

$$① \quad 1.1) (f \circ h)\left(\frac{1}{2}\right) = f\left(h\left(\frac{1}{2}\right)\right) = f(3) = 2$$

$$1.2) f^{-1}(2) + (g \circ h)(\sqrt{2}) = f^{-1}(2) + g(h(\sqrt{2})) = 3 + g(1) = 3 + 3 = 6$$

$g(1) = 2 \cdot 1 + 1 = 3$

$$② \quad (g \circ f)(a) = 4 \Leftrightarrow g(f(a)) = 4 \Leftrightarrow f(a) = 0 \Leftrightarrow -2 + \frac{a}{3} = 0 \Leftrightarrow -6 + a = 0 \Leftrightarrow a = 6$$

$$③ \quad D_f = \{u \in \mathbb{R} : 4 - 2u > 0 \wedge u^2 + 3u \neq 0\} =]-\infty, 2[\setminus \{-3, 0\}$$

3.1) C.A.

$$\begin{cases} 4 - 2u > 0 \\ -2u > -4 \\ 2u < 4 \\ u < 2 \end{cases} \quad \begin{cases} u^2 + 3u \neq 0 \\ u(u+3) \neq 0 \\ u \neq 0 \wedge u \neq -3 \end{cases}$$

$$D_g = \mathbb{R}$$

$$D_h = \{u \in \mathbb{R} : u + 6 \neq 0\} = \mathbb{R} \setminus \{-6\}$$

C.A.

$$\begin{cases} u + 6 \neq 0 \\ u \neq -6 \end{cases}$$

$$3.2) \quad a) \quad \left. \begin{aligned} f(-6) &= \frac{\sqrt{4 - 2(-6)}}{(-6)^2 + 3(-6)} = \frac{\sqrt{16}}{36 - 18} = \frac{4}{18} = \frac{2}{9} \\ g^{-1}(2) &= ? \\ 2u + 3 &= 2 \Leftrightarrow 2u = -6 \Leftrightarrow u = -3 \end{aligned} \right\} \quad \left. \begin{aligned} f(-6) + g^{-1}(2) &= \\ \frac{2}{9} - 3 &= \frac{2}{9} - \frac{27}{9} = -\frac{25}{9} \end{aligned} \right\}$$

$$g^{-1}(2) = -3$$

$$b) (f \circ g)(-3) = f(g(-3)) = f(2) = \frac{\sqrt{4 - 2 \cdot 2}}{2^2 + 3 \cdot 2} = 0$$

$$d) (h \circ h)(0) = h(h(0)) = h\left(\frac{12}{8}\right) = \frac{3}{2}$$

C.A.

$$g(-3) = 2 \cdot (-3) + 3 = 2$$

$$e) (g^{-1} \circ h)(-3) = g^{-1}(h(-3)) = g^{-1}(4) = -2$$

C.A.

$$\frac{h(0)}{h(0)} = \frac{12}{0+6} = 2$$

C.A.

$$h(-3) = \frac{12}{-3+6} = \frac{12}{3} = 4 \quad \left\{ \begin{aligned} g(u) &= 4 \\ 2u + 3 &= 4 \Leftrightarrow 2u = 1 \Leftrightarrow u = \frac{1}{2} \end{aligned} \right.$$

$$3.3) \quad a) (g \circ h)(u) = g(h(u)) = g\left(\frac{12}{u+6}\right) = 2 \times \frac{12}{u+6} + 3 = \frac{24}{u+6} + 3$$

$$b) g(u) = y \Leftrightarrow 2u + 3 = y \Leftrightarrow 2u = y - 3 \Leftrightarrow u = \frac{y-3}{2}, \text{ luego } g^{-1}(u) = \frac{u-3}{2} (*)$$

$$c) h(u) = y \Leftrightarrow \frac{12}{u+6} = y \Leftrightarrow y(u+6) = 12 \Leftrightarrow yu + 6y = 12 \Leftrightarrow yu = 12 - 6y \Leftrightarrow u = \frac{12-6y}{y}$$

$$d) (f \circ g)(u) = f(g(u)) = f(2u+3) = \frac{\sqrt{4 - 2(2u+3)}}{(2u+3)^2 + 3(2u+3)} = \frac{\sqrt{4 - 4u - 6}}{4u^2 + 32u + 64 + 6u + 24} = \frac{\sqrt{-12 - 4u}}{4u^2 + 38u + 88}$$

$\text{ luego } h^{-1}(u) = \frac{12-6u}{u}$

$$3.4) \quad D'_{f \circ h} = \mathbb{R} \quad D'_h = \mathbb{R} \setminus \{0\}$$

$$4.1) D_f = \{ u \in \mathbb{R} : u^2 - 9 \neq 0 \} = \mathbb{R} \setminus \{-3, 3\}$$

$$\text{c.a. } \frac{1}{u} \neq 0 \Leftrightarrow u \neq 0 \Leftrightarrow u \neq \pm\sqrt{9} \Leftrightarrow u \neq \pm 3$$

$$4.2) D_g =]-\infty, 5[\setminus 4-5 \} \quad \mid \quad D'_g = [-4, 4]$$

$$4.3) \text{ Zeros de } f: \{-8\} \quad \mid \quad \text{Zeros de } g: \{-4; 0; 5\}$$

$$\text{c.a. } 8+u=0 \quad \mid$$

$$u=-8$$

$$4.4) a) 3 \quad b) \text{ infinites}$$

$$4.5) a) K = -4 \vee K = 4 \quad b) K \in]-4, -2[\cup]0, 3[$$

$$4.6) \text{ Monotonie}$$

$$f \nearrow \text{ sur }]-5, -4[\text{ et sur } [-2, 2[\text{ et sur } [2, 3]$$

$$f \searrow \text{ sur } [-4, -2] \text{ et sur } [3, 5]$$

$$g \rightarrow \text{ sur }]-\infty, -5[$$

Extremums

$$0 \text{ est max relat pour } u = -4$$

$$-4 \text{ est min relat pour } u = -2$$

$$3 \text{ est min " " } u = 2$$

$$4 \text{ est max " " } u = 3$$

$$0 \text{ est min " " } u = 5$$

$$-4 \text{ est min absolu pour } u = -2$$

$$4 \text{ est max " " } u = 3$$

$$4.7) a) [3, 5[\quad b)]-5, -4[\text{ ou } [-2, 0[\text{ ou } \dots$$

$$4.8) \begin{array}{c|cccccc} u & -\infty & -5 & -4 & 0 & 3 & 5 \\ \hline f & - & \text{max} & - & 0 & - & 0 \\ g & - & \text{min} & - & 0 & - & 0 \end{array}$$

$$4.9) a) g(u) + f(2) - 2 = 0 \Leftrightarrow g(u) - 2 - 2 = 0 \Leftrightarrow g(u) = 4 \Leftrightarrow u = 3$$

$$\text{c.a. } f(x) = \frac{8+2}{x^2-9} = \frac{10}{-5} = -2$$

$$b) g(x) \times f(1) < 0 \Leftrightarrow g(x) > 0 \Leftrightarrow x \in]0, 5[$$

$$\text{c.a. } f(1) = \frac{8+1}{1^2-9} = -\frac{9}{8} < 0$$

4.10) a) $p(x) > 0$; logo $D_h =]0, 5[$

b) $q(x) \neq 0 \Leftrightarrow x \neq -4 \wedge x \neq 0 \wedge x \neq 5$ logo $D_i = \mathbb{R} \setminus \{-4, 0, 5\}$

4.11) A(0,0) B(2,4)

$\vec{AB} = B - A = (2,4) - (0,0) = (2,4)$

$m = \frac{4}{2} = 2$

utilizo AB: $y = 2x$
ordenada 3 $\rightarrow 3 = 2x \Leftrightarrow x = \frac{3}{2}$ ponto $(\frac{3}{2}, 3)$

4.12) $f(-x) = \frac{3-x}{(-x)^2-9} = \frac{3-x}{x^2-9} \neq f(x)$ logo f não é par nem ímpar
 $\neq -f(x)$

O gráfico de f não é simétrico relativamente à origem
nem relativamente ao eixo dos yy , logo f não
é par nem ímpar.

4.13) g é não injetiva porque existem retas horizontais que
interseccionam o gráfico em mais do que um ponto.