .  $|z| = \sqrt{3+1} = 2$

$$\begin{cases} \tan(\text{Arg}(z)) = \frac{-1}{\sqrt{3}} = -\frac{\sqrt{3}}{3} \Rightarrow -\frac{\pi}{6} \text{ é um argumento de } z. \\ (\sqrt{3}, -1) \in 4.^\circ\text{Q} \end{cases}$$

Logo,  $z = 2e^{-i\frac{\pi}{6}}$ .

$$\frac{8i \times (2i)^8}{(\sqrt{3}-i)^{10}} = \frac{2^3 e^{i\frac{\pi}{2}} \times 2^8 \times e^{i\frac{8\pi}{2}}}{\left(2e^{-i\frac{\pi}{6}}\right)^{10}} = \frac{2^{11} \times e^{i(\frac{\pi}{2}+4\pi)}}{2^{10} e^{-i\frac{10\pi}{6}}} = 2 \frac{e^{i\frac{\pi}{2}}}{e^{-i\frac{5\pi}{3}}}$$

$$= 2e^{i(\frac{\pi}{2}+\frac{5\pi}{3})} = 2e^{i\frac{13\pi}{6}} = 2e^{i\frac{\pi}{6}} = 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right) =$$

$$= 2\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right) = \sqrt{3} + i$$

29.3.  $\frac{(-2i)^3}{(1+i)^6} = \frac{-8i^3}{(1-i)^6} = \frac{8i}{(1-i)^6}$

Seja  $z = 1 - i$ .  $|z| = \sqrt{1+1} = \sqrt{2}$

$$\begin{cases} \tan(\text{Arg } z) = \frac{1}{-1} = -1 \Rightarrow -\frac{\pi}{4} \text{ é um argumento de } z. \\ (1, -1) \in 4.^\circ\text{Q} \end{cases}$$

Logo,  $z = \sqrt{2}e^{-i\frac{\pi}{4}}$ .

$$z^6 = \left(\sqrt{2}e^{-i\frac{\pi}{4}}\right)^6 = (\sqrt{2})^6 e^{-i\frac{6\pi}{4}} = 8e^{-i\frac{3\pi}{2}} = 8e^{i\frac{\pi}{2}} = 8i$$

$$\frac{(-2i)^3}{(1+i)^6} = \frac{8i}{8i} = 1$$

30.1.  $(\sqrt{3}-i)^2 + 2e^{-i\frac{\pi}{3}} =$

$$= 3 - 2\sqrt{3}i - 1 + 2\left(\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)\right) =$$

$$= 2 - 2\sqrt{3}i + 2\left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) = 2 - 2\sqrt{3}i + 1 - \sqrt{3}i =$$

$$= 3 - 3\sqrt{3}i$$

$$|3 - 3\sqrt{3}i| = \sqrt{9+27} = \sqrt{36} = 6$$

$$\begin{cases} \tan(\text{Arg}(3-3\sqrt{3}i)) = -\sqrt{3} \\ (3, -3\sqrt{3}) \in 4.^\circ\text{Q} \end{cases} \Rightarrow -\frac{\pi}{3} \text{ é um argumento de } 3-3\sqrt{3}i.$$

Logo,  $(\sqrt{3}-i)^2 + 2e^{-i\frac{\pi}{3}} = 6e^{-i\frac{\pi}{3}}$ .

30.2.  $z = \overline{1+i} + 2\sqrt{2}e^{i\frac{5\pi}{6}} \times e^{-i\frac{\pi}{12}} = 1-i + 2\sqrt{2}e^{i(\frac{5\pi}{6}-\frac{\pi}{12})} =$

$$= 1-i + 2\sqrt{2}e^{i\frac{3\pi}{4}} = 1-i + 2\sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$$

$$= 1-i + 2\sqrt{2}\left(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right) = 1-i-2+2i = -1+i$$

$$|-1+i| = \sqrt{1+1} = \sqrt{2}$$

$$\begin{cases} \tan(\text{Arg } z) = \frac{1}{-1} = -1 \Rightarrow \pi - \frac{\pi}{4} = \frac{3\pi}{4} \text{ é um argumento de } z. \\ (-1, 1) \in 2.^\circ\text{Q} \end{cases}$$

Logo,  $z = \sqrt{2}e^{i\frac{3\pi}{4}}$ .

30.3. Seja  $z = 2 - 2i$ .  $|z| = \sqrt{4+4} = \sqrt{8}$

$$\begin{cases} \tan(\text{Arg}(z)) = -1 \Rightarrow -\frac{\pi}{4} \text{ é um argumento de } z. \\ (2, -2) \in 4.^\circ\text{Q} \end{cases}$$

Logo,  $z = \sqrt{8}e^{-i\frac{\pi}{4}}$ . Assim:

$$(2-2i) \times \sqrt{8}e^{-i\frac{\pi}{6}} = \sqrt{8}e^{-i\frac{\pi}{4}} \times \sqrt{8}e^{-i\frac{\pi}{6}} = 8e^{i(-\frac{\pi}{4}-\frac{\pi}{6})} = 8e^{-i\frac{5\pi}{12}}$$

31.1.  $z_1 \times \bar{z}_2 = 2e^{i\frac{\pi}{4}} \times 4\left(\cos\frac{\pi}{8} + i\sin\frac{\pi}{8}\right) = 2e^{i\frac{\pi}{4}} \times 4e^{i\frac{\pi}{8}} =$

$$= 8e^{i\frac{\pi}{4}} \times e^{-i\frac{\pi}{8}} = 8e^{i(\frac{\pi}{4}-\frac{\pi}{8})} = 8e^{i\frac{\pi}{8}}$$

31.2.  $-z_1 \times z_2^2 = -2e^{i\frac{\pi}{4}} \times \left(4e^{i\frac{2\pi}{8}}\right)^2 \times 2 \times 16e^{i\frac{5\pi}{4}} \times e^{i\frac{\pi}{4}} =$

$$= -32e^{i(\frac{5\pi}{4}+\frac{\pi}{4})} = -32e^{i\frac{3\pi}{2}} = -32e^{-i\frac{\pi}{2}}$$

31.3.  $\left(\frac{i\bar{z}_1^2}{z_2^2}\right) = \left[\frac{e^{i\frac{\pi}{2}} \times \left(2e^{-i\frac{\pi}{4}}\right)^2}{\left(4e^{i\frac{\pi}{8}}\right)^2}\right]^2 = \left[\frac{e^{i\frac{\pi}{2}} \times 2^2 e^{-i\frac{2\pi}{4}}}{4^2 e^{i\frac{2\pi}{8}}}\right]^2 =$

$$= \left[\frac{4e^{i(\frac{\pi}{2}-\frac{\pi}{2})}}{16e^{i\frac{\pi}{4}}}\right]^2 = \left(\frac{1}{4} \times \frac{e^{i0}}{e^{i\frac{\pi}{4}}}\right)^2 = \left(\frac{1}{4} \times e^{-i\frac{\pi}{4}}\right)^2 = \frac{1}{16}e^{-i\frac{\pi}{2}}$$

32.1.  $z = \left(\frac{\sqrt{3}+i}{2+2i}\right)^4$ .  $z_1 = \sqrt{3}+i$  e  $|z_1| = \sqrt{3+1} = 2$

$$\begin{cases} \tan(\arg(z_1)) = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} \Rightarrow \frac{\pi}{6} \text{ é um argumento de } z_1 \\ (\sqrt{3}, 1) \in 1.^\circ\text{Q} \end{cases}$$

Logo,  $z_1 = 2e^{i\frac{\pi}{6}}$ .  $z_2 = 2+2i$ ;  $|z_2| = \sqrt{4+4} = \sqrt{8}$

$$\begin{cases} \tan(\text{Arg}(z_2)) = \frac{2}{2} = 1 \Rightarrow \frac{\pi}{4} \text{ é um argumento de } z_2 \\ (2, 2) \in 1.^\circ\text{Q} \end{cases}$$

Logo,  $z_2 = \sqrt{8}e^{i\frac{\pi}{4}}$ .

$$\left(\frac{\sqrt{3}+i}{2+2i}\right)^4 = \left(\frac{2e^{i\frac{\pi}{6}}}{\sqrt{8}e^{i\frac{\pi}{4}}}\right)^4 = \left(\frac{2e^{i\frac{\pi}{6}}}{2\sqrt{2}e^{i\frac{\pi}{4}}}\right)^4 = \left(\frac{1}{\sqrt{2}}e^{i(\frac{\pi}{6}-\frac{\pi}{4})}\right)^4 =$$

$$= \left(\frac{1}{\sqrt{2}}e^{-i\frac{\pi}{12}}\right)^4 = \left(\frac{1}{\sqrt{2}}\right)^4 e^{-i\frac{4\pi}{12}} = \frac{1}{4}e^{-i\frac{\pi}{3}}$$

Logo,  $|z| = \frac{1}{4}$  e  $\text{Arg}(z) = -\frac{\pi}{3}$ .

32.2. 
$$z = \left(\frac{i^{19}-1}{3+\sqrt{3}i}\right)^4 \times 6e^{i\frac{\pi}{3}} = \left(\frac{i^3-1}{3+\sqrt{3}i}\right)^4 \times 6e^{i\frac{\pi}{3}} = \left(\frac{-1-i}{3+\sqrt{3}i}\right)^4 \times 6e^{i\frac{\pi}{3}}$$

Seja  $z_1 = -1-i$ .  $|z_1| = \sqrt{1+1} = \sqrt{2}$   $|19=4 \times 4+3$

$$\begin{cases} \tan(\text{Arg}(z_1)) = \frac{-1}{-1} = 1 \Rightarrow -\pi + \frac{\pi}{4} = -\frac{3\pi}{4} \text{ é um argumento de } z_1. \\ (-1, -1) \in 3.^\circ\text{Q} \end{cases}$$

Logo,  $z_1 = \sqrt{2}e^{-i\frac{3\pi}{4}}$ . Seja  $z_2 = 3+\sqrt{3}i$ .  $|z_2| = \sqrt{9+3} = \sqrt{12}$

$$\begin{cases} \tan(\text{Arg}(z_2)) = \frac{\sqrt{3}}{3} \Rightarrow \frac{\pi}{6} \text{ é um argumento de } z_2. \\ (3, \sqrt{3}) \in 1.^\circ\text{Q} \end{cases}$$

Logo,  $z_2 = \sqrt{12}e^{i\frac{\pi}{6}}$ .

$$z = \left(\frac{\sqrt{2}e^{-i\frac{3\pi}{4}}}{\sqrt{12}e^{i\frac{\pi}{6}}}\right)^4 \times 6e^{i\frac{\pi}{3}} = \left(\frac{1}{\sqrt{6}}e^{i\left(-\frac{3\pi}{4}-\frac{\pi}{6}\right)}\right)^4 \times 6e^{i\frac{\pi}{3}} = \left(\frac{\sqrt{6}}{6}e^{-i\frac{11\pi}{12}}\right)^4 \times 6e^{i\frac{\pi}{3}} = \frac{1}{36}e^{-i\frac{44\pi}{12}} \times 6e^{i\frac{\pi}{3}} = \frac{1}{6}e^{-i\frac{10\pi}{3}} = \frac{1}{6}e^{i\frac{2\pi}{3}}$$

Portanto,  $|z| = \frac{1}{6}$  e  $\text{Arg}(z) = \frac{2\pi}{3}$ .

32.3. 
$$\frac{4(i-1)^2}{(i^5-\sqrt{3})^4} = \frac{4(-1-2i+1)}{(-\sqrt{3}-i)^4} = \frac{-8i}{(2e^{-i\frac{5\pi}{6}})^4} \begin{cases} z_1 = |z_1|e^{i\theta_1} = -\sqrt{3}-i \\ |z_1| = \sqrt{3+1} = 2 \\ \tan \theta_1 = \frac{-1}{-\sqrt{3}} = \frac{\sqrt{3}}{3} \Rightarrow (-\sqrt{3}, -1) \in 3.^\circ\text{Q} \\ \Rightarrow -\pi + \frac{\pi}{6} \text{ é um argumento de } z_1 \end{cases}$$

$$= \frac{8e^{-i\frac{\pi}{2}}}{2^4 e^{-i\frac{20\pi}{6}}} = \frac{1}{2}e^{-i\left(\frac{\pi}{2}-\frac{10\pi}{3}\right)} = \frac{1}{2}e^{-i\frac{23\pi}{6}} = \frac{1}{2}e^{i\frac{\pi}{6}}$$

Logo,  $|z| = \frac{1}{2}$  e  $\text{Arg}(z) = \frac{\pi}{6}$ .

32.4. 
$$z = \frac{3-5i+2i^{17}}{\left(\sqrt[8]{2}e^{i\frac{\pi}{16}}\right)^4} = \frac{3-5i+2i}{\sqrt{2}e^{i\frac{\pi}{4}}} = \frac{3-3i}{\sqrt{2}e^{i\frac{\pi}{4}}} \quad |17=4 \times 4+1$$

$z_1 = 3-3i$ ;  $|z_1| = \sqrt{9+9} = \sqrt{18}$

$$\begin{cases} \tan(\text{Arg}(z_1)) = \frac{-3}{3} = -1 \Rightarrow -\frac{\pi}{4} \text{ é um argumento de } z_1 \\ (3, -3) \in 4.^\circ\text{Q} \end{cases}$$

$z_1 = \sqrt{18}e^{-i\frac{\pi}{4}}$

$$z = \frac{\sqrt{18}e^{-i\frac{\pi}{4}}}{\sqrt{2}e^{i\frac{\pi}{4}}} = \frac{\sqrt{18}}{\sqrt{2}}e^{i\left(-\frac{\pi}{4}-\frac{\pi}{4}\right)} = \sqrt{9}e^{-i\frac{\pi}{2}} = 3e^{-i\frac{\pi}{2}}$$

$|z| = 3$  e  $\text{Arg}(z) = -\frac{\pi}{2}$

32.5. 
$$z = 1 + e^{i\frac{\pi}{3}} = 1 + \cos\frac{\pi}{3} + i\sin\frac{\pi}{3} = 1 + \frac{1}{2} + \frac{\sqrt{3}}{2}i = \frac{3}{2} + \frac{\sqrt{3}}{2}i$$

$|z| = \sqrt{\frac{9}{4} + \frac{3}{4}} = \sqrt{\frac{12}{4}} = \sqrt{3}$

$$\begin{cases} \tan(\text{Arg}(z)) = \frac{\sqrt{3}}{3} \\ \left(\frac{3}{2}, \frac{\sqrt{3}}{2}\right) \in 1.^\circ\text{Q} \end{cases} \Rightarrow \frac{\pi}{6} \text{ é um argumento de } z$$

$z = \sqrt{3}e^{i\frac{\pi}{6}}$ ;  $|z| = \sqrt{3}$  e  $\text{Arg}(z) = \frac{\pi}{6}$

33.1. 
$$\frac{\cos\theta + i\sin\theta}{\cos\theta - i\sin\theta} = \frac{e^{i\theta}}{\cos(-\theta) + i\sin(-\theta)} = \frac{e^{i\theta}}{e^{-i\theta}} = e^{i\theta-(-i\theta)} = e^{i\theta+i\theta} = e^{2i\theta}$$

33.2. 
$$\frac{\cos\theta + i\sin\theta}{-\cos\theta + i\sin\theta} = \frac{e^{i\theta}}{-(\cos\theta - i\sin\theta)} = \frac{e^{i\theta}}{-(\cos(-\theta) + i\sin(-\theta))}$$

$$= \frac{e^{i\theta}}{-e^{-i\theta}} = \frac{e^{i\theta}}{e^{i(\pi-\theta)}} = e^{i(\theta-\pi+\theta)} = e^{i(2\theta-\pi)}$$

33.3. 
$$\frac{\sin\theta + i\cos\theta}{\cos\theta + i\sin\theta} = \frac{\cos\left(\frac{\pi}{2}-\theta\right) + i\sin\left(\frac{\pi}{2}-\theta\right)}{e^{i\theta}} = \frac{e^{i\left(\frac{\pi}{2}-\theta\right)}}{e^{i\theta}} = e^{i\left(\frac{\pi}{2}-\theta-\theta\right)} = e^{i\left(\frac{\pi}{2}-2\theta\right)}$$

34. 
$$z = \frac{1+i\tan\theta}{1-i\tan\theta} = \frac{1+i\frac{\sin\theta}{\cos\theta}}{1-i\frac{\sin\theta}{\cos\theta}} = \frac{\cos\theta + i\sin\theta}{\cos\theta - i\sin\theta} = \frac{e^{i\theta}}{\cos(-\theta) + i\sin(-\theta)} = \frac{e^{i\theta}}{e^{-i\theta}} = e^{i(\theta+\theta)} = e^{i2\theta}$$

$|z| = 1$  e  $\text{Arg}(z) = 2\theta$

35. 
$$z = \left(\frac{i-1}{i-\sqrt{3}}\right)^n = \left(\frac{-1+i}{-\sqrt{3}+i}\right)^n$$

$z_1 = -1+i = |z_1|e^{i\theta_1}$ ;  $|z_1| = \sqrt{1+1} = \sqrt{2}$

$$\begin{cases} \tan\theta_1 = \frac{1}{-1} = -1 \Rightarrow \pi - \frac{\pi}{4} = \frac{3\pi}{4} \text{ é um argumento de } z_1. \\ (-1, 1) \in 2.^\circ\text{Q} \end{cases}$$

$z_1 = \sqrt{2}e^{i\frac{3\pi}{4}}$ ;  $z_2 = -\sqrt{3}+i = |z_2|e^{i\theta_2}$ ;  $|z_2| = \sqrt{3+1} = 2$

$$\begin{cases} \tan\theta_2 = \frac{1}{-\sqrt{3}} = -\frac{\sqrt{3}}{3} \Rightarrow \pi - \frac{\pi}{6} = \frac{5\pi}{6} \text{ é um argumento de } z_2. \\ (-\sqrt{3}, 1) \in 2.^\circ\text{Q} \end{cases}$$

$z_2 = 2e^{i\frac{5\pi}{6}}$

$$z = \left(\frac{\sqrt{3}e^{i\frac{3\pi}{4}}}{2e^{i\frac{5\pi}{6}}}\right)^n = \left(\frac{\sqrt{2}}{2}e^{i\left(\frac{3\pi}{4}-\frac{5\pi}{6}\right)}\right)^n = \left(\frac{\sqrt{2}}{2}e^{-i\frac{\pi}{12}}\right)^n = \left(\frac{\sqrt{2}}{2}\right)^n e^{-i\frac{n\pi}{12}}$$

35.1.  $z \in \mathbb{R} \Leftrightarrow -\frac{n\pi}{12} = k\pi, k \in \mathbb{Z} \Leftrightarrow \frac{n}{12} = -k, k \in \mathbb{Z}; n = 12$

35.2.  $z = bi$  com  $b \in \mathbb{R}^- \Leftrightarrow \frac{n\pi}{12} = -\frac{\pi}{2} + 2k\pi, k \in \mathbb{Z} \Leftrightarrow n = 6 + 12k, k \in \mathbb{Z}; n = 6$

36. 
$$\begin{aligned} 1-z &= 1-e^{i\theta} = 1-\cos\theta - i\sin\theta = \\ &= 1-\cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2} - 2i\sin\frac{\theta}{2}\cos\frac{\theta}{2} = \\ &= \sin^2\frac{\theta}{2} + \sin^2\frac{\theta}{2} - 2i\sin\frac{\theta}{2}\cos\frac{\theta}{2} = \\ &= 2\sin^2\frac{\theta}{2} - 2i\sin\frac{\theta}{2}\cos\frac{\theta}{2} = 2\sin\frac{\theta}{2}\left(\sin\frac{\theta}{2} - i\cos\frac{\theta}{2}\right) = \end{aligned}$$

$$= 2 \sin \frac{\theta}{2} \left[ \cos \left( -\frac{\pi}{2} + \frac{\theta}{2} \right) + i \sin \left( -\frac{\pi}{2} + \frac{\theta}{2} \right) \right] = 2 \sin \frac{\theta}{2} e^{i \left( -\frac{\pi}{2} + \frac{\theta}{2} \right)}$$

Como  $\frac{\theta}{2} \in \left] 0, \frac{\pi}{2} \right[$ ,  $2 \sin \theta > 0$  e  $-\frac{\pi}{2} < \frac{\theta}{2} - \frac{\pi}{2} < 0$ .

Assim,  $|1-z| = 2 \sin \frac{\theta}{2}$  e  $\text{Arg}(1-z) = \frac{\theta}{2} - \frac{\pi}{2}$ .

**37.1.**  $1 + e^{i\frac{\pi}{4}} = 1 + \cos \frac{\pi}{4} + i \sin \frac{\pi}{4} =$   
 $= 1 + \cos^2 \frac{\pi}{8} - \sin^2 \frac{\pi}{8} + 2i \sin \frac{\pi}{8} \cos \frac{\pi}{8} =$   
 $= 2 \cos^2 \frac{\pi}{8} + 2i \sin \frac{\pi}{8} \cos \frac{\pi}{8} =$   
 $= 2 \cos \frac{\pi}{8} \left( \cos \frac{\pi}{8} + i \sin \frac{\pi}{8} \right) = 2 \cos \frac{\pi}{8} e^{i\frac{\pi}{8}} = \sqrt{2 + \sqrt{2}} e^{i\frac{\pi}{8}}$

Cálculo auxiliar

$$\cos \frac{\pi}{4} = \cos^2 \frac{\pi}{8} - \sin^2 \frac{\pi}{8} \Leftrightarrow \frac{\sqrt{2}}{2} = \cos^2 \frac{\pi}{8} - 1 + \cos^2 \frac{\pi}{8} \Leftrightarrow \frac{\sqrt{2}}{2} + 1 = 2 \cos^2 \frac{\pi}{8}$$

$$\Leftrightarrow \cos^2 \frac{\pi}{8} = \frac{\sqrt{2} + 2}{4} \Leftrightarrow \cos \frac{\pi}{8} = \frac{\sqrt{\sqrt{2} + 2}}{2} \Leftrightarrow 2 \cos \frac{\pi}{8} = \sqrt{\sqrt{2} + 2}$$

**37.2.**  $1 + e^{i\frac{\pi}{6}} = 1 + \cos \frac{\pi}{6} + i \sin \frac{\pi}{6} =$   
 $= 1 + \cos^2 \frac{\pi}{12} - \sin^2 \frac{\pi}{12} + 2i \sin \frac{\pi}{12} \cos \frac{\pi}{12} =$   
 $= 2 \cos^2 \frac{\pi}{12} + 2i \sin \frac{\pi}{12} \cos \frac{\pi}{12} =$   
 $= 2 \cos \frac{\pi}{12} \left( \cos \frac{\pi}{12} + i \sin \frac{\pi}{12} \right) = \sqrt{2 + \sqrt{3}} e^{i\frac{\pi}{12}}$

Cálculo auxiliar

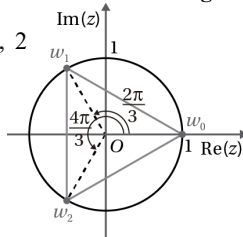
$$\cos \frac{\pi}{6} = \cos^2 \frac{\pi}{12} - \sin^2 \frac{\pi}{12} \Leftrightarrow \frac{\sqrt{3}}{2} = \cos^2 \frac{\pi}{12} - 1 + \cos^2 \frac{\pi}{12}$$

$$\Leftrightarrow \frac{\sqrt{3}}{2} + 1 = 2 \cos^2 \frac{\pi}{12} \Leftrightarrow 2 \cos^2 \frac{\pi}{12} = \frac{\sqrt{3} + 2}{2} \Leftrightarrow \cos^2 \frac{\pi}{12} = \frac{\sqrt{3} + 2}{4}$$

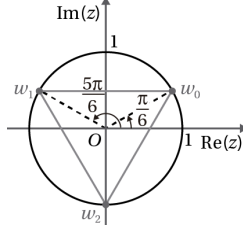
$$\Leftrightarrow \cos \frac{\pi}{12} = \frac{\sqrt{\sqrt{3} + 2}}{2} \Leftrightarrow 2 \cos \frac{\pi}{12} = \sqrt{2 + \sqrt{3}}$$

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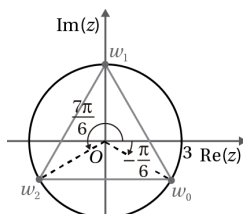
**38.1.**  $\sqrt[3]{w} = \sqrt[3]{1} = \sqrt[3]{e^{i0}} = e^{i\frac{2k\pi}{3}}$ ,  $k = 0, 1, 2$   
 $k = 0, w_0 = e^{i \cdot 0} = 1$   
 $k = 1, w_1 = e^{i\frac{2\pi}{3}}$   
 $k = 2, w_2 = e^{i\frac{4\pi}{3}}$



**38.2.**  $\sqrt[3]{w} = \sqrt[3]{i} = \sqrt[3]{e^{i\frac{\pi}{2}}} = e^{i\left(\frac{\pi}{6} + \frac{2k\pi}{3}\right)} = e^{i\left(\frac{\pi}{6} + \frac{4k\pi}{6}\right)}$ ,  $k = 0, 1, 2$   
 $k = 0, w_0 = e^{i\frac{\pi}{6}}$   
 $k = 1, w_1 = e^{i\left(\frac{\pi}{6} + \frac{4\pi}{6}\right)} = e^{i\frac{5\pi}{6}}$   
 $k = 2, w_2 = e^{i\left(\frac{\pi}{6} + \frac{8\pi}{6}\right)} = e^{i\frac{9\pi}{6}} = e^{i\frac{3\pi}{2}}$



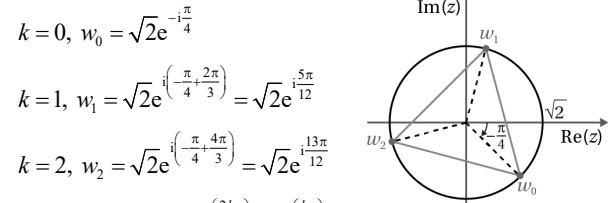
**38.3.**  $\sqrt[3]{27e^{-i\frac{\pi}{2}}} = \sqrt[3]{27} e^{-i\frac{\pi}{6}} = 3e^{-i\frac{\pi}{6}}$ ,  $k = 0, 1, 2$   
 $k = 0, w_0 = 3e^{-i\frac{\pi}{6}}$   
 $k = 1, w_1 = 3e^{i\left(-\frac{\pi}{6} + \frac{4\pi}{3}\right)} = 3e^{i\frac{7\pi}{6}}$   
 $k = 2, w_2 = 3e^{i\left(-\frac{\pi}{6} + \frac{8\pi}{3}\right)} = 3e^{i\frac{15\pi}{6}} = 3e^{i\frac{5\pi}{2}}$



**38.4.**  $w = -2 - 2i$ ;  $|w| = \sqrt{4+4} = \sqrt{8}$   
 $\left\{ \begin{array}{l} \tan(\text{Arg } w) = \frac{-2}{-2} = 1 \Rightarrow -\pi + \frac{\pi}{4} = -\frac{3\pi}{4} \text{ é argumento de } w \\ (-2, -2) \in 3.^\circ\text{Q} \end{array} \right.$

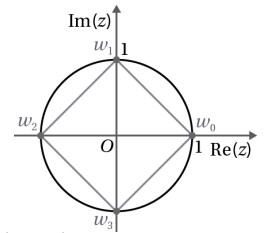
Logo,  $w = \sqrt{8} e^{-i\frac{3\pi}{4}}$ .

$$\sqrt[3]{w} = \sqrt[3]{\sqrt{8} e^{-i\frac{3\pi}{4}}} = \sqrt[3]{\sqrt{8}} e^{-i\frac{\pi}{4}} = \sqrt[3]{8} e^{-i\frac{\pi}{4}} = 2e^{-i\frac{\pi}{4}}$$



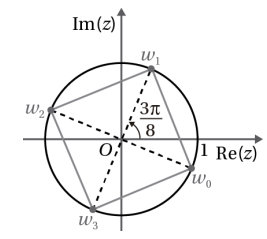
**39.1.**  $\sqrt[4]{w} = \sqrt[4]{1} = \sqrt[4]{e^{i0}} = e^{i\frac{2k\pi}{4}} = e^{i\frac{k\pi}{2}}$ ,  $k = 0, 1, 2, 3$

$k = 0, w_0 = e^{i0} = 1$   
 $k = 1, w_1 = e^{i\frac{\pi}{2}} = i$   
 $k = 2, w_2 = e^{i\pi} = -1$   
 $k = 3, w_3 = e^{i\frac{3\pi}{2}} = -i$



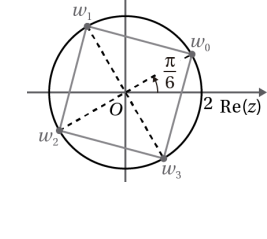
**39.2.**  $\sqrt[4]{w} = \sqrt[4]{-i} = \sqrt[4]{e^{-i\frac{\pi}{2}}} = e^{-i\frac{\pi}{8}} = e^{i\left(-\frac{\pi}{8} + \frac{2k\pi}{4}\right)} = e^{i\left(-\frac{\pi}{8} + \frac{k\pi}{2}\right)}$ ,  $k = 0, 1, 2, 3$

$k = 0, w_0 = e^{-i\frac{\pi}{8}}$   
 $k = 1, w_1 = e^{i\frac{3\pi}{8}}$   
 $k = 2, w_2 = e^{i\frac{7\pi}{8}}$   
 $k = 3, w_3 = e^{i\frac{11\pi}{8}}$



**39.3.**  $\sqrt[4]{w} = \sqrt[4]{16e^{i\frac{2\pi}{3}}} = \sqrt[4]{16} e^{i\frac{2\pi}{12}} = 2e^{i\frac{\pi}{6}} = 2e^{i\left(\frac{\pi}{6} + \frac{k\pi}{2}\right)} = 2e^{i\left(\frac{\pi}{6} + \frac{3k\pi}{6}\right)}$   
 com  $k = 0, 1, 2, 3$ .

$k = 0, w_0 = 2e^{i\frac{\pi}{6}}$   
 $k = 1, w_1 = 2e^{i\frac{4\pi}{6}} = e^{i\frac{2\pi}{3}}$   
 $k = 2, w_2 = 2e^{i\frac{7\pi}{6}}$   
 $k = 3, w_3 = 2e^{i\frac{10\pi}{6}} = 2e^{i\frac{5\pi}{3}}$



**39.4.**  $w = -\frac{1+i\sqrt{3}}{2} = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$ ;  $|w| = \sqrt{\frac{1}{4} + \frac{3}{4}} = 1$

$$\left\{ \begin{array}{l} \tan(\text{Arg } w) = \frac{-\sqrt{3}}{-1} = \sqrt{3} \\ \left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right) \in 3.^\circ\text{Q} \end{array} \right. \Rightarrow$$

$\Rightarrow -\pi + \frac{\pi}{3} = -\frac{2\pi}{3}$  é um argumento de  $w$ . Logo,  $w = e^{-i\frac{2\pi}{3}}$ .

$$\sqrt[4]{w} = \sqrt[4]{e^{-i\frac{2\pi}{3}}} = e^{-i\frac{2\pi}{12}} = e^{-i\frac{\pi}{6}} = e^{i\left(-\frac{\pi}{6} + \frac{k\pi}{2}\right)} = e^{i\left(-\frac{\pi}{6} + \frac{3k\pi}{6}\right)}$$

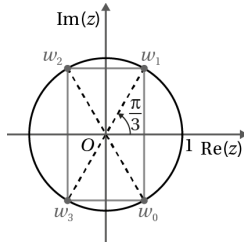
$k = 0, 1, 2, 3$ .

$$k = 0, w_0 = e^{-i\frac{\pi}{6}}$$

$$k = 1, w_1 = e^{i\frac{\pi}{3}}$$

$$k = 2, w_2 = e^{i\frac{5\pi}{6}}$$

$$k = 3, w_3 = e^{i\frac{4\pi}{3}}$$



40.1.  $\sqrt[6]{w} = \sqrt[6]{-1} = \sqrt[6]{e^{i\pi}} = e^{i(\frac{\pi}{6} + \frac{2k\pi}{6})}, k = 0, 1, 2, 3, 4, 5$

$$k = 0, w_0 = e^{i\frac{\pi}{6}}$$

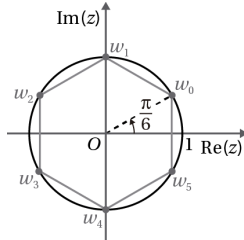
$$k = 1, w_1 = e^{i\frac{\pi}{2}}$$

$$k = 2, w_2 = e^{i\frac{5\pi}{6}}$$

$$k = 3, w_3 = e^{i\frac{7\pi}{6}}$$

$$k = 4, w_4 = e^{i\frac{3\pi}{2}}$$

$$k = 5, w_5 = e^{i\frac{11\pi}{6}}$$



40.2.  $\sqrt[6]{w} = \sqrt[6]{i} = \sqrt[6]{e^{i\frac{\pi}{2}}} = e^{i(\frac{\pi}{12} + \frac{2k\pi}{6})} = e^{i(\frac{\pi}{12} + \frac{4k\pi}{12})}, k = 0, 1, 2, 3, 4, 5$

$$k = 0, w_0 = e^{i\frac{\pi}{12}}$$

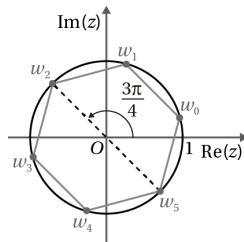
$$k = 1, w_1 = e^{i(\frac{\pi}{12} + \frac{4\pi}{12})} = e^{i\frac{5\pi}{12}}$$

$$k = 2, w_2 = e^{i(\frac{\pi}{12} + \frac{8\pi}{12})} = e^{i\frac{9\pi}{12}} = e^{i\frac{3\pi}{4}}$$

$$k = 3, w_3 = e^{i(\frac{\pi}{12} + \frac{12\pi}{12})} = e^{i\frac{13\pi}{12}}$$

$$k = 4, w_4 = e^{i(\frac{5\pi}{12} + \frac{16\pi}{12})} = e^{i\frac{17\pi}{12}}$$

$$k = 5, w_5 = e^{i(\frac{\pi}{12} + \frac{20\pi}{12})} = e^{i\frac{21\pi}{12}} = e^{i\frac{7\pi}{4}}$$



40.3.  $\sqrt[6]{w} = \sqrt[6]{8e^{-i\frac{\pi}{8}}} = \sqrt[6]{8}e^{i(\frac{-\pi}{6} + \frac{2k\pi}{6})} = \sqrt{2}e^{i(\frac{-\pi}{48} + \frac{16k\pi}{48})}, k = 0, 1, 2, 3, 4, 5, 6$

$$k = 0, w_0 = \sqrt{2}e^{i\frac{\pi}{48}}$$

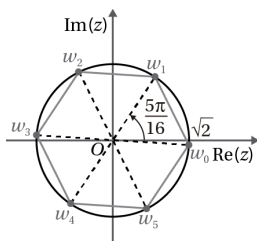
$$k = 1, w_1 = \sqrt{2}e^{i(\frac{-\pi}{48} + \frac{16\pi}{48})} = \sqrt{2}e^{i\frac{15\pi}{48}} = \sqrt{2}e^{i\frac{5\pi}{16}}$$

$$k = 2, w_2 = \sqrt{2}e^{i(\frac{-\pi}{48} + \frac{32\pi}{48})} = \sqrt{2}e^{i\frac{31\pi}{48}}$$

$$k = 3, w_3 = \sqrt{2}e^{i(\frac{-\pi}{48} + \frac{48\pi}{48})} = \sqrt{2}e^{i\frac{47\pi}{48}}$$

$$k = 4, w_4 = \sqrt{2}e^{i(\frac{-\pi}{48} + \frac{64\pi}{48})} = \sqrt{2}e^{i\frac{63\pi}{48}} = \sqrt{2}e^{i\frac{21\pi}{16}}$$

$$k = 5, w_5 = \sqrt{2}e^{i(\frac{-\pi}{48} + \frac{80\pi}{48})} = \sqrt{2}e^{i\frac{79\pi}{48}}$$



40.4.  $\sqrt[6]{w} = \sqrt[6]{\frac{1}{8}i} = \frac{1}{\sqrt[6]{8}}e^{i(\frac{\pi}{6} + \frac{2k\pi}{6})} = \frac{\sqrt{2}}{2}e^{i(\frac{\pi}{12} + \frac{4k\pi}{12})}, k = 0, 1, 2, 3, 4, 5$

$$k = 0, w_0 = \frac{\sqrt{2}}{2}e^{i\frac{\pi}{12}}$$

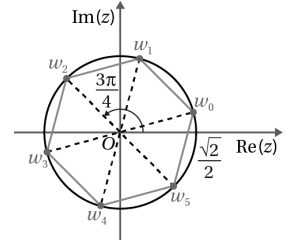
$$k = 1, w_1 = \frac{\sqrt{2}}{2}e^{i\frac{5\pi}{12}}$$

$$k = 2, w_2 = \frac{\sqrt{2}}{2}e^{i\frac{\pi}{12}} = \frac{\sqrt{2}}{2}e^{i\frac{3\pi}{4}}$$

$$k = 3, w_3 = \frac{\sqrt{2}}{2}e^{i\frac{13\pi}{12}}$$

$$k = 4, w_4 = \frac{\sqrt{2}}{2}e^{i\frac{17\pi}{12}}$$

$$k = 5, w_5 = \frac{\sqrt{2}}{2}e^{i\frac{21\pi}{12}} = \frac{\sqrt{2}}{2}e^{i\frac{7\pi}{4}}$$



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41.  $\sqrt[6]{-64} = \sqrt[6]{64e^{i\pi}} = \sqrt[6]{64}e^{i(\frac{\pi}{6} + \frac{2k\pi}{6})} = 2e^{i(\frac{\pi}{6} + \frac{2k\pi}{6}), k = 0, 1, 2, 3, 4, 5$

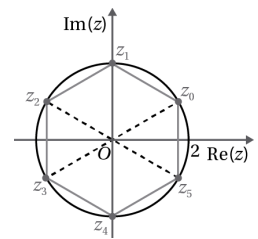
$$k = 0, z_0 = 2e^{i\frac{\pi}{6}}$$

$$k = 1, z_1 = 2e^{i\frac{3\pi}{6}} = 2e^{i\frac{\pi}{2}}$$

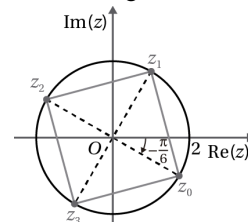
$$k = 2, z_2 = 2e^{i\frac{5\pi}{6}}$$

$$k = 3, z_3 = 2e^{i\frac{7\pi}{6}}$$

$$k = 4, z_4 = 2e^{i\frac{\pi}{6}} = 2e^{i\frac{3\pi}{2}}; k = 5, z_5 = 2e^{i\frac{11\pi}{6}}$$



42.1. Os afixos das raízes quartas de  $z$  são vértices de um quadrado de centro na origem.



42.2.  $z_0 = 2e^{-i\frac{\pi}{6}} = 2\left(\cos\left(-\frac{\pi}{6}\right) + i\sin\left(-\frac{\pi}{6}\right)\right) = \quad (4.^\circ Q)$

$$= 2\left(\frac{\sqrt{3}}{2} - \frac{1}{2}i\right) = \sqrt{3} - i$$

$$z_1 = 2e^{i(\frac{\pi}{6} + \frac{\pi}{2})} = 2e^{i\frac{\pi}{3}} = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right) =$$

$$= 2\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) = 1 + \sqrt{3}i$$

$$z_2 = 2e^{i(\frac{\pi}{3} + \frac{\pi}{2})} = 2e^{i\frac{5\pi}{6}} = 2\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right) = \quad (2.^\circ Q)$$

$$= 2\left(-\frac{\sqrt{3}}{2} + \frac{1}{2}i\right) = -\sqrt{3} + i$$

$$z_3 = 2e^{i(\frac{5\pi}{6} + \frac{\pi}{2})} = 2e^{i\frac{4\pi}{3}} = 2\left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right) = \quad (3.^\circ Q)$$

$$= 2\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) = -1 - \sqrt{3}i$$

42.3.  $z = z_0^4 = \left(2e^{-i\frac{\pi}{6}}\right)^4 = 2^4e^{-i\frac{4\pi}{6}} = 16e^{-i\frac{2\pi}{3}} =$

$$= 16\left(\cos\left(-\frac{2\pi}{3}\right) + i\sin\left(-\frac{2\pi}{3}\right)\right) = -8 - 8\sqrt{3}i$$

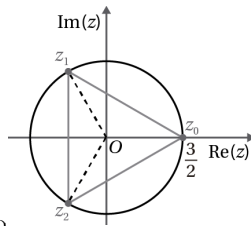
43.1.  $8z^3 = 27 \Leftrightarrow z^3 = \frac{27}{8} \Leftrightarrow z = \sqrt[3]{\frac{27}{8}} \Leftrightarrow z = \sqrt[3]{\frac{27}{8}} e^{i0} \Leftrightarrow$

$\Leftrightarrow z = \sqrt[3]{\frac{27}{8}} e^{i\frac{2k\pi}{3}} \Leftrightarrow z = \frac{3}{2} e^{i\frac{2k\pi}{3}}, k = 0, 1, 2$

$k = 0, z_0 = \frac{3}{2} e^{i0}$

$k = 1, z_1 = \frac{3}{2} e^{i\frac{2\pi}{3}}$

$k = 2, z_2 = \frac{3}{2} e^{i\frac{4\pi}{3}}$



Logo,  $S = \left\{ \frac{3}{2} e^{i0}, \frac{3}{2} e^{i\frac{2\pi}{3}}, \frac{3}{2} e^{i\frac{4\pi}{3}} \right\}$

As imagens geométricas das soluções da equação definem, no plano complexo, um triângulo equilátero inscrito numa circunferência de raio  $\frac{3}{2}$ .

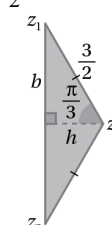
43.2. O triângulo equilátero é composto por três triângulo isósceles, cada um com dois lados de medida  $\frac{3}{2}$  que

formam um ângulo de  $\frac{\pi}{3}$  rad.

$\cos \frac{\pi}{3} = \frac{h}{\frac{3}{2}} \Leftrightarrow h = \frac{3}{2} \cos \frac{\pi}{3} \Leftrightarrow$

$\Leftrightarrow h = \frac{3}{2} \times \frac{1}{2} \Leftrightarrow h = \frac{3}{4}$

$\sin \frac{\pi}{3} = \frac{b}{\frac{3}{2}} \Leftrightarrow b = \frac{3}{2} \sin \frac{\pi}{3} \Leftrightarrow b = \frac{3}{2} \times \frac{\sqrt{3}}{2} \Leftrightarrow b = \frac{3\sqrt{3}}{4}$



Área do triângulo  $[z_0 z_1 z_2]$ :

$A = 3 \times \frac{2 \times \frac{3\sqrt{3}}{4} \times \frac{3}{4}}{2} = \frac{27\sqrt{3}}{16}$

A área do triângulo é igual a  $\frac{27\sqrt{3}}{16}$  u.a.

44.1. Para que os afixos de  $z_1$  e  $z_2$  sejam vértices de um hexágono,  $\text{Arg}(z_2) - \text{Arg}(z_1) = \frac{2\pi}{6} = \frac{\pi}{3}$  e  $|z_1| = |z_2|$ .

$|z_1| = |-1 - i\sqrt{3}| = \sqrt{1+3} = 2; |z_2| = |1 - i\sqrt{3}| = \sqrt{1+3} = 2$

$\left\{ \begin{aligned} \tan(\text{Arg } z_2) &= \frac{-\sqrt{3}}{1} = -\sqrt{3} \Rightarrow -\frac{\pi}{3} \text{ é um argumento de } z_2 \\ (1, -\sqrt{3}) &\in 4^\circ\text{Q} \end{aligned} \right.$

$\text{Arg}(z_2) = -\frac{\pi}{3}$

$\text{Arg}(z_2) - \text{Arg}(z_1) = -\frac{\pi}{3} - \left(\frac{2\pi}{3}\right) = -\frac{\pi}{3} + \frac{2\pi}{3} = \frac{\pi}{3} = \frac{2\pi}{6}$

Logo, o polígono é um hexágono.

44.2. Os argumentos dos afixos que são vértices consecutivos do hexágono diferem entre si  $\frac{\pi}{3}$  rad.

$z_2 = 2e^{-i\frac{\pi}{3}}; z_3 = 2e^{i\left(-\frac{\pi}{3} + \frac{\pi}{3}\right)} = 2e^{i0} = 2$

$z_4 = 2e^{i\left(0 + \frac{\pi}{3}\right)} = 2e^{i\frac{\pi}{3}} = 2\left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}\right) = 2\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) = 1 + \sqrt{3}i$

$z_5 = 2e^{i\left(\frac{\pi}{3} + \frac{\pi}{3}\right)} = 2e^{i\frac{2\pi}{3}} = 2\left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}\right) =$

$= 2\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) = -1 + \sqrt{3}i$

$z_6 = 2e^{i\left(\frac{2\pi}{3} + \frac{\pi}{3}\right)} = 2e^{i\pi} = -2$

As coordenadas dos vértices do hexágono são:

$(-1, -\sqrt{3}), (1, -\sqrt{3}), (2, 0), (1, \sqrt{3}), (-1, \sqrt{3})$  e  $(-2, 0)$

44.3. Por exemplo,  $2^6 = 64$ . Logo, a equação  $z^6 = 64$ , sendo  $z$  um número complexo, é um exemplo de equação pedida.

45.1.  $z^4 = z \Leftrightarrow z^4 - z = 0 \Leftrightarrow z(z^3 - 1) = 0 \Leftrightarrow$

$\Leftrightarrow z = 0 \vee z^3 - 1 = 0 \Leftrightarrow z = 0 \vee z^3 = 1 \Leftrightarrow$

$\Leftrightarrow z = 0 \vee z = \sqrt[3]{1} \Leftrightarrow z = 0 \vee z = e^{i\frac{2k\pi}{3}}, k = 0, 1, 2 \Leftrightarrow$

$\Leftrightarrow z = 0 \vee z = e^{i0} = 1 \vee z = e^{i\frac{2\pi}{3}} \vee z = e^{i\frac{4\pi}{3}}$

$S = \left\{ 0, 1, e^{i\frac{2\pi}{3}}, e^{i\frac{4\pi}{3}} \right\}$

45.2.  $z^4 = \bar{z}^2 \Leftrightarrow (re^{i\theta})^4 = (re^{-i\theta})^2 \Leftrightarrow |z = e^{i\theta}$

$\Leftrightarrow r^4 e^{i4\theta} = r^2 e^{-i2\theta} \Leftrightarrow \begin{cases} r^4 = r^2 \\ 4\theta = -2\theta + 2k\pi, k \in \mathbb{Z} \end{cases} \Leftrightarrow$

$\Leftrightarrow \begin{cases} r^2(r^2 - 1) = 0 \\ 6\theta = 2k\pi, k \in \mathbb{Z} \end{cases} \Leftrightarrow \begin{cases} r = 0 \vee r = 1 \\ \theta = \frac{2k\pi}{6}, k \in \mathbb{Z} \end{cases} \Leftrightarrow$

$\Leftrightarrow z = 0 \vee z = e^{i0} \vee z = e^{i\frac{\pi}{3}} \vee z = e^{i\frac{2\pi}{3}} \vee$

$\vee z = e^{i\pi} \vee z = e^{i\frac{4\pi}{3}} \vee z = e^{i\frac{5\pi}{3}}$

$S = \left\{ 0, 1, e^{i\frac{\pi}{3}}, e^{i\frac{2\pi}{3}}, e^{i\pi}, e^{i\frac{4\pi}{3}}, e^{i\frac{5\pi}{3}} \right\}$

45.3.  $z^3 = (1-i)^3$ . Seja  $w = 1-i$ .  $|w| = \sqrt{1+1} = \sqrt{2}$

$\left\{ \begin{aligned} \tan(\text{Arg}(w)) &= \frac{-1}{1} = -1 \Rightarrow -\frac{\pi}{4} \text{ é um argumento de } w. \\ (1, -1) &\in 4^\circ\text{Q} \end{aligned} \right.$

Logo,  $w = \sqrt{2}e^{-i\frac{\pi}{4}}$  e  $w^3 = \sqrt{8}e^{-i\frac{3\pi}{4}}$ .

$z^3 = (1-i)^3 \Leftrightarrow z^3 = \sqrt{8}e^{-i\frac{3\pi}{4}} \Leftrightarrow z = \sqrt[3]{\sqrt{8}e^{-i\frac{3\pi}{4}}} \Leftrightarrow$

$\Leftrightarrow z = \sqrt{2}e^{i\left(\frac{-\frac{3\pi}{4}}{3} + \frac{2k\pi}{3}\right)}, k = 0, 1, 2 \Leftrightarrow$

$\Leftrightarrow z = \sqrt{2}e^{i\left(\frac{-\frac{3\pi}{4} + 8k\pi}{12}\right)}, k = 0, 1, 2$

$z_0 = \sqrt{2}e^{-i\frac{3\pi}{12}} = e^{-i\frac{\pi}{4}}; z_1 = \sqrt{2}e^{i\frac{5\pi}{12}}; z_2 = \sqrt{2}e^{i\frac{13\pi}{12}}$

$S = \left\{ \sqrt{2}e^{-i\frac{\pi}{4}}, \sqrt{2}e^{i\frac{5\pi}{12}}, \sqrt{2}e^{i\frac{13\pi}{12}} \right\}$

45.4.  $\bar{z}^3 - z = 0$ . Seja  $z = re^{i\theta}$ .

$\bar{z}^3 - z = 0 \Leftrightarrow \bar{z}^3 = z \Leftrightarrow r^3 e^{-i3\theta} = re^{i\theta} \Leftrightarrow$

$\Leftrightarrow \begin{cases} r^3 = r \wedge r \geq 0 \\ -3\theta = \theta + 2k\pi, k \in \mathbb{Z} \end{cases} \Leftrightarrow \begin{cases} r^3 - r = 0 \wedge r \geq 0 \\ -4\theta = 2k\pi, k \in \mathbb{Z} \end{cases} \Leftrightarrow$

$$\Leftrightarrow \begin{cases} r(r^2 - 1) = 0 \wedge r \geq 0 \\ \theta = \frac{k\pi}{2}, k \in \mathbb{Z} \end{cases} \Leftrightarrow \begin{cases} r = 0 \vee r = 1 \\ \theta = \frac{k\pi}{2}, k \in \mathbb{Z} \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} r = 0 \\ \theta = \frac{k\pi}{2}, k \in \mathbb{Z} \end{cases} \vee \begin{cases} r = 1 \\ \theta = \frac{k\pi}{2}, k \in \mathbb{Z} \end{cases}$$

Logo,  $z_0 = 0$ ;  $z_1 = e^{i0} = 1$ ;  $z_2 = e^{i\frac{\pi}{2}}$ ;  $z_3 = e^{i\pi}$ ;  $z_4 = e^{i\frac{3\pi}{2}}$

$$S = \left\{ 0, 1, e^{i\frac{\pi}{2}}, e^{-i\frac{\pi}{2}} \right\} = \{0, 1, i, -1, -i\}$$

45.5.  $z^4 + i = 1 \Leftrightarrow z^4 = 1 - i$ ;  $|1 - i| = \sqrt{1+1} = \sqrt{2}$

$$\begin{cases} \tan(\text{Arg}(1-i)) = \frac{-1}{1} = -1 \Rightarrow -\frac{\pi}{4} \text{ é um argumento de } 1-i \\ (1, -1) \in 4.^\circ\text{Q} \end{cases}$$

Assim, tem-se:

$$z^4 = 1 - i \Leftrightarrow z^4 = \sqrt{2}e^{-i\frac{\pi}{4}} \Leftrightarrow z = \sqrt[4]{\sqrt{2}}e^{-i\frac{\pi}{4}} \Leftrightarrow$$

$$\Leftrightarrow z = \sqrt[8]{2}e^{i\left(\frac{-\pi}{4} + \frac{2k\pi}{4}\right)}, k = 0, 1, 2, 3 \Leftrightarrow$$

$$\Leftrightarrow z = \sqrt[8]{2}e^{i\left(\frac{-\pi}{16} + \frac{8k\pi}{16}\right)}, k = 0, 1, 2, 3$$

Logo:  $z_0 = \sqrt[8]{2}e^{-i\frac{\pi}{16}}$ ;  $z_1 = \sqrt[8]{2}e^{i\frac{7\pi}{16}}$ ;  $z_2 = \sqrt[8]{2}e^{i\frac{15\pi}{16}}$ ;  $z_3 = \sqrt[8]{2}e^{i\frac{23\pi}{16}}$

$$S = \left\{ \sqrt[8]{2}e^{-i\frac{\pi}{16}}; \sqrt[8]{2}e^{i\frac{7\pi}{16}}; \sqrt[8]{2}e^{i\frac{15\pi}{16}}; \sqrt[8]{2}e^{i\frac{23\pi}{16}} \right\}$$

46.1.  $z_0 = \frac{\sqrt{3}}{4} - \frac{1}{4}i$ ;  $|z_0| = \sqrt{\frac{3}{16} + \frac{1}{16}} = \sqrt{\frac{4}{16}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$

$$\begin{cases} \tan(\text{Arg } z_0) = \frac{-1}{\sqrt{3}} = -\frac{\sqrt{3}}{3} \Rightarrow -\frac{\pi}{6} \text{ é um argumento de } z_0 \\ \left(\frac{\sqrt{3}}{4}, -\frac{1}{4}\right) \in 4.^\circ\text{Q} \end{cases}$$

Assim,  $z_0 = \frac{1}{2}e^{-i\frac{\pi}{6}}$ .

$$z = z_0^4 = \left(\frac{1}{2}e^{-i\frac{\pi}{6}}\right)^4 = \frac{1}{16}e^{-i\frac{4\pi}{6}} = \frac{1}{16}e^{-i\frac{2\pi}{3}}$$

Logo,  $|z| = \frac{1}{16}$  e  $\arg(z) = -\frac{2\pi}{3}$ .

46.2.  $\sqrt[4]{z} = \sqrt[4]{\frac{1}{16}e^{-i\frac{2\pi}{3}}} = \sqrt[4]{\frac{1}{16}}e^{i\left(\frac{-2\pi}{4} + \frac{2k\pi}{4}\right)}, k = 0, 1, 2, 3$

$$= \frac{1}{2}e^{i\left(\frac{-2\pi}{12} + \frac{6k\pi}{12}\right)}, k = 0, 1, 2, 3$$

Logo,  $z_0 = \frac{1}{2}e^{-i\frac{\pi}{6}}$ ;  $z_1 = \frac{1}{2}e^{i\frac{\pi}{3}}$ ;  $z_2 = \frac{1}{2}e^{i\frac{5\pi}{6}}$ ;  $z_3 = \frac{1}{2}e^{i\frac{4\pi}{3}}$

As raízes quartas de  $z$  são:  $\frac{1}{2}e^{-i\frac{\pi}{6}}$ ,  $\frac{1}{2}e^{i\frac{\pi}{3}}$ ,  $\frac{1}{2}e^{i\frac{5\pi}{6}}$  e  $\frac{1}{2}e^{i\frac{4\pi}{3}}$

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47.1.  $z^2 + 4 = 0 \Leftrightarrow z^2 = -4 \Leftrightarrow z = \pm\sqrt{-4} \Leftrightarrow z = \pm 2\sqrt{-1} \Leftrightarrow$   
 $\Leftrightarrow z = \pm 2i$

$$S = \{-2i, 2i\}$$

47.2.  $z^2 + 10z + 26 = 0 \Leftrightarrow z = \frac{-10 \pm \sqrt{100 - 104}}{2} \Leftrightarrow$

$$\Leftrightarrow z = \frac{-10 \pm \sqrt{-4}}{2} \Leftrightarrow z = \frac{-10 \pm 2\sqrt{-1}}{2} \Leftrightarrow z = -5 \pm i$$

$$S = \{-5 - i, -5 + i\}$$

47.3.  $16z^2 + 8z + 5 = 0 \Leftrightarrow z = \frac{-8 \pm \sqrt{64 - 320}}{32} \Leftrightarrow$

$$\Leftrightarrow z = \frac{-8 \pm \sqrt{-256}}{32} \Leftrightarrow z = \frac{-8 \pm 16i}{32} \Leftrightarrow z = -\frac{1}{4} \pm \frac{1}{2}i$$

$$S = \left\{ -\frac{1}{4} - \frac{1}{2}i, -\frac{1}{4} + \frac{1}{2}i \right\}$$

47.4.  $z^3 + 16z = 0 \Leftrightarrow z(z^2 + 16) = 0 \Leftrightarrow z = 0 \vee z^2 + 16 = 0 \Leftrightarrow$

$$\Leftrightarrow z = 0 \vee z^2 = -16 \Leftrightarrow z = 0 \vee z = \pm\sqrt{-16} \Leftrightarrow$$

$$\Leftrightarrow z = 0 \vee z = \pm 4i$$

$$S = \{-4i, 0, 4i\}$$

47.5.  $z^3 + 6z^2 + 13z = 0 \Leftrightarrow z(z^2 + 6z + 13) = 0 \Leftrightarrow$

$$\Leftrightarrow z = 0 \vee z^2 + 6z + 13 = 0 \Leftrightarrow z = 0 \vee z = \frac{-6 \pm \sqrt{36 - 52}}{2}$$

$$\Leftrightarrow z = 0 \vee z = \frac{-6 \pm \sqrt{-16}}{2} \Leftrightarrow$$

$$\Leftrightarrow z = 0 \vee z = \frac{-6 \pm 4i}{2} \Leftrightarrow z = 0 \vee z = -3 \pm 2i \Leftrightarrow$$

$$\Leftrightarrow z = 0 \vee z = -3 - 2i \vee z = -3 + 2i$$

$$S = \{0, -3 - 2i, -3 + 2i\}$$

47.6.  $z^3 + 6iz^2 = 10z \Leftrightarrow z^3 + 6iz^2 - 10z = 0 \Leftrightarrow$

$$\Leftrightarrow z(z^2 + 6iz - 10) = 0 \Leftrightarrow z = 0 \vee z^2 + 6iz - 10 = 0$$

$$\Leftrightarrow z = 0 \vee z = \frac{-6i \pm \sqrt{-36 + 40}}{2} \Leftrightarrow$$

$$\Leftrightarrow z = 0 \vee z = \frac{-6i \pm 2}{2} \Leftrightarrow$$

$$\Leftrightarrow z = 0 \vee z = -1 - 3i \vee z = 1 - 3i$$

$$S = \{0, -1 - 3i, 1 - 3i\}$$

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48.1. As raízes quadradas de  $4 - 3i$  são as soluções da equação  $z^2 = 4 - 3i$ . Seja  $z = x + yi$ .

$$z^2 = 4 - 3i \Leftrightarrow (x + yi)^2 = 4 - 3i \Leftrightarrow x^2 - y^2 + 2xyi = 4 - 3i$$

$$\Leftrightarrow \begin{cases} x^2 - y^2 = 4 \\ 2xy = -3 \end{cases} \Leftrightarrow \begin{cases} x^2 - \left(\frac{-3}{2x}\right)^2 = 4 \\ y = -\frac{3}{2x} \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} x^2 - \frac{9}{4x^2} = 4 \\ 4x^4 - 16x^2 - 9 = 0 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} x^2 = \frac{16 \pm \sqrt{256 + 144}}{8} \\ x^2 = \frac{16 \pm \sqrt{400}}{8} \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} x^2 = \frac{16 \pm 20}{8} \\ x^2 = \frac{36}{8} \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} x = \frac{-3\sqrt{2}}{2} \\ y = \frac{\sqrt{2}}{2} \end{cases} \vee \begin{cases} x = \frac{3\sqrt{2}}{2} \\ y = -\frac{\sqrt{2}}{2} \end{cases} \quad \left| y = -\frac{3}{2} \times \left(-\frac{2}{3\sqrt{2}}\right) = \frac{\sqrt{2}}{2} \right.$$

As raízes quadradas de  $4 - 3i$  são  $-\frac{3\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$  e  $\frac{3\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$ .

48.2. As raízes quadradas de  $\frac{3}{4} - i$  são as soluções da equação

$$z^2 = \frac{3}{4} - i.$$

Seja  $z = x + yi$ .

$$z^2 = \frac{3}{4} - i \Leftrightarrow (x + yi)^2 = \frac{3}{4} - i \Leftrightarrow x^2 - y^2 + 2xyi = \frac{3}{4} - i \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} x^2 - y^2 = \frac{3}{4} \\ 2xy = -1 \end{cases} \Leftrightarrow \begin{cases} x^2 - \frac{1}{4x^2} = \frac{3}{4} \\ y = -\frac{1}{2x} \end{cases} \quad \begin{matrix} x \neq 0 \\ \Leftrightarrow \end{matrix}$$

$$\Leftrightarrow \begin{cases} 4x^4 - 3x^2 - 1 = 0 \\ \end{cases} \Leftrightarrow \begin{cases} x^2 = \frac{3 \pm \sqrt{9+16}}{8} \\ \end{cases} \quad \begin{matrix} x \in \mathbb{R} \\ \Leftrightarrow \end{matrix}$$

$$\Leftrightarrow \begin{cases} x^2 = 1 \\ \end{cases} \quad \begin{matrix} x \geq 0 \\ \Leftrightarrow \end{matrix} \begin{cases} x = 1 \\ y = -\frac{1}{2} \end{cases} \vee \begin{cases} x = -1 \\ x = \frac{1}{2} \end{cases}$$

As raízes quadradas de  $\frac{3}{4} - i$  são  $1 - \frac{1}{2}i$  e  $-1 + \frac{1}{2}i$ .

**48.3.** As raízes quadradas de  $5 - 12i$  são as soluções da equação  $z^2 = 5 - 12i$ . Seja  $z = x + yi$ .

$$z^2 = 5 - 12i \Leftrightarrow (x + yi)^2 = 5 - 12i \Leftrightarrow$$

$$\Leftrightarrow x^2 - y^2 + 2xyi = 5 - 12i \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} x^2 - y^2 = 5 \\ 2xy = -12 \end{cases} \Leftrightarrow \begin{cases} x^2 - \frac{36}{x^2} = 5 \\ y = -\frac{6}{x} \end{cases} \quad \begin{matrix} x \in \mathbb{R} \\ \Leftrightarrow \end{matrix}$$

$$\Leftrightarrow \begin{cases} x^4 - 5x^2 - 36 = 0 \\ \end{cases} \Leftrightarrow \begin{cases} x^2 = \frac{5 \pm \sqrt{25+144}}{2} \\ \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} x^2 = 9 \\ \end{cases} \quad \begin{matrix} x \in \mathbb{R} \\ \Leftrightarrow \end{matrix} \begin{cases} x = 3 \\ y = -2 \end{cases} \vee \begin{cases} x = -3 \\ y = 2 \end{cases}$$

As raízes quadradas de  $5 - 12i$  são  $3 - 2i$  e  $-3 + 2i$ .

**49.1.**  $z^2 + z + 1 = 0 \Leftrightarrow z = \frac{-1 \pm \sqrt{1-4}}{2} \Leftrightarrow z = \frac{-1 \pm \sqrt{3}}{2} \Leftrightarrow$

$$\Leftrightarrow z = -\frac{1}{2} - \frac{\sqrt{3}i}{2} \vee z = -\frac{1}{2} + \frac{\sqrt{3}i}{2}$$

$$S = \left\{ -\frac{1}{2} - \frac{\sqrt{3}i}{2}, -\frac{1}{2} + \frac{\sqrt{3}i}{2} \right\}$$

**49.2.**  $z^2 - (1+2i)z + i - 1 = 0 \Leftrightarrow$

$$\Leftrightarrow z = \frac{1+2i \pm \sqrt{(-1-2i)^2 - 4 \times 1 \times (i-1)}}{2} \Leftrightarrow$$

$$\Leftrightarrow z = \frac{1+2i \pm \sqrt{1-4+4i-4i+4}}{2} \Leftrightarrow z = \frac{1+2i \pm 1}{2} \Leftrightarrow$$

$$\Leftrightarrow z = i \vee z = 1 + i$$

$$S = \{i, 1 + i\}$$

**49.3.**  $4z^2 + 3 - 4i = 0 \Leftrightarrow 4z^2 = -3 + 4i \Leftrightarrow z^2 = -\frac{3}{4} + i \Leftrightarrow$

$$\Leftrightarrow z^2 = -\frac{3}{4} + 1i + \left(\frac{1}{2}\right)^2 - \frac{1}{4} \Leftrightarrow z^2 = \left(\frac{1}{2} + i\right)^2$$

$$\Leftrightarrow z^2 = \left(\frac{1}{2} + i\right)^2 \Leftrightarrow z = \pm \left(\frac{1}{2} + i\right) \Leftrightarrow$$

$$\Leftrightarrow z = \frac{1}{2} + i \vee z = -\frac{1}{2} - i$$

$$S = \left\{ -\frac{1}{2} - i, \frac{1}{2} + i \right\}$$

**49.4.**  $z^2 - (3+4i)z - 1 + 5i = 0 \Leftrightarrow$

$$\Leftrightarrow z = \frac{3+4i \pm \sqrt{(-3-4i)^2 - 4(-1+5i)}}{2} \Leftrightarrow$$

$$\Leftrightarrow z = \frac{3+4i \pm \sqrt{9-16+24i+4-20i}}{2} \Leftrightarrow$$

$$\Leftrightarrow z = \frac{3+4i \pm \sqrt{-3+4i}}{2} \Leftrightarrow$$

$$\Leftrightarrow z = \frac{3+4i \pm \sqrt{(1+2i)^2}}{2} \Leftrightarrow$$

$$\Leftrightarrow z = \frac{3+4i+1+2i}{2} \vee z = \frac{3+4i-1-2i}{2} \Leftrightarrow$$

$$\Leftrightarrow z = 2 + 3i \vee z = 1 + i$$

$$S = \{2 + 3i, 1 + i\}$$

**50.1.**  $24 - 10i = 24 - 2 \times 5i + 25 - 25 = 25 - 2 \times 5i - 1 =$   
 $= 5^2 - 2 \times 5i + i^2 = (5 - i)^2$

$$\pm \sqrt{(5-i)^2} = \pm(5-i)$$

$$S = \{5-i, -5+i\}$$

**50.2.**  $8 - 6i = 8 - 2 \times 3i + 9 - 9 = 9 - 2 \times 3i - 1 =$   
 $= 3^2 - 2 \times 3i + i^2 = (3 - i)^2$

$$\pm \sqrt{(3-i)^2} = \pm(3-i)$$

$$S = \{3-i, -3+i\}$$

**51.1.**  $z^4 + 10z^2 + 169 = 0 \Leftrightarrow$

$$\Leftrightarrow z^2 = \frac{-10 \pm \sqrt{100 - 4 \times 169}}{2} \Leftrightarrow$$

$$\Leftrightarrow z^2 = \frac{-10 \pm \sqrt{-576}}{2} \Leftrightarrow$$

$$\Leftrightarrow z^2 = \frac{-10 \pm 24i}{2} \Leftrightarrow$$

$$\Leftrightarrow z^2 = -5 - 12i \vee z^2 = -5 + 12i \Leftrightarrow$$

$$\Leftrightarrow z^2 = (2-3i)^2 \vee z^2 = (2+3i)^2 \Leftrightarrow$$

$$\Leftrightarrow z = 2 - 3i \vee z = -2 + 3i \vee z = 2 + 3i \vee z = -2 - 3i$$

$$S = \{-2 - 3i; -2 + 3i; 2 - 3i; 2 + 3i\}$$

**51.2.**  $z^4 + (3-6i)z^2 - 8 - 6i = 0 \Leftrightarrow$

$$\Leftrightarrow z^2 = \frac{-3+6i \pm \sqrt{(3-6i)^2 - 4(-8-6i)}}{2} \Leftrightarrow$$

$$\Leftrightarrow z^2 = \frac{-3+6i \pm \sqrt{9-36-36i+32+24i}}{2} \Leftrightarrow$$

$$\Leftrightarrow z^2 = \frac{-3+6i \pm \sqrt{5-12i}}{2} \Leftrightarrow$$

$$\Leftrightarrow z^2 = \frac{-3+6i \pm \sqrt{(3-2i)^2}}{2} \Leftrightarrow$$

$$\Leftrightarrow z^2 = \frac{-3+6i \pm (3-2i)}{2} \Leftrightarrow$$

$$\Leftrightarrow z^2 = 2i \vee z^2 = -3 + 4i \Leftrightarrow$$

$$\Leftrightarrow z^2 = (1+i)^2 \vee z^2 = (1+2i)^2 \Leftrightarrow$$

$$\Leftrightarrow z = 1+i \vee z = -1-i \vee$$
  
 $\vee z = 1+2i \vee z = -1-2i$

$$S = \{1+i, -1-i, -1-2i, 1+2i\}$$

$$\begin{cases} -3+4i = 1+4i-4 = \\ = 1+4i+(2i)^2 = (1+2i)^2 \end{cases}$$

$$\begin{cases} -5-12i = 4-12i-9 = \\ = (2-3i)^2 \\ -5+12i = 4+12i-9 = \\ = (2+3i)^2 \end{cases}$$

$$\begin{cases} 5-12i = 9-12i-4 = \\ = (3-2i)^2 \\ 2i = 1+2i-1 = \\ = (1+i)^2 \\ -3+4i = 1+4i-4 = \\ = (1+2i)^2 \end{cases}$$

$$51.3. z^4 + 3z^2 + 2 = 0 \Leftrightarrow z^2 = \frac{-3 \pm \sqrt{9-8}}{2} \Leftrightarrow$$

$$\Leftrightarrow z^2 = -2 \vee z^2 = -1 \Leftrightarrow z = \pm \sqrt{-2} \vee z = \pm \sqrt{-1} \Leftrightarrow$$

$$\Leftrightarrow z = \pm \sqrt{2}i \vee z = \pm i$$

$$S = \{-\sqrt{2}i, -i, i, \sqrt{2}i\}$$

$$51.4. z^4 - 32z^2 - 144 = 0 \Leftrightarrow z^2 = \frac{32 \pm \sqrt{(-32)^2 - 4 \times (-144)}}{2}$$

$$\Leftrightarrow z^2 = \frac{32 \pm 40}{2} \Leftrightarrow z^2 = -4 \vee z^2 = 36 \Leftrightarrow$$

$$\Leftrightarrow z = \pm \sqrt{-4} \vee z = \pm 6 \Leftrightarrow z = \pm 2i \vee z = \pm 6$$

$$S = \{-6, 6, -2i, 2i\}$$

$$51.5. z^4 + 10z - 11 = 0 \Leftrightarrow z^2 = \frac{-10 \pm \sqrt{100 - 4 \times (-11)}}{2} \Leftrightarrow$$

$$\Leftrightarrow z^2 = \frac{-10 \pm \sqrt{144}}{2} \Leftrightarrow z^2 = \frac{-10 \pm 12}{2} \Leftrightarrow$$

$$\Leftrightarrow z^2 = -11 \vee z^2 = 1 \Leftrightarrow z = \pm \sqrt{11}i \vee z = \pm 1$$

$$S = \{-1, 1, -\sqrt{11}i, \sqrt{11}i\}$$

$$51.6. z^4 + 2z^2 + 4 = 0 \Leftrightarrow z^2 = \frac{-2 \pm \sqrt{4-16}}{2} \Leftrightarrow$$

$$\Leftrightarrow z^2 = \frac{-2 \pm \sqrt{-12}}{2} \Leftrightarrow z^2 = \frac{-2 \pm 2\sqrt{3}i}{2} \Leftrightarrow$$

$$\Leftrightarrow z^2 = -1 - \sqrt{3}i \vee z^2 = -1 + \sqrt{3}i \Leftrightarrow z^2 = 2^{-\frac{2\pi}{3}} \vee z^2 = 2e^{i\frac{2\pi}{3}}$$

$$\Leftrightarrow z = \sqrt{2}e^{i\left(\frac{2\pi}{3} + \frac{2k\pi}{2}\right)} \vee z = \sqrt{2}e^{i\left(\frac{2\pi}{3} + \frac{2k\pi}{2}\right)}, k = 0, 1$$

$$\Leftrightarrow z = \sqrt{2}e^{i\left(\frac{\pi}{3} + k\pi\right)} \vee z = \sqrt{2}e^{i\left(\frac{\pi}{3} + k\pi\right)}, k = 0, 1$$

$$\Leftrightarrow z = \sqrt{2}e^{-i\frac{\pi}{3}} \vee z = \sqrt{2}e^{i\frac{2\pi}{3}} \vee z = \sqrt{2}e^{i\frac{\pi}{3}} \vee z = \sqrt{2}e^{i\frac{4\pi}{3}}$$

$$S = \left\{ \sqrt{2}e^{-i\frac{\pi}{3}}, \sqrt{2}e^{i\frac{2\pi}{3}}, \sqrt{2}e^{i\frac{\pi}{3}}, \sqrt{2}e^{i\frac{4\pi}{3}} \right\}$$

$$52.1. A(z) = z^3 + iz^2 + 2i$$

Como  $A(i) = i^3 + i \cdot i^2 + 2i = -i - i + 2i = 0$ :

$$\begin{array}{c|ccc} i & 1 & i & 0 & 2i \\ & & i & -2 & -2i \\ \hline & 1 & 2i & -2 & 0 \end{array}$$

Logo,  $A(z) = (z-i)(z^2 + 2iz - 2)$

$$52.2. A(z) = 0 \Leftrightarrow (z-i)(z^2 + 2iz - 2) = 0 \Leftrightarrow$$

$$\Leftrightarrow z - i = 0 \vee z^2 + 2iz - 2 = 0 \Leftrightarrow$$

$$\Leftrightarrow z = i \vee z = \frac{-2i \pm \sqrt{(2i)^2 + 8}}{2} \Leftrightarrow$$

$$\Leftrightarrow z = i \vee z = \frac{-2i \pm \sqrt{-4 + 8}}{2} \Leftrightarrow$$

$$\Leftrightarrow z = i \vee z = \frac{-2i - 2}{2} \vee z = \frac{-2i + 2}{2} \Leftrightarrow$$

$$\Leftrightarrow z = i \vee z = -1 - i \vee z = 1 - i$$

$$S = \{i, -1 - i, 1 - i\}$$

$$53.1. (-i)^3 + 3i(-i)^2 - 7 \times (-i) - 5i = 0 \Leftrightarrow i - 3i + 7i - 5i = 0$$

Logo,  $-i$  é solução da equação dada.

$$53.2. z^2 = 2iz - 5 = 0 \Leftrightarrow$$

$$\Leftrightarrow z = \frac{-2i \pm \sqrt{(2i)^2 + 20}}{2} \Leftrightarrow$$

$$\Leftrightarrow z = \frac{-2i \pm \sqrt{16}}{2} \Leftrightarrow z = \frac{-2i - 4}{2} \vee z = \frac{-2i + 4}{2} \Leftrightarrow$$

$$\Leftrightarrow z = -2 - i \vee z = 2 - i$$

$$S = \{-i, -2 - i, 2 - i\}$$

$$\begin{array}{c|ccc} -i & 1 & 3i & -7 & -5i \\ & & -i & 2 & 5i \\ \hline & 1 & 2i & -5 & 0 \end{array}$$

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$$54.1. (2-3i)\bar{z} = 8+i \Leftrightarrow \bar{z} = \frac{8+i}{2-3i} \Leftrightarrow$$

$$\Leftrightarrow \bar{z} = \frac{(8+i)(2+3i)}{4+9} \Leftrightarrow \bar{z} = \frac{16+24i+2i-3}{13} \Leftrightarrow$$

$$\Leftrightarrow \bar{z} = \frac{13+26i}{13} \Leftrightarrow \bar{z} = 1+2i \Leftrightarrow z = 1-2i$$

$$S = \{1-2i\}$$

$$54.2. (z+1-i)(\bar{z}-1+3i) = 0 \Leftrightarrow z+1-i = 0 \vee \bar{z}-1+3i = 0 \Leftrightarrow$$

$$\Leftrightarrow z = -1+i \vee \bar{z} = 1-3i \Leftrightarrow z = -1+i \vee z = 1+3i$$

$$S = \{-1+i, 1+3i\}$$

$$54.3. \text{Seja } z = x + yi.$$

$$z^2 + 2\bar{z} + 1 = 0 \Leftrightarrow (x+yi)^2 + 2(x-yi) + 1 = 0 \Leftrightarrow$$

$$\Leftrightarrow x^2 - y^2 + 2xyi + 2x - 2yi + 1 = 0 \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} x^2 - y^2 + 2x + 1 = 0 \\ 2xy - 2y = 0 \end{cases} \Leftrightarrow \begin{cases} \text{---} \\ 2y(x-1) = 0 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} x^2 + 2x + 1 = 0 \\ y = 0 \end{cases} \vee \begin{cases} 1 - y^2 + 2 + 1 = 0 \\ x = 1 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{matrix} x \in \mathbb{R} \wedge y \in \mathbb{R} \\ \Leftrightarrow \begin{cases} (x+1)^2 = 0 \\ y = 0 \end{cases} \vee \begin{cases} y^2 = 4 \\ x = 1 \end{cases} \end{matrix} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} x = -1 \\ y = 0 \end{cases} \vee \begin{cases} y = 2 \\ x = 1 \end{cases} \vee \begin{cases} y = -2 \\ x = 1 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow z = -1 \vee z = 1 + 2i \vee z = 1 - 2i$$

$$S = \{-1, 1 + 2i, 1 - 2i\}$$

$$55. (z+i)(\bar{z}+i) = 4+2i \Leftrightarrow z\bar{z} + zi + \bar{z}i - 1 = 4+2i \Leftrightarrow$$

$$\Leftrightarrow x^2 + y^2 + (x+yi)i + (x-yi)i = 5+2i \Leftrightarrow$$

$$\Leftrightarrow x^2 + y^2 + xi - y + xi + y = 5+2i \Leftrightarrow$$

$$\Leftrightarrow x^2 + y^2 + 2xi = 5+2i \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} x^2 + y^2 = 5 \\ 2x = 2 \end{cases} \Leftrightarrow \begin{cases} y^2 = 4 \\ x = 1 \end{cases} \Leftrightarrow \begin{cases} y = -2 \\ y = 2 \\ x = 1 \end{cases}$$

$$\Leftrightarrow z = 1 - 2i \vee z = 1 + 2i; S = \{1 - 2i, 1 + 2i\}$$

$$56. z^2 - 2z \cos \alpha + 1 = 0 \Leftrightarrow z = \frac{2 \cos \alpha \pm \sqrt{4 \cos^2 \alpha - 4}}{2} \Leftrightarrow$$

$$\Leftrightarrow z = \frac{2 \cos \alpha \pm \sqrt{-4(1 - \cos^2 \alpha)}}{2} \Leftrightarrow$$

$$\Leftrightarrow z = \frac{2 \cos \alpha \pm 2i \sqrt{1 - \cos^2 \alpha}}{2} \Leftrightarrow$$

$$\Leftrightarrow z = \frac{2 \cos \alpha \pm 2i \sqrt{\sin^2 \alpha}}{2} \Leftrightarrow z = \frac{2 \cos \alpha \pm 2i \sin \alpha}{2}$$

$$\Leftrightarrow z = \cos \alpha + i \sin \alpha \vee z = \cos \alpha - i \sin \alpha \Leftrightarrow$$

$$\Leftrightarrow z = e^{i\alpha} \vee z = \cos(-\alpha) + i \sin(-\alpha) \Leftrightarrow z = e^{i\alpha} \vee z = e^{-i\alpha}$$

$$S = \{e^{i\alpha}, e^{-i\alpha}\}$$

57.  $z = z' \times e^{i\theta}$  Pág. 51

$$\begin{aligned} (2+4i) &= (-\sqrt{2}+3\sqrt{2}i)e^{i\theta} \Leftrightarrow \frac{2+4i}{-\sqrt{2}+3\sqrt{2}i} = e^{i\theta} \Leftrightarrow \\ &\Leftrightarrow \frac{(2+4i)(-\sqrt{2}-3\sqrt{2}i)}{2+9 \times 2} = e^{i\theta} \Leftrightarrow \\ &\Leftrightarrow \frac{-2\sqrt{2}-6\sqrt{2}i-4\sqrt{2}i+12\sqrt{2}}{20} = e^{i\theta} \Leftrightarrow \\ &\Leftrightarrow \frac{10\sqrt{2}-10\sqrt{2}i}{20} = e^{i\theta} \Leftrightarrow \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i = e^{i\theta} \end{aligned}$$

$$\begin{cases} \tan \theta = -1 \\ \left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right) \in 4.^\circ Q \Rightarrow \theta = -\frac{\pi}{4} + 2k\pi \end{cases}$$

$$\theta = -\frac{\pi}{4} + 2k\pi, k \in \mathbb{Z}$$

58.1.  $z_A = \sqrt{3} + i$ ;  $z_B = 2i$  Pág. 52

$$\begin{aligned} z_{A'} &= z_A \times 2e^{i\frac{\pi}{3}} = (\sqrt{3} + i) \times 2e^{i\frac{\pi}{3}} = 2e^{i\frac{\pi}{6}} \times 2e^{i\frac{\pi}{3}} = \\ &= 4e^{i\left(\frac{\pi}{6} + \frac{\pi}{3}\right)} = 4e^{i\frac{\pi}{2}} = 4i \\ A'(0, 4) \end{aligned}$$

$$\begin{aligned} z_{B'} &= z_B \times 2e^{i\frac{\pi}{3}} = 3i \times 2e^{i\frac{\pi}{3}} = 3e^{i\frac{\pi}{2}} \times 2e^{i\frac{\pi}{3}} = \\ &= 6e^{i\left(\frac{\pi}{2} + \frac{\pi}{3}\right)} = 6e^{i\frac{5\pi}{6}} = 6\left(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6}\right) \quad (2.^\circ Q) \\ &= 6\left(-\frac{\sqrt{3}}{2} + \frac{1}{2}i\right) = -3\sqrt{3} + 3i \end{aligned}$$

$$B'(-3\sqrt{3}, 3)$$

$$O' \equiv O, A'(0, 4), B'(-3\sqrt{3}, 3)$$

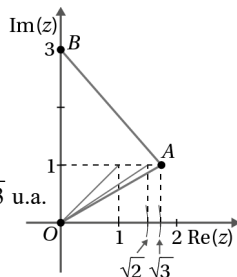
58.2. Área de  $[OAB]$ : Pág. 53

$$A(\sqrt{3}, 1); O(0, 0) \text{ e } B(0, 3)$$

$$A_{[OAB]} = \frac{OB \times \text{abscissa de } A}{2} = \frac{3\sqrt{3}}{2}$$

Como  $r = 2$ :

$$A_{[O'A'B']} = A_{[OAB]} \times r^2 = \frac{3\sqrt{3}}{2} \times 2^2 = 6\sqrt{3} \text{ u.a.}$$



59.1.  $f(z) = -iz \Leftrightarrow f(z) = z \times e^{-i\frac{\pi}{2}}$

O afixo de  $f(z)$  é a imagem do afixo de  $z$  por uma rotação de centro  $O$  e ângulo  $-\frac{\pi}{2} + 2k\pi, k \in \mathbb{Z}$ .

59.2.  $f(z) = z + i - 1 \Leftrightarrow f(z) = z + (-1 + i)$

O afixo de  $f(z)$  é a imagem do afixo de  $z$  pela translação de vetor  $(-1, 1)$ .

59.3.  $f(z) = 2z \Leftrightarrow f(z) = z \times 2e^{i \times 0}$

O afixo de  $f(z)$  é a imagem do afixo de  $z$  por uma homotetia de centro  $O$  e razão 2.

59.4.  $f(z) = \frac{iz}{i+\sqrt{3}} \Leftrightarrow f(z) = z \times \frac{i}{\sqrt{3}+i} \Leftrightarrow$

$$\Leftrightarrow f(z) = z \times \frac{e^{i\frac{\pi}{2}}}{2e^{i\frac{\pi}{6}}} \Leftrightarrow f(z) = z \times \frac{1}{2}e^{i\left(\frac{\pi}{2}-\frac{\pi}{6}\right)}$$

$$\Leftrightarrow f(z) = z \times \frac{1}{2}e^{i\frac{\pi}{3}}$$

$$\begin{aligned} \sqrt{3} + i &= r e^{i\theta} \\ r &= |\sqrt{3} + i| = 2 \\ \theta &\in 1.^\circ Q \\ \text{Logo, um valor de } \theta &\text{ é} \\ \theta &= \arctan \frac{1}{\sqrt{3}} = \frac{\pi}{6} \end{aligned}$$

O afixo de  $f(z)$  é a imagem do afixo de  $z$  por uma rotação de centro na origem  $O$  e amplitude  $\frac{\pi}{3}$  composta com uma homotetia de centro  $O$  e razão  $\frac{1}{2}$ .

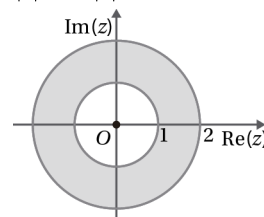
59.5.  $f(z) = -\bar{z} + 2i$

O afixo de  $f(z)$  é a imagem do afixo de  $z$  por uma simetria de eixo real seguida de uma reflexão central de centro  $O$  e uma translação de vetor  $(0, 2)$ . Trata-se de uma reflexão deslizante de eixo imaginário e vetor  $(0, 2)$ .

60.1.  $|z-1+2i| \leq 3 \Leftrightarrow |z-(1-2i)| \leq 3$

$$C(1, -2), r=3$$

60.2.  $|z| \geq 1 \wedge |z| \leq 2$

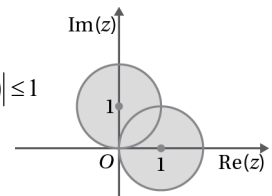


60.3.  $|z-i| \leq 1 \vee |z-1| \leq 1$

$$\Leftrightarrow |z-(0+i)| \leq 1 \vee |z-(1+0i)| \leq 1$$

$$C_1(0, 1), r=1$$

$$C_2(1, 0), r=1$$



61.  $z_0 = r e^{i\frac{\pi}{3}}$

$|z| = |z_0|$  define uma circunferência de centro na origem e raio  $|z_0|$ .

Como a medida do seu perímetro é  $8\pi$ , temos:

$$2\pi|z_0| = 8\pi \Leftrightarrow |z_0| = 4$$

Logo,  $r = 4$  e  $z_0 = 4e^{i\frac{\pi}{3}}$ .

$$z_0 = 4\left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}\right) = 4\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) = 2 + 2\sqrt{3}i$$

62. Fronteiras:

Circunferência de centro na origem e que passa em  $A(-2, 2)$ :

$$r = \overline{CA} = \sqrt{(-2)^2 + 2^2} = \sqrt{4+4} = 2\sqrt{2}$$

$$|z| = 2\sqrt{2}$$

Circunferência de centro  $A(-2, 2)$  e raio 2:

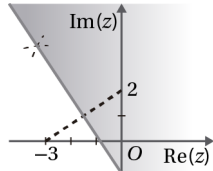
$$|z - (-2 + 2i)| = 2 \Leftrightarrow |z + 2 - 2i| = 2$$

Mediatriz do segmento  $[BC]$  com  $B(-2, 0)$  e  $C(0, 3)$ :

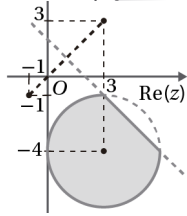
$$|z - (-2 + 0i)| = |z - (0 + 3i)| \Leftrightarrow |z + 2| = |z - 3i|$$

$$\text{Condição: } |z| \leq 2\sqrt{2} \wedge |z + 2 - 2i| = 2 \wedge |z + 2| \geq |z - 3i|$$

63.1.  $|z - 2i| \leq |z + 3| \Leftrightarrow$   
 $\Leftrightarrow |z - (0 + 2i)| \leq |z - (-3 + 0i)|$   
 $A(0, 2)$   
 $B(-3, 0)$

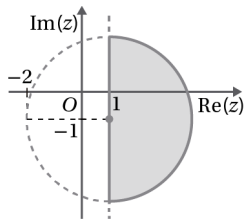


63.2.  $|z - 3 + 4i| \leq 3 \wedge |z + 1 + i| \leq |z - 3 - 3i|$   
 $\Leftrightarrow |z - (3 - 4i)| \leq 3 \wedge$   
 $\wedge |z - (-1 - i)| \leq |z - (3 + 3i)|$   
 $A(3, -4), r = 3$   
 $B(-1, -1), C(3, 3)$

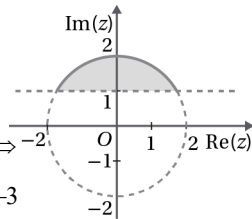


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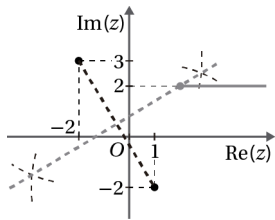
64.1.  $|z - 1 + i| \leq 3 \wedge \text{Re}(z - 3 - i) \geq -2$   
 $|z - 1 + i| \leq 3 \Leftrightarrow |z - (1 - i)| \leq 3$   
 $C(1, -1), r = 3$   
 $\text{Re}(z - 3 - i) \geq -2 \Leftrightarrow$   
 $\Leftrightarrow \text{Re}(z + yi - 3 - i) \geq -2$   
 $\Leftrightarrow \text{Re}(x - 3 + (y - 1)i) \geq -2$   
 $\Leftrightarrow x - 3 \geq -2 \Leftrightarrow x \geq 1$



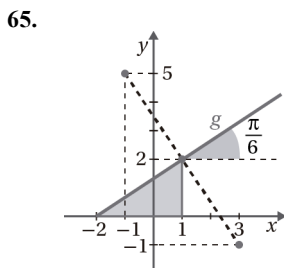
64.2.  $\text{Im}(-z + 2 - 2i) < -3 \wedge |z| \leq 2$   
 $\text{Im}(-z + 2 - 2i) < -3 \Leftrightarrow$   
 $\Leftrightarrow \text{Im}(-x - yi + 2 - 2i) < -3 \Leftrightarrow$   
 $\Leftrightarrow \text{Im}[-x + 2 + (-y - 2)i] < -3$   
 $\Leftrightarrow -y - 2 < -3 \Leftrightarrow y + 2 > 3$   
 $\Leftrightarrow y > 1$



64.3.  $|z + 2 - 3i| \geq |z - 1 + 2i| \wedge \text{Re}(iz + 2) = 0$   
 $|z + 2 - 3i| \geq |z - 1 + 2i| \Leftrightarrow |z - (-2 + 3i)| \geq |z - (1 - 2i)|$   
 $A(-2, 3), B(1, -2)$   
 $\text{Re}(iz + 2) = 0 \Leftrightarrow$   
 $\Leftrightarrow \text{Re}(i(x + yi) + 2) = 0$   
 $\Leftrightarrow \text{Re}(xi - y + 2) = 0$   
 $\Leftrightarrow -y + 2 = 0 \Leftrightarrow y = 2$



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Por exemplo:

$$\left( |z - (-1 + 5i)| \geq z - (3 - i) \wedge \frac{\pi}{2} < \text{Arg}(z - 1) \leq \pi \right) \vee$$

$$\vee 0 < \text{Arg}(z - (1 + 2i)) \leq \frac{\pi}{6}$$

66.1. É uma elipse e o seu interior:

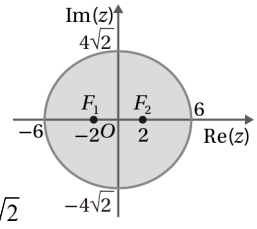
$$F_1(-2, 0), F_2(2, 0)$$

$$c = 2$$

$$2a = 12 \Leftrightarrow a = 6$$

$$b^2 + c^2 = a^2$$

$$b^2 + 4 = 36 \Leftrightarrow b^2 = 32 \Leftrightarrow b = 4\sqrt{2}$$



66.2.  $F_1(0, -3), F_2(0, 3)$

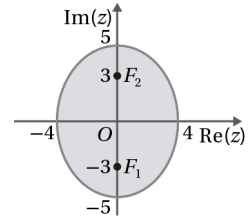
$$c = 3$$

$$2b = 10 \Leftrightarrow b = 5$$

$$c^2 + a^2 = b^2$$

$$9 + a^2 = 25 \Leftrightarrow a^2 = 16 \Leftrightarrow$$

$$\Leftrightarrow a = 4$$



67.1.  $\left| \frac{z - i}{z + 2i} \right| \leq 2 \Leftrightarrow \left| \frac{z + yi - i}{x + yi + 2i} \right| \leq 2 \Leftrightarrow$

$$\Leftrightarrow x^2 + (y - 1)^2 \leq 4[x^2 + (y + 2)^2] \wedge (x, y) = (0, -2) \Leftrightarrow$$

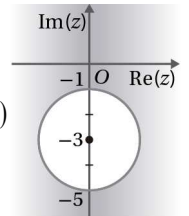
$$\Leftrightarrow z^2 + y^2 - 2y + 1 \leq 4x^2 + 4y^2 + 16y + 16 \wedge (x, y) = (0, -2)$$

$$\Leftrightarrow 3x^2 + 3y^2 + 18y + 15 \geq 0 \wedge (x, y) = (0, -2)$$

$$\Leftrightarrow x^2 + y^2 + 6y + 5 \geq 0 \wedge (x, y) = (0, -2)$$

$$\Leftrightarrow x^2 + y^2 + 6y + 9 - 9 + 5 \geq 0 \wedge (x, y) = (0, -2)$$

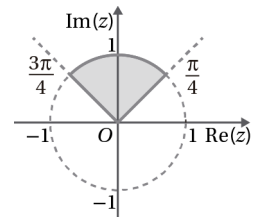
$$\Leftrightarrow x^2 + (y + 3)^2 \geq 4 \wedge (x, y) = (0, -2)$$



67.2.  $z \bar{z} = \left| e^{i\frac{\pi}{4}} \right| \wedge \frac{\pi}{4} \leq \text{Arg}(z) \leq \frac{3\pi}{4}$

$$\Leftrightarrow |z|^2 \leq 1 \wedge \frac{\pi}{4} \leq \text{Arg}(z) \leq \frac{3\pi}{4}$$

$$\Leftrightarrow |z| \leq 1 \wedge \frac{\pi}{4} \leq \text{Arg}(z) \leq \frac{3\pi}{4}$$



Atividades complementares

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68.1.  $(2 - i) - 2 \times \left( \frac{1}{2} - 2i \right) = 2 - i - 1 + 4i = 1 + 3i$

68.2.  $(3 - i) \times (2 - 3i) + 10i = 6 - 9i - 2i - 3 + 10i = 3 - i$

68.3.  $\frac{1}{5}(2 - i)^2 \times (1 - 2i) = \frac{1}{5}(4 - 4i - 1) \times (1 - 2i) =$   
 $= \frac{1}{5}(3 - 4i) \times (1 - 2i) = \frac{1}{5}(3 - 6i - 4i - 8)$   
 $= \frac{1}{5}(-5 - 10i) = -1 - 2i$

68.4.  $(1 - i^{27})^2 \times (1 - i^{14})^3 = (1 - i^3)^2 \times (1 - i^2)^3 =$  27 = 6\*4 + 3  
14 = 3\*4 + 2  
 $= (1 + i)^2 \times (1 + 1)^3 = (1 + 2i - 1) \times 8 = 16i$

69.  $(a - 4i) \times (2 - 3i) = 10 + bi \Leftrightarrow$   
 $\Leftrightarrow 2a - 3ai - 8i - 12 = 10 + bi \Leftrightarrow$   
 $\Leftrightarrow 2a - 12 = 10 \wedge -3a - 8 = b \Leftrightarrow a = 11 \wedge b = -41$   
 Logo,  $a = 11$  e  $b = -41$ .

70.1.  $z = (1 - ai)^2 + (a + 1)i^5 = 1 - 2ai - ai - a^2 + (a + 1)i =$   
 $= 1 - 2ai - a^2 + ai + i = (1 - a^2) + (1 - a)i$

$z$  é um número real se  $\text{Im}(z) = 0$ .

$$1 - a = 0 \Leftrightarrow a = 1$$

Logo,  $z$  é um número real se  $a = 1$ .

70.2.  $z = (1 - a^2) + (1 - a)i$

$z$  é um número imaginário puro se  $\text{Re}(z) = 0$  e

$\text{Im}(z) \neq 0$ .

$1 - a^2 = 0 \wedge 1 - a \neq 0 \Leftrightarrow a^2 = 1 \wedge a \neq 1 \Leftrightarrow$

$\Leftrightarrow a = \pm 1 \wedge a \neq 1 \Leftrightarrow a = -1$

Logo,  $z$  é um imaginário puro se  $a = -1$ .

70.3.  $z = (1 - a^2) + (1 - a)i$

$z = -3 - i \Leftrightarrow (1 - a^2) + (1 - a)i = -3 - i \Leftrightarrow$

$\Leftrightarrow 1 - a^2 = -3 \wedge 1 - a = -1 \Leftrightarrow a^2 = 4 \wedge a = 2 \Leftrightarrow$

$\Leftrightarrow a = \pm 2 \wedge a = 2 \Leftrightarrow a = 2$

Logo,  $z = -3 - i$  se  $a = 2$ .

71.1.  $z_1 = 3 - 2i^3 = -3 + 2i$ ;  $\bar{z}_1 = -3 + 2i = -3 - 2i$

71.2.  $\bar{z}_2 = \overline{1 + 2i} = 1 - 2i$

71.3.  $\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2 = -3 - 2i + 1 - 2i = -2 - 4i$

71.4.  $\overline{z_1 - z_2} = -3 - 2i - (1 - 2i) = -3 - 2i - 1 + 2i = -4$

71.5.  $\overline{z_1 \times z_2} = \bar{z}_1 \times \bar{z}_2 = (-3 - 2i)(1 - 2i) = 3 + 6i - 2i - 4 = -7 + 4i$

71.6.  $i \bar{z}_1 \times \bar{z}_2 = i(-3 + 2i) \times (1 - 2i) = (-3i - 2)(1 - 2i) =$   
 $= -3i - 6 - 2 + 4i = -8 + i = -8 - i$

72.1.  $f(z) = z + \overline{2 - i} = z + 2 + i$

O afixo de  $f(z)$  é a imagem do afixo de  $z$  pela translação de vetor  $\vec{v}(2, 1)$ .

$g(z) = \bar{z} - 3 = \bar{z} + (-3 + 0i)$

O afixo de  $g(z)$  é a imagem do afixo de  $z$  pela reflexão de eixo real seguida de translação de vetor  $\vec{v}(-3, 0)$ , ou seja, pela reflexão deslizante de eixo real e vetor  $\vec{v}(-3, 0)$ .

$h(z) = -(\overline{z + i}) = -(\bar{z} - i) = -\bar{z} + i = -\bar{z} + (0 + i)$

O afixo de  $h(z)$  é a imagem do afixo de  $z$  pela reflexão de eixo real, seguida da reflexão central de centro  $O$  e pela translação de vetor  $\vec{v}(0, 1)$ , ou seja, pela reflexão deslizante de eixo imaginário e vetor  $\vec{v}(0, 1)$ .

72.2. Seja:

- $A(-1, 0)$  o afixo de  $z_1$
- $B(0, 1)$  o afixo de  $z_2$
- $O(0, 0)$  o afixo de  $z_0$

$f(z_1) = -1 + 2 + i = 1 + i$

$f(z_2) = i + 2 + i = 2i$

$f(z_0) = 2 + i$

Considerando  $A_f$  o afixo de  $f(z_1)$ ,  $B_f$  o afixo de  $f(z_2)$  e  $O_f$  o afixo de  $f(z_0)$ , a imagem por  $f$  do triângulo  $[ABO]$  é o triângulo  $[A_f B_f O_f]$ , sendo

$A_f(1, 1), B_f(2, 2)$  e  $O_f(2, 1)$ .

$g(z_1) = \overline{(-1 + 0i)} - 3 = -1 - 3 = -4$

$g(z_2) = \overline{(0 + i)} - 3 = -3 - i$

$g(z_0) = \overline{(0 + 0i)} - 3 = -3$

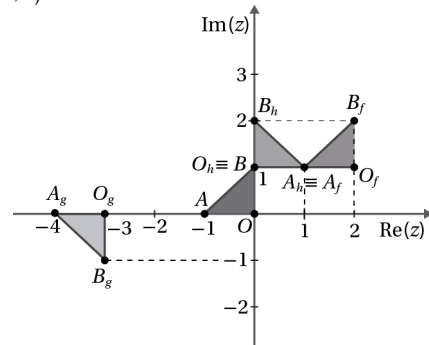
Considerando  $A_g$  o afixo de  $g(z_1)$ ,  $B_g$  o afixo de  $g(z_2)$  e  $O_g$  o afixo de  $g(z_0)$ , a imagem por  $g$  do triângulo  $[ABO]$  é o triângulo  $[A_g B_g O_g]$ , sendo  $A_g(-4, 0)$ ,  $B_g(-3, -1)$  e  $O_g(-3, 0)$ .

$h(z_1) = \overline{-(-1 + 0i)} + i = -(-1) + i = 1 + i$

$h(z_2) = \overline{-(0 + i)} + i = -(-i) + i = i + i = 2i$

$h(z_0) = \overline{-(0 + 0i)} + i = i$

Considerando  $A_h$  o afixo de  $h(z_1)$ ,  $B_h$  o afixo de  $h(z_2)$  e  $O_h$  o afixo de  $h(z_0)$ , a imagem por  $h$  do triângulo  $[ABO]$  é o triângulo  $[A_h B_h O_h]$ , sendo  $A_h(1, 1)$ ,  $B_h(0, 2)$  e  $O_h(0, 1)$ , ou seja, pela reflexão deslizante de eixo imaginário e vetor  $\vec{v}(0, 1)$ .



73.1.  $|z_1| = |3| = 3$

$|z_2| = |-3 + 2i| = \sqrt{9 + 4} = \sqrt{13}$

$|z_3| = |1 + 4i| = \sqrt{1 + 16} = \sqrt{17}$

$|z_4| = |3 + 5i| = \sqrt{9 + 25} = \sqrt{34}$

73.2.  $|z_3 - z_2| = |1 + 4i - (-3 + 2i)| = |4 + 2i| = \sqrt{16 + 4} = \sqrt{20} = 2\sqrt{5}$

$|z_4 - z_3| = |3 + 5i - (1 + 4i)| = |2 + i| = \sqrt{4 + 1} = \sqrt{5}$

$|z_4 - z_2| = |3 + 5i - (-3 + 2i)| = |6 + 3i| = \sqrt{36 + 9} = 3\sqrt{5}$

Como  $\overline{BC} + \overline{CD} = \overline{BD}$ , pois  $2\sqrt{5} + \sqrt{5} = 3\sqrt{5}$ , os pontos  $A, B$  e  $C$  são colineares.

73.3. Já vimos que  $\overline{BC} = \sqrt{20}$ .

$\overline{AB} = |z_2 - z_1| = |-3 + 2i - 3| = |-6 + 2i| = \sqrt{36 + 4} = \sqrt{40}$

$\overline{AC} = |z_3 - z_1| = |1 + 4i - 3| = |-2 + 4i| = \sqrt{4 + 16} = \sqrt{20}$

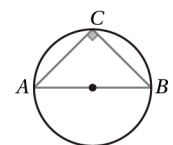
Como  $\overline{AC} = \overline{BC}$ , o triângulo  $[ABC]$  é isósceles.

$\overline{AB}^2 = (\sqrt{40})^2 = 40$

$\overline{AC}^2 + \overline{BC}^2 = (\sqrt{20})^2 + (\sqrt{20})^2 = 20 + 20 = 40$

Como  $\overline{AB}^2 = \overline{AC}^2 + \overline{BC}^2$ , o triângulo  $[ABC]$  é retângulo em  $C$ , uma vez que o ângulo  $ACB$  se opõe ao lado  $[AB]$ , o lado de maior comprimento.

73.4. Tendo em conta o esquema ao lado, dado que o triângulo  $[ABC]$  é retângulo em  $C$  o lado  $[AB]$  é um diâmetro da circunferência que passa nos pontos  $A, B$  e  $C$ . Logo, a circunferência que contém os pontos  $A, B$  e  $C$  tem centro no ponto  $M$ , ponto



médio de  $[AB]$ , e raio  $r = \frac{\overline{AB}}{2}$ .

$$r = \frac{\sqrt{40}}{2} = \frac{\sqrt{4 \times 10}}{2} = \frac{2\sqrt{10}}{2} = \sqrt{10}$$

$$A(3, 0) \text{ e } B(-3, 2); M\left(\frac{3-3}{2}, \frac{0-12}{2}\right) = (0, 1)$$

Uma equação da circunferência pedida é:

$$x^2 + (y-1)^2 = 10$$

$$74. zw = (2+3i)(1+i) = 2+2i+3i-3 = -1+5i$$

$$\frac{1}{w} = \frac{1}{1+i} = \frac{1-i}{(1+i)(1-i)} = \frac{1-i}{1+1} = \frac{1-i}{2} = \frac{1}{2} - \frac{1}{2}i$$

$$\frac{\bar{w}}{z} = \frac{\overline{1+i}}{2+3i} = \frac{1-i}{2+3i} = \frac{(1-i)(2-3i)}{(2+3i)(2-3i)} = \frac{2-3i-2i-3}{4+9} =$$

$$= \frac{-1-5i}{13} = -\frac{1}{13} - \frac{5}{13}i$$

$$75.1. \frac{10-15i}{1+2i} \times (7i-4) = \frac{(10-15i)(1-2i)}{(1+2i)(1-2i)} \times (7i-4) =$$

$$= \frac{10-20i-15i-30}{1+4} \times (7i-4) =$$

$$= \left(\frac{20}{5} - \frac{35}{5}i\right) \times (7i-4) = (-4-7i) \times (-4+7i) =$$

$$= 16+49 = 65+0i$$

$$75.2. \frac{(2-i)^3 - i^3}{1-i} = \frac{(2-i)^2(2-i) - (-i)}{1-i} =$$

$$= \frac{(4-1-4i)(2-i)+i}{1-i} = \frac{(3-4i)(2-i)+i}{1-i} =$$

$$= \frac{6-3i-8i-4+i}{1-i} = \frac{2-10i}{1-i} = \frac{(2-10i)(1+i)}{(1-i)(1+i)} =$$

$$= \frac{2+2i-10+10i}{1+1} = \frac{12}{2} - \frac{8}{2}i = 6-4i$$

$$75.3. \frac{(1-2i^3)(3-i)+i^{13}+i^{14}}{2i} = \frac{(1+2i)(3-i)+i-1}{2i} =$$

$$= \frac{3-i+6i+2+i-1}{2i} = \frac{4+6i}{2i} = \frac{2+3i}{i} =$$

$$= \frac{(2+3i) \times (-i)}{i \times (-i)} = \frac{-2i+3}{1} = 3-2i$$

$$76.1. \frac{5 \times \bar{z}_1}{z_2} = \frac{5 \times (1-i)}{3-4i} = \frac{5(1+i)}{(3-4i)(3+4i)} =$$

$$= \frac{15+20i+15i-20}{9+16} = \frac{5+35i}{25} =$$

$$= -\frac{5}{25} + \frac{35}{25}i = -\frac{1}{5} + \frac{7}{5}i$$

$$76.2. \frac{iz_3}{|z_2|+z_2} = \frac{i \times 2i}{|3-4i|+3-4i} = \frac{2i^2}{\sqrt{9+16}+3-4i} =$$

$$= \frac{-2}{5+3-4i} = \frac{-2}{8-4i} = \frac{-1}{4-2i} = \frac{-(4+2i)}{(4-2i)(4+2i)} =$$

$$= \frac{-4-2i}{16+4} = \frac{-4-2i}{20} = -\frac{1}{5} - \frac{1}{10}i$$

$$76.3. \left(\frac{z_2}{z_3} + i\right) \times \frac{z_1}{i} = \left(\frac{3-4i}{2i} + i\right) \times \frac{1-i}{i} =$$

$$= \left(\frac{3-4i}{2i} + i\right) \times \frac{(1-i) \times (-i)}{i \times (-i)} = \left(\frac{5}{2} + i\right) \times (-i-1) =$$

$$= -\frac{5}{2}i - \frac{5}{2} + 1 - i = -\frac{3}{2} - \frac{7}{2}i$$

$$76.4. \frac{|2z_3 - z_2|}{(1+i)^4} = \frac{|2 \times 2i - (3-4i)|}{(1+i)^2(1+i)^2} = \frac{|4i-3+4i|}{(1+2i-1)(1+2i-1)}$$

$$= \frac{|-3+8i|}{2i \times 2i} = \frac{\sqrt{9+64}}{-4} = -\frac{\sqrt{73}}{4}$$

$$76.5. \frac{(z_3-1)^{11} + |z_3|}{2+z_1} = \frac{(2i-i)^{11} + 2}{2+1-i} = \frac{i^{11} + 2}{3-i} = \left| \begin{array}{l} 11=4 \times 2 + 3 \\ i^{11} = i^3 = -i \end{array} \right.$$

$$= \frac{-i+2}{3-i} = \frac{(2-i)(3+i)}{(3-i)(3+i)} = \frac{6+2i-3i+1}{9+1} = \frac{7-i}{10} = \frac{7}{10} - \frac{1}{10}i$$

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77.1. Seja  $z = x + yi$ .

$$w = \frac{z-1}{z-3} = \frac{z+yi-1}{x+yi-3} = \frac{(x-1+yi)(x-3-yi)}{(x-3+yi)(x-3-yi)} =$$

$$= \frac{x^2-3x-xyi-x+3+yi+xyi-3yi+y^2}{(x-3)^2+y^2} =$$

$$= \frac{x^2-4x+y^2-2yi+3}{(x-3)^2+y^2} =$$

$$= \frac{x^2-4x+y^2+3}{(x-3)^2+y^2} - \frac{2y}{(x-3)^2+y^2}i$$

$w$  é um número real se  $\text{Im}(w) = 0$ .

$$-2y = 0 \Leftrightarrow y = 0$$

Logo,  $w$  é um número real se  $y = 0$ .

$$77.2. w = \frac{x^2-4x+y^2+3}{(x-3)^2+y^2} - \frac{2y}{(x-3)^2+y^2}$$

$w$  é um número imaginário puro se  $\text{Re}(w) = 0 \wedge \text{Im}(w) \neq 0$ .

$$x^4-4x+y^2+3=0 \wedge -2y \neq 0 \Leftrightarrow$$

$$\Leftrightarrow (x^2-4x+4) - 4 + y^2 + 3 = 0 \wedge y \neq 0 \Leftrightarrow$$

$$\Leftrightarrow (x-2)^2 + y^2 = 1 \wedge y \neq 0$$

Logo,  $w$  é um número imaginário puro se os afixos de  $w$

pertencerem à circunferência de centro no ponto de coordenadas  $(2, 0)$  e raio 1 exceto os pontos de coordenadas  $(1, 0)$  e  $(3, 0)$ .

78.1. Seja  $z = x + yi$ .

$$w = \frac{iz+1}{z+i} = \frac{i(x+yi)+1}{x+yi+i} =$$

$$= \frac{(-y+1+xi)(x-(y+1)i)}{(x+(y+1)i)(x-(y+1)i)} =$$

$$= \frac{-xy+(y^2+y)i+x-(y+1)i+x^2i+xy+x}{x^2+(y+1)^2} =$$

$$= \frac{2x+(x^2+y^2-1)i}{x^2+(y+1)^2} = \frac{2x}{x^2+(y+1)^2} + \frac{x^2+y^2-1}{x^2+(y+1)^2}i$$

$w$  é um número real se  $\text{Im}(w) = 0$ .

$$x^2+y^2-1=0 \Leftrightarrow x^2+y^2=1$$

Logo,  $w$  é um número real se os afixos de  $w$  pertencerem à circunferência de centro na origem do referencial e raio 1, exceto o ponto de coordenadas  $(0, -1)$ .

78.2.  $w$  é um número imaginário puro se  $\text{Re}(w) = 0 \wedge \text{Im}(w) \neq 0$

$$2x = 0 \wedge x^2 + y^2 - 1 \neq 0 \Leftrightarrow x = 0 \wedge y^2 - 1 \neq 0 \Leftrightarrow$$

$$\Leftrightarrow x = 0 \wedge y^2 \neq 1 \Leftrightarrow x = 0 \wedge y \neq -1 \wedge y \neq 1$$

Logo,  $w$  é um número imaginário puro se a abscissa dos afixos de  $w$  for nula e a ordenada diferente de  $-1$  e  $1$ , ou seja, é o eixo imaginário exceto os pontos de coordenadas  $(0, 1)$  e  $(0, -1)$ .

79.1.  $z_1 = 2\sqrt{2} + 2\sqrt{2}i$ ;  $|z_1| = \sqrt{(2\sqrt{2})^2 + (2\sqrt{2})^2} = \sqrt{8+8} = \sqrt{16} = 4$

Seja  $\theta$  um argumento de  $z_1$ .

$$\left. \begin{aligned} \tan \theta = \frac{2\sqrt{2}}{2\sqrt{2}} = 1 \\ (2\sqrt{2}, 2\sqrt{2}) \in 1.^\circ Q \end{aligned} \right\} \Rightarrow \theta = \frac{\pi}{4} \text{ é um argumento de } z_1$$

Logo,  $z_1 = 4e^{i\frac{\pi}{4}}$ .

79.2.  $z_2 = -3\sqrt{3} + 3i$

$$|z_2| = \sqrt{(-3\sqrt{3})^2 + 3^2} = \sqrt{27+9} = \sqrt{36} = 6$$

Seja  $\theta$  um argumento de  $z_2$ .

$$\left. \begin{aligned} \tan \theta = \frac{3}{-3\sqrt{3}} = -\frac{\sqrt{3}}{3} \\ (-3\sqrt{3}, 3) \in 2.^\circ Q \end{aligned} \right\} \Rightarrow \theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6} \text{ é um argumento de } z_2$$

Logo,  $z_2 = 6e^{i\frac{5\pi}{6}}$ .

79.3.  $z_3 = -\frac{3}{2} - \frac{3\sqrt{3}}{2}i$

$$|z_3| = \sqrt{\left(-\frac{3}{2}\right)^2 + \left(-\frac{3\sqrt{3}}{2}\right)^2} = \sqrt{\frac{9+27}{4}} = \sqrt{\frac{36}{4}} = \sqrt{9} = 3$$

$$\left. \begin{aligned} \tan \theta = \frac{-3}{-3\sqrt{3}} = \frac{\sqrt{3}}{3} \\ \left(-\frac{3}{2}, -\frac{3\sqrt{3}}{2}\right) \in 3.^\circ Q \end{aligned} \right\} \Rightarrow -\pi + \frac{\pi}{3} = -\frac{2\pi}{3} \text{ é um argumento de } z_3$$

Logo,  $z_3 = 3e^{-i\frac{2\pi}{3}}$ .

79.4.  $z_4 = 2 - 2i$ ;  $|z_4| = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

Seja  $\theta$  um argumento de  $z_4$ .

$$\left. \begin{aligned} \tan \theta = \frac{-2}{2} = -1 \\ (2, -2) \in 4.^\circ Q \end{aligned} \right\} \Rightarrow -\frac{\pi}{4} \text{ é um argumento de } z_4$$

Logo,  $z_4 = 2\sqrt{2}e^{-i\frac{\pi}{4}}$ .

79.5.  $z_5 = -\frac{3\sqrt{3}}{4} - \frac{3}{4}i$

$$|z_5| = \sqrt{\left(-\frac{3\sqrt{3}}{4}\right)^2 + \left(-\frac{3}{4}\right)^2} = \sqrt{\frac{27+9}{16}} = \sqrt{\frac{36}{16}} = \frac{6}{4} = \frac{3}{2}$$

Seja  $\theta$  um argumento de  $z_5$ .

$$\left. \begin{aligned} \tan \theta = \frac{-3}{-3\sqrt{3}} = \frac{\sqrt{3}}{3} \\ \left(-\frac{3\sqrt{3}}{4}, -\frac{3}{4}\right) \in 3.^\circ Q \end{aligned} \right\} \Rightarrow -\pi + \frac{\pi}{6} = -\frac{5\pi}{6} \text{ é um argumento de } z_5$$

Logo,  $z_5 = \frac{3}{2}e^{-i\frac{5\pi}{6}}$ .

79.6.  $z_6 = -2e^{i\frac{2\pi}{3}} = 2e^{i(\frac{2\pi}{3} + \pi)} = 2e^{-i\frac{\pi}{3}}$

79.7.  $z_7 = 2e^{-i\frac{\pi}{6}} = 2e^{i\frac{\pi}{6}}$

80.1.  $z_1 = 3e^{i\frac{\pi}{6}}$ ;  $z_2 = 3e^{i\frac{11\pi}{6}} = 3e^{-i\frac{\pi}{6}}$

$$\frac{\pi}{6} = -\frac{11\pi}{6} + 2k\pi \Leftrightarrow 2k\pi = \frac{12\pi}{6} \Leftrightarrow k = 1 \in \mathbb{Z}$$

Como  $|z_1| = |z_2| = 3$  e  $\frac{\pi}{6} = -\frac{11\pi}{6} + 2\pi$ , temos  $z_1 = z_2$ .

80.2.  $z_1 = -2e^{i\frac{2\pi}{3}} = 2e^{i(\frac{2\pi}{3} + \pi)} = 2e^{i\frac{5\pi}{3}}$ ;  $z_2 = 2e^{-i\frac{7\pi}{3}}$

$$\frac{5\pi}{3} = -\frac{7\pi}{3} + 2k\pi \Leftrightarrow 5 = -7 + 6k, k \in \mathbb{Z} \Leftrightarrow$$

$$\Leftrightarrow 12 = 6k, k \in \mathbb{Z} \Leftrightarrow k = 2 \in \mathbb{Z}$$

Como  $|z_1| = |z_2| = 2$  e  $\frac{5\pi}{3} = -\frac{7\pi}{3} + 2\pi$ , temos  $z_1 = z_2$ .

81.1.  $\frac{z}{w} = \frac{1+i\sqrt{3}}{1-i} = \frac{(1+i\sqrt{3})(1+i)}{(1-i)(1+i)} =$

$$= \frac{1+i+i\sqrt{3}-\sqrt{3}}{1+1} = \frac{1-\sqrt{3}}{2} + \frac{1+i\sqrt{3}}{2}i$$

81.2.  $\left|\frac{z}{w}\right| = \frac{|z|}{|w|} = \frac{|1+i\sqrt{3}|}{|1-i|} = \frac{\sqrt{1+3}}{1+1} = \frac{2}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$

Seja  $\theta_1$  um argumento de  $z$ .

$$\left. \begin{aligned} \tan \theta_1 = \frac{\sqrt{3}}{1} = \sqrt{3} \\ (1, \sqrt{3}) \in 1.^\circ Q \end{aligned} \right\} \Rightarrow \theta_1 = \frac{\pi}{3} \text{ é um argumento de } z$$

Seja  $\theta_2$  um argumento de  $w$ .

$$\left. \begin{aligned} \tan \theta_2 = \frac{-1}{1} = -1 \\ (1, -1) \in 4.^\circ Q \end{aligned} \right\} \Rightarrow \theta_2 = -\frac{\pi}{4} \text{ é um argumento de } w$$

$$\theta_1 = \text{Arg } z = \frac{\pi}{3}; \theta_2 = \text{Arg } w = -\frac{\pi}{4}$$

$$\theta_1 - \theta_2 = \frac{\pi}{3} - \left(-\frac{\pi}{4}\right) = \frac{\pi}{3} + \frac{\pi}{4} = \frac{7\pi}{12} = \text{Arg}\left(\frac{z}{w}\right)$$

Logo,  $\left|\frac{z}{w}\right| = \sqrt{2}$  e  $\text{Arg}\left(\frac{z}{w}\right) = \frac{7\pi}{12}$ .

81.3.  $\frac{z}{w} = \frac{1-\sqrt{3}}{2} + \frac{1+\sqrt{3}}{2}i = \sqrt{2}\left(\cos\frac{7\pi}{12} + i\sin\frac{7\pi}{12}\right) \Leftrightarrow$

$$\Leftrightarrow \sqrt{2}\cos\frac{7\pi}{12} = \frac{1-\sqrt{3}}{2} \wedge \sqrt{2}\sin\frac{7\pi}{12} = \frac{1+\sqrt{3}}{2} \Leftrightarrow$$

$$\Leftrightarrow \cos\frac{7\pi}{12} = \frac{1-\sqrt{3}}{2\sqrt{2}} \wedge \sin\frac{7\pi}{12} = \frac{1+\sqrt{3}}{2\sqrt{2}} \Leftrightarrow$$

$$\Leftrightarrow \cos\frac{7\pi}{12} = \frac{\sqrt{2}-\sqrt{6}}{4} \wedge \sin\frac{7\pi}{12} = \frac{\sqrt{6}+\sqrt{2}}{4}$$

82.1.  $w = \frac{1}{1-z} = \frac{1}{1-\sqrt{2}e^{i\theta}} = \frac{1}{1-\sqrt{2}(\cos\theta + i\sin\theta)} =$

$$= \frac{1}{1-\sqrt{2}\cos\theta - i\sqrt{2}\sin\theta} =$$

$$= \frac{1-\sqrt{2}\cos\theta + i\sqrt{2}\sin\theta}{(1-\sqrt{2}\cos\theta - i\sqrt{2}\sin\theta)(1-\sqrt{2}\cos\theta + i\sqrt{2}\sin\theta)}$$

$$= \frac{1-\sqrt{2}\cos\theta + i\sqrt{2}\sin\theta}{(1-\sqrt{2}\cos\theta)^2 + (\sqrt{2}\sin\theta)^2} =$$

$$= \frac{1-\sqrt{2}\cos\theta + i\sqrt{2}\sin\theta}{1-2\sqrt{2}\cos\theta + 2\cos^2\theta + 2\sin^2\theta} =$$

$$= \frac{1-\sqrt{2}\cos\theta}{3-2\sqrt{2}\cos\theta} + \frac{\sqrt{2}\sin\theta}{3-2\sqrt{2}\cos\theta}i$$

Logo,  $\text{Re}(w) = \frac{1-\sqrt{2}\cos\theta}{3-2\sqrt{2}\cos\theta} = \frac{\sqrt{2}\cos\theta - 1}{2\sqrt{2}\cos\theta - 3}$ .

82.2.  $\text{Im}(w) = \frac{\sqrt{2}\sin\theta}{3-2\sqrt{2}\cos\theta}$

82.3.  $\text{Re}(w) = \text{Im}(w) \Leftrightarrow \frac{1-\sqrt{2}\cos\theta}{3-2\sqrt{2}\cos\theta} = \frac{\sqrt{2}\sin\theta}{3-2\sqrt{2}\cos\theta} \Leftrightarrow$   
 $\Leftrightarrow \sqrt{2}\sin\theta = 1-\sqrt{2}\cos\theta \Leftrightarrow \sqrt{2}\sin\theta + \sqrt{2}\cos\theta = 1 \Leftrightarrow$   
 $\Leftrightarrow \frac{\sqrt{2}}{2}\sin\theta + \frac{\sqrt{2}}{2}\cos\theta = \frac{1}{2} \Leftrightarrow$   
 $\Leftrightarrow \sin\theta\cos\frac{\pi}{4} + \sin\frac{\pi}{4}\cos\theta = \frac{1}{2} \Leftrightarrow$   
 $\Leftrightarrow \sin\left(\theta + \frac{\pi}{4}\right) = \frac{1}{2} \Leftrightarrow \sin\left(\theta + \frac{\pi}{4}\right) = \sin\left(\frac{\pi}{6}\right) \Leftrightarrow$   
 $\Leftrightarrow \theta + \frac{\pi}{4} = \frac{\pi}{6} + 2k\pi \vee \theta + \frac{\pi}{4} = \frac{5\pi}{6} + 2k\pi, k \in \mathbb{Z} \Leftrightarrow$   
 $\Leftrightarrow \theta = -\frac{\pi}{12} + 2k\pi \vee \theta = \frac{7\pi}{12} + 2k\pi, k \in \mathbb{Z}$   
 Como  $\theta \in ]-\pi, \pi[$ ,  $\theta = -\frac{\pi}{12} \vee \theta = \frac{7\pi}{12}$ .

Logo,  $\theta \in \left\{-\frac{\pi}{12}, \frac{7\pi}{12}\right\}$ .

83.1.  $|z| = \left|\frac{1-i\sqrt{3}}{4}\right| = \left|\frac{1}{4} - \frac{\sqrt{3}}{4}i\right| = \sqrt{\frac{1}{16} + \frac{3}{16}} = \sqrt{\frac{4}{16}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$

$\tan\theta = \frac{-\sqrt{3}}{1} = -\sqrt{3} \Rightarrow -\frac{\pi}{3}$  é um argumento de  $z$   
 $\left(\frac{1}{4}, -\frac{\sqrt{3}}{4}\right) \in 4.^\circ\text{Q}$

$|z| = \frac{1}{2}$  e um argumento de  $z$  é  $-\frac{\pi}{3}$ .

83.2.  $z^7 = \left(\frac{1}{2}e^{-i\frac{\pi}{3}}\right)^7 = \left(\frac{1}{2}\right)^7 e^{-i\frac{7\pi}{3}} = \frac{1}{128}e^{-i\frac{\pi}{3}} \quad \left|-\frac{7}{3}\pi + 2\pi = -\frac{\pi}{3}\right|$

84.  $z_1 = 1 + \sqrt{3}i^{15} = 1 + \sqrt{3}i^3 = 1 - \sqrt{3}i \quad |i^{15} = i^{4 \cdot 3 + 3} = i^3 = i$   
 $|z_1| = \sqrt{1+3} = 2$

$\tan\alpha = -\sqrt{3} \Rightarrow -\frac{\pi}{3}$  é um argumento de  $z_1$   
 $(1, -\sqrt{3}) \in 4.^\circ\text{Q}$

$z_3 = \left(\frac{z_2}{z_1}\right)^3 = \left(\frac{e^{i\theta}}{2e^{-i\frac{\pi}{3}}}\right)^3 = \frac{1}{8}e^{3i\left(\theta + \frac{\pi}{3}\right)} = \frac{1}{4}e^{i(3\theta + \pi)}$

Como  $\frac{3\pi}{4}$  é um argumento de  $z_3$ :

$3\theta + \pi = \frac{3\pi}{4} + 2k\pi, k \in \mathbb{Z} \Leftrightarrow 3\theta = \frac{3\pi}{4} - \pi + 2k\pi, k \in \mathbb{Z}$

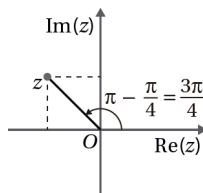
$\Leftrightarrow \theta = -\frac{\pi}{12} + \frac{2k\pi}{3}, k \in \mathbb{Z}$

Se  $k=1, \theta = -\frac{\pi}{12} + \frac{2\pi}{3} = \frac{7\pi}{12}$

Se  $k=2, \theta = -\frac{\pi}{12} + \frac{4\pi}{3} = \frac{15\pi}{12} = \frac{5\pi}{4}$

Como  $\theta \in ]\pi, \frac{3\pi}{2}[$ ,  $\theta = \frac{5\pi}{4} = \pi + \frac{\pi}{4}$ .

Logo,  $z_2 = e^{i\frac{5\pi}{4}} = \cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4} = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$ .



85.1.  $z_1 = -1 + i\sqrt{3}; z_2 = e^{-i\frac{\pi}{2}}$

$|z_1| = \sqrt{1+3} = 2$

$\tan\theta = \frac{\sqrt{3}}{-1} = -\sqrt{3} \Rightarrow \pi - \frac{\pi}{3} = \frac{2\pi}{3}$  é um argumento de  $z_1$   
 $(-1, \sqrt{3}) \in 2.^\circ\text{Q}$

$z_1 = 2e^{i\frac{2\pi}{3}}$

$z = (z_1 \times z_2)^n = \left(2e^{i\frac{2\pi}{3}} \times e^{-i\frac{\pi}{2}}\right)^n = \left(2e^{i\left(\frac{2\pi}{3} - \frac{\pi}{2}\right)}\right)^n =$   
 $= \left(2e^{i\frac{\pi}{6}}\right)^n = 2^n e^{i\frac{n\pi}{6}}$

$z$  é um número real positivo se:

$\frac{n\pi}{6} = 2k\pi, k \in \mathbb{Z} \wedge n \in \mathbb{N} \Leftrightarrow n = 12k, k \in \mathbb{Z} \wedge n \in \mathbb{N}; n = 12$

85.2.  $z = 2^n e^{i\frac{n\pi}{6}}$

$z$  é um número real negativo se:

$\frac{n\pi}{6} = \pi + 2k\pi, k \in \mathbb{Z} \wedge n \in \mathbb{N} \Leftrightarrow n = 6 + 12k, k \in \mathbb{Z} \wedge n \in \mathbb{N}$

$n = 6$

85.3.  $z$  é um número imaginário puro se:

$\frac{n\pi}{6} = \frac{\pi}{2} + k\pi, k \in \mathbb{Z} \wedge n \in \mathbb{N} \Leftrightarrow n = 3 + 6k, k \in \mathbb{Z} \wedge n \in \mathbb{N}$

O menor valor de  $n$  é  $n = 3$  (para  $k = 0$ ).

86. Seja  $\theta = \text{Arg}(z), -\pi < \theta \leq \pi$ .

$z + 1 = e^{i\theta} + 1 = \cos\theta + i\sin\theta + 1 =$   
 $= \cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2} + 2i\sin\frac{\theta}{2}\cos\frac{\theta}{2} + 1 =$   
 $= \cos^2\frac{\theta}{2} + \cos^2\frac{\theta}{2} + 2i\sin\frac{\theta}{2}\cos\frac{\theta}{2} = \left|1 - \sin^2\frac{\theta}{2} = \cos^2\frac{\theta}{2}\right|$   
 $= 2\cos^2\frac{\theta}{2} + 2i\sin\frac{\theta}{2}\cos\frac{\theta}{2} =$   
 $= 2\cos\frac{\theta}{2}\left(\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}\right) =$   
 $= 2\cos\frac{\theta}{2}e^{i\frac{\theta}{2}}$

Como  $-\frac{\pi}{2} < \frac{\theta}{2} \leq \frac{\pi}{2}$ ,  $\cos\frac{\theta}{2} \geq 0$ .

Logo,  $\text{Arg}(z + 1) = \frac{\theta}{2} = \frac{\text{Arg}(z)}{2}$ .

87.1.  $\sqrt[4]{z} = \sqrt[4]{8\sqrt{2} - 8\sqrt{2}i} =$   
 $= \sqrt[4]{16e^{-i\frac{\pi}{4}}} =$   
 $= 2e^{i\left(\frac{-\frac{\pi}{4} + 2k\pi}{4}\right)}, k = 0, 1, 2, 3$   
 $= 2e^{i\left(\frac{-\pi + 8k\pi}{16}\right)}, k = 0, 1, 2, 3$

Cálculos auxiliares

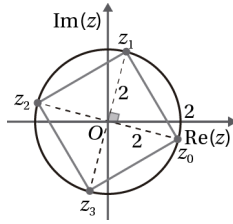
$|z| = \sqrt{(8\sqrt{2})^2 + (-8\sqrt{2})^2} =$   
 $= \sqrt{128 + 128} = \sqrt{256} = 16$   
 $\tan\theta = \frac{-8\sqrt{2}}{8\sqrt{2}} = -1 \Rightarrow$   
 $(8\sqrt{2}, -8\sqrt{2}) \in 4.^\circ\text{Q}$   
 $\Rightarrow -\frac{\pi}{4}$  é um argumento de  $z$

$k = 0, z_0 = 2e^{-i\frac{\pi}{16}}; k = 1, z_1 = 2e^{i\frac{7\pi}{16}}$

$k = 2, z_2 = 2e^{i\frac{15\pi}{16}}; k = 3, z_3 = 2e^{i\frac{23\pi}{16}}$

As raízes índice 4 de  $z$  são:  $2e^{-i\frac{\pi}{16}}, 2e^{i\frac{7\pi}{16}}, 2e^{i\frac{15\pi}{16}}, 2e^{i\frac{23\pi}{16}}$

87.2. Os afixos das raízes quartas de  $z$  são vértices de um quadrado inscrito numa circunferência de centro na origem do referencial e raio 2.



As diagonais do quadrado medem 4.

Seja  $l$  a medida do lado do quadrado

$$l^2 + l^2 = 4^2 \Leftrightarrow 2l^2 = 16 \Leftrightarrow l^2 = 8$$

A medida da área do quadrado é igual a 8 u.a.

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88.1.  $2z + 1 = i \Leftrightarrow z = -\frac{1}{2} + \frac{1}{2}i$ ;  $S = \left\{ -\frac{1}{2} + \frac{1}{2}i \right\}$

88.2.  $z + i = 2 - iz \Leftrightarrow z + iz = 2 - i \Leftrightarrow z(1+i) = 2 - i \Leftrightarrow z = \frac{2-i}{1+i}$   
 $\Leftrightarrow z = \frac{(2-i)(1-i)}{(1+i)(1-i)} \Leftrightarrow z = \frac{2-2i-i-1}{1+1} \Leftrightarrow z = \frac{1-3i}{2}$ ;  $S = \left\{ \frac{1}{2} - \frac{3}{2}i \right\}$

88.3.  $2z^2 - 10z + 17 = 0 \Leftrightarrow z = \frac{10 \pm \sqrt{100 - 136}}{4}$   
 $\Leftrightarrow z = \frac{10 \pm \sqrt{-36}}{4} \Leftrightarrow z = \frac{10 \pm 6i}{4}$   
 $\Leftrightarrow z = \frac{5}{2} - \frac{3}{2}i \vee z = \frac{5}{2} + \frac{3}{2}i$   
 $S = \left\{ \frac{5}{2} - \frac{3}{2}i, \frac{5}{2} + \frac{3}{2}i \right\}$

88.4.  $z^3 - 2z^2 + 10z = 0 \Leftrightarrow z(z^2 - 2z + 10) = 0 \Leftrightarrow z = 0 \vee z^2 - 2z + 10 = 0 \Leftrightarrow z = 0 \vee z = \frac{2 \pm \sqrt{4 - 40}}{2} \Leftrightarrow z = 0 \vee z = \frac{2 \pm 6i}{2} \Leftrightarrow z = 0 \vee z = 1 - 3i \vee z = 1 + 3i$   
 $S = \{0, 1 - 3i, 1 + 3i\}$

88.5.  $z^4 - 2z^2 - 8 = 0 \Leftrightarrow z^2 = \frac{2 \pm \sqrt{4 + 32}}{2} \Leftrightarrow z^2 = \frac{2 \pm 6}{2} \Leftrightarrow z^2 = 4 \vee z^2 = -2 \Leftrightarrow z = \pm\sqrt{4} \vee z = \pm\sqrt{-2} \Leftrightarrow z = -2 \vee z = 2 \vee z = -\sqrt{2}i \vee z = \sqrt{2}i$   
 $S = \{-2, 2, -\sqrt{2}i, \sqrt{2}i\}$

88.6.  $z^5 + iz = z \Leftrightarrow z^5 + iz - z = 0 \Leftrightarrow z(z^4 + i - 1) = 0 \Leftrightarrow z = 0 \vee z^4 + i - 1 = 0 \Leftrightarrow z = \sqrt[4]{1 - i} \Leftrightarrow z = 0 \vee z = \sqrt[4]{\sqrt{2} e^{-i\frac{\pi}{4}}} \Leftrightarrow z = 0 \vee z = \sqrt[4]{\sqrt{2}} e^{-i\frac{\pi}{16}}$   
 $\left. \begin{array}{l} |1-i| = \sqrt{1+1} = \sqrt{2} \\ \tan \theta = -1 \\ (1, -1) \in 4^o Q \end{array} \right\} \Rightarrow \theta = -\frac{\pi}{4}$   
 $\Leftrightarrow z = 0 \vee z = \sqrt[4]{2} e^{-i\frac{\pi}{16}}$ ,  $k = 0, 1, 2, 3 \Leftrightarrow z = 0 \vee z = \sqrt[4]{2} e^{-i\left(\frac{\pi}{16} + \frac{2k\pi}{4}\right)}$ ,  $k = 0, 1, 2, 3$

$$k = 0, z_0 = \sqrt[4]{2} e^{-i\frac{\pi}{16}}, k \in 0, 1, 2, 3$$

$$k = 1, z_1 = \sqrt[4]{2} e^{-i\frac{7\pi}{16}}$$

$$k = 2, z_2 = \sqrt[4]{2} e^{-i\frac{15\pi}{16}}$$

$$k = 3, z_3 = \sqrt[4]{2} e^{-i\frac{23\pi}{16}}$$

$$S = \left\{ 0, \sqrt[4]{2} e^{-i\frac{\pi}{16}}, \sqrt[4]{2} e^{-i\frac{7\pi}{16}}, \sqrt[4]{2} e^{-i\frac{15\pi}{16}}, \sqrt[4]{2} e^{-i\frac{23\pi}{16}} \right\}$$

88.7. Seja  $z = x + yi$

$$\bar{z} + 2z - 1 = 0 \Leftrightarrow z - yi - 1 = 0 \Leftrightarrow 3x - 1 + yi = 0$$

$$\Leftrightarrow 3x - 1 = 0 \wedge y = 0 \Leftrightarrow x = \frac{1}{3} \wedge y = 0 \Leftrightarrow z = \frac{1}{3}$$

$$S = \left\{ \frac{1}{3} \right\}$$

88.8. Seja  $z = x + yi$ .

$$2z\bar{z} + z^2 - 3 = 0 \Leftrightarrow 2|z|^2 + z^2 - 3 = 0$$

$$\Leftrightarrow 2(x^2 + y^2) + (x + yi)^2 - 3 = 0 \Leftrightarrow$$

$$\Leftrightarrow 2x^2 + 2y^2 + x^2 + 2xyi - y^2 - 3 = 0 \Leftrightarrow$$

$$\Leftrightarrow 3x^2 + y^2 - 3 + 2xyi = 0 \Leftrightarrow$$

$$\Leftrightarrow 3x^2 + y^2 - 3 = 0 \wedge 2xy = 0 \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} x = 0 \\ y^2 = 3 \end{cases} \vee \begin{cases} y = 0 \\ 3x^2 = 3 \end{cases} \Leftrightarrow \begin{cases} x = 0 \\ y = \pm\sqrt{3} \end{cases} \vee \begin{cases} y = 0 \\ x = \pm 1 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow z = -\sqrt{3}i \vee z = \sqrt{3}i \vee z = -1 \vee z = 1$$

$$S = \{-1, 1, -\sqrt{3}i, \sqrt{3}i\}$$

89.1.  $P(z) = z^4 - 2z^3 + 6z^2 - 2z + 5$

$$\overline{P(z)} = P(\bar{z})$$

$$\overline{P(z)} = \overline{z^4 - 2z^3 + 6z^2 - 2z + 5} =$$

$$= \bar{z}^4 - 2\bar{z}^3 + 6\bar{z}^2 - 2\bar{z} + 5 =$$

$$= \bar{z}^4 - 2\bar{z}^3 + 6\bar{z}^2 - 2\bar{z} + 5 = P(\bar{z})$$

89.2. a)  $P(1 - 2i) =$

$$= (1 - 2i)^4 - 2(1 - 2i)^3 + 6(1 - 2i)^2 - 2(1 - 2i) + 5 =$$

$$= (-3 - 4i)^2 - 2(3 - 4i)(1 - 2i) + \left. \begin{array}{l} (1 - 2i)^2 = \\ = 1 - 4i - 4 = \\ = -3 - 4i \end{array} \right\}$$

$$+ 6(-3 - 4i) - 2 + 4i + 5 =$$

$$= 9 + 24i - 16 + 6 - 12i + 8i +$$

$$+ 16 - 18 - 24i + 4i + 3 = 0$$

Logo,  $1 - 2i$  é uma solução da equação  $P(z) = 0$ .

b)  $P(1 - 2i) = 0 + 0i$

Pela propriedade de 89.1.:

$$\overline{P(1 - 2i)} = P(\overline{1 - 2i}) \Leftrightarrow 0 = P(1 + 2i)$$

Logo,  $1 + 2i$  também é solução da equação  $P(z) = 0$ .

c)

1 - 2i	1	-2	6	-2	5	$\left. \begin{array}{l} (1 - 2i)(-1 - 2i) = \\ = -1 - 2i + 2i - 4 = -5 \\ (1 - 2i)(11 - 24i) = \\ = 11 - 24i - 22i \end{array} \right\}$
		1 - 2i	-5	1 - 2i	-5	
1 + 2i	1	-1 - 2i	1	-1 - 2i	0	
		1 + 2i	0	1 + 2i		
	1	0	1	0		

$$z^2 + 1 = 0 \Leftrightarrow z^2 = -1 \Leftrightarrow z = \pm\sqrt{-1} \Leftrightarrow z = \pm i$$

$$S = \{1 - 2i, 1 + 2i, -i, i\}$$

90.1.  $z_1^n = z_2^n$ ;  $|z_1| = \sqrt{(-\sqrt{2})^2 + (-\sqrt{2})^2} = \sqrt{2+2} = 2$   
 $\left. \begin{aligned} \tan \theta = \frac{-\sqrt{2}}{-\sqrt{2}} = 1 \\ (-\sqrt{2}, -\sqrt{2}) \in 3.^\circ Q \end{aligned} \right\} \Rightarrow -\pi + \frac{\pi}{4} = -\frac{3\pi}{4} \text{ é um argumento de } z_1$

Logo,  $z_1 = 2e^{-i\frac{3\pi}{4}}$   
 $\text{Arg } z_2 - \text{Arg } z_1 = -\frac{5\pi}{12} - \left(-\frac{3\pi}{4}\right) =$   
 $= -\frac{5\pi}{12} + \frac{3\pi}{4} = \frac{\pi}{3} = \frac{2\pi}{6} \quad \left| \frac{2\pi}{6} = \frac{\pi}{3} \right.$

Logo,  $n = 6$ .  
 Os afixo de  $z_1$  e  $z_2$  são vértices de um hexágono regular inscrito na circunferência centrada na origem e de raio 2.

90.2.  $z_1 = 2e^{-i\frac{3\pi}{4}} \quad z_2 = 2e^{-i\frac{5\pi}{12}}$   
 $z_3 = 2e^{i\left(-\frac{5\pi}{12} + \frac{\pi}{3}\right)} = 2e^{-i\frac{\pi}{12}} \quad z_4 = 2e^{i\left(-\frac{\pi}{12} + \frac{\pi}{3}\right)} = 2e^{i\frac{3\pi}{12}} = 2e^{i\frac{\pi}{4}}$   
 $z_5 = 2e^{i\left(\frac{\pi}{4} + \frac{\pi}{3}\right)} = 2e^{i\frac{7\pi}{12}} \quad z_6 = 2e^{i\left(\frac{7\pi}{12} + \frac{\pi}{3}\right)} = 2e^{i\frac{11\pi}{12}}$

Além de  $z_1$  e  $z_2$ , as restantes raízes índice 6 de  $z$  são:

$2e^{-i\frac{\pi}{12}}, 2e^{i\frac{\pi}{4}}, 2e^{i\frac{7\pi}{12}}, 2e^{i\frac{11\pi}{12}}$

90.3. Por exemplo:  $z = z_3^6 = \left(2e^{-i\frac{\pi}{12}}\right)^6 = 2^6 e^{-i\frac{6\pi}{12}} = 64e^{-i\frac{\pi}{2}} = -64i$

91.1.  $z_A = i$ ;  $z_B = e^{i\frac{\pi}{6}}$ ;  $z_C = 3e^{i\frac{\pi}{2}}$ . Sejam  $z'_A$ ,  $z'_B$  e  $z'_C$  os números complexos cujos afijos são  $A'$ ,  $B'$  e  $C'$ , respetivamente.

$z'_A = e^{i\frac{2\pi}{3}} \times z_A = e^{i\frac{2\pi}{3}} \times i = e^{i\frac{3\pi}{3}} \times e^{i\frac{\pi}{2}} = e^{i\left(\frac{2\pi}{3} + \frac{\pi}{2}\right)} = e^{i\frac{7\pi}{6}} =$   
 $= \cos \frac{7\pi}{6} + i \sin \frac{7\pi}{6} = -\frac{\sqrt{3}}{2} - \frac{1}{2}i$   
 $z'_B = e^{i\frac{2\pi}{3}} \times z_B = e^{i\frac{2\pi}{3}} \times e^{i\frac{\pi}{6}} = e^{i\left(\frac{2\pi}{3} + \frac{\pi}{6}\right)} = e^{i\frac{5\pi}{6}} =$   
 $= \cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6} = -\frac{\sqrt{3}}{2} + \frac{1}{2}i$   
 $z'_C = e^{i\frac{2\pi}{3}} \times z_C = e^{i\frac{2\pi}{3}} \times e^{i\frac{\pi}{2}} = 3e^{i\frac{7\pi}{6}} = -\frac{3\sqrt{3}}{2} - \frac{3}{2}i$

Logo,  $A'\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$ ;  $B'\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$  e  $C'\left(-\frac{3\sqrt{3}}{2}, -\frac{3}{2}\right)$

91.2. Sejam  $z''_A$ ,  $z''_B$  e  $z''_C$  os números complexos cujos afijos são  $A''$ ,  $B''$  e  $C''$ , respetivamente.

$z''_A = z_A \times 2e^{-i\frac{5\pi}{6}} = 2 \times e^{i\left(\frac{\pi}{2} - \frac{5\pi}{6}\right)} = 2e^{-i\frac{\pi}{3}} =$   
 $= 2\left(\cos\left(-\frac{\pi}{3}\right) + i \sin\left(-\frac{\pi}{3}\right)\right) = 2\left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) = 1 - \sqrt{3}i$   
 $z''_B = z_B \times 2e^{-i\frac{5\pi}{6}} = 2 \times e^{i\left(\frac{\pi}{6} - \frac{5\pi}{6}\right)} = 2e^{-i\frac{2\pi}{3}} =$   
 $= 2\left(\cos\left(-\frac{2\pi}{3}\right) + i \sin\left(-\frac{2\pi}{3}\right)\right) = 2\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) = -1 - \sqrt{3}i$   
 $z''_C = z_C \times 2e^{-i\frac{5\pi}{6}} = 2 \times 3e^{i\left(\frac{\pi}{2} - \frac{5\pi}{6}\right)} = 6e^{-i\frac{\pi}{3}} =$   
 $= 6\left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) = 3 - 3\sqrt{3}i$

Logo,  $A''(1, -\sqrt{3})$ ;  $B''(-1, -\sqrt{3})$  e  $C''(3, -3\sqrt{3})$ .

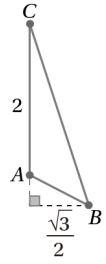
91.3.  $z_B = e^{i\frac{\pi}{6}} = \cos \frac{\pi}{6} + i \sin \frac{\pi}{6} = \frac{\sqrt{3}}{2} + \frac{1}{2}i$ ;  $z_C = 3e^{i\frac{\pi}{2}} = 3i$

$A = \frac{b \times h}{2}$

Como os pontos  $A$  e  $C$  têm abscissa 0.

$b = 3 - 1 = 2$  e  $h = \frac{\sqrt{3}}{2}$

$A_{[ABC]} = \frac{2 \times \frac{\sqrt{3}}{2}}{2} = \frac{\sqrt{3}}{2}$  u.a.



O triângulo  $[A'B'C']$  é a imagem do triângulo  $[ABC]$  por uma rotação. Como se trata de uma isometria, têm as

mesmas áreas. Logo,  $A_{[A'B'C']} = \frac{\sqrt{3}}{2}$  u.a. O triângulo

$[A''B''C'']$  é a imagem do triângulo  $[ABC]$  por uma rotação composta com uma homotetia de raio 2, logo os dois triângulos são semelhantes, sendo a razão das áreas da

ampliação igual a 4. Assim,  $A_{[A''B''C'']} = 4 \times \frac{\sqrt{3}}{2} = 2\sqrt{3}$  u.a.

92.1. O ponto  $B$  é o afixo de  $z_2 = e^{i\frac{2\pi}{3}} \times z_1$ .

$e^{i\frac{2\pi}{3}} = \cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3} = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$

$z_2 = e^{i\frac{2\pi}{3}} \times z_1 = \left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) \times (-\sqrt{2} - \sqrt{2}i) =$

$= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i - \frac{\sqrt{6}}{2}i + \frac{\sqrt{6}}{2} =$

$= \frac{\sqrt{6} + \sqrt{2}}{2} + \frac{-\sqrt{6} + \sqrt{2}}{2}i$

Logo,  $B\left(\frac{\sqrt{6} + \sqrt{2}}{2}, \frac{-\sqrt{6} + \sqrt{2}}{2}\right)$ .

92.2.  $z = z_1^3 = (-\sqrt{2} - i\sqrt{2})^3 = (-\sqrt{2} - i\sqrt{2})^2 (-\sqrt{2} - i\sqrt{2}) =$

$= (2 - 2 + 2 \times 2i)(-\sqrt{2} - i\sqrt{2}) =$

$= 4i(-\sqrt{2} - i\sqrt{2}) = 4\sqrt{2} - 4\sqrt{2}i$

A equação pedida é, por exemplo,  $z^3 = 4\sqrt{2} - 4\sqrt{2}i$ .

92.3.  $OA = |z_1| = \sqrt{2+2} = 2$

$\frac{h}{2} = \cos \frac{\pi}{3} \Leftrightarrow h = 2 \times \frac{1}{2} \Leftrightarrow h = 1$

$\frac{b}{2} = \sin \frac{\pi}{3} \Leftrightarrow b = 2 \times \frac{\sqrt{3}}{2} \Leftrightarrow b = \sqrt{3}$

$A_{[AOB]} = \frac{2b \times h}{2} = \sqrt{3} \times 1 = \sqrt{3}$

$A_{[ABC]} = 3\sqrt{3}$

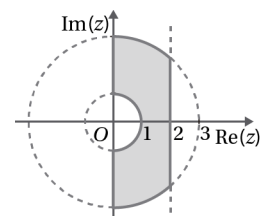
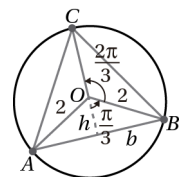
93.1.  $1 < \bar{z} < 9 \wedge 0 \leq z + \bar{z} \leq 4$

$\Leftrightarrow 1 < |z|^2 < 9 \wedge 0 \leq x + yi + (x - yi) \leq 4$

$\Leftrightarrow 1 < |z| < 3 \wedge 0 \leq 2x < 4$

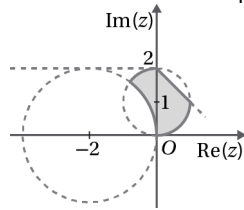
$\Leftrightarrow 1 < |z| < 3 \wedge 0 \leq x < 2$

$|z = x + yi$



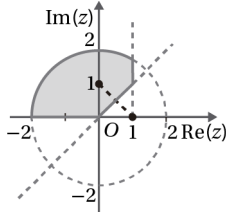
93.2.  $|z+2| \leq 2 \wedge |z-i| \leq 1 \wedge -\pi \leq \text{Arg}(z-2i) \leq -\frac{\pi}{4}$

$C_1(-2, 0), r=2$   
 $C_2(0, 1), r=1$   
 $V(0, 2)$



93.3.  $|z-i| \leq |z-1| \wedge \frac{\pi}{2} \leq \text{Arg}(z-1) \leq \pi \wedge |z+iz| \leq 2\sqrt{2}$

$|z+iz| \leq 2\sqrt{2} \Leftrightarrow |z(1+i)| \leq 2\sqrt{2} \Leftrightarrow$   
 $\Leftrightarrow |z| |1+i| \leq 2\sqrt{2} \Leftrightarrow |z| \times \sqrt{2} \leq 2\sqrt{2} \Leftrightarrow |z| \leq 2$

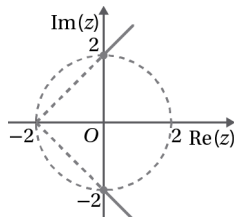


93.4.  $\left| \frac{4}{z} \right| \leq 2 \wedge \left| \text{Arg}(z+2) \right| = \frac{\pi}{4} \Leftrightarrow$

$\Leftrightarrow \left| \frac{4}{z} \right| \leq 2 \wedge \left( \text{Arg}(z+2) = -\frac{\pi}{4} \vee \text{Arg}(z+2) = \frac{\pi}{4} \right) \Leftrightarrow$

$2|z| \geq 4 \wedge \left( \text{Arg}(z+2) = -\frac{\pi}{4} \vee \text{Arg}(z+2) = \frac{\pi}{4} \right) \Leftrightarrow$

$|z| \geq 2 \wedge \left( \text{Arg}(z+2) = -\frac{\pi}{4} \vee \text{Arg}(z+2) = \frac{\pi}{4} \right)$



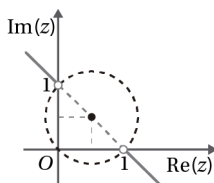
93.5.  $\text{Re}(z+iz) = 1 \wedge \left| z - \frac{1+i}{2} \right| > \frac{\sqrt{2}}{2}$

$\text{Re}(z-iz) = 1 \Leftrightarrow \text{Re}[x+yi-i(x+yi)] = 1 \Leftrightarrow$

$\Leftrightarrow \text{Re}(x+yi-xi+y) = 1 \Leftrightarrow x+y=1 \Leftrightarrow y=-x+1$

$\left| z - \frac{1+i}{2} \right| > \frac{\sqrt{2}}{2} \Leftrightarrow \left| z - \left( \frac{1}{2} + \frac{1}{2}i \right) \right| > \frac{\sqrt{2}}{2}$

$C\left(\frac{1}{2}, \frac{1}{2}\right), r = \frac{\sqrt{2}}{2}$



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94.1. Se  $[OABC]$  é um paralelogramo, então  $w = z + (-\bar{z})$ .

$w = z + (-\bar{z}) = e^{i\alpha} - e^{-i\alpha} =$

$= \cos \alpha + i \sin \alpha - (\cos(-\alpha) + i \sin(-\alpha)) =$

$= \cos \alpha + i \sin \alpha - (\cos \alpha - i \sin \alpha) =$

$= \cos \alpha + i \sin \alpha - \cos \alpha + i \sin \alpha =$

$= 2i \sin \alpha = 2 \sin \alpha \times e^{i\frac{\pi}{2}}$

Como  $\alpha \in \left] 0, \frac{\pi}{2} \right[$ ,  $\sin \alpha > 0$  e, dado que  $w = 2 \sin \alpha \times e^{i\frac{\pi}{2}}$ , então  $|w| = 2 \sin \alpha$ .

94.2.  $i \times z^3 = e^{i\frac{\pi}{2}} \times (e^{i\alpha})^3 = e^{i\frac{\pi}{2}} \times e^{i3\alpha} = e^{i(\frac{\pi}{2}+3\alpha)}$

$i \times z^3 \in \mathbb{R} \Leftrightarrow \frac{\pi}{2} + 3\alpha = k\pi, k \in \mathbb{Z} \Leftrightarrow 3\alpha = -\frac{\pi}{2} + k\pi, \mathbb{Z} \Leftrightarrow$

$\Leftrightarrow \alpha = -\frac{\pi}{6} + \frac{k\pi}{3}, k \in \mathbb{Z}$

Como  $\alpha \in \left] 0, \frac{\pi}{2} \right[$ , vem  $\alpha = \frac{\pi}{6}$  (para  $k=0$ )

Assim,  $z = e^{i\alpha} = e^{i\frac{\pi}{6}} = \cos \frac{\pi}{6} + i \sin \frac{\pi}{6} = \frac{\sqrt{3}}{2} + \frac{1}{2}i$ .

95.1.  $\text{Arg}(z) = \theta$ ;  $\text{Arg}(w) = \alpha$

$z = e^i = \cos \theta + i \sin \theta$

$1+z = 1 + \cos \theta + i \sin \theta =$

$= 1 + \cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2} + 2i \sin \frac{\theta}{2} \cos \frac{\theta}{2} =$

$= 1 - \sin^2 \frac{\theta}{2} + \cos^2 \frac{\theta}{2} + 2i \sin \frac{\theta}{2} \cos \frac{\theta}{2} =$

$= \cos^2 \frac{\theta}{2} + \cos^2 \frac{\theta}{2} + 2i \sin \frac{\theta}{2} \cos \frac{\theta}{2} =$

$= 2 \cos^2 \frac{\theta}{2} + 2i \sin \frac{\theta}{2} \cos \frac{\theta}{2} =$

$= 2 \cos \frac{\theta}{2} \left( \cos \frac{\theta}{2} + i \sin \frac{\theta}{2} \right) = 2 \cos \frac{\theta}{2} e^{i\frac{\theta}{2}}$

95.2. Seja  $\text{Arg} w = \alpha, -\pi < \alpha \leq \pi$

$w = e^{i\alpha} = \cos \alpha + i \sin \alpha, -\pi < \alpha \leq \pi$

$1+z = 2 \cos \frac{\theta}{2} e^{i\frac{\theta}{2}}, -\pi < \theta \leq \pi$  (95.1.)

$1+z+w=0 \Leftrightarrow 1+z=-w \Leftrightarrow 2 \cos \frac{\theta}{2} e^{i\frac{\theta}{2}} = -e^{i\alpha} \Leftrightarrow$

$\Leftrightarrow 2 \cos \frac{\theta}{2} e^{i\frac{\theta}{2}} = e^{i(\pi+\alpha)} \Leftrightarrow \begin{cases} 2 \cos \frac{\theta}{2} = 1 \\ \frac{\theta}{2} = \pi + \alpha + 2k\pi, k \in \mathbb{Z} \end{cases} \Leftrightarrow$

$\Leftrightarrow \begin{cases} \cos \frac{\theta}{2} = \frac{1}{2} \\ \frac{\theta}{2} = \pi + \alpha + 2k\pi, k \in \mathbb{Z} \end{cases} \Leftrightarrow$

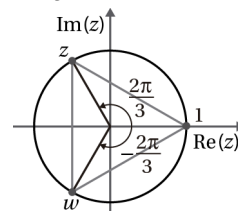
$\Leftrightarrow \begin{cases} \frac{\theta}{2} = \frac{\pi}{3} \\ \frac{\pi}{3} = \pi + \alpha + 2k\pi, k \in \mathbb{Z} \end{cases} \Leftrightarrow$

$\begin{cases} \text{Se } -\pi < \theta \leq \pi, \\ -\frac{\pi}{2} < \frac{\theta}{2} \leq \frac{\pi}{2} \end{cases}$

$\Leftrightarrow \begin{cases} \theta = \frac{2\pi}{3} \\ \alpha = -\frac{2\pi}{3} + 2k\pi, k \in \mathbb{Z} \end{cases} \Leftrightarrow$

$|\alpha| < \pi$

$\Leftrightarrow \theta = \frac{2\pi}{3} \wedge \alpha = -\frac{2\pi}{3}$



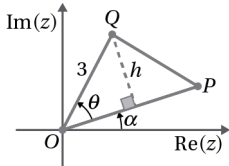
Para além de  $|z| = |w| = |1|$ , se  $\text{Arg } 1 = 0$ ,  $\text{Arg } z = \frac{2\pi}{3}$  e

$\text{Arg } w = -\frac{2\pi}{3}$ , podemos concluir que os afixos de 1,  $z$  e  $w$

são vértices de um triângulo equilátero inscrito numa circunferência de centro  $O$  e raio 1.

96.  $w = 4w^{i\alpha} \curvearrowright P$ ;  $z = 4e^{i\alpha} \times \frac{3}{4}e^{i\theta} \curvearrowright Q$

96.1.



$$\overline{OQ} = \frac{3}{4}\overline{OP} = \frac{3}{4} \times 4 = 3$$

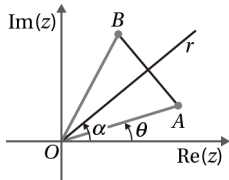
$$\frac{h}{3} = \sin \theta \Leftrightarrow h = 3 \sin \theta$$

$$A_{\{OPQ\}} = \frac{\overline{OP} \times h}{2} = \frac{4 \times \sin \theta}{2} = 6 \sin \theta$$

96.2.  $w = 4e^{i\alpha}$ ;  $z = 4e^{i\alpha} \times \frac{3}{4}e^{i\theta} = 3e^{i(\alpha+\theta)}$

$$\begin{aligned} z\bar{w} - \bar{z}w &= 3e^{i(\alpha+\theta)} \times 4e^{-i\alpha} - 3e^{i(-\alpha-\theta)} \times 4e^{i\alpha} = \\ &= 12e^{i(\alpha+\theta-\alpha)} - 12e^{i(-\alpha-\theta+\alpha)} = 12e^{i\theta} - 12e^{i(-\theta)} = 12(e^{i\theta} - e^{-i\theta}) \\ &= 12[\cos \theta + i \sin \theta - (\cos(-\theta) + i \sin(-\theta))] = \\ &= 12(\cos \theta + i \sin \theta - \cos \theta + i \sin \theta) = 12 \times 2i \sin \theta = \\ &= 24i \sin \theta = 4i(6 \sin \theta) = 4iA \end{aligned}$$

97.



97.1.  $\text{Arg}(z_1) = \theta$ ;  $\text{Arg}(z_2) = \theta + 2(\alpha - \theta)$ ;

$\text{Arg}(z_2) = 2\alpha - \theta$ ;  $\text{Arg}(z_2) = 2\alpha - \text{Arg}(z_1)$

97.2.  $\overline{OA} = \overline{OB} \Leftrightarrow |z_1| = |z_2|$

$$z_1 = |z_1|e^{i\theta}; \quad z_2 = |z_2|e^{i(2\alpha-\theta)}$$

$$z_1 \times z_2 = |z_1| \times |z_2| e^{i(\theta+2\alpha-\theta)} = |z_1|^2 e^{i(2\alpha)}$$

97.3.  $\alpha = \frac{\pi}{4}$ ;  $z_1^6 = z_2^6$

$\text{Arg}(z_2) - \text{Arg}(z_1) = \frac{2\pi}{6}$  porque um hexágono regular de centro na origem tem, no máximo, dois vértices no primeiro quadrante pelo que  $A$  e  $B$  são vértices consecutivos.  
 $\text{Arg}(z_1) = \theta$

$$\text{Arg}(z_2) = 2\alpha - \theta = 2 \times \frac{\pi}{4} - \theta = \frac{\pi}{2} - \theta$$

$$\left(\frac{\pi}{2} - \theta\right) - \theta = \frac{2\pi}{6} \Leftrightarrow \frac{\pi}{2} - 2\theta = \frac{\pi}{3} \Leftrightarrow$$

$$\Leftrightarrow 2\theta = \frac{\pi}{2} - \frac{\pi}{3} \Leftrightarrow 2\theta = \frac{\pi}{6} \Leftrightarrow \theta = \frac{\pi}{12}$$

$$z = z_1^6 \left( |z_1| e^{i\frac{\pi}{12}} \right)^6 = |z_1|^6 e^{i\frac{\pi}{2}}$$

Logo,  $z$  é um imaginário puro.

98.1.  $\overline{OA} = \overline{OB} = \overline{OC}$

Dado que  $C$  é o afixo de  $z + w$ , vem

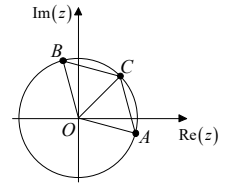
$\overline{OC} = \overline{OA} + \overline{OB}$  e, como  $[OACB]$  é

um paralelogramo,  $\overline{AC} = \overline{OB}$  e

$\overline{BC} = \overline{OA}$ .

$\overline{AC} = \overline{OB}$  e  $\overline{BC} = \overline{OA}$  pelo que os

triângulos  $[OAC]$  e  $[OCB]$  são equiláteros.



Logo,  $\widehat{AOC} = \frac{\pi}{3} = \widehat{COB}$  pelo que  $\widehat{AOB} = \frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3}$ .

98.2.  $|z| = |w| = r$ ;  $z = r e^{i\theta}$ ;  $w = r e^{i(\theta+\frac{2\pi}{3})}$

$$w^3 = \left( r e^{i(\theta+\frac{2\pi}{3})} \right)^3 = r^3 e^{i(3\theta+2\pi)} = e^3 e^{i(3\theta)} = (r e^{i\theta})^3 = z^3$$

98.3.  $z = r e^{i\theta}$ ,  $w = r e^{i(\theta+\frac{2\pi}{3})}$

$$\begin{aligned} z^3 + w^2 + zw &= r^2 e^{i(2\theta)} + r^2 e^{i(2\theta+\frac{4\pi}{3})} + r^2 e^{i(\theta+\theta+\frac{2\pi}{3})} = \\ &= r^2 e^{i(2\theta)} \left( 1 + e^{i\frac{4\pi}{3}} + e^{i\frac{2\pi}{3}} \right) = \\ &= r^2 e^{i(2\theta)} \left( 1 + \cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3} + \cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3} \right) = \\ &= r^2 e^{i(2\theta)} \left( 1 - \frac{1}{2} - \frac{\sqrt{3}}{2}i - \frac{1}{2} + \frac{\sqrt{3}}{2}i \right) = r^2 e^{i(2\theta)} \times 0 = 0 \end{aligned}$$

**Avaliação**

1.  $z = 2e^{i\frac{\pi}{6}}$

$$\frac{-z}{z} = \frac{2e^{i(\frac{\pi}{6}+\pi)}}{2e^{-i\frac{\pi}{6}}} = e^{i(\frac{5\pi}{6}+\frac{\pi}{6})} = e^{-i\frac{4\pi}{6}} = e^{-i\frac{2\pi}{3}}$$

Resposta: (A)

2.  $\cos\left(\theta + \frac{\pi}{2}\right) + i \sin\left(-\theta - \frac{3\pi}{2}\right) = -\sin \theta + i \cos \theta =$

$$= \cos\left(\frac{\pi}{2} + \theta\right) + i \sin\left(\frac{\pi}{2} + \theta\right) = e^{i(\frac{\pi}{2}+\theta)}$$

Resposta: (A)

3.  $|iz + 1| = \left| i \left( z + \frac{1}{i} \right) \right| = |i| |z - i| = |z - i|$

Resposta: (C)

4. Seja  $\alpha$  um argumento de  $1 - \sqrt{3}i$ .

$$\left. \begin{aligned} \tan \alpha = \frac{-\sqrt{3}}{1} = -\sqrt{3} \\ (1, -\sqrt{3}) \in 4.^\circ Q \end{aligned} \right\} \Rightarrow -\frac{\pi}{3} \text{ é um argumento de } 1 - \sqrt{3}i$$

Um argumento de  $-\bar{z}$  é  $-\theta + \pi$ .

$$\text{Um argumento de } \frac{1 - \sqrt{3}i}{-\bar{z}} \text{ é } -\frac{\pi}{3} - (-\theta + \pi) = -\frac{4\pi}{3} + \theta.$$

$$-\frac{4\pi}{3} + 2\pi + \theta = \frac{2\pi}{3} + \theta$$

Resposta: (B)

5. Seja  $\theta$  um argumento de  $1 + i$ .

$$\left. \begin{aligned} \tan \theta = 1 \\ (1, 1) \in 1.^\circ Q \end{aligned} \right\} \frac{\pi}{4} \text{ é um argumento de } 1 + i.$$

Um argumento de  $(1+i)^n$  é  $n\frac{\pi}{4}$ ,  $n \in \mathbb{N}$ .

Como  $(1+i)^n$  é um número real negativo:

$$n\frac{\pi}{4} = \pi + 2k\pi, k \in \mathbb{Z} \Leftrightarrow n = 4 + 8k, k \in \mathbb{Z}$$

Para  $k = 7$ ,  $n = 60$ .

**Resposta: (A)**

$$6. (z-i)(\bar{z}-i) = 4 \Leftrightarrow (z-i)(\overline{z-i}) = 4 \Leftrightarrow \begin{cases} \bar{z}+i = \overline{z-i} \\ \Leftrightarrow |z-i|^2 = 4 \Leftrightarrow |z-i| = 2 \\ |z\bar{z}| = |z|^2 \end{cases}$$

**Resposta: (C)**

$$7. -\cos\frac{\pi}{5} + i\sin\frac{\pi}{5} = -\left(\cos\frac{\pi}{5} - i\sin\frac{\pi}{5}\right) = -\left[\cos\left(-\frac{\pi}{5}\right) + i\sin\left(-\frac{\pi}{5}\right)\right] = -e^{i\left(-\frac{\pi}{5}\right)} = e^{i\left(-\frac{\pi}{5}\right)} = e^{i\frac{4\pi}{5}}$$

**Resposta: (D)**

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$$8.1. z^3 + \frac{1}{z^2} = 0 \Leftrightarrow z^5 + 1 = 0 \wedge z \neq 0 \Leftrightarrow z = \sqrt[5]{-1} \wedge z \neq 0 \Leftrightarrow$$

$$\Leftrightarrow z = \sqrt[5]{e^{i\pi}} \wedge z \neq 0 \Leftrightarrow z = e^{i\frac{2k\pi}{5}}, k = \{0, 1, 2, 3, 4\} \Leftrightarrow$$

$$\Leftrightarrow z = e^{i\frac{\pi}{5}} \vee z = e^{i\frac{3\pi}{5}} \vee z = e^{i\pi} \vee z = e^{i\frac{7\pi}{5}} \vee z = e^{i\frac{9\pi}{5}}$$

$$S = \left\{ e^{i\frac{\pi}{5}}, e^{i\frac{3\pi}{5}}, e^{i\pi}, e^{i\frac{7\pi}{5}}, e^{i\frac{9\pi}{5}} \right\}$$

$$8.2. (1-i)\bar{z} = \frac{1+i}{3-i} \Leftrightarrow \bar{z} = \frac{1+i}{(3-i)(1-i)} \Leftrightarrow$$

$$\Leftrightarrow \bar{z} = \frac{(1+i)^2}{(3-i)(1-i)(1+i)} \Leftrightarrow \bar{z} = \frac{2i}{(3-i) \times 2} \Leftrightarrow$$

$$\Leftrightarrow \bar{z} = \frac{i(3+i)}{(3-i)(3+i)} \Leftrightarrow \bar{z} = \frac{-1+3i}{9+1} \Leftrightarrow z = -\frac{1}{10} - \frac{3}{10}i$$

$$S = \left\{ -\frac{1}{10} - \frac{3}{10}i \right\}$$

9.1. Seja  $z = x + yi$ .

$$3z^2 + 5z\bar{z} - 2 = 0 \Leftrightarrow 3(x+yi)^2 + 5(x^2+y^2) - 2 = 0 \Leftrightarrow$$

$$\Leftrightarrow 3x^2 - 2y^2 + 6xyi + 5x^2 + 5y^2 - 2 = 0 \Leftrightarrow$$

$$\Leftrightarrow 8x^2 + 2y^2 - 2 + 6xyi = 0 \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} 8x^2 + 2y^2 - 2 = 0 \\ 6xy = 0 \end{cases} \Leftrightarrow \begin{cases} 2y^2 = 2 \\ x = 0 \vee y = 0 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} y = \pm 1 \\ x = 0 \end{cases} \vee \begin{cases} x = \pm \frac{1}{2} \\ y = 0 \end{cases}$$

Logo,  $-\frac{1}{2}, \frac{1}{2}, -i$  e  $i$  são soluções da equação.

$$S = \left\{ -\frac{1}{2}, \frac{1}{2}, -i, i \right\}$$

9.2. Sejam  $A, B, C$  e  $D$  os afixos dos números complexos

$-\frac{1}{2}, \frac{1}{2}, -i$  e  $i$ , respectivamente.

$$A\left(-\frac{1}{2}, 0\right), B\left(\frac{1}{2}, 0\right), C(0, -1) \text{ e } D(0, 1)$$

$$\overline{AC} = \left| -i - \left(-\frac{1}{2}\right) \right| = \left| \frac{1}{2} - i \right| = \sqrt{\frac{1}{4} + 1} = \sqrt{\frac{5}{4}} = \frac{\sqrt{5}}{2}$$

$$\overline{CB} = \left| \frac{1}{2} - (-i) \right| = \left| \frac{1}{2} + i \right| = \sqrt{\frac{1}{4} + 1} = \sqrt{\frac{5}{4}} = \frac{\sqrt{5}}{2}$$

$$\overline{BD} = \left| i - \frac{1}{2} \right| = \sqrt{1 + \frac{1}{4}} = \sqrt{\frac{5}{4}} = \frac{\sqrt{5}}{2}$$

$$\overline{AD} = \left| i - \left(-\frac{1}{2}\right) \right| = \left| i + \frac{1}{2} \right| = \sqrt{1 + \frac{1}{4}} = \frac{5}{4} = \frac{\sqrt{5}}{2}$$

Como os lados do quadrilátero  $[ABCD]$  têm o mesmo comprimento, os afixo das soluções da equação dada são vértices de um losango.

10.

$$z = e^{i\theta}$$

$$z + \frac{1}{z} = e^{i\theta} + \frac{1}{e^{i\theta}} = e^{i\theta} + e^{-i\theta} =$$

$$= (\cos\theta + i\sin\theta) + (\cos(-\theta) + i\sin(-\theta)) =$$

$$= \cos\theta + i\sin\theta + \cos\theta - i\sin(-\theta) = 2\cos\theta \in \mathbb{R}$$

11.

$$\frac{(z+w)^2}{zw} = \frac{z^2 + w^2 + 2zw}{zw} = \frac{z^2}{zw} + \frac{w^2}{zw} + 2 =$$

$$= \frac{z}{w} + \frac{w}{z} + 2 = \frac{z\bar{w}}{w\bar{w}} + \frac{w\bar{z}}{z\bar{z}} + 2 = \frac{z\bar{w}}{|w|^2} + \frac{z\bar{w}}{|z|^2} + 2 =$$

$$= \frac{z\bar{w} + w\bar{z}}{|z|^2} + 2 = \frac{z\bar{w} + \overline{z\bar{w}}}{|z|^2} + 2 \in \mathbb{R} \text{ porque } u + \bar{u} \in \mathbb{R}$$

$$= \frac{2\operatorname{Re}(z\bar{w})}{|z|^2} + 2 \in \mathbb{R}$$

12.

$$\bar{z} = \frac{9}{z} \Leftrightarrow \bar{z}z = 9 \wedge z \neq 0 \Leftrightarrow |z|^2 = 9 \Leftrightarrow |z| = 3$$

Circunferência de centro na origem e raio 3.

13.

$$1 + z + z^2 + z^3 + z^4 =$$

Progressão geométrica de razão  $z$

$$= 1 \times \frac{1 - z^5}{1 - z} = \frac{1 - \left(e^{i\frac{4\pi}{5}}\right)^5}{1 - e^{i\frac{4\pi}{5}}} = \frac{1 - e^{i4\pi}}{1 - e^{i\frac{4\pi}{5}}} = \frac{1 - e^{i \times 0}}{1 - e^{i\frac{4\pi}{5}}} = \frac{1 - 1}{1 - e^{i\frac{4\pi}{5}}} = 0$$

14.

$$2i\bar{z} = z^2 = 0 \Leftrightarrow 2i(x-yi) - (x+yi)^2 = 0 \Leftrightarrow (z = x + yi)$$

$$\Leftrightarrow 2xi + 2y - (x^2 - y^2 + 2xyi) = 0 \Leftrightarrow$$

$$\Leftrightarrow -x^2 + y^2 + 2y = 0 \wedge 2x - 2xy = 0 \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} -x^2 + y^2 + 2y = 0 \\ 2x(1-y) = 0 \end{cases} \Leftrightarrow \begin{cases} y(y+2) = 0 \\ x = 0 \end{cases} \vee \begin{cases} -x^2 + 3 = 0 \\ y = 1 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} y = 0 \vee y = -2 \\ x = 0 \end{cases} \vee \begin{cases} x = -\sqrt{3} \vee x = \sqrt{3} \\ y = 1 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow z = 0 \vee z = -2i \vee z = \sqrt{3} + i \vee z = \sqrt{3} + i \vee z = -\sqrt{3} + i$$

$$(-2i) = \left(2e^{-i\frac{\pi}{2}}\right)^3 = 8e^{-i\frac{3\pi}{2}} = 8i$$

$$(\sqrt{3} + i)^3 = \left(2e^{i\frac{\pi}{6}}\right)^3 = 8e^{i\frac{\pi}{2}} = 8i$$

$$(-\sqrt{3} + i)^3 = \left(2e^{i\frac{5\pi}{6}}\right)^3 = 8e^{i\frac{5\pi}{2}} = 8i$$

15.

$$z^2 - \left(2\cos\frac{\pi}{5}\right)z + 1 = 0 \Leftrightarrow z = \frac{2\cos\frac{\pi}{5} \pm \sqrt{4\cos^2\frac{\pi}{5} - 4}}{2} \Leftrightarrow$$

$$\Leftrightarrow z = \frac{2\cos\frac{\pi}{5} \pm \sqrt{-4\sin^2\frac{\pi}{5}}}{2} \Leftrightarrow z = \frac{2\cos\frac{\pi}{5} \pm 2i\sin\frac{\pi}{5}}{2} \Leftrightarrow$$

$$\Leftrightarrow z = \cos\frac{\pi}{5} + i\sin\frac{\pi}{5} \vee z = \cos\frac{\pi}{5} - i\sin\frac{\pi}{5} \Leftrightarrow$$

$$\Leftrightarrow z = e^{i\frac{\pi}{5}} \vee z = e^{-i\frac{\pi}{5}}$$

$$S = \left\{ e^{i\frac{\pi}{5}}, e^{-i\frac{\pi}{5}} \right\}$$

$$16.1. \overline{P(z)} = \overline{z^4 - 2z^3 + 3z^2 - 2z + 2} = \overline{z^4} - \overline{2z^3} + \overline{3z^2} - \overline{2z} + \overline{2} = \\ = \overline{z}^4 - 2\overline{z}^3 + 3\overline{z}^2 - 2\overline{z} + 2 = \\ = \overline{z^4 - 2z^3 + 3z^2 - 2z + 2} = P(\overline{z})$$

Como  $\overline{P(z)} = P(\overline{z})$  e  $P(1-i) = 0$ :

$$\overline{P(1-i)} = P(\overline{1-i}) \Leftrightarrow 0 = P(1+i)$$

Logo,  $1+i$  é outro zero de  $P(z)$ .

$$16.2. P(1-i) = (1-i)^4 - 2(1-i)^3 + 3(1-i)^2 - (1-i) + 2 = \\ = (-2i)^2 - 2(1-i)(-2i) + 3(-2i) - 2 + 2i + 2 = \begin{cases} (1-i)^2 = \\ = 1-2i-1 \\ = -2i \end{cases} \\ = -4 + 4i + 4 - 6i + 2i = 0$$

Como  $\overline{P(z)} = P(\overline{z})$  e  $P(1-i) = 0$ , temos:

$$\overline{P(1-i)} = P(\overline{1-i}) = P(1+i)$$

$1+i$  é outro zero de  $P(z)$

$$16.3. P = z^4 - 2z^3 + 3z^2 - 2z + 2$$

$$P(1-i) = P(1+i) = 0$$

	1	-2	3	-2	2	
1-i		1-i	-2	1-i	-2	$\left. \begin{aligned} (1-i)(-1-i) = \\ = -1-i+i-1 = -2 \end{aligned} \right\}$
	1	-1-i	1	-1-i	0	
1+i		1+i	0	1+i		
	1	0	1	0		

$$z^2 + 1 = 0 \Leftrightarrow z^2 = -1 \Leftrightarrow z = \pm\sqrt{-1} \Leftrightarrow z = \pm i$$

$$S = \{-i, i, 1-i, 1+i\}$$

$$17.1. z^2 = \left( \sqrt{2-\sqrt{2}} + i\sqrt{\sqrt{2}+2} \right)^2 \\ z^2 = 2 - \sqrt{2} - \sqrt{2} - 2 + 2i\sqrt{2-\sqrt{2}}\sqrt{\sqrt{2}+2} = \\ = -2\sqrt{2} + 2i\sqrt{4-2} = \\ = -2\sqrt{2} + 2\sqrt{2}i$$

$$17.2. |z^2| = \sqrt{8+8} = 4$$

$$\tan(\text{Arg}(z^2)) = -1$$

$$\text{Arg}(z^2) \in 2.^\circ \text{Q}$$

$$z^2 = 4e^{i\frac{3\pi}{4}}$$

$$z = e^{i\left(\frac{3\pi}{8} + \frac{2k\pi}{2}\right)}$$

$$z_0 = 2e^{i\frac{3\pi}{8}}; z_1 = 2e^{i\frac{11\pi}{8}}$$

Como o afixo de  $z$  é do 1.º quadrante,  $z = 2e^{i\frac{3\pi}{8}}$ .

$$18.1. a) w = 1 + i\frac{\sqrt{3}}{3} = re^{i\theta}$$

$$r = \sqrt{1 + \left(\frac{\sqrt{3}}{3}\right)^2} = \sqrt{1 + \frac{1}{3}} = \sqrt{\frac{4}{3}} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$\left. \begin{aligned} \tan \theta = \frac{\sqrt{3}}{3} \\ \left(1, \frac{\sqrt{3}}{3}\right) \in 1.^\circ \text{Q} \end{aligned} \right\} \Rightarrow \frac{\pi}{6} \text{ é um argumento de } w$$

$$w = \frac{2\sqrt{3}}{3} e^{i\frac{\pi}{6}}$$

$$b) z_2 = \left(1 + \frac{\sqrt{3}}{3}\right) \times z_1 = \frac{2\sqrt{3}}{3} e^{i\frac{\pi}{6}} \times 1 = \frac{2\sqrt{3}}{3} e^{i\frac{\pi}{6}}$$

$$c) z_3 = \left(1 + i\frac{\sqrt{3}}{3}\right) \times z_2 = \frac{2\sqrt{3}}{3} e^{i\frac{\pi}{6}} \times \frac{2\sqrt{3}}{3} e^{i\frac{\pi}{6}} \\ = \frac{2\sqrt{3} \times 2\sqrt{3}}{9} e^{i\left(\frac{\pi}{6} + \frac{\pi}{6}\right)} = \frac{4}{3} e^{i\frac{\pi}{3}}$$

18.2. Da fórmula de recorrência:

$$z_{n+1} = \left(1 + i\frac{\sqrt{3}}{3}\right) z_n, \forall n \in \mathbb{N} \Leftrightarrow \frac{z_{n+1}}{z_n} = 1 + i\frac{\sqrt{3}}{3}, \forall n \in \mathbb{N}$$

$$\Leftrightarrow \frac{z_{n+1}}{z_n} = \frac{2\sqrt{3}}{3} e^{i\frac{\pi}{6}}$$

Logo,  $z_n$  é uma progressão geométrica  $r = \frac{2\sqrt{3}}{3} e^{i\frac{\pi}{6}}$

Então,  $z_n = z_1 \times r^{n-1}$ .

$$z_n = 1 \times \left(\frac{2\sqrt{3}}{3} e^{i\frac{\pi}{6}}\right)^{n-1} \Leftrightarrow z_n = \left(\frac{2\sqrt{3}}{3}\right)^{n-1} e^{i\frac{(n-1)\pi}{6}}$$

18.3.  $\text{Arg}(z_n) = \text{Arg}(z_1) + \pi$

$$\text{Arg}(z_1) = 0$$

$$(n-1)\frac{\pi}{6} = 0 + k\pi, k \in \mathbb{Z} \wedge n \in \mathbb{N} \Leftrightarrow$$

$$\Leftrightarrow n-1 = 6k, k \in \mathbb{Z} \wedge n \in \mathbb{N} \Leftrightarrow$$

$$\Leftrightarrow n = 1 + 6k, k \in \mathbb{Z} \wedge n \in \mathbb{N}$$

$n$  é um número natural da forma  $n = 1 + 6k$ , com  $k \in \mathbb{N}_0$ .

#### Avaliação global

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$$1. \bullet z = \frac{1}{2} + 3i$$

$$z-1 = \frac{1}{2} + 3i - 1 = -\frac{1}{2} + 3i; \left(-\frac{1}{2}, 3\right) \in 2.^\circ \text{Q}$$

$$\bullet z = 3 + \frac{1}{2}i$$

$$z-1 = 3 + \frac{1}{2}i - 1 = 2 + \frac{1}{2}i; \text{Im}(z-1) = \frac{1}{2} < 1$$

$$\bullet z = 3e^{i\frac{\pi}{3}} = 3\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right) = 3\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) = \frac{3}{2} + \frac{3\sqrt{3}}{2}i$$

$$z-1 = \frac{3}{2} + \frac{3\sqrt{3}}{2}i - 1 = \frac{1}{2} + \frac{3\sqrt{3}}{2}i$$

$$\text{Im}(z-1) = \frac{3\sqrt{3}}{2} > 1$$

O afixo de  $z-1$  é do 1.º quadrante

$$\text{Arg}(z-1) = \arctan\left(\frac{3\sqrt{3}}{1}\right) = \arctan(3\sqrt{3})$$

$$\text{Como } 3\sqrt{3} > \sqrt{3}, \frac{\pi}{6} \leq \text{Arg}(z-1) \leq \frac{\pi}{2}$$

$$\bullet z = \sqrt{3}e^{i\frac{\pi}{2}} = \sqrt{3}i$$

$$z-1 = -1 + \sqrt{3}i$$

$$(-1, \sqrt{3}) \in 2.^\circ \text{Q}$$

Resposta: (C)

$$2. z = 1 + i, |z| = \sqrt{1+1} = \sqrt{2}$$

$$(1, 1) \in 1.^\circ \text{Q}, \text{Arg}(z) = \arctan(1) = \frac{\pi}{4}$$

$$\text{Logo, } z = \sqrt{2}e^{i\frac{\pi}{4}}$$

$$w = z^n = \left(\sqrt{2} e^{i\frac{\pi}{4}}\right)^n = \sqrt{2}^n e^{in\frac{\pi}{4}}$$

$w$  é um número real se  $n\frac{\pi}{4} = k\pi, k \in \mathbb{Z} \Leftrightarrow n = 4k, k \in \mathbb{Z}$

Como  $n$  é um múltiplo de 4,  $n = 8$ .

**Resposta: (C)**

3.  $z = (1+i)^2 = 1 + 2i - 1 = 2i$

- $z$  é um número imaginário puro
- $|z| = 2$

- $w^2 = z \Leftrightarrow w^2 = 2i \Leftrightarrow w^2 = (1+i)^2 \Leftrightarrow w = \pm\sqrt{(1+i)^2}$   
 $\Leftrightarrow w = \pm(1+i) \Leftrightarrow w = -1-i \vee w = 1+i$

A equação  $w^2 = z$  tem duas soluções.

- $z = 2i = 2e^{i\frac{\pi}{2}}$

$$z^n = \left(2e^{i\frac{\pi}{2}}\right)^n = 2^n e^{in\frac{\pi}{2}}$$

$z^n$  é um número real negativo se

$$\frac{n\pi}{2} = \pi + 2k\pi, k \in \mathbb{Z} \Leftrightarrow n = 2 + 4k, k \in \mathbb{Z}$$

Logo,  $n$  é um número natural que é termo da sucessão  $u_k = 2 + 4k, k \in \mathbb{N}$ .

Portanto:

$$z^n = (2i)^{2+4k}, k \in \mathbb{N} = (2i)^2 \times (2i)^{4k} = -4 \times ((2i)^4)^k = -4 \times 16^k, k \in \mathbb{N}$$

$-4 \times 16^k \rightarrow -\infty$  quando  $k \rightarrow +\infty$ .

**Resposta: (D)**

4.  $z^2 - 2z + 3 = 0 \Leftrightarrow z = \frac{2 \pm \sqrt{4-12}}{2} \Leftrightarrow z = \frac{2 \pm \sqrt{-8}}{2} \Leftrightarrow$

$$\Leftrightarrow z = \frac{2 \pm 2\sqrt{2}i}{2} \Leftrightarrow z_1 = 1 - \sqrt{2}i \vee z_2 = 1 + \sqrt{2}i$$

$P(1, -\sqrt{2}); Q(1, \sqrt{2}), O(0, 0)$

- $\overline{OP} = (1, -\sqrt{2}), \overline{OQ} = (1, \sqrt{2})$

$$\overline{OP} \cdot \overline{OQ} = (1, -\sqrt{2}) \cdot (1, \sqrt{2}) = 1 - 2 = -1$$

- $z_1 + z_2 = 1 - \sqrt{2}i + 1 + \sqrt{2}i = 2$

- $M_{[PQ]} \left(\frac{1+1}{2}, \frac{-\sqrt{2}+\sqrt{2}}{2}\right) = (1, 0)$

$M$  pertence ao eixo real.

- $PQ$  é a reta de equação  $x = 1$ .

Logo, a reta  $PQ$  é paralela ao eixo imaginário.

**Resposta: (A)**

5.  $z_3 = (1-i)^2 = -1 - 2i - 1 = -2i$

$$z_1 \times z_3 = (1-i\sqrt{3}) \times (-2i) = -2\sqrt{3} - 2i$$

Como  $(-2\sqrt{3}, -2) \in 3.^\circ Q$ ,  $\frac{\pi}{6}$  não é um argumento de

$$z_1 \times z_3.$$

- $\frac{z_2}{z_3} = \frac{4+4i}{-2i} = \frac{(4+4i) \times 2i}{-2i \times 2i} = \frac{-8+8i}{4} = -2+2i$

$$\left|\frac{z_2}{z_3}\right| = |-2+2i| = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

Seja  $\theta$  um argumento de  $-2+2i$ .

$$\left. \begin{aligned} \tan \theta = \frac{2}{-2} = -1 \\ (-2, 2) \in 2.^\circ Q \end{aligned} \right\} \Rightarrow \pi - \frac{\pi}{4} = \frac{3\pi}{4} \text{ é um argumento de } -2+2i$$

Logo,  $\frac{z_2}{z_3} = 2\sqrt{2} e^{i\frac{3\pi}{4}}$

- $z_1 \times z_2 = (1-i\sqrt{3})(4+4i) = 4+4i-4\sqrt{3}i+4\sqrt{3} = 4+4\sqrt{3}+(4-4\sqrt{3})i$

$$\begin{aligned} |z_1 \times z_2| &= \sqrt{(4+4\sqrt{3})^2 + (4-4\sqrt{3})^2} = \\ &= \sqrt{16+32\sqrt{3}+48+16-32\sqrt{3}+48} = \\ &= \sqrt{128} = 8\sqrt{2} \neq 4\sqrt{2} \end{aligned}$$

- $\frac{z_3}{z_4} = \frac{-2i}{1-i\sqrt{3}} = \frac{-2i(1+i\sqrt{3})}{(1-i\sqrt{3})(1+i\sqrt{3})} = \frac{2\sqrt{3}-2i}{1+3} = \frac{\sqrt{3}}{2} - \frac{1}{2}i$

Como  $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right) \in 4.^\circ Q$ , o argumento de  $\frac{z_3}{z_4}$  é

diferente de  $\frac{\pi}{6}$

**Resposta: (B)**

6.  $z = 1 - \tan^2 \alpha + 2i \tan \alpha = 1 - \frac{\sin^2 \alpha}{\cos^2 \alpha} + 2i \frac{\sin \alpha}{\cos \alpha}$

$$= \frac{\cos^2 \alpha - \sin^2 \alpha}{\cos^2 \alpha} + i \frac{2 \sin \alpha \cos \alpha}{\cos^2 \alpha}$$

$$= \frac{1}{\cos^2 \alpha} (\cos(2\alpha) + i \sin(2\alpha)) = \frac{1}{\cos^2 \alpha} e^{i(2\alpha)}$$

**Resposta: (C)**

7.  $z^2 - 4\sqrt{3}z + 16 = 0 \Leftrightarrow z = \frac{4\sqrt{3} \pm \sqrt{48-64}}{2} \Leftrightarrow$

$$\Leftrightarrow z = \frac{4\sqrt{3} \pm \sqrt{-16}}{2} \Leftrightarrow z = \frac{4\sqrt{3}-4i}{2} \vee z = \frac{4\sqrt{3}+4i}{2}$$

$$\Leftrightarrow z = 2\sqrt{3} - 2i \vee z = 2\sqrt{3} + 2i$$

$$|2\sqrt{3} - 2i| = \sqrt{12+4} = \sqrt{16} = 4$$

Seja  $\theta$  um argumento de  $2\sqrt{3} - 2i$

$$\left. \begin{aligned} \tan \theta = \frac{-2}{2\sqrt{3}} = -\frac{\sqrt{3}}{3} \\ (2\sqrt{3}, -2) \in 4.^\circ Q \end{aligned} \right\} \Rightarrow -\frac{\pi}{6} \text{ é um argumento de } 2\sqrt{3} - 2i$$

Logo,  $2\sqrt{3} - 2i = 4e^{-i\frac{\pi}{6}}$  e  $2\sqrt{3} + 2i = \overline{2\sqrt{3} - 2i} = 4e^{i\frac{\pi}{6}}$

$$S = \left\{ 4e^{-i\frac{\pi}{6}}, 4e^{i\frac{\pi}{6}} \right\}$$

8.1.  $z_2 = e^{i\frac{5\pi}{6}} = \cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6} = -\frac{\sqrt{3}}{2} + \frac{1}{2}i$

8.2.  $O(0, 0); A(2\sqrt{3}, -2); B\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$$\overline{OA}(2\sqrt{3}, -2), \overline{OB}\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$$

$$\frac{2\sqrt{3}}{-\sqrt{3}} = -4; \frac{-2}{\frac{1}{2}} = -4$$

Logo, os vetores  $\overline{OA}$  e  $\overline{OB}$  são colineares e, portanto, os pontos  $O, A$  e  $B$  são colineares.

$$9. \frac{\sqrt{2} \times i^{4n-9} + 2e^{\frac{\pi}{4}}}{\sqrt{2}e^{\frac{\pi}{5}}} = \left| \begin{array}{l} i^{4n-9} = i^{4n} \times i^{-9} = \\ = 1 \times i^{-9+3 \times 4} = i^3 = -i \end{array} \right.$$

$$= \frac{\sqrt{2} \times (-i) + 2 \left( \cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right)}{\sqrt{2}e^{\frac{\pi}{5}}} = \frac{-\sqrt{2}i + \frac{2\sqrt{2}}{2} + \frac{2\sqrt{2}}{2}i}{\sqrt{2}e^{\frac{\pi}{5}}} = \frac{\sqrt{2} + \sqrt{2}i - \sqrt{2}i}{\sqrt{2}e^{\frac{\pi}{5}}} = \frac{\sqrt{2}}{\sqrt{2}e^{\frac{\pi}{5}}} = \frac{1}{e^{\frac{\pi}{5}}} = e^{-\frac{\pi}{5}}$$

$$10.1. \overline{AC} = 2|z_1| = 2\sqrt{9+4} = 2\sqrt{13} \Leftrightarrow$$

Seja  $h$  a medida do lado do quadrado  $l^2 + l^2 = \overline{AC}^2$ .

$$2l^2 = (2\sqrt{13})^2 \Leftrightarrow l^2 = \frac{52}{2} \Leftrightarrow l^2 = 26$$

A área do quadrado  $[ABCD]$  é  $l^2$ , ou seja, é igual a 26 u.a.

$$10.2. z_3 \text{ é o simétrico de } z_1.$$

$$z_3 = -z_1 = -(3 + 2i) = -3 - 2i$$

$$11. F_1(-2, 0) \text{ é o afixo de } z_1 = -2.$$

$$F_2(2, 0) \text{ é o afixo de } z_2 = 2.$$

Por definição de elipse,  $|z - z_1| + |z - z_2| = 2a$ , sendo  $z$  um número complexo cujo afixo é  $P$ , um ponto qualquer da elipse, e  $2a$  a medida do comprimento do eixo maior.

$$2a = \overline{PF_2} + \overline{PF_1} = |2 - (2 + 3i)| + |-2 - (2 + 3i)| =$$

$$= |-3i| + |-4 - 3i| = 3 + \sqrt{16 + 9} = 3 + 5 = 8$$

Logo, a condição que define a região sombreada da figura é  $|z - 2| + |z + 2| \leq 8 \wedge \text{Im}(z) \geq 0 \wedge \text{Re}(z) \geq 2$

$$12.1. \overline{OA} = |z_1| = |2 - 5i| = \sqrt{4 + 25} = \sqrt{29}$$

$$\overline{OB} = |z_2| = |7 - 3i| = \sqrt{49 + 9} = \sqrt{58}$$

$$\overline{AB} = |z_2 - z_1| = |7 - 3i - (2 - 5i)| = |5 + 2i| = \sqrt{25 + 4} = \sqrt{29}$$

Como  $\overline{OA} = \overline{AB}$ , o triângulo  $[OAB]$  é isósceles.

Por outro lado  $(\sqrt{58})^2 = (\sqrt{29})^2 + (\sqrt{29})^2$ , pelo que o triângulo é retângulo.

Logo, a proposição é verdadeira.

$$12.2. |z - i| = |z + 2i| \Leftrightarrow |z - (0 + i)| = |z - (0 - 2i)|$$

A condição define a mediatriz do segmento de reta cujos extremos são os pontos de coordenadas  $(0, 1)$  e  $(0, -2)$ , ou seja, a reta de equação  $y = -1$ , a qual é paralela ao eixo real.

Logo, a proposição é verdadeira.

$$12.3. z_1 = 3 + \sqrt{3}i$$

Por exemplo  $z_1^1 = (3 + \sqrt{3}i)^1 = 3 + \sqrt{3}i$  não é imaginário puro. Logo, a proposição é falsa.

$$12.4. \text{ Se } \text{Arg}(z) = \frac{\pi}{2}, \text{ então } z = yi \text{ com } y > 0.$$

$$|i + z| = |i + yi| = |i(1 + y)| = |i||1 + y| = |1 + y| = 1 + y$$

porque  $y > 0$

$$= 1 + |z| \quad \text{porque } |yi| = y$$

A proposição é verdadeira

$$12.5. |z| = 1; z = e^{i\theta}$$

$$z^2 + \frac{1}{z^2} = e^{i(2\theta)} + \frac{1}{e^{i(2\theta)}} = e^{i2\theta} + e^{-i2\theta} =$$

$$= \cos(2\theta) + i \sin(2\theta) + \cos(-2\theta) + i \sin(-2\theta) =$$

$$= \cos(2\theta) + i \sin(2\theta) + \cos(2\theta) - i \sin(2\theta) =$$

$$= 2 \cos(2\theta) \in \mathbb{R}$$

A proposição é verdadeira.

$$13. z = \frac{1}{2} + \frac{\sqrt{3}}{2}i, |z| = \sqrt{\frac{1}{4} + \frac{3}{4}} = \sqrt{1} = 1$$

Seja  $\theta$  um argumento de  $z$

$$\tan \theta = \frac{\sqrt{3}}{1} = \sqrt{3} \left. \begin{array}{l} \\ \\ \end{array} \right\} \Rightarrow \text{Arg}(z) = \frac{\pi}{3}$$

$$\left( \frac{1}{2}, \frac{\sqrt{3}}{2} \right) \in 1.^\circ \text{Q}$$

$$\left( \frac{1}{2} + \frac{\sqrt{3}}{2}i \right)^{100} = \left( e^{i\frac{\pi}{3}} \right)^{100} = e^{100i\frac{\pi}{3}} = e^{i\frac{4\pi}{3}} = \left| \frac{100\pi}{3} = 32\pi + \frac{4\pi}{3} \right.$$

$$= \cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3} = \cos \left( \pi - \frac{\pi}{3} \right) + i \sin \left( \pi - \frac{\pi}{3} \right) =$$

$$= -\cos \frac{\pi}{3} - i \sin \frac{\pi}{3} = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$$