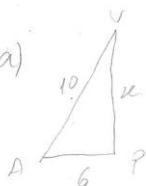


① a) 

$$u^2 + 6^2 = 10^2$$

$$u^2 = 100 - 36$$

$$u^2 = 64$$

$$u = \pm 8$$

$$u = 8$$

$$\text{Logo } V(6, 6, 8)$$

b) $B(12, 6, 0)$ $M_{[BV]} = \left(\frac{12+6}{2}, \frac{6+6}{2}, \frac{0+8}{2} \right) = (9, 6, 4)$
 $V(6, 6, 8)$
 $C(6, 12, 0)$ $\vec{CM} = M - C = (3, -6, 4)$
 $(x, y, z) = (9, 6, 4) + K(3, -6, 4); K \in \mathbb{R}$

c) $P(6, 6, 0)$
 $D(0, 6, 0)$ $\vec{DV} = (6, 0, 8)$
 $V(6, 6, 8)$

$$6x + 8z + D = 0$$

$(6, 6, 0)$ $6 \times 6 + 8 \times 0 + D = 0$
 $D = -36$ Logo = equação do plano
 $6x + 8z - 36 = 0$
 $\Rightarrow 3x + 4z - 18 = 0$

② $4x - z + 1 = 0 \perp (4, 0, -1) \parallel \pi$ (D)

③ $A(3, 0, 0)$ $\vec{HE} = (-3, -1, -3)$ $\cos \alpha = \frac{|\vec{HE} \cdot \vec{HA}|}{\|\vec{HE}\| \cdot \|\vec{HA}\|} = \frac{7}{\sqrt{19} \times \sqrt{13}}$
 $H(3, -2, 3)$ $\vec{HA} = (0, 2, -3)$
 $C(0, -3, 0)$ $\|\vec{HE}\| = \sqrt{9+1+9} = \sqrt{19}$
 $\|\vec{HA}\| = \sqrt{4+9} = \sqrt{13}$
 $\vec{HE} \cdot \vec{HA} = -3 \times 0 - 1 \times 2 - 3 \times (-3) = 7$
 $\cos^2 \alpha = \frac{49}{19 \times 13} = \frac{49}{247}$

$$\sin^2 \alpha = 1 - \cos^2 \alpha =$$

$$= 1 - \frac{49}{247} =$$

$$= \frac{198}{247}$$

(4) $A(1, 0, 3)$

$\alpha \hookrightarrow 3x + 2y - 4 = 0 \quad \vec{n}^0(3, 2, 0)$

(A) $3x + 2y - 3 = 0 \perp (3, 2, 0)$ paralelo a α

(B) $2x - 3y - z + 1 = 0 \perp (2, -3, -1)$ perpendicular a α

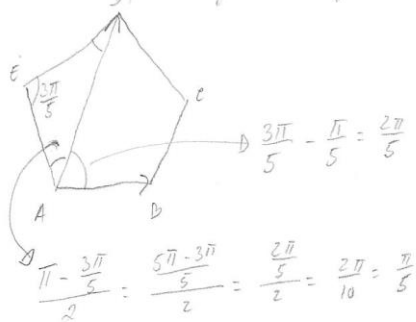
(C) $2x - 3y + z = 0 \perp (2, -3, 1)$ perpendicular a α

(D) $3x + 2y = 0 \perp (3, 2, 0)$ paralelo a α

$A(1, 0, 3) \in \alpha$ plano $2x - 3y - z + 1 = 0$ porque
 $2 \cdot 1 - 3 \cdot 0 - 3 + 1 = 0$

(B)

(5) Num polígono regular, $\angle$ interno = $\frac{\pi \times 3}{5} = \frac{3\pi}{5} \left(\frac{\pi \times (n-2)}{n} \right)$



$$\begin{aligned} \frac{\vec{AB} \cdot \vec{AD}}{\|\vec{AB}\| \|\vec{AD}\|} &= \frac{\|\vec{AB}\| \|\vec{AD}\| \cos\left(\frac{2\pi}{5}\right)}{\|\vec{AB}\| \|\vec{AD}\|} = \cos\left(\frac{2\pi}{5}\right) = \\ &= \cos\left(2 \times \frac{\pi}{5}\right) = \cos^2\left(\frac{\pi}{5}\right) - \sin^2\left(\frac{\pi}{5}\right) = \\ &= \underbrace{1 - \sin^2\left(\frac{\pi}{5}\right)}_{=1} - \sin^2\left(\frac{\pi}{5}\right) = \\ &= 1 - 2\sin^2\left(\frac{\pi}{5}\right) \end{aligned}$$

(6) $\alpha \hookrightarrow x - y - 2z = 3 \perp (1, -1, -2) \parallel (-1, 1, 2) \parallel \vec{n}^0$ (B)

(7) Note $AD \hookrightarrow (x, y, z) = (3, 0, 0) + K(3, 0, -5), K \in \mathbb{R}$

Como a recta AD passa em $D(0, 0, z)$, então

$(0, 0, z) = (3, 0, 0) + K(3, 0, -5)$

$\Leftrightarrow \begin{cases} 0 = 3 + 3K \\ z = -5K \end{cases} \begin{cases} K = -1 \\ z = 5 \end{cases} \quad \text{Assim } D(0, 0, 5)$

Seja $E(0, y, 0)$ o $D(0, 0, 5)$ então $\vec{ED} = (0, -y, 5)$

Assim $\|\vec{ED}\|^2 = (-y)^2 + 5^2 = y^2 + 25 = 41$. Logo $y^2 = 16 \Leftrightarrow y = \pm 4$ $E(0, 4, 0)$

$B(3, -3, 0)$ $\vec{BE} = (-3, -1, 0)$
 $\vec{BD} = (-3, 3, 5)$

Seja $\vec{m}$ o vetor normal ao plano BCD

$$\begin{cases} \vec{m} \cdot \vec{BD} = 0 \\ \vec{m} \cdot \vec{CD} = 0 \end{cases} \begin{cases} (a, b, c) \cdot (-3, -1, 1) = 0 \\ (a, b, c) \cdot (-3, 3, 5) = 0 \end{cases} \begin{cases} -3a - b = 0 \\ -3a + 3b + 5c = 0 \end{cases} \begin{cases} \vec{m}(a, b, c) \\ b = -3a \\ -3a + 3(-3a) + 5c = 0 \end{cases}$$

$$\begin{cases} -3a - 9a + 5c = 0 \\ -12a + 5c = 0 \end{cases} \begin{cases} c = \frac{12}{5}a \end{cases}$$

$$\vec{m}(a, -3a, \frac{12}{5}a)$$

Se $a=5$ então $\vec{m}(5, -15, 12)$

⑧ $m: \tan 60^\circ = \sqrt{3}$. Assim a equação da reta AB será $y = \sqrt{3}x + \frac{b}{\sqrt{3}}$
negativo ⑨

⑨ $A(0, 0, 2)$

a) $1(x-0) - 2(y-0) + 1(z-2) = 0$

$$x - 2y + z - 2 = 0$$

b) $n_{[AB]} = \left(\frac{0+4}{2}, \frac{0+0}{2}, \frac{2+0}{2} \right) = (2, 0, 1) = \text{Cenário} = e$

$\overline{AC} = \text{raio} = \sqrt{(0-2)^2 + (0-0)^2 + (2-1)^2} = \sqrt{5}$
 $(x-2)^2 + y^2 + (z-1)^2 = 5$

c) $P(4, y, 0)$ $A(0, 0, 2)$ $B(4, 0, 0)$

$$\frac{1}{2} = \cos\left(\frac{\pi}{3}\right) = \frac{\vec{AB} \cdot \vec{AP}}{\|\vec{AB}\| \|\vec{AP}\|} = \frac{20}{\sqrt{20} \times \sqrt{y^2 + 20}} = \frac{20}{\sqrt{20y^2 + 400}} \Leftrightarrow \sqrt{20y^2 + 400} = 40$$

$$\left. \begin{aligned} \vec{AB} &= (4, 0, -2) \quad \|\vec{AB}\| = \sqrt{16+4} = \sqrt{20} \\ \vec{AP} &= (4, y, -2) \quad \|\vec{AP}\| = \sqrt{16+y^2+4} = \sqrt{y^2+20} \end{aligned} \right\} \begin{aligned} \Leftrightarrow 20y^2 + 400 &= 1600 \\ \Leftrightarrow 20y^2 &= 1200 \\ \Leftrightarrow y^2 &= 60 \\ \Leftrightarrow y &= \sqrt{60} \end{aligned}$$

$$\vec{AB} \cdot \vec{AP} = 16 + 4 = 20$$

$$\Leftrightarrow y = \sqrt{60}$$

$$(10) \quad x^2 + (y-1)^2 = 2$$

$$(x, 0) \in x^2 + (y-1)^2 = 2$$

$$x^2 + (0-1)^2 = 2$$

$$x^2 + 1 = 2$$

$$x^2 = 1$$

$$x = \pm 1$$

$$A(1, 0)$$

$$A(1, 0) \quad C(0, 1) = \text{centro de circunferencia}$$

$$\vec{AC} = (-1, 1) \quad m_{\vec{AC}} = \frac{1}{-1} = -1$$

$$m_{\perp} = -\frac{1}{-1} = 1$$

$$y - y_0 = 1(x - x_0)$$

$$y - 0 = 1(x - 1)$$

$$y = x - 1 \quad (3)$$

$$(11) \quad V(1, 1, z) \in \text{plano } 6x + z - 12 = 0$$

a)

$$6 \cdot 1 + z - 12 = 0$$

$$6 + z - 12 = 0$$

$$z = 6$$

$$V(1, 1, 6)$$

$$b) \quad P(2, 0, 0)$$

$$R(2, 2, 2)$$

$$\vec{OR} = (2, 2, 2)$$

$$2(x-2) + 2(y-0) + 2(z-0) = 0$$

$$2x - 4 + 2y + 2z = 0$$

$$2x + 2y + 2z - 4 = 0$$

$$x + y + z - 2 = 0$$

$$c) \quad A(x, 2, x^3)$$

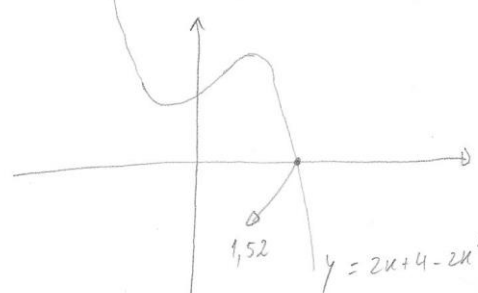
$$T(0, 0, 2)$$

$$\vec{OA} = (x, 2, x^3)$$

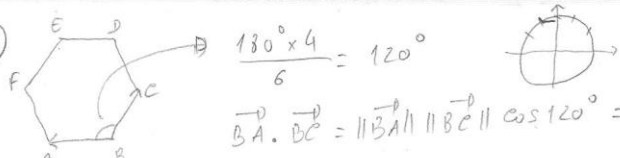
$$Q(2, 2, 0)$$

$$\vec{TQ} = (2, 2, -2)$$

$$\vec{OA} \cdot \vec{TQ} = 0 \Rightarrow (x, 2, x^3) \cdot (2, 2, -2) = 0 \Rightarrow 2x + 4 - 2x^3 = 0$$



$$R: x = 1,52$$

(12)  $\frac{180^\circ \times 4}{6} = 120^\circ$
 $\vec{BA} \cdot \vec{BC} = \|\vec{BA}\| \|\vec{BC}\| \cos 120^\circ = 2 \times 2 \times (-\frac{1}{2}) = -2$ (B)

(13) a) $\vec{OP} = (-2, 1, 3a) \parallel (2, -1, 1)$

$$\frac{-2}{2} = \frac{1}{-1} = \frac{3a}{1} \Rightarrow 3a = -1 \Rightarrow a = -\frac{1}{3}$$

b) $P(x, 0, 0)$ é plano β , logo $2x - 0 + 0 - 4 = 0 \Rightarrow x = 2$ $B(2, 0, 0)$

$A(1, 2, 3)$

$C(-2, 0, 0)$

$\vec{AB} = (1, -2, -3) \quad \|\vec{AB}\| = \sqrt{1+4+9} = \sqrt{14}$

$\vec{AC} = (-3, -2, -3) \quad \|\vec{AC}\| = \sqrt{9+4+9} = \sqrt{22}$

$\vec{AB} \cdot \vec{AC} = -3 + 4 + 9 = 10$

$\cos \alpha = \frac{10}{\sqrt{14} \times \sqrt{22}} = \frac{10}{\sqrt{308}} \Rightarrow \alpha = \arccos\left(\frac{10}{\sqrt{308}}\right) \approx 55^\circ$

c) Equação da reta $\perp$ plano β e que passe na origem:

$(x, y, z) = (0, 0, 0) + K(2, -1, 1); K \in \mathbb{R}$

Interseção da reta com o plano β

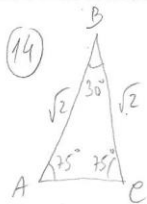
$$\begin{cases} x = 2K \\ y = -K \\ z = K \\ 2x - y + z - 4 = 0 \end{cases} \Rightarrow \begin{cases} x = 2K \\ y = -K \\ z = K \\ 4K + K + K - 4 = 0 \end{cases} \Rightarrow \begin{cases} x = \frac{4}{3} \\ y = -\frac{2}{3} \\ z = \frac{2}{3} \\ K = \frac{2}{3} \end{cases}$$

Ponto de tangência da reta $\perp$ o plano $\beta = T = \left(\frac{4}{3}, -\frac{2}{3}, \frac{2}{3}\right)$

Raio $= \overline{OT} = \sqrt{\frac{16}{9} + \frac{4}{9} + \frac{4}{9}} = \sqrt{\frac{24}{9}}$

Centro $(0, 0, 0)$

$x^2 + y^2 + z^2 = \frac{24}{9}$



$$\begin{aligned}\overrightarrow{BA} \cdot \overrightarrow{BC} &= \sqrt{2} \times \sqrt{2} \times \cos 30^\circ = \\ &= 2 \times \frac{\sqrt{3}}{2} = \\ &= \sqrt{3} \quad \text{(C)}\end{aligned}$$

(15)

a) $\text{dist} = 1 \quad (x+1)^2 + (y-1)^2 + (z-1)^2 = 1$
 Centro $(-1, 1, 1)$

b) $M_{[AE]} = \left(\frac{-1-3}{2}, \frac{1+3}{2}, \frac{1+1}{2} \right) = (-2, 2, 1)$ logo $V(-2, 2, z)$

$V(-2, 2, z) \in \text{plano } 3y + z - 10 = 0$

$3 \times 2 + z - 10 = 0$

$6 + z - 10 = 0$

$z = 4$

logo $V(-2, 2, 4)$

c) Equações do plano α :

$\overrightarrow{AE} = (-2, 2, 0)$

$A(-1, 1, 1)$

$E(-3, 3, 1)$

$P(1, -2, -1) \in \alpha$

$-2(x-1) + 2(y+2) + 0(z+1) = 0$

$-2x + 2 + 2y + 4 = 0$

$-2x + 2y + 6 = 0$

$x - y - 3 = 0$

Interseção dos planos:

$$\begin{cases} x - y - 3 = 0 \\ 3y + z - 10 = 0 \end{cases} \Rightarrow \begin{cases} y = x - 3 \\ 3y = -z + 10 \end{cases}$$

$$\begin{cases} y = \frac{-z+10}{3} \\ y = \frac{z-10}{-3} \end{cases}$$

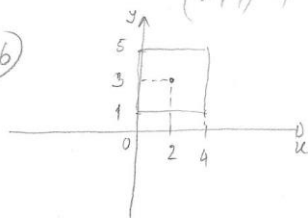
Equações cartesianas da reta

$x - 3 = y = \frac{z-10}{-3}$

Equação vetorial da reta

$(x, y, z) = (3, 0, 10) + K(1, 1, -3); K \in \mathbb{R}$

(16)



(C)

(17)

a) $(x, y, z) = (2, 1, 4) + K(3, 2, 4); K \in \mathbb{R}$

b) Reta OD

$O(0, 0, 0) \quad \overrightarrow{OD} = (4, 2, 2)$

$D(4, 2, 2)$

$(x, y, z) = (0, 0, 0) + K(4, 2, 2); K \in \mathbb{R}$

Interseccão de reta OD e o plano α :

$$\begin{cases} x = 4K \\ y = 2K \\ z = 2K \\ 3x + 2y + 4z - 12 = 0 \end{cases} \quad \begin{cases} 3 \times 4K + 2 \times 2K + 4 \times 2K - 12 = 0 \\ 12K + 4K + 8K - 12 = 0 \end{cases} \quad \begin{cases} K = 2 \\ y = 1 \\ z = 1 \\ K = \frac{1}{2} \end{cases}$$

Ponto de interseccão = $(2, 1, 1)$

c) $A(\overset{+}{\textcircled{1}}, 0, 0) \quad \overrightarrow{PA} = (x, 0, -z) \quad \overrightarrow{PA} \cdot \overrightarrow{PB} = 0 + 0 + z^2 = z^2 > 0$
 $B(0, \overset{+}{\textcircled{2}}, 0) \quad \overrightarrow{PB} = (0, y, -z) \quad \cos \alpha = \frac{\overrightarrow{PA} \cdot \overrightarrow{PB}}{\|\overrightarrow{PA}\| \|\overrightarrow{PB}\|} > 0, \text{ logo } \alpha \text{ é um } \neq \text{ agudo}$
 $P(0, 0, \overset{+}{\textcircled{3}})$

$$(18) 1 + \tan^2 \alpha = \frac{1}{\cos^2 \alpha} \Leftrightarrow 1 + \tan^2 \alpha = \frac{1}{\left(\frac{1}{\sqrt{5}}\right)^2} \Leftrightarrow 1 + \tan^2 \alpha = \frac{1}{\frac{1}{5}} \Leftrightarrow 1 + \tan^2 \alpha = 5 \Leftrightarrow \tan^2 \alpha = 4$$

$$\Leftrightarrow \tan \alpha = \pm 2, \text{ Assim, } m = \pm 2 \quad \textcircled{c}$$

$$\text{Note } m = \tan \alpha$$

$$(19.1) T(0,0,3) \text{ Centro } (0,0,0) \quad x^2 + y^2 + z^2 = 9$$

$$T'(0,0,-3) \quad r=3$$

$$(19.2) \vec{OP} \cdot \vec{RS} = \|\vec{OP}\| \|\vec{RS}\| \cos \pi = 3 \times 3 \times (-1) = -9$$

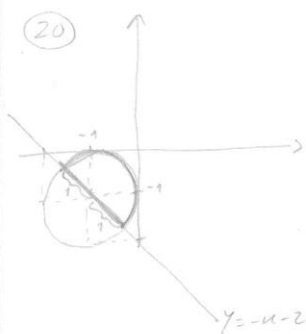
$$(19.3) Q(0,y,0) \in \text{plano } x+y=2, \text{ logo } 0+y=2 \Leftrightarrow y=2$$

$$\text{Assim } Q(0,2,0)$$

$$\vec{TQ} = Q - T = (0,2,0) - (0,0,3) = (0,2,-3)$$

$$\text{equação vetorial: } (x,y,z) = (0,0,3) + K(0,2,-3); K \in \mathbb{R}$$

$$(20) \begin{cases} (x+1)^2 + (y+1)^2 \leq 1 \\ x+y+2 \geq 0 \\ y \geq -x-2 \end{cases}$$



$$P = \frac{2\pi r}{2} + 2 = \pi \times 1 + 2 = \pi + 2 \quad \textcircled{c}$$

$$(21.1) A(x,4,0) \in \text{plano } x+y-z-6=0, \text{ logo } x+4-0-6=0 \Leftrightarrow x=2 \text{ c.p.d.}$$

$$(21.2) r: (x,y,z) = (1,1,0) + K(1,-1,1), K \in \mathbb{R}$$

$$\Leftrightarrow \begin{cases} x = 1+K \\ y = 1-K \\ z = K \end{cases}, K \in \mathbb{R}$$

$$\text{Interseção de } r \text{ e } o \text{ plano } x+y-z-6=0$$

$$1+K+1-K-K-6=0$$

$$-K=4$$

$$K=-4$$

$$\text{Ponto de interseção } (1-4, 1+4, -4) = (-3, 5, -4)$$

$$(21.3) V_{\text{pirâmide}} = 4 \text{ m}^3$$

$$\frac{1}{3} \times 2 \times 2 \times h = 4 \text{ m}^3$$

$$\Leftrightarrow \frac{4}{3} h = 4 \text{ m}$$

$$\Leftrightarrow h = 3$$

$$P(1,5,5)$$

$$G(2,6,2)$$

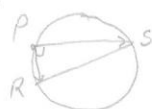
$$O(0,0,0)$$

$$\vec{GO} = (-2, -6, -2) \quad \|\vec{GO}\| = \sqrt{(-2)^2 + (-6)^2 + (-2)^2} = \sqrt{44}$$

$$\vec{GP} = (-1, -1, 3) \quad \|\vec{GP}\| = \sqrt{(-1)^2 + (-1)^2 + 3^2} = \sqrt{11}$$

$$\cos \alpha = \frac{\vec{GO} \cdot \vec{GP}}{\|\vec{GO}\| \|\vec{GP}\|} = \frac{2+6-6}{\sqrt{44} \times \sqrt{11}} = \frac{2}{11} \Leftrightarrow \alpha = \arccos\left(\frac{2}{11}\right) \Leftrightarrow \alpha \approx 85^\circ$$

(22)



$$\vec{PR} \cdot \vec{PS} = 0$$

(D)

(23.1)



$$A = 2 \times 3 = 6$$

(23.2)

$$B(1, 3, 0)$$

$$C(1, 2, 3)$$

$$\vec{BC} = (0, -1, 3)$$

$$\text{reta } BC: (x, y, z) = (1, 3, 0) + K(0, -1, 3); K \in \mathbb{R}$$

$$\begin{cases} x = 1 \\ y = 3 - K \\ z = 3K \end{cases}; K \in \mathbb{R}$$

Interseções de reta BC e do plano xOz ($y=0$)

$$3 - K = 0 \Rightarrow K = 3$$

$$\text{Ponto de interseção } (1, 3-3, 3 \times 3) = (1, 0, 9)$$

(23.3)

$$\text{reta: } \begin{cases} x = K \\ y = K \\ z = 1 - K \end{cases}; K \in \mathbb{R} \rightarrow \text{vetor } (1, 1, -1) // \mathcal{R}$$

Plano α :vetor normal $(1, 1, -1)$ Ponto $A(1, 2, 0)$

$$Ax + By + Cz + D = 0$$

$$x + y - z + D = 0$$

$$0 \cdot 1 + 2 - 0 + D = 0$$

$$D = -3$$

$$\alpha: x + y - z - 3 = 0$$

$$P(2, 2, z) \in \text{plano } x + y - z - 3 = 0, \text{ logo}$$

$$2 + 2 - z - 3 = 0 \Rightarrow z = 1$$

Assim, $P(2, 2, 1)$

$$O(0, 0, 0)$$

$$C(1, 2, 3)$$

$$\vec{OP} = (2, 2, 1) \parallel \vec{OP} \parallel = \sqrt{2^2 + 2^2 + 1^2} = \sqrt{9} = 3$$

$$\vec{OC} = (1, 2, 3) \parallel \vec{OC} \parallel = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{14}$$

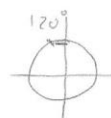
$$\vec{OP} \cdot \vec{OC} = 2 \times 1 + 2 \times 2 + 1 \times 3 = 2 + 4 + 3 = 9$$

$$\text{Seja } \alpha = \angle POC, \cos \alpha = \frac{\vec{OP} \cdot \vec{OC}}{\parallel \vec{OP} \parallel \parallel \vec{OC} \parallel} = \frac{9}{3\sqrt{14}} = \frac{3}{\sqrt{14}}$$

$$\alpha = \arccos\left(\frac{3}{\sqrt{14}}\right) \approx 37^\circ$$



$$\vec{OP} \cdot \vec{OR} = \|\vec{OP}\| \|\vec{OR}\| \cos 120^\circ = 4 \times 4 \times \left(-\frac{1}{2}\right) = -8$$



$$\frac{(6-2) \times 120^\circ}{6} = 120^\circ$$

24.b) Reta PS: $(u, y, z) = (14, 5, 0) + K(2, 3, -1)$, $K \in \mathbb{R}$

Interseção de reta PS com o plano POR:

$$\begin{cases} u = 14 + 2K \\ y = 5 + 3K \\ z = -K \\ 2u + 3y - z - 15 = 0 \end{cases} \Rightarrow \begin{cases} 2(14 + 2K) + 3(5 + 3K) - (-K) - 15 = 0 \\ 4K + 9K + K = 15 - 28 - 15 \end{cases}$$

$$\begin{cases} u = 10 \\ y = -1 \\ z = 2 \\ K = -2 \end{cases} \text{ Assim } P(10, -1, 2). \text{ Logo } \overline{PS} = \sqrt{(4-10)^2 + (5+1)^2 + (0-2)^2} = \sqrt{56}$$

Conclui-se que a área lateral = $4 \times \sqrt{56} \times 6 = 24\sqrt{56} \approx 179,6$

25.a) $P(1, 3, z)$, $z < 0$

Como P é sup. esférica, então $(1-1)^2 + (3-2)^2 + (z+1)^2 = 10$ e $(z+1)^2 = 9 \Rightarrow z+1 = 3 \vee z+1 = -3 \Rightarrow z = 2 \vee z = -4$. Como $z < 0$, logo $P(1, 3, -4)$

Plano pedido: $4u + y - 2z + d = 0$

$$(1, 3, -4) \in \text{plano} \Rightarrow 4 \times 1 + 3 - 2 \times (-4) + d = 0$$

$$4 + 3 + 8 + d = 0$$

$$d = -15$$

Assim, a equação do plano pedido é $4u + y - 2z - 15 = 0$

25.b) $C(1, 2, -1)$

$$\vec{OA} = (1, 2, 1)$$

$$\vec{OA} \cdot \vec{OC} = 1 + 4 - 1 = 4$$

$A(1, 2, 1)$

$$\vec{OC} = (1, 2, -1)$$

$$\cos \alpha = \frac{\vec{OA} \cdot \vec{OC}}{\|\vec{OA}\| \|\vec{OC}\|} = \frac{4}{6} = \frac{2}{3}$$

$O(0, 0, 0)$

$$\|\vec{OA}\| = \sqrt{1^2 + 2^2 + 1^2} = \sqrt{6}$$

$$\|\vec{OC}\| = \sqrt{1^2 + 2^2 + (-1)^2} = \sqrt{6}$$

$$\alpha = \arccos\left(\frac{2}{3}\right) \approx 48^\circ$$

26) $\vec{OA} = (2, 1)$

$$m = \frac{1}{2} \text{ logo } m_R = -2$$

Reta R: $y = -2x + b$

Como $A(2, 1) \in R$, $1 = -2 \times 2 + b$ e $b = 5$ (B)

27.a) $\vec{OP} = (1, 1, 1)$

$\vec{u} = -2\vec{OP} = (-2, -2, -2)$

$Q = P + \vec{u} = (1, 1, 1) + (-2, -2, -2) = (-1, -1, -1)$

Como $Q = P - 2\vec{OP} = P + 2\vec{PO}$,
então Q é o simétrico de
 P , relativo à origem. Sendo
um ponto de sup. esf.,
[OQ] será um diâmetro da
sup. esf.

27.b) $R(0, y, 0)$ com $y < 0$

Como $R \in$ sup. esf., então $0^2 + y^2 + 0^2 = 3 \Rightarrow y = \pm\sqrt{3}$. Como $y < 0$, $R(0, -\sqrt{3}, 0)$

$\vec{OR} = (0, -\sqrt{3}, 0)$ $\|\vec{OR}\| = \sqrt{0^2 + (-\sqrt{3})^2 + 0^2} = \sqrt{3}$

$\vec{OP} = (1, 1, 1)$ $\|\vec{OP}\| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$

$\vec{OR} \cdot \vec{OP} = 0 - \sqrt{3} + 0 = -\sqrt{3}$

$\cos \alpha = \frac{-\sqrt{3}}{\sqrt{3} \times \sqrt{3}} = -\frac{\sqrt{3}}{3}$

$\alpha = \arccos\left(-\frac{\sqrt{3}}{3}\right) \approx 125^\circ$

(28) $m_T = -\frac{a}{2}$

$m_S = \frac{2a}{a} = 2$

$ax + 2y + 1 = 0$

$2y = -ax - 1$

$y = \left(-\frac{a}{2}\right)x - \frac{1}{2}$

Então, $-\frac{a}{2} = 2 \Rightarrow a = -4$ (A)

(29)a) $A(2, 1, 0)$

$\vec{AV} = V - A = (3, -1, 2) - (2, 1, 0) = (1, -2, 2)$

$E(0, -1, 2)$

$\vec{AE} = E - A = (0, -1, 2) - (2, 1, 0) = (-2, -2, 2)$

$V(3, -1, 2)$

$\cos \alpha = \frac{\vec{AV} \cdot \vec{AE}}{\|\vec{AV}\| \|\vec{AE}\|} = \frac{(1, -2, 2) \cdot (-2, -2, 2)}{\sqrt{1^2 + (-2)^2 + 2^2} \times \sqrt{(-2)^2 + (-2)^2 + 2^2}} = \frac{-2 + 4 + 4}{\sqrt{9} \times \sqrt{12}}$

$\cos \alpha = \frac{6}{3\sqrt{12}} \Rightarrow \alpha = \arccos\left(\frac{6}{3\sqrt{12}}\right) \approx 55^\circ$

b) Seja M o centro de S_{ACE} e ponto médio de (AE)

$M = \left(\frac{2+0}{2}, \frac{1-1}{2}, \frac{0+2}{2}\right) = (1, 0, 1)$

$\vec{VM} = M - V = (1, 0, 1) - (3, -1, 2) = (-2, 1, -1) \perp$ plano que contém a S_{ACE}

$-2x + y - z + d = 0$

Como $(1, 0, 1) \in$ plano, logo $-2 \times 1 + 0 - 1 + d = 0 \Rightarrow d = 3$

Assim, o plano tem equação $-2x + y - z + 3 = 0$

30) O ponto $(3, 4)$ pertence à reta $y = \frac{4}{3}x$ e a sua distância à origem é $\sqrt{9+16} = 5$

O ponto $(5, 0)$ pertence ao eixo das abscissas e a sua distância à origem é 5

Assim, a reta r será a mediatriz do segmento de reta formado por estes 2 pontos

$$(x-3)^2 + (y-4)^2 = (x-5)^2 + (y-0)^2 \Rightarrow x^2 - 6x + 9 + y^2 - 8y + 16 = x^2 - 10x + 25 + y^2$$

$$\Leftrightarrow -2y = 6x - 10x - 9 + 16 + 25 \Rightarrow y = \frac{1}{2}x$$

31.a) $A(x, 0, 0) \in$ plano $3x + 4y - 12 = 0$, logo $3x + 0 - 12 = 0 \Rightarrow x = 4$ $A(4, 0, 0)$
 $B(0, y, 0) \in$ " " " " $0 + 4y - 12 = 0 \Rightarrow y = 3$ $B(0, 3, 0)$

$$AB = \sqrt{4^2 + 3^2 + 0^2} = 5$$

$$V = 5 \times 6 \times 10 = 300$$

$$BE = \sqrt{0^2 + 0^2 + 6^2} = 6$$

$$BE = \sqrt{6^2 + 3^2 + 0^2} = 10$$

31.b) $P(1, -4, 3)$

reta $r \Rightarrow (x, y, z) = (1, -4, 3) + K(3, 4, 0)$, $K \in \mathbb{R}$

$$\begin{cases} x = 1 + 3K \\ y = -4 + 4K \\ z = 3 \end{cases} \begin{cases} 3 + 9K = 16 + 16K - 12 = 0 \\ 25K = 25 \end{cases} \begin{cases} K = 1 \end{cases} \begin{cases} x = 4 \\ y = 0 \\ z = 3 \end{cases}$$

Ponto de interseção $(4, 0, 3)$

32) $A(x, 0) \in$ reta $y = 2x + 4$, logo $0 = 2x + 4 \Rightarrow x = -2$ $A(-2, 0)$
 $B(0, y) \in$ reta $y = 2x + 4$, logo $y = 2 \times 0 + 4 \Rightarrow y = 4$ $B(0, 4)$
 $M = \left(\frac{-2+0}{2}, \frac{0+4}{2} \right) = (-1, 2)$ (B)

33.a) Ponto $A(x, 4, z) \in$ reta r , logo $(x, 4, z) = (1, 2, 1) + K(0, 1, 5)$, $K \in \mathbb{R}$

$$\Leftrightarrow \begin{cases} x = 1 \\ 4 = 2 + K \\ z = 1 + 5K \end{cases} \Leftrightarrow \begin{cases} K = 2 \\ z = 11 \end{cases} \Leftrightarrow \begin{cases} x = 1 \\ z = 11 \end{cases} \text{ Assim, } A(1, 4, 11)$$

O plano é de forma $2x + 3y - z + d = 0$.

Como $A(1, 4, 11)$ pertence ao plano, então $2 \times 1 + 3 \times 4 - 11 + d = 0 \Rightarrow$

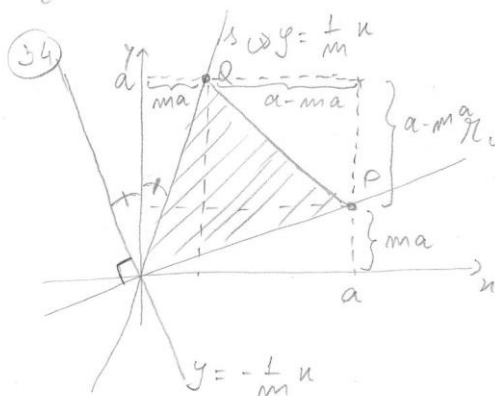
$$\Leftrightarrow 2 + 12 - 11 + d = 0 \Rightarrow 3 + d = 0 \Rightarrow d = -3$$

Equação do plano:
 $2x + 3y - z - 3 = 0$

33.5

$$\begin{cases} x=1 \\ y=2+K \\ z=1+5K \\ 2x+3y-z-9=0 \end{cases} \Leftrightarrow \begin{cases} \text{---} \\ \text{---} \\ \text{---} \\ 2 \cdot 1 + 3(2+K) - (1+5K) - 9 = 0 \end{cases} \begin{cases} \text{---} \\ \text{---} \\ \text{---} \\ 2+6+3K-1-5K-9=0 \end{cases}$$

$$\begin{cases} \text{---} \\ \text{---} \\ -2K=2 \end{cases} \begin{cases} x=1 \\ y=1 \\ z=-4 \\ K=-1 \end{cases} \text{Ponto intersecc\~ao } (1, 1, -4)$$



$$P(a, ma)$$

$$Q(ma, a)$$

$$A_{\nabla} = \frac{ma \times a}{2} = \frac{ma^2}{2}$$

$$A_{\triangle} = A_{\nabla} = \frac{ma^2}{2}$$

$$A_{\triangle} = \frac{(a-ma)^2}{2} = \frac{a^2 - 2ma^2 + m^2a^2}{2} = \frac{a^2}{2} - ma^2 + \frac{m^2a^2}{2}$$

$$A_{\square} = a \times a = a^2$$

$$A_{\text{perdida}} = a^2 - \frac{a^2}{2} + ma^2 - \frac{m^2a^2}{2} - \frac{ma^2}{2} - \frac{ma^2}{2} =$$

$$= \frac{a^2}{2} + \cancel{ma^2} - \frac{m^2a^2}{2} - \frac{\cancel{2ma^2}}{2} =$$

$$= \frac{a^2}{2} - \frac{m^2a^2}{2} = \frac{a^2}{2} (1 - m^2) \text{ e.g.d.}$$

35.a) $A(0, y, 0) \in \text{plano } ABC, \text{ logo } 3x + 4y + 4x - 12 = 0 \Leftrightarrow y = 3$

$$A(0, 3, 0)$$

$B(x, 0, 0) \in \text{plano } ABC, \text{ logo } 3x + 4x + 4x - 12 = 0 \Leftrightarrow x = 4$

$$B(4, 0, 0)$$

$h = \text{altura do cilindro} = \overline{AB} = \sqrt{(0-4)^2 + (3-0)^2 + (0-0)^2} = 5$

$V_{\text{cil}} = 10\pi \text{ cm}^3 \Rightarrow \pi \times \overline{BC}^2 \times 5 = 10\pi \text{ cm}^3 \Rightarrow \overline{BC}^2 = 2 \Rightarrow \overline{BC} = \sqrt{2}$

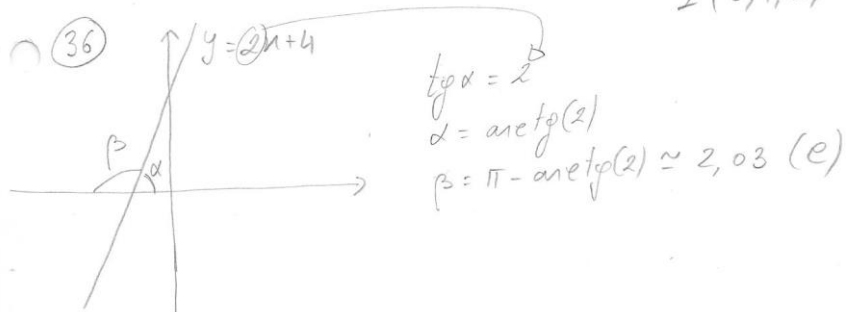
35.6) Seja π a reta que contém P e que é perpendicular ao plano ABE . Pretende-se obter as coordenadas do ponto de interseccção, I , de π com o plano ABE

$$\pi \hookrightarrow (u, y, z) = (3, 5, 6) + K(3, 4, 4), K \in \mathbb{R}$$

$$\begin{cases} u = 3 + 3K \\ y = 5 + 4K \\ z = 6 + 4K \\ 3u + 4y + 4z - 12 = 0 \end{cases} \Leftrightarrow \begin{cases} 3(3 + 3K) + 4(5 + 4K) + 4(6 + 4K) - 12 = 0 \\ 41K = -41 \\ K = -1 \end{cases}$$

$$\Leftrightarrow \begin{cases} u = 3 - 3 \\ y = 5 - 4 \\ z = 6 - 4 \end{cases} \Leftrightarrow \begin{cases} u = 0 \\ y = 1 \\ z = 2 \end{cases}$$

$$I(0, 1, 2)$$



37.a) $\vec{M}(3, -6, 2)$

$$3x - 6y + 2z + d = 0$$

$(5, 3, 6) \hookrightarrow 3 \times 5 - 6 \times 3 + 2 \times 6 + d = 0 \Rightarrow 15 - 18 + 12 + d = 0 \Rightarrow d = -9$

eq. do plano EFT: $3x - 6y + 2z - 9 = 0$

37.b) $A(7, 1, 4)$ $M[A, G] = \left(\frac{7+5}{2}, \frac{1+3}{2}, \frac{4+6}{2}\right) = (6, 2, 5) = \text{centro}$

$G(5, 3, 6)$

$$raio = \overline{AM} = \sqrt{(7-6)^2 + (1-2)^2 + (4-5)^2} = \sqrt{3}$$

equação de sup. esf. $(x-6)^2 + (y-2)^2 + (z-5)^2 = 3$

38.a) $\vec{BE} = (2, 2, -2) \Rightarrow E - B = (2, 2, -2) \Rightarrow E = (0, 2, 4) + (2, 2, -2) \Rightarrow E = (2, 4, 2)$

$\vec{BO} = (0, -2, -4)$

$\vec{BE} = (2, 2, -2)$

$$\|\vec{BO}\| = \sqrt{0^2 + (-2)^2 + (-4)^2} = \sqrt{20}$$

$$\|\vec{BE}\| = \sqrt{2^2 + 2^2 + (-2)^2} = \sqrt{12}$$

$$\vec{BO} \cdot \vec{BE} = 0 \times 2 + 2 \times (-2) + (-2) \times (-4) = 4$$

$$\cos \alpha = \frac{\vec{BO} \cdot \vec{BE}}{\|\vec{BO}\| \|\vec{BE}\|} = \frac{4}{\sqrt{20} \sqrt{12}}$$

$$\cos \alpha = \frac{4}{\sqrt{240}}$$

$$\alpha = \arccos\left(\frac{4}{\sqrt{240}}\right) \approx 75^\circ$$

38.b) $\vec{OE} = (2, 4, 2)$

$$2x + 4y + 2z + d = 0$$

$G(0, 2, 2) \hookrightarrow 2 \times 0 + 4 \times 2 + 2 \times 2 + d = 0 \Rightarrow d = -12$

$\alpha \hookrightarrow 2x + 4y + 2z - 12 = 0$

$P(x, 0, 0) \hookrightarrow 2x + 4 \times 0 + 2 \times 0 - 12 = 0 \Rightarrow 2x - 12 = 0 \Rightarrow x = 6$
 $P(6, 0, 0)$

$Q(0, y, 0) \dots Q(0, 3, 0)$

$R(0, 0, z) \dots R(0, 0, 6)$

$$A_{bcu} = \frac{6 \times 3}{2} = 9$$

altura = 6

$$V = \frac{9 \times 6}{3} = 18$$

39) $\angle \text{interno do hexágono} = \frac{4 \times 180^\circ}{6} = 120^\circ$

$\angle NMO = 60^\circ \Rightarrow m = \frac{1}{2} 120^\circ = 60^\circ$

$$y = -\sqrt{3}x + b$$

$(1, 0) \hookrightarrow 0 = -\sqrt{3} \times 1 + b$

$$\sqrt{3} = b$$

(A)

40.9) (c) $(x, y, z) = (7, -10, 3) + K(0, 3, 3), K \in \mathbb{R}$

$\hookrightarrow (0, 3, 3) \cdot (-3, -2, 2) = 0 - 6 + 6 = 0$

logo a reta r $\perp$ $\hat{=}$ a reta EF

$E(7, 2, 15) \in$ a reta deste opcoes

$(7, 2, 15) = (7, -10, 3) + K(0, 3, 3)$

$$\begin{cases} 7 = 7 \\ 2 = -10 + 3K \\ 15 = 3 + 3K \end{cases} \begin{cases} - \\ 3K = 12 \\ 3K = 12 \end{cases} \begin{cases} K = 4 \\ K = 4 \end{cases}$$

Nas restantes opcoes, a reta não $\perp$ $\hat{=}$ a reta EF ou não contém o ponto E

40.6) Plano ABF $\perp (-3, -2, 2)$

$-3x - 2y + 2z + d = 0$

$S(6, 10, 13) \hookrightarrow -3 \times 6 - 2 \times 10 + 2 \times 13 + d = 0 \dots \Rightarrow d = 12$

Plano ABF: $-3x - 2y + 2z + 12 = 0$

$B(0, y, 0) \in$ plano ABF, logo $-3 \times 0 - 2y + 2 \times 0 + 12 = 0$
 $\Rightarrow \dots \Rightarrow y = 6$

Assim, $B(0, 6, 0)$

Logo: $\overline{BD} = \overline{EF} = \sqrt{(7-6)^2 + (-10-0)^2 + (3-13)^2} = \dots = \sqrt{69}$

equação pedida: $x^2 + (y-6)^2 + z^2 = 69$

41.8) $\overline{RD} = \sqrt{(-2+5)^2 + (1-5)^2 + (1+3)^2} = \sqrt{9+16+16} = \sqrt{41}$, logo opcao c

41.5) $\overrightarrow{RS} \parallel \overrightarrow{PO}$

$\overrightarrow{PO} = O - P = (-2, 1, 1) - (1, -1, 2) = (-3, 2, -1)$

Plano: $-3x + 2y - z + d = 0$

$P(1, -1, 2) \hookrightarrow -3 \times 1 + 2 \times (-1) - 2 + d = 0$

$-3 - 2 - 2 + d = 0$

$d = 7$

Plano pedido: $-3x + 2y - z + 7 = 0$

$\Leftrightarrow 3x - 2y + z - 7 = 0$

- 42.a) opcao A não contém o ponto $(1, 0, 1)$
 opcao B não é $\perp$ a reta BE pois $(1, 6, 2) \cdot (-1, 0, 2) = -1 + 36 + 4 = 39 \neq 0$
 opcao D não contém o ponto $(1, 0, 1)$
 opcao E é a correta pois contém $(1, 0, 1)$ e é $\perp$ a reta BE

42.b) seja I o centro de base do pirâmide.

$$I(-2, 0, 2)$$

$$EI = \sqrt{(-2+2)^2 + (6-0)^2 + (2-2)^2} = \sqrt{0+36+0} = 6$$

$$\frac{AB \times 6}{3} = 20 \Leftrightarrow AB^2 = 10 \Leftrightarrow AB = \sqrt{10}$$

$$B = E + \vec{EB} = (-2, 6, 2) + (1, -6, -2) = (-1, 0, 0)$$



$$OA^2 + 1^2 = (\sqrt{10})^2$$

$$\Leftrightarrow OA^2 = 9$$

$$\Leftrightarrow OA = 3$$

$$\text{Então } A(0, 0, 3) \text{ e } B(-1, 0, 0)$$

$$\text{Assim } \vec{AB} = B - A = (-1, 0, 0) - (0, 0, 3) = (-1, 0, -3)$$

43.a) $A(0, y, 0) \in$ plano de equação $4y - 3z = 16$
 Assim $4y - 3 \cdot 0 = 16 \Leftrightarrow y = 4$. Então $A(0, 4, 0)$

opcao D: $x + 3y + 4z = 3 \perp$ plano definido por $4y - 3z = 16$ pois
 $(1, 3, 4) \cdot (0, 4, -3) = 1 \cdot 0 + 3 \cdot 4 + 4 \cdot (-3) = 0$
 $(1, 2, -1) \in$ plano definido por $x + 3y + 4z = 3$ pois
 $1 + 3 \cdot 2 + 4 \cdot (-1) = 3$ Verdade

43.b) reta AV: $(x, y, z) = (0, 4, 0) + K(0, 4, -3), K \in \mathbb{R}$

$$V(0, 0, z) \in \text{reta AV, logo}$$

$$(0, 0, z) = (0, 4, 0) + K(0, 4, -3) \Leftrightarrow \begin{cases} 0 = 0 \\ 0 = 4 + 4K \\ z = 0 - 3K \end{cases} \Rightarrow \begin{cases} K = -1 \\ z = 3 \end{cases}$$

$$\text{Então } V(0, 0, 3) \text{ e } A(0, 4, 0)$$

$$\overline{VA} = \sqrt{0^2 + (-4)^2 + 3^2} = \sqrt{25} = 5$$

$$\text{Volume} = \frac{\pi \cdot 3^2 \cdot 5}{3} = 15\pi$$

(44) $(x+2)^2 + (y-1)^2 = 9$ $\leftarrow$ Centro $(-2, 1)$
 raio $= \sqrt{9} = 3$

Comprimento do arco $AB = 2\pi r \cdot \alpha \times 3 = 2\pi \cdot \alpha \Rightarrow \alpha = \frac{2\pi}{3}$

$\vec{CA} \cdot \vec{CB} = \|\vec{CA}\| \|\vec{CB}\| \cos\left(\frac{2\pi}{3}\right) = 3 \times 3 \times \left(-\frac{1}{2}\right) = -\frac{9}{2}$



(45.a) $\vec{AB} \cdot \vec{AE} = \|\vec{AB}\| \|\vec{AE}\| \cos 90^\circ = 0$ logo op. $\perp$ B

(45.b) Como E é rta definida por $(x, y, z) = (0, 0, 3) + K(1, -1, -1)$, $K \in \mathbb{R}$

então $E(K, -K, 3-K)$; $K \in \mathbb{R}$

Como $\vec{AB} \perp \vec{AE}$ então $\vec{AB} \cdot \vec{AE} = 0 \Rightarrow$

$$\left\{ \begin{array}{l} \vec{AB} = B - A = (1, -1, 2) - (-2, 5, 0) = (3, -6, 2) \\ \vec{AE} = E - A = (K, -K, 3-K) - (-2, 5, 0) = (K+2, -K-5, 3-K) \end{array} \right.$$

$\Rightarrow (3, -6, 2) \cdot (K+2, -K-5, 3-K) = 0$

$\Rightarrow 3K + 6 + 6K + 30 + 6 - 2K = 0$

$\Rightarrow 7K + 42 = 0$

$\Rightarrow 7K = -42$

$\Rightarrow K = -\frac{42}{7} = -6$ Assim, $E(-6, 6, 9)$

(46.a) Como o resto de M é 2, então $z = 2 \times 2$ ou seja: $z = 4$ (B)

(46.b) plano BCJ // plano LEF

plano LEF:

$3x - \sqrt{3}y + d = 0$

$B(x, 0, 0) \in$ plano BCJ, logo $3x - 6 = 0 \Rightarrow x = 2$. Assim $B(2, 0, 0)$

$\vec{BM} = M - B = \left(\frac{4}{3}, \frac{2\sqrt{3}}{3}, 2\right) - (2, 0, 0) = \left(-\frac{2}{3}, \frac{2\sqrt{3}}{3}, 2\right)$

$L = B + 2\vec{BM} = (2, 0, 0) + 2\left(-\frac{2}{3}, \frac{2\sqrt{3}}{3}, 2\right) = (2, 0, 0) + \left(-\frac{4}{3}, \frac{4\sqrt{3}}{3}, 4\right) = \left(\frac{2}{3}, \frac{4\sqrt{3}}{3}, 4\right)$

Como L $\in$ plano LEF, logo

$3 \times \frac{2}{3} - \sqrt{3} \times \frac{4\sqrt{3}}{3} + d = 0$

$\Rightarrow 2 - 4 + d = 0$

$\Rightarrow d = 2$

Então a equação do plano LEF será $3x - \sqrt{3}y + 2 = 0$