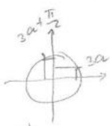
(1) $A_{\triangle ODC} = \frac{1 \times tg\alpha}{2} = \frac{tg\alpha}{2}$ $\Bigg\{$ $A_{\triangle OAB} = \frac{\cos\alpha \times sen\alpha}{2} = \frac{2\cos\alpha\, sen\alpha}{4} = \frac{sen(2\alpha)}{4}$

$A = \frac{tg\alpha}{2} - \frac{sen(2\alpha)}{4}$ (B)



(2) $f'(x) = 3sen(3x)$ $\Bigg\{$ $g'(x) = 3\cos(3x)$

$m_r = f'(a) = 3sen(3a)$ $\Bigg|$ $m_s = g'(a+\frac{\pi}{6}) = 3\cos(3a+\frac{3\pi}{6}) = 3\cos(3a+\frac{\pi}{2}) = -3sen(3a)$

$r \perp s$ logo $m_r = -\frac{1}{m_s} \Leftrightarrow 3sen(3a) = -\frac{1}{-3sen(3a)} \Leftrightarrow 9sen^2(3a) = 1 \Leftrightarrow$

$\Leftrightarrow sen^2(3a) = \frac{1}{9} \Leftrightarrow sen(3a) = \pm\frac{1}{3}$ Como $\pi < 3a < \frac{3\pi}{2}$, então $sen(3a) = -\frac{1}{3}$

$\frac{\pi}{3} < a < \frac{\pi}{2}$

$\pi < 3a < \frac{3\pi}{2}$

(3) $f'(x) = 3 \times \underbrace{2\, sen x \cos x}_{sen(2x)} = 3 sen(2x)$

$f''(x) = 3 \times 2\cos(2x) = 6\cos(2x)$ (C)

(4)

a) $d(t) = d(0) \Leftrightarrow 1 + \frac{1}{2} sen(\pi t + \frac{\pi}{6}) = \frac{5}{4} \Leftrightarrow$ $\Bigg\{$ c.A.: $d(0) = 1 + \frac{1}{2} sen(0 + \frac{\pi}{6}) =$

$\Leftrightarrow 4 + 2sen(\pi t + \frac{\pi}{6}) = 5 \Leftrightarrow 2sen(\pi t + \frac{\pi}{6}) = 1 \Leftrightarrow sen(\pi t + \frac{\pi}{6}) = \frac{1}{2} \Leftrightarrow$ $\Bigg|$ $= 1 + \frac{1}{2} \times \frac{1}{2} = \frac{5}{4}$

$\Leftrightarrow \pi t + \frac{\pi}{6} = \frac{\pi}{6} + 2K\pi \vee \pi t + \frac{\pi}{6} = \pi - \frac{\pi}{6} + 2K\pi,\ K \in \mathbb{Z} \Leftrightarrow$

$\Leftrightarrow 6\pi t + \cancel{\pi} = \cancel{\pi} + 12K\pi \vee 6\pi t + \pi = 6\pi - \pi + 12K\pi,\ K \in \mathbb{Z} \Leftrightarrow$

$\Leftrightarrow \frac{6\cancel{\pi}t}{\cancel{\pi}} = \frac{12K\cancel{\pi}}{\cancel{\pi}} \vee \frac{6\cancel{\pi}t}{\cancel{\pi}} = \frac{4\cancel{\pi}}{\cancel{\pi}} + \frac{12K\cancel{\pi}}{\cancel{\pi}},\ K \in \mathbb{Z}$

$\Leftrightarrow 6t = 12K \vee 6t = 4 + 12K,\ K \in \mathbb{Z} \Leftrightarrow t = 2K \vee t = \frac{4+12K}{6},\ K \in \mathbb{Z}$

$K = 0 \rightarrow t = 0 \vee t = \frac{2}{3}$

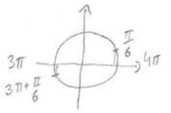
$K = 1 \rightarrow t = 2 \vee t = \frac{8}{3}$

$K = 2 \rightarrow t = 4 \vee t = \frac{14}{3}$

Nos primeiros 3 s, para além de $t = 0$, o ponto P passou pelo ponto A nos instantes $t = \frac{2}{3}$, $t = 2$ e $t = \frac{8}{3}$

b) d é cont. em $[3, 4]$

$d(3) = 1 + \frac{1}{2} sen(3\pi + \frac{\pi}{6}) = 1 - \frac{1}{2} sen(\frac{\pi}{6}) = 1 - \frac{1}{2} \times \frac{1}{2} = 1 - \frac{1}{4} = \frac{3}{4}$



$d(4) = 1 + \frac{1}{2} sen(4\pi + \frac{\pi}{6}) = 1 + \frac{1}{2} sen(\frac{\pi}{6}) = 1 + \frac{1}{2} \times \frac{1}{2} = 1 + \frac{1}{4} = \frac{5}{4}$

$d(3) < 1{,}1 < d(4)$

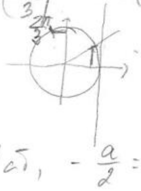
Assim, pelo Teor. de Bolzano-Cauchy, concluímos que houve, pelo menos, um instante, entre os 3 s e os 4 s após o início, em que a distância de P a O foi igual a 1,1 m

⑤ $f(x) = a \operatorname{sen} x$

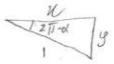
Sabe-se que $m_r = f'\left(\frac{2\pi}{3}\right)$ e que $m_r = tg\left(\frac{\pi}{6}\right)$

$f'(x) = a \cos x$

$f'\left(\frac{2\pi}{3}\right) = a \cos\left(\frac{2\pi}{3}\right) = a \times \left(-\frac{1}{2}\right) = -\frac{a}{2}$ $\left\{ tg\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{3} \right.$

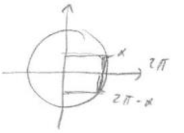


Temos, então, $-\frac{a}{2} = \frac{\sqrt{3}}{3} \Leftrightarrow 3a = -2\sqrt{3} \Leftrightarrow a = -\frac{2\sqrt{3}}{3}$

⑥ 

$\cos(2\pi-\alpha) = \frac{x}{1} \Leftrightarrow x = \cos(2\pi-\alpha) \Leftrightarrow x = \cos\alpha$

$\operatorname{sen}(2\pi-\alpha) = \frac{y}{1} \Leftrightarrow y = \operatorname{sen}(2\pi-\alpha) \Leftrightarrow y = -\operatorname{sen}\alpha$



$\left.\begin{array}{l} B = 1 + y = 1 - \operatorname{sen}\alpha \\ b = 1 \\ h = x = \cos\alpha \end{array}\right\} A = \frac{1 - \operatorname{sen}\alpha + 1}{2} \times \cos\alpha = \frac{2 - \operatorname{sen}\alpha}{2} \times \cos\alpha =$

$= \frac{\cancel{2}\cos\alpha}{\cancel{2}} - \frac{\operatorname{sen}\alpha\cos\alpha}{2} = \cos\alpha - \frac{\operatorname{sen}\alpha\cos\alpha}{2}$ (D)

⑦-a) $h'(t) = 20' + \left(\frac{1}{2\pi}\cos(2\pi t)\right)' + t' \operatorname{sen}(2\pi t) + t(\operatorname{sen}(2\pi t))' =$

$= 0 - \frac{2\pi}{2\pi}\operatorname{sen}(2\pi t) + \operatorname{sen}(2\pi t) + t\,(2\pi\cos(2\pi t)) =$

$= \cancel{-\operatorname{sen}(2\pi t)} + \cancel{\operatorname{sen}(2\pi t)} + 2\pi t\cos(2\pi t) = 2\pi t \cos(2\pi t)$

$h'(t) = 0 \Leftrightarrow 2\pi t\cos(2\pi t) = 0 \Leftrightarrow 2\pi t = 0 \vee \cos(2\pi t) = 0 \Leftrightarrow t = 0 \vee \underset{(2)}{2\pi t} = \frac{\pi}{2} + k\pi, k \in \underset{(2)}{\mathbb{Z}}$

$\Leftrightarrow t = 0 \vee \frac{4\pi t}{\pi} = \frac{\pi}{\pi} + \frac{2k\pi}{\pi}, k \in \mathbb{Z} \Leftrightarrow 4t = 1 + 2k \overset{\vee t=0}{\Leftrightarrow} t = \frac{1+2k}{4}, k \in \mathbb{Z} \vee t = 0$

$k = 0 \rightsquigarrow t = \frac{1}{4} \in [0,1]$

$k = 1 \rightsquigarrow t = \frac{3}{4} \in [0,1]$

$k = 2 \rightsquigarrow t = \frac{5}{4} \notin [0,1]$

$k = -1 \rightsquigarrow t = -\frac{1}{4} \notin [0,1]$

	0		$\frac{1}{4}$		$\frac{3}{4}$		1
$2\pi t$	0	+	+	+	+	+	+
$\cos(2\pi t)$	+	+	0	−	0	+	+
h'	0	+	0	−	0	+	+
h	mín	↗	máx	↘	mín	↗	máx

$h(0) = 20 + \frac{1}{2\pi}\underbrace{\cos(0)}_{1} + \underbrace{0 \times \operatorname{sen}(0)}_{0} =$

$= 20 + \frac{1}{2\pi} \approx 20{,}2$

$h\left(\frac{1}{4}\right) = 20 + \frac{1}{2\pi}\underbrace{\cos\left(\frac{\pi}{2}\right)}_{0} + \frac{1}{4}\underbrace{\operatorname{sen}\left(\frac{\pi}{2}\right)}_{1} =$

$= 20 + \frac{1}{4} = \frac{81}{4} = 20{,}25$

$h\left(\frac{3}{4}\right) = 20 + \frac{1}{2\pi}\underbrace{\cos\left(\frac{3\pi}{2}\right)}_{0} + \frac{3}{4}\underbrace{\operatorname{sen}\left(\frac{3\pi}{2}\right)}_{-1} =$

$= 20 - \frac{3}{4} = \frac{77}{4} = 19{,}25$

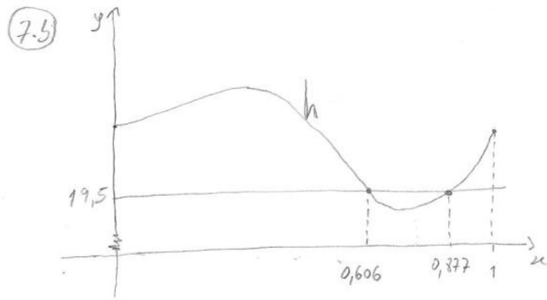
$h(1) = 20 + \frac{1}{2\pi}\underbrace{\cos(2\pi)}_{1} + \underbrace{\operatorname{sen}(2\pi)}_{0} =$

$= 20 + \frac{1}{2\pi} \approx 20{,}2$

Assim $m = \frac{77}{4}$ e $M = \frac{81}{4}$

$A = M - m = \frac{81}{4} - \frac{77}{4} = \frac{4}{4} = 1$

7.b)



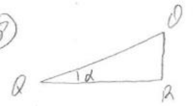
$a \approx 0{,}606$

$b \approx 0{,}877$

$b-a \approx 0{,}877 - 0{,}606 \approx 0{,}27$

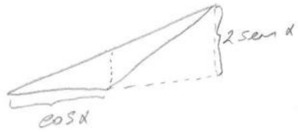
Durante 0,27 minutos, a distância de um ponto P do tabuleiro a um ponto fixo do vale foi inferior a 19,5 m

8)



$\operatorname{sen}\alpha = \frac{\overline{OR}}{1} \Leftrightarrow \boxed{\overline{OR} = \operatorname{sen}\alpha}$

$\cos\alpha = \frac{\overline{QR}}{1} \Leftrightarrow \boxed{\overline{QR} = \cos\alpha}$



$$A = \frac{\cos\alpha \times 2\operatorname{sen}\alpha}{2} = \frac{2\operatorname{sen}\alpha\cos\alpha}{2} = \frac{\operatorname{sen}(2\alpha)}{2}$$

(D)

9.a) Apenas tem sentido $x \to +\infty$, pois $D = \left]-\frac{\pi}{2}, +\infty\right[$

$$m = \lim_{x\to+\infty} \frac{f(x)}{x} = \lim_{x\to+\infty}\left(\frac{x}{x} - \frac{\ln x}{x}\right) = \lim_{x\to+\infty}\left(1 - \underbrace{\frac{\ln x}{x}}_{\text{L.N.}\ 0}\right) = 1 - 0 = 1$$

Assim, concluímos que não há A.O.

$$b = \lim_{x\to+\infty}(x - \ln x - x) = \lim_{x\to+\infty}(-\ln x) = -(+\infty) = -\infty$$

9.b) Em $\left]-\frac{\pi}{2}, 0\right[$:

$$f'(x) = \frac{(2+\operatorname{sen}x)'\cos x - (2+\operatorname{sen}x)(\cos x)'}{\cos^2 x} = \frac{\cos x \times \cos x - (2+\operatorname{sen}x)(\operatorname{sen}x)}{\cos^2 x} =$$

$$= \frac{\cos^2 x + \operatorname{sen}^2 x + 2\operatorname{sen}x}{\cos^2 x} = \frac{1 - \cancel{\operatorname{sen}^2 x} + \cancel{\operatorname{sen}^2 x} + 2\operatorname{sen}x}{\cos^2 x} = \frac{1 + 2\operatorname{sen}x}{\cos^2 x}$$

$f'(x) = 0 \Leftrightarrow 1 + 2\operatorname{sen}x = 0 \wedge \underbrace{\cos^2 x \neq 0}_{\text{Cond. universal em }]-\frac{\pi}{2}, 0[} \Rightarrow 1 + 2\operatorname{sen}x = 0 \Leftrightarrow \operatorname{sen}x = -\frac{1}{2} \Leftrightarrow$

$\Leftrightarrow x = -\frac{\pi}{6} + 2k\pi \vee x = \pi - \left(-\frac{\pi}{6}\right) + 2k\pi, k \in \mathbb{Z} \Leftrightarrow x = -\frac{\pi}{6} + 2k\pi \vee x = \frac{7\pi}{6} + 2k\pi, k \in \mathbb{Z}$

$k = 0 \Rightarrow x = -\frac{\pi}{6}$ ✓ $\vee\ x = \frac{7\pi}{6}$ ✗

$k = 1 \Rightarrow x = \frac{11\pi}{6}$ ✗ $\vee\ x = \frac{19\pi}{6}$ ✗

$k = -1 \Rightarrow x = -\frac{13\pi}{6}$ ✗ $\vee\ x = -\frac{5\pi}{6}$ ✗

	$-\frac{\pi}{2}$		$-\frac{\pi}{6}$		0
$1+2\operatorname{sen}x$		$-$	0	$+$	
$\cos^2 x$		$+$	$+$	$+$	
f'		$-$	0	$+$	
f		↘	mín	↗	

f é decrescente em $\left]-\frac{\pi}{2}, -\frac{\pi}{6}\right]$

f é crescente em $\left[-\frac{\pi}{6}, 0\right[$

f tem um mínimo relativo, para $x = -\frac{\pi}{6}$

9.c) $x = \frac{1}{2}$

$f'(x) = x' - (\ln x)' = 1 - \frac{1}{x}$

$m_r = f'\left(\frac{1}{2}\right) = 1 - \frac{1}{\frac{1}{2}} = 1 - 2 = -1$

$T\left(\frac{1}{2}, f\left(\frac{1}{2}\right)\right) = \left(\frac{1}{2}, \frac{1}{2} + \ln 2\right)$

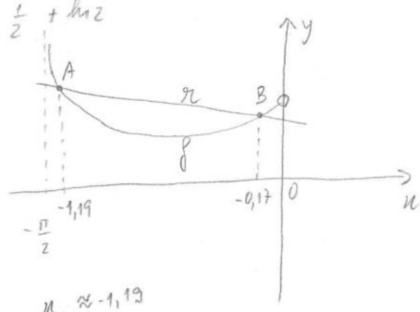
$f\left(\frac{1}{2}\right) = \frac{1}{2} - \ln\left(\frac{1}{2}\right) = \frac{1}{2} - (\underbrace{\ln 1}_{0} - \ln 2) = \frac{1}{2} + \ln 2$

$y = mx + b$

$y = -x + b$

$\frac{1}{2} + \ln 2 = -\frac{1}{2} + b \Leftrightarrow b = \frac{1}{2} + \frac{1}{2} + \ln 2 = 1 + \ln 2$

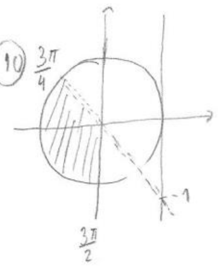
Equação da reta r: $y = -x + 1 + \ln 2$



$x_A \approx -1{,}19$

$x_B \approx -0{,}17$

10)



$A = \left]\frac{3\pi}{4}, \frac{3\pi}{2}\right[$ (B)

11.1) $\lim_{x \to 1^-} \frac{1-x^2}{1-e^{x-1}} = \lim_{x \to 1^-} \frac{1-x}{1-e^{x-1}} \times \lim_{x \to 1} (1+x) = \lim_{x-1 \to 0} \left(-\frac{x-1}{1-e^{x-1}}\right) \times 2 = 2 \lim_{y \to 0} \left(-\frac{y}{1-e^y}\right) =$ $\boxed{y = x-1 \to 0}$

$= 2 \lim_{y \to 0} \frac{y}{e^y - 1} = 2 \lim_{y \to 0} \frac{1}{\underbrace{\frac{e^y-1}{y}}_{L.N.\ 1}} = 2 \times \frac{1}{1} = 2$

$\lim_{x \to 1^+} \left(3 + \frac{\text{sen}(x-1)}{1-x}\right) = 3 + \lim_{x-1 \to 0} \left(-\frac{\text{sen}(x-1)}{x-1}\right) \underset{y = x-1 \to 0}{=} 3 + \lim_{y \to 0} \left(-\underbrace{\frac{\text{sen}\, y}{y}}_{L.N.\ 1}\right) = 3 - 1 = 2$

Como $\lim_{x \to 1^+} g(x) = \lim_{x \to 1^-} g(x) = g(1) = 2$, então $\lim_{x \to 1} g(x)$ existe. Assim, g é cont. p/ $x = 1$

11.2) Em $]4,5[$, $g(x) = 3 \Leftrightarrow \cancel{3} + \frac{\text{sen}(x-1)}{1-x} = \cancel{3} \Leftrightarrow \frac{\text{sen}(x-1)}{1-x} = 0 \Leftrightarrow \text{sen}(x-1) = 0 \wedge \underbrace{1-x \neq 0}_{\text{cond. univ. em }]4,5[}$

$\Leftrightarrow \text{sen}(x-1) = 0 \Leftrightarrow x - 1 = k\pi, k \in \mathbb{Z} \Leftrightarrow x = 1 + k\pi, k \in \mathbb{Z}$

$k = 1 \hookrightarrow x = 1 + \pi \in]4,5[$

$k = 2 \hookrightarrow x = 1 + 2\pi \notin]4,5[$

$k = 0 \hookrightarrow x = 1 \notin]4,5[$

$S = \{1 + \pi\}$

(12) $\left(2x \operatorname{sen}\alpha + \frac{\cos\alpha}{x}\right)^2 = 4x^2\operatorname{sen}^2\alpha + \underbrace{\frac{2\times 2\cancel{x}\operatorname{sen}\alpha\cos\alpha}{\cancel{x}}} + \frac{\cos^2\alpha}{x^2}$

Termo independente de x: $4\operatorname{sen}\alpha\cos\alpha$

$4\operatorname{sen}\alpha\cos\alpha = 1 \Leftrightarrow 2\times\underbrace{2\operatorname{sen}\alpha\cos\alpha}_{\operatorname{sen}(2\alpha)} = 1 \Leftrightarrow \operatorname{sen}(2\alpha) = \frac{1}{2} \Leftrightarrow$

$\Leftrightarrow 2\alpha = \frac{\pi}{6} + 2k\pi \vee 2\alpha = \pi - \frac{\pi}{6} + 2k\pi,\ k\in\mathbb{Z} \Leftrightarrow \alpha = \frac{\pi}{12} + k\pi \vee \alpha = \frac{5\pi}{12} + k\pi,\ k\in\mathbb{Z}$

$k=0 \rightsquigarrow \alpha = \frac{\pi}{12}$ X $\vee\ \alpha = \frac{5\pi}{12}$ X

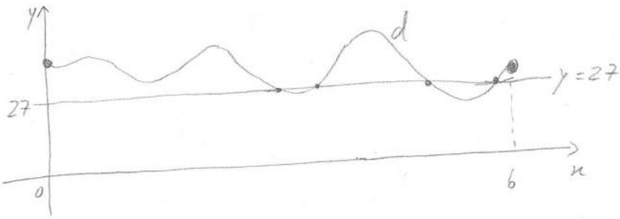
$k=1 \rightsquigarrow \alpha = \frac{13\pi}{12}$ ✓ $\vee\ \alpha = \frac{17\pi}{12}$ ✓

$k=2 \rightsquigarrow \alpha = \frac{25\pi}{12}$ X $\vee\ \alpha = \frac{29\pi}{12}$ X

$S = \left\{\frac{13\pi}{12}, \frac{17\pi}{12}\right\}$

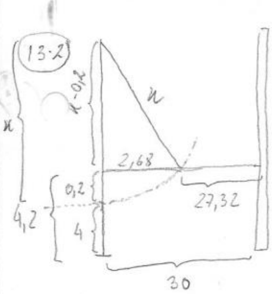
(13) $d(t) = \begin{cases} 30 + t\operatorname{sen}(\pi t) & \text{se } 0 \leq t < 12 \\ 30 + 12e^{12-t}\operatorname{sen}(\pi t) & \text{se } t \geq 12 \end{cases}$

(13.1)



N.º de soluções: 4
Nos primeiros 6 segundos, a cadeira esteve 4 vezes a 27dm do muro

(13.2)



$d(0) = 30$

$d(13{,}5) = 30 + 12e^{12-13{,}5}\operatorname{sen}(13{,}5\pi) \approx 27{,}32$

$30 - 27{,}32 = 2{,}68$

$x^2 = (x-0{,}2)^2 + 2{,}68^2 \Leftrightarrow \cancel{x^2} = \cancel{x^2} - 0{,}4x + 0{,}04 + 7{,}1824 \Leftrightarrow$

$\Leftrightarrow 0{,}4x = 7{,}2224 \Leftrightarrow x = \frac{7{,}2224}{0{,}4} \Leftrightarrow x \approx 18$

O Comprimento da haste é 18.

(14) $\arccos x: [-1, 1] \longrightarrow [0, \pi]$

$\arccos(2x): \left[-\frac{1}{2}, \frac{1}{2}\right] \longrightarrow [0, \pi]$

$\arccos(-2x): \left[-\frac{1}{2}, \frac{1}{2}\right] \longrightarrow [0, \pi]$

$1 + \arccos(-2x): \underbrace{\left[-\frac{1}{2}, \frac{1}{2}\right]}_{D} \longrightarrow \underbrace{[1, 1+\pi]}_{D'}$ (A)

(15) $f = \frac{\frac{\pi}{3}}{2\pi} = \frac{\pi}{6\pi} = \frac{1}{6}$ (D)

(16) a) • $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} \frac{2x-2}{\text{sen}(x-1)} = \boxed{\begin{array}{l} x-1 = y \to 0^- \\ x = y+1 \end{array}} \lim_{y \to 0^-} \frac{2(y+1)-2}{\text{sen}\, y} = \lim_{y \to 0^-} \frac{2y+2-2}{\text{sen}\, y} =$

$= \lim_{y \to 0^-} \frac{2y}{\text{sen}\, y} = \lim_{y \to 0^-} \frac{2}{\underbrace{\frac{\text{sen}\, y}{y}}_{\text{Lim. } \to 1}} = \frac{2}{1} = 2$

• $\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} \left(e^{-2x+4} + \ln(x-1)\right) = e^2 + \ln(0^+) = e^2 + (-\infty) = -\infty$

• $f(1) = 2$

Como $f(1) = \lim_{x \to 1^-} f(x) \neq \lim_{x \to 1^+} f(x)$, conclui-se que

f é cont. à esquerda no ponto 1 mas não é cont. à direita nesse ponto.

A afirmação é verdadeira

(16.b) $f'(x) = \frac{(2x-2)' \text{sen}(x-1) - (2x-2)(\text{sen}(x-1))'}{\text{sen}^2(x-1)} = \frac{2\,\text{sen}(x-1) - (2x-2)\cos(x-1)}{\text{sen}^2(x-1)}$

$m = f'\left(1 - \frac{\pi}{2}\right) = \frac{2\,\text{sen}\left(-\frac{\pi}{2}\right) - (2-\pi-2)\cos\left(-\frac{\pi}{2}\right)}{\text{sen}^2\left(-\frac{\pi}{2}\right)} = \frac{2 \times (-1) - (-\pi) \times 0}{(-1)^2} = -2$

$f\left(1 - \frac{\pi}{2}\right) = \frac{2 - \pi - 2}{\text{sen}\left(-\frac{\pi}{2}\right)} = \frac{-\pi}{-1} = \pi$

$P\left(1 - \frac{\pi}{2}, \pi\right) \to y = -2x + b \Leftrightarrow \pi = -2\left(1 - \frac{\pi}{2}\right) + b \Leftrightarrow$

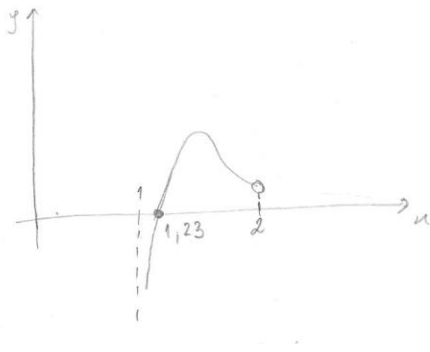
$\Leftrightarrow \pi = -2 + \pi + b \Leftrightarrow b = 2$

$y = -2x + 2$

(16.c) em $]1,2[$, $f(x) = e^{-2x+4} + \ln(x-1)$

$f'(x) = (-2x+4)' e^{-2x+4} + \frac{(x-1)'}{x-1} = -2e^{-2x+4} + \frac{1}{x-1}$

$f''(x) = -2(-2x+4)' e^{-2x+4} + \frac{1'(x-1) - 1(x-1)'}{(x-1)^2} = 4e^{-2x+4} - \frac{1}{(x-1)^2}$



$f''(x) = 0$

$x \approx 1{,}23$

(17) $\overline{CD} = tg\alpha$
$\overline{OB} = sen(2\alpha)$ } altura $= tg\alpha - sen(2\alpha)$

base $= \overline{AB} = -\cos(2\alpha)$

$$A = \frac{-\cos(2\alpha)\times(tg\alpha - sen(2\alpha))}{2} = \frac{-\cos(2\alpha)\,tg\alpha\left(1-\frac{sen(2\alpha)}{tg\alpha}\right)}{2}$$

$$= \frac{-\cos(2\alpha)\,tg\alpha\cdot\left(1-\frac{2\,sen\alpha\cos\alpha}{\frac{sen\alpha}{\cos\alpha}}\right)}{2} = \frac{-\cos(2\alpha)\,tg\alpha\left(1-2\cos^2\alpha\right)}{2}$$

$$= \frac{-\cos(2\alpha)\,tg\alpha\left(sen^2\alpha+\cos^2\alpha-2\cos^2\alpha\right)}{2} = \frac{-\cos(2\alpha)\,tg\alpha\left(sen^2\alpha-\cos^2\alpha\right)}{2}$$

$$= \frac{-\cos(2\alpha)\,tg\alpha\left(-\cos(2\alpha)\right)}{2} = \frac{\cos^2(2\alpha)\,tg\alpha}{2}$$ c.q.d.

(18) $arcsen(1) + arccos\left(-\frac{1}{2}\right) = \frac{\pi}{2} + \frac{2\pi}{3} = \frac{7\pi}{6}$ (A)

(19.a) $\frac{e^{2x}-1}{4x} = 0 \wedge x < 0$

$e^{2x}-1 = 0 \wedge 4x \neq 0 \wedge x < 0$

$e^{2x} = 1 \wedge x \neq 0 \wedge x < 0$

$2x = 0 \wedge x \neq 0 \wedge x < 0$

$x = 0 \wedge x \neq 0 \wedge x < 0$

para $x < 0$, não tem zeros

$\underbrace{\frac{1}{2-sen(2x)} = 0}_{\text{imp.}} \wedge 0 \leq x \leq \pi$

Assim, g não tem zeros (A)

(19.b) g cont. p/ $x=0 \Leftrightarrow g(0) = \lim_{x\to 0^-} g(x) = \lim_{x\to 0^+} g(x)$

$g(0) = \frac{1}{2-sen(2\times 0)} = \frac{1}{2-sen\,0} = \frac{1}{2-0} = \frac{1}{2}$

$\lim_{x\to 0^+} g(x) = \frac{1}{2-sen(2\times 0)} = \frac{1}{2}$

$\lim_{x\to 0^-} g(x) = \lim_{x\to 0^-}\frac{e^{2x}-1}{4x} = \lim_{y\to 0^-}\frac{e^{y}-1}{4\times\frac{y}{2}} = \lim_{y\to 0^-}\frac{e^{y}-1}{2y} = \frac{1}{2}\times 1 = \frac{1}{2}$

$\boxed{\begin{array}{l} 2x = y \to 0^- \\ x = \frac{y}{2}\end{array}}$

L.N. $\to 1$

Como $g(0) = \lim_{x\to 0^-} g(x) = \lim_{x\to 0^+} g(x)$, então g é cont. no ponto 0.

(19.c) em $]0,\pi]$ $g(x) = \dfrac{1}{2-\operatorname{sen}(2x)}$

$$g'(x) = \frac{1'(2-\operatorname{sen}(2x)) - 1\times(2-\operatorname{sen}(2x))'}{(2-\operatorname{sen}(2x))^2} = \frac{0-(0-(2x)'\cos(2x))}{(2-\operatorname{sen}(2x))^2} =$$

$$= \frac{2\cos(2x)}{(2-\operatorname{sen}(2x))^2}$$

Zeros de g':

$2\cos(2x) = 0 \wedge (2-\operatorname{sen}(2x))^2 \neq 0$

$\Leftrightarrow \cos(2x) = 0 \wedge \underbrace{\operatorname{sen}(2x) \neq 2}_{\text{Condição universal}}$

$\Leftrightarrow 2x = \frac{\pi}{2} + K\pi,\ K\in\mathbb{Z}$

$\Leftrightarrow x = \frac{\pi}{4} + \frac{K\pi}{2},\ K\in\mathbb{Z}$

$K=0 \hookrightarrow x = \frac{\pi}{4} \in]0,\pi]$

$K=1 \hookrightarrow x = \frac{3\pi}{4} \in]0,\pi]$

$K=2 \hookrightarrow x = \frac{5\pi}{4} \notin]0,\pi]$

$K=-1 \hookrightarrow x = -\frac{\pi}{4} \notin]0,\pi]$

	0		$\frac{\pi}{4}$		$\frac{3\pi}{4}$		π
g'		$+$	0	$-$	0	$+$	$+$
g		$\nearrow$	max	$\searrow$	mín	$\nearrow$	max

$g\left(\frac{\pi}{4}\right) = \dfrac{1}{2-\operatorname{sen}\left(\frac{\pi}{2}\right)} = \dfrac{1}{2-1} = 1$

$g\left(\frac{3\pi}{4}\right) = \dfrac{1}{2-\operatorname{sen}\left(\frac{3\pi}{2}\right)} = \dfrac{1}{2+1} = \dfrac{1}{3}$

$g(\pi) = \dfrac{1}{2-\operatorname{sen}(2\pi)} = \dfrac{1}{2-0} = \dfrac{1}{2}$

g crescente em $]0,\frac{\pi}{4}]$ e em $[\frac{3\pi}{4},\pi]$

g decrescente em $[\frac{\pi}{4},\frac{3\pi}{4}]$

1 é max relat p/ $x = \frac{\pi}{4}$

$\frac{1}{2}$ é " " " $x = \pi$

$\frac{1}{3}$ é mín. " " $x = \frac{3\pi}{4}$

(20) $\lim\limits_{x\to\pi} \dfrac{x}{\operatorname{sen} x} = \dfrac{\pi}{0} = \infty$ logo a reta de equação $x=\pi$ é A.V.
(B)

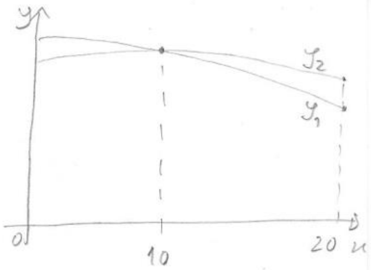
(21) $\hat{C} = 180 - 81 - 57 = 42°$

$\dfrac{\operatorname{sen} 42°}{\overline{AB}} = \dfrac{\operatorname{sen} 81°}{5} \Leftrightarrow \overline{AB} = \dfrac{5\times\operatorname{sen}42°}{\operatorname{sen}81°} \Leftrightarrow \overline{AB} \approx 3{,}39$ (C)

(22) $0° < \alpha < 20°$

$$d(3\alpha) = 0{,}97\, d(\alpha)$$

$$\underbrace{\frac{555}{10 - 2{,}06\cos(3\alpha)}}_{y_1} = \underbrace{0{,}97 \times \frac{555}{10 - 2{,}06\cos\alpha}}_{y_2}$$



R: $\alpha \approx 10°$

(23) $f''(x) = (3x)' - (tg\, x)' = 3 - \frac{x'}{\cos^2 x} = 3 - \frac{1}{\cos^2 x}$

$f''(x) = 0 \Leftrightarrow 3 = \frac{1}{\cos^2 x} \Leftrightarrow \cos^2 x = \frac{1}{3} \Leftrightarrow \cos x = \pm\sqrt{\frac{1}{3}} \Leftrightarrow \underset{x \in]0, \frac{\pi}{2}[}{\cos x = \frac{\sqrt{3}}{3}}$

$\Leftrightarrow x = arc\cos\left(\frac{\sqrt{3}}{3}\right) \Leftrightarrow x \approx 0{,}96$ (D)

(24) $g'(x) = 2\cos x + 2\, sen\, x \cos x$

Pretende-se o máximo de g'

$g''(x) = -2\, sen\, x + 2(\cos x \cos x + sen\, x(-sen\, x)) =$

$= -2\, sen\, x + 2(\cos^2 x - sen^2 x)$

$= -2\, sen\, x + 2\cos^2 x - 2\, sen^2 x =$

$= -2\, sen\, x + 2(1 - sen^2 x) - 2\, sen^2 x =$

$= -2\, sen\, x + 2 - 2\, sen^2 x - 2\, sen^2 x =$

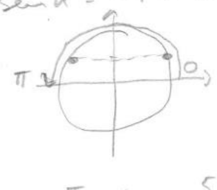
$= -4\, sen^2 x - 2\, sen\, x + 2$

Zeros de g'': $4\, sen^2 x + 2\, sen\, x - 2 = 0$

$\Rightarrow sen\, x = \frac{-2 \pm \sqrt{4 - 4 \times 4 \times (-2)}}{2 \times 4}$

(...)

$\Leftrightarrow sen\, x = -1 \vee sen\, x = \frac{1}{2}$



$\Leftrightarrow x = \frac{\pi}{6} \vee x = \frac{5\pi}{6}$ em $[0, \pi]$

	0		$\frac{\pi}{6}$		$\frac{5\pi}{6}$		π
g''	+	+	0	−	0	+	+
g'	mín	↗	máx	↘	mín	↗	máx

$g'\left(\frac{\pi}{6}\right) = 2\cos\left(\frac{\pi}{6}\right) + 2\, sen\left(\frac{\pi}{6}\right)\cos\left(\frac{\pi}{6}\right) =$

$= 2 \times \frac{\sqrt{3}}{2} + 2 \times \frac{1}{2} \times \frac{\sqrt{3}}{2} = \sqrt{3} + \frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{2}$

$g'(\pi) = 2\cos\pi + 2\, sen\,\pi \cos\pi =$

$= 2 \times (-1) + 2 \times 0 \times (-1) = -2$

Assim, o declive máximo da reta é $\frac{3\sqrt{3}}{2}$

(25) $\operatorname{sen}(\pi t) + \cos(\pi t) = \sqrt{2} \times \frac{\sqrt{2}}{2}\left(\operatorname{sen}(\pi t) + \cos(\pi t)\right) =$

$= \sqrt{2}\left(\frac{\sqrt{2}}{2}\operatorname{sen}(\pi t) + \frac{\sqrt{2}}{2}\cos(\pi t)\right) =$

$= \sqrt{2}\left(\cos\left(\frac{\pi}{4}\right)\operatorname{sen}(\pi t) + \operatorname{sen}\left(\frac{\pi}{4}\right)\cos(\pi t)\right) =$

$= \sqrt{2}\operatorname{sen}\left(\pi t + \frac{\pi}{4}\right) = \sqrt{2}\cos\left(\pi t + \frac{\pi}{4} - \frac{\pi}{2}\right) = \sqrt{2}\cos\left(\pi t - \frac{\pi}{4}\right)$

$= \sqrt{2}\cos\left(\pi t - \frac{\pi}{4} + 2\pi\right) = \sqrt{2}\cos\left(\pi t + \frac{7\pi}{4}\right)$

↳ expressa de forma $x(t) = A\cos(\omega t + \varphi)$

Assim, amplitude A do oscilador é igual a $\sqrt{2}$ (B)

Em alternativa, recorrer à calculadora gráfica, visualizar o gráfico e concluir

(26)

a) h cont p/ $x=0 \Leftrightarrow h(0) = \lim_{x\to 0^-} h(x) = \lim_{x\to 0^+} h(x)$

$h(0) = \frac{e^0}{0+1} = \frac{1}{1} = 1$

$\lim_{x\to 0^-} h(x) = \lim_{x\to 0^-} \frac{\operatorname{sen}^2 x}{\operatorname{sen}(x^2)} \overset{\left(\frac{0}{0}\right)}{=} \lim_{x\to 0^-} \frac{\overbrace{\frac{\operatorname{sen} x}{x}}^{\text{L.N.} \to 1} \times \overbrace{\frac{\operatorname{sen} x}{x}}^{\text{L.N.} \to 1}}{\frac{\operatorname{sen}(x^2)}{x^2}} = \frac{1 \times 1}{1} = 1$

Nota: $\lim_{x\to 0^-} \frac{\operatorname{sen}(x^2)}{x^2} = \lim_{y\to 0} \frac{\operatorname{sen} y}{y} \overset{\text{L.N.}}{=} 1$ $\boxed{x^2 = y \to 0}$

$\lim_{x\to 0^+} h(x) = \lim_{x\to 0^+} \frac{e^x}{x+1} = \frac{e^0}{0+1} = \frac{1}{1} = 1$

Assim, como $h(0) = \lim_{x\to 0^-} h(x) = \lim_{x\to 0^+} h(x)$, h é cont no ponto 0

(26.b) h é cont em $\left[-\frac{\pi}{3}, +\infty\right[$. Assim, a única A.V. possível seria a reta de equação $x = -\frac{\pi}{3}$. Ora $\lim_{x\to -\frac{\pi}{3}^+} h(x) = \frac{\operatorname{sen}^2\left(-\frac{\pi}{3}\right)}{\operatorname{sen}\left(\left(-\frac{\pi}{3}\right)^2\right)} \neq \infty$, logo

h não tem A.V.

A.N.V.?

$m = \lim_{x\to +\infty} \frac{\frac{e^x}{x+1}}{x} = \lim_{x\to +\infty} \frac{e^x}{x^2+x} = \lim_{x\to +\infty} \frac{\frac{e^x}{x^2}}{\frac{x^2}{x^2} + \frac{x}{x^2}} = \lim_{x\to +\infty} \frac{\overbrace{\frac{e^x}{x^2}}^{\text{L.N.} \to +\infty}}{1 + \frac{1}{x}}$

$= \frac{+\infty}{1 + \frac{1}{+\infty}} = \frac{+\infty}{1+0} = +\infty$ logo não há A.N.V.

Assim, não há qq assíntota.

(26.c) Em $[0,+\infty[$, $h(x)=\frac{e^x}{x+1}$

$$h'(x)=\frac{(e^x)'(x+1)-e^x(x+1)'}{(x+1)^2}=\frac{e^x(x+1)-e^x}{(x+1)^2}$$

Zeros de h':

$e^x(x+1-1)=0 \wedge (x+1)^2\neq 0$

$\Leftrightarrow \underbrace{e^x=0}_{\text{imp.}} \vee x=0 \wedge x+1\neq 0$

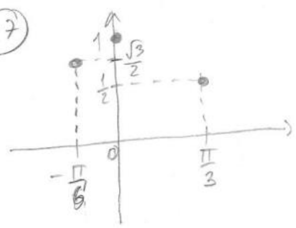
$\Leftrightarrow x=0 \wedge x\neq -1$

$\Leftrightarrow x=0$

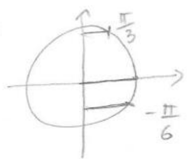
	0		$+\infty$
h'	0	+	
h	min	↗	

(B)

(27)

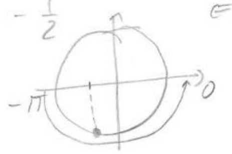


$D'=\left[\frac{1}{2},1\right]$



(C)

(28) $2\cos x+1=0 \Leftrightarrow \cos x=-\frac{1}{2} \Leftrightarrow x=-\frac{2\pi}{3}$ em $[-\pi,0]$



(B)

(29) $P=\frac{2\pi}{b}=\frac{2\pi}{\pi}=2$ (A)

(30.a) Para $x=1$, $f(x)=\frac{x}{x-\ln x}$

$$f'(x)=\frac{x'(x-\ln x)-x(x-\ln x)'}{(x-\ln x)^2}=\frac{x-\ln x-x\left(1-\frac{1}{x}\right)}{(x-\ln x)^2}=\frac{1-\ln x}{(x-\ln x)^2}$$

$$m=f'(1)=\frac{1-\ln 1}{(1-\ln 1)^2}=\frac{1-0}{(1-0)^2}=1$$

$$f(1)=\frac{1}{1-\ln 1}=\frac{1}{1-0}=1$$

$P(1,1) \hookrightarrow y=x+b$

$1=1+b$

$b=0$

Logo, $y=x$.

30.b) f cont p/ $x=0 \Leftrightarrow f(0) = \lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x)$

- $f(0) = 0$
- $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \frac{1-\cos x}{x} \left(\frac{0}{0}\right) = \lim_{x\to 0^-} \frac{(1-\cos x)(1+\cos x)}{x(1+\cos x)} = \lim_{x\to 0^-} \frac{1-\cos^2 x}{x(1+\cos x)}$

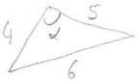
$= \lim_{x\to 0^-} \frac{\operatorname{sen}^2 x}{x(1+\cos x)} = \lim_{x\to 0^-} \left(\underbrace{\frac{\operatorname{sen} x}{x}}_{1 \text{ L.N.}} \times \frac{\operatorname{sen} x}{1+\cos x} \right) = 1 \times \frac{\operatorname{sen} 0}{1+\cos 0} =$

$= 1 \times \frac{0}{1+1} = 1 \times 0 = 0$

- $\lim_{x\to 0^+} \frac{x}{x - \ln x} = \frac{0^+}{0^+ - \ln 0^+} = \frac{0^+}{0^+ - (-\infty)} = \frac{0^+}{+\infty} = 0$

Assim, f é cont. no ponto 0

31)



$6^2 = 4^2 + 5^2 - 2\times 4\times 5 \times \cos\alpha \Leftrightarrow 36 = 16 + 25 - 40\cos\alpha$

$\Leftrightarrow \cos\alpha = \frac{5}{40} \Leftrightarrow \cos\alpha = \frac{1}{8}$

$\operatorname{sen}^2\alpha = 1 - \left(\frac{1}{8}\right)^2 = \frac{63}{64} \Leftrightarrow \operatorname{sen}\alpha = \sqrt{\frac{63}{64}} \approx 0{,}992$ (B)

32) $\operatorname{sen}\left(3 \arccos \frac{1}{2}\right) = \operatorname{sen}\left(3 \times \frac{\pi}{3}\right) = \operatorname{sen}\pi = 0$ (C)

33.a) $g'(x) = -\frac{1}{4} \times 2\operatorname{sen}(2x) + \operatorname{sen} x = -\frac{1}{2}\operatorname{sen}(2x) + \operatorname{sen} x$

$g''(x) = -\frac{1}{2} \times 2\cos(2x) + \cos x = -\cos(2x) + \cos x$

Zeros de g''

$-\cos(2x) = -\cos x \Leftrightarrow$

$\Leftrightarrow \cos(2x) = \cos x$

$\Leftrightarrow 2x = x + 2k\pi \lor 2x = -x + 2k\pi, k\in\mathbb{Z}$

$\Leftrightarrow x = 2k\pi \lor 3x = 2k\pi, k\in\mathbb{Z}$

$\Leftrightarrow x = 2k\pi \lor x = \frac{2k\pi}{3}$

$k=0 \to x = 0 \lor x = 0$

$k=+1 \to x = 2\pi \lor x = \frac{2\pi}{3}$

$k=-1 \to x = -2\pi \lor x = -\frac{2\pi}{3}$

	0		$\frac{2\pi}{3}$		π
g''		+	0	−	
g		∪	P.I	∩	

$g\left(\frac{2\pi}{3}\right) = \frac{1}{4}\cos\left(\frac{4\pi}{3}\right) - \cos\left(\frac{2\pi}{3}\right)$

$= \frac{1}{4} \times \left(-\frac{1}{2}\right) - \left(-\frac{1}{2}\right) = -\frac{1}{8} + \frac{1}{2} = \frac{3}{8}$

Concav. voltada p/ baixo em $\left[\frac{2\pi}{3}, \pi\right[$

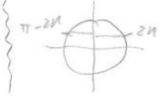
" " " cima em $\left]0, \frac{2\pi}{3}\right]$

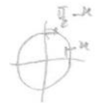
P.I $\to \left(\frac{2\pi}{3}, \frac{3}{8}\right)$

33.b $g(-x) = \frac{1}{4}\cos(-2x) - \cos(-x) = \frac{1}{4}\cos(2x) - \cos x$

$g\left(\frac{\pi}{2}-x\right) = \frac{1}{4}\cos\left[2\left(\frac{\pi}{2}-x\right)\right] - \cos\left(\frac{\pi}{2}-x\right) = \frac{1}{4}\underbrace{\cos(\pi-2x)}_{-\cos(2x)} - \underbrace{\cos\left(\frac{\pi}{2}-x\right)}_{\text{sen}\, x}$

$= -\frac{1}{4}\cos(2x) - \text{sen}\, x$

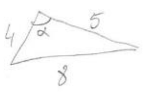




$g(-x)+g\left(\frac{\pi}{2}-x\right) = \frac{1}{4}\cancel{\cos(2x)} - \cos x - \frac{1}{4}\cancel{\cos(2x)} - \text{sen}\, x$

$= -\cos x - \text{sen}\, x$ (B)

34



$8^2 = 4^2+5^2-2\times4\times5\cos\alpha \Leftrightarrow 64 = 16+25-40\cos\alpha \Leftrightarrow$

$\Leftrightarrow \cos\alpha = -\frac{23}{40} \Leftrightarrow \alpha = \arccos\left(-\frac{23}{40}\right) \Leftrightarrow \alpha \approx 125^\circ$ (D)

35.a

a) $f(\pi-x) = \frac{\text{sen}(\pi-x)}{2+\cos(\pi-x)} = \frac{\text{sen}\, x}{2-\cos x}$

$\lim\limits_{x\to 0} \frac{\text{sen}\, x}{x(2-\cos x)} = \lim\limits_{x\to 0}\left(\underbrace{\frac{\text{sen}\, x}{x}}_{1\ \text{L.N.}} \times \frac{1}{2-\cos x}\right) = 1\times\frac{1}{2-\cos 0} = 1\times\frac{1}{2-1} = 1\times1 = 1$

35.b $f'(x) = \frac{(\text{sen}\, x)'(2+\cos x) - \text{sen}\, x(2+\cos x)'}{(2+\cos x)^2} = \frac{\cos x(2+\cos x) - \text{sen}\, x(-\text{sen}\, x)}{(2+\cos x)^2}$

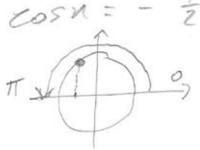
Zeros de f'

$\cos x(2+\cos x) + \text{sen}^2 x = 0 \wedge (2+\cos x)^2 \neq 0$

$\Leftrightarrow 2\cos x + \cancel{\cos^2 x} + 1 - \cancel{\cos^2 x} = 0 \wedge 2+\cos x \neq 0$

$\wedge \underbrace{\cos x \neq -2}_{\text{cond. univ.}}$

$\Leftrightarrow \cos x = -\frac{1}{2}$



$\Leftrightarrow x = \frac{2\pi}{3}$ em $]0,\pi[$

	0		$\frac{2\pi}{3}$		π
f'		+	0	−	
f		↗	máx	↘	

$f\left(\frac{2\pi}{3}\right) = \frac{\text{sen}\left(\frac{2\pi}{3}\right)}{2+\cos\left(\frac{2\pi}{3}\right)} = \frac{\frac{\sqrt{3}}{2}}{2+\left(-\frac{1}{2}\right)} =$

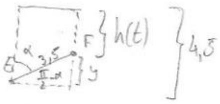
$= \frac{\frac{\sqrt{3}}{2}}{\frac{3}{2}} = \frac{\sqrt{3}}{3}$

f crescente em $\left]0, \frac{2\pi}{3}\right]$

f decrescente em $\left[\frac{2\pi}{3}, \pi\right[$

máx. relativo $= \frac{\sqrt{3}}{3}$ para $x = \frac{2\pi}{3}$

36

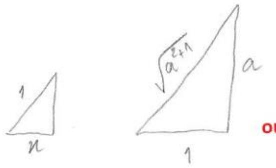


$sen\left(\frac{\pi}{2}-\alpha\right) = \frac{y}{3,5} \Leftrightarrow y = 3,5\, sen\left(\frac{\pi}{2}-\alpha\right) \Leftrightarrow y = 3,5\cos\alpha = 3,5\cos\left(\frac{\pi}{6}t\right)$

1 hora — $\frac{\pi}{6}$ rad
t horas — α rad $\Rightarrow \alpha = \frac{\pi}{6}t$

Assim $h(t) = 4,5 - 3,5\cos\left(\frac{\pi}{6}t\right)$

37



$1 — \sqrt{a^2+1}$
$x — 1$ $\Leftrightarrow x = \frac{1}{\sqrt{a^2+1}}$ (A)

ou, usando fórmulas trigonométricas....

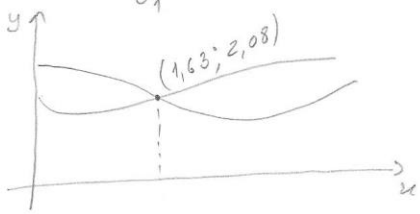
$tg\,\alpha = a$ $\quad 1+tg^2\alpha = \frac{1}{\cos^2\alpha}$ $(\ldots)$ $x = \cos\alpha =$

38.a $d(0) - 1 = \cos 0 + \sqrt{9-(sen\, 0)^2} - 1 = 1 + \sqrt{9-0} - 1 = 3$ (B)

38.b

$d(t+2) = 0,75\, d(t) \Leftrightarrow$

$\Leftrightarrow \underbrace{\cos(t+2) + \sqrt{9 - sen^2(t+2)}}_{y_1} = \underbrace{0,75\left(\cos t + \sqrt{9 - sen^2 t}\right)}_{y_2}$



$d(1,63) = \cos(1,63) + \sqrt{9 - sen^2(1,63)}$
$\approx 2,8$
R: 2,8 cm

39.a g é cont em $x=0 \Leftrightarrow g(0) = \lim_{x\to 0^-} g(x) = \lim_{x\to 0^+} g(x)$

• $g(0) = 0$

• $\lim_{x\to 0^-} g(x) = \lim_{x\to 0^-}\left(1 + \frac{sen\, x}{1-e^x}\right) = 1 + \lim_{x\to 0^-}\frac{\frac{sen\, x}{x}}{\frac{1-e^x}{x}} = 1 + \lim_{x\to 0^-}\frac{\overbrace{\frac{sen\, x}{x}}^{L.N.\ 1}}{-\underbrace{\frac{e^x-1}{x}}_{L.N.\ 1}} = 1 + \frac{1}{-1} = 0$

• $\lim_{x\to 0^+} g(x) = \lim_{x\to 0^+}(x^2 \ln x) = \lim_{x\to 0^+}\frac{\ln x}{\frac{1}{x^2}} = \lim_{y\to +\infty}\frac{\ln\left(\frac{1}{y}\right)^{\frac{1}{2}}}{y} = \lim_{y\to +\infty}\left(\frac{1}{2}\times\frac{\ln 1 - \ln y}{y}\right)$

$\boxed{\begin{array}{l}\frac{1}{x^2} = y \to +\infty \\ x^2 = \frac{1}{y} \\ x = \left(\frac{1}{y}\right)^{\frac{1}{2}}\end{array}}$ $= \lim_{y\to +\infty}\left(\frac{1}{2}\times\frac{-\ln y}{y}\right) =$

$= -\frac{1}{2}\times\lim_{y\to +\infty}\underbrace{\frac{\ln y}{y}}_{L.N.\ 0} = -\frac{1}{2}\times 0 = 0$

Assim, como $g(0) = \lim_{x\to 0^-} g(x) = \lim_{x\to 0^+} g(x)$, conclui-se que g é cont em $x=0$

(39.b) Em $]0,+\infty[$, $g(x) = x^2 \ln x$

$g'(x) = (x^2)' \ln x + x^2 (\ln x)' = 2x \ln x + x^2 \times \dfrac{x'}{x} = 2x \ln x + \dfrac{x^2}{x} \underset{x \neq 0}{=}$

$= 2x \ln x + x$

Zeros de g':

$2x \ln x + x = 0$

$\Leftrightarrow x(2\ln x + 1) = 0$

$\Leftrightarrow x = 0 \lor 2\ln x = -1$

$\Leftrightarrow x = 0 \lor \ln x = -\frac{1}{2}$

$\Leftrightarrow x = 0 \lor x = e^{-\frac{1}{2}}$

$\Leftrightarrow x = 0 \lor x = \dfrac{1}{\sqrt{e}}$

	0		$e^{-\frac{1}{2}}$	$+\infty$
x		$+$	$+$	$+$
$2\ln x+1$		$-$	0	$+$
g'		$-$	0	$+$
g		$\searrow$	mín	$\nearrow$

$g\left(e^{-\frac{1}{2}}\right) = \left(e^{-\frac{1}{2}}\right)^2 \times \ln\left(e^{-\frac{1}{2}}\right) = e^{-1} \times \left(-\frac{1}{2}\right) = -\frac{1}{2e}$

decrescente em $\left]0, \frac{1}{\sqrt{e}}\right]$

crescente em $\left[\frac{1}{\sqrt{e}}, +\infty\right[$

mín. relat $= -\frac{1}{2e}$ para $x = \frac{1}{\sqrt{e}}$

(40.a) $(f \circ g)(x) = f(g(x)) = f(\cos x) = \cos^2 x$

$(f \circ g)'(x) = 2\cos x (\cos x)' = 2\cos x(-\operatorname{sen} x) = -2 \operatorname{sen} x \cos x =$

$= -\operatorname{sen}(2x)$

$m = (f \circ g)'\left(\frac{\pi}{4}\right) = -\operatorname{sen}\left(2 \times \frac{\pi}{4}\right) = -\operatorname{sen}\left(\frac{\pi}{2}\right) = -1$ (B)

(40.b) $f(x) = g(x) \Leftrightarrow f(x) - g(x) = 0$

Seja $h(x) = f(x) - g(x) = x^2 - \cos x$

$h(0) = 0^2 - \cos 0 = 0 - 1 = -1 < 0$

$h\left(\frac{\pi}{3}\right) = \left(\frac{\pi}{3}\right)^2 - \cos\left(\frac{\pi}{3}\right) = \frac{\pi^2}{9} - \frac{1}{2} > 0$

h é cont em $\left[0, \frac{\pi}{3}\right]$

Pelo Teorema de Bolzano-Cauchy, conclui-se que $h(x) = 0$ tem, pelo menos, uma solução em $\left]0, \frac{\pi}{3}\right[$, isto é, $f(x) = g(x)$ tem, pelo menos, uma solução no intervalo $\left]0, \frac{\pi}{3}\right[$ c.q.d.

(41.a) $r = \frac{3}{5} R$

Percentagem pedida $= 50\left(1 - \sqrt{1 - \left(\dfrac{\frac{3}{5}\cancel{R}}{\cancel{R}}\right)^2}\right) = 50\left(1 - \sqrt{1 - \left(\frac{3}{5}\right)^2}\right) =$

$= 50\left(1 - \sqrt{1 - \frac{9}{25}}\right) = 50\left(1 - \sqrt{\frac{16}{25}}\right) =$

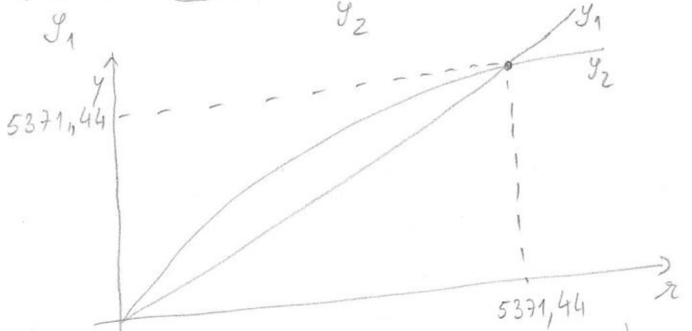
$= 50\left(1 - \frac{4}{5}\right) = 50 \times \frac{1}{5} = 10\%$ (C)

41.b) $R = 6400$

$h = r$

Substituindo na igualdade $r = \frac{R}{h+R}\sqrt{h^2+2hR}$, teremos

$$\underbrace{r}_{y_1} = \underbrace{\frac{6400}{r+6400}\sqrt{r^2+2\times 6400\, r}}_{y_2}$$



$$\text{Percentagem} = 50\left(1-\sqrt{1-\left(\frac{5371,44}{6400}\right)^2}\right) \approx 23\%$$

42) h é cont. em $x=1 \Leftrightarrow \lim\limits_{x\to 1^-} h(x) = \lim\limits_{x\to 1^+} h(x) = h(1)$

- $h(1) = 1+1e^{1-1} = 1+1 = 2$
- $\lim\limits_{x\to 1^-} h(x) = 1+1e^{1-1} = 2$
- $\lim\limits_{x\to 1^+} h(x) = \lim\limits_{x\to 1^+} \frac{\sqrt{x}-1}{\operatorname{sen}(x-1)} \overset{\left(\frac{0}{0}\right)}{=} \lim\limits_{x\to 1^+} \frac{(\sqrt{x}-1)(\sqrt{x}+1)}{\operatorname{sen}(x-1)(\sqrt{x}+1)} =$

$= \lim\limits_{x-1\to 0} \frac{x-1}{\operatorname{sen}(x-1)} \times \lim\limits_{x\to 1} \frac{1}{\sqrt{x}+1} = 1\times\frac{1}{\sqrt{1}+1} = \frac{1}{2}$

C.A.:

$$\lim\limits_{x-1\to 0} \frac{x-1}{\operatorname{sen}(x-1)} = \lim\limits_{y\to 0} \frac{y}{\operatorname{sen} y} = \lim\limits_{y\to 0} \frac{1}{\underbrace{\frac{\operatorname{sen} y}{y}}_{\to 1 \text{ L.N.}}} = \frac{1}{1} = 1$$

$\boxed{x-1=y\to 0}$

Assim, conclui-se que h não é contínua em $x=1$

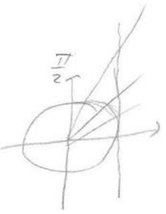
(43) Como o domínio é limitado, não tem A.N.V.

A.V

Como f é cont. para $0<x<\frac{\pi}{2}$, as A.V., caso existam, serão para $x=0^+$ ou $x=\frac{\pi}{2}^-$

$$\lim_{x\to 0^+} \frac{e^{2x}-1}{tg\, x} \overset{\left(\frac{0}{0}\right)}{=} \lim_{x\to 0^+}\left(\frac{e^{2x}-1}{sen\, x}\times \cos x\right) = \lim_{x\to 0^+} \frac{\frac{e^{2x}-1}{x}}{\frac{sen\, x}{x}} \times \lim_{x\to 0^+}(\cos x)$$

$$= \frac{\underbrace{2\lim_{2x\to 0^+}\frac{e^{2x}-1}{2x}}_{L.N.:\ 1}}{\underbrace{\lim_{x\to 0^+}\frac{sen\, x}{x}}_{L.N.\ 1}} \times \lim_{x\to 0^+}(\cos x) = \frac{2\times 1}{1}\times \cos 0 = 2\times 1 = 2$$

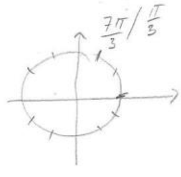


$$\lim_{x\to \frac{\pi}{2}^-} \frac{e^{2x}-1}{tg\, x} = \frac{e^{\pi}-1}{tg\left(\frac{\pi}{2}^-\right)} = \frac{e^{\pi}-1}{+\infty} = 0$$

Assim, não há A.V.

Conclusão: o gráfico de f não tem assíntotas

(44.a) $$t.m.v_{\left[\frac{\pi}{6}, \frac{7\pi}{6}\right]} = \frac{h\left(\frac{7\pi}{6}\right)-h\left(\frac{\pi}{6}\right)}{\frac{7\pi}{6}-\frac{\pi}{6}} = \frac{\frac{10}{11}-\frac{10}{11}}{\pi} = 0 \quad (C)$$



$$h\left(\frac{7\pi}{6}\right) = \frac{5}{4+3\cos\left(\frac{14\pi}{6}\right)} = \frac{5}{4+3\cos\left(\frac{7\pi}{3}\right)} = \frac{5}{4+3\times\frac{1}{2}} = \frac{10}{11}$$

$$h\left(\frac{\pi}{6}\right) = \frac{5}{4+3\cos\left(\frac{2\pi}{6}\right)} = \frac{5}{4+3\cos\frac{\pi}{3}} = \frac{5}{4+3\times\frac{1}{2}} = \frac{10}{11}$$

(44.b) $h(x)=2 \Leftrightarrow \frac{5}{4+3\cos(2x)} = \frac{2}{1} \Leftrightarrow 8+6\cos(2x)=5 \Leftrightarrow 6\cos(2x)=-3 \Leftrightarrow$

$\Leftrightarrow \cos(2x) = -\frac{1}{2} \Leftrightarrow 2x = \frac{2\pi}{3}+2k\pi \vee 2x = -\frac{2\pi}{3}+2k\pi,\ k\in\mathbb{Z} \Leftrightarrow$

$\Leftrightarrow x = \frac{\pi}{3}+k\pi \vee x = -\frac{\pi}{3}+k\pi,\ k\in\mathbb{Z}$

$k=0 \hookrightarrow x=\frac{\pi}{3}\ ✓ \vee x=-\frac{\pi}{3}\ ✓$

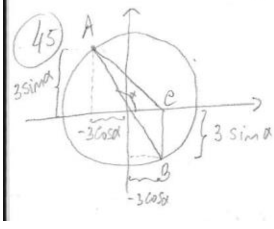
$k=1 \hookrightarrow x=\frac{4\pi}{3}\ ✗ \vee x=\frac{2\pi}{3}\ ✓$

$k=-1 \hookrightarrow x=-\frac{2\pi}{3}\ ✓ \vee x=-\frac{8\pi}{3}\ ✗$

$k=2 \hookrightarrow x=\frac{7\pi}{3}\ ✗ \vee x=\frac{5\pi}{3}\ ✗$

R: $x=-\frac{2\pi}{3};\ x=-\frac{\pi}{3};\ x=\frac{\pi}{3};\ x=\frac{2\pi}{3}$

(45)



$A(3\cos\alpha;\ 3\sin\alpha)$

$$A = \frac{b\times h}{2} = \frac{3\sin\alpha \times(-6\cos\alpha)}{2} = \frac{-18\sin\alpha\cos\alpha}{2} = -9\sin\alpha\cos\alpha$$

(46) $m = -\frac{1}{2}$

$g'(x) = -\frac{1}{2}$

seja $h(x) = g'(x) = (x\cos x)' + (\operatorname{sen} x)' = \cos x + x(-\operatorname{sen} x) + \cos x$

$h(x) = \cos x - x\operatorname{sen} x + \cos x$

$h(x) = 2\cos x - x\operatorname{sen} x$

h é cont em $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$

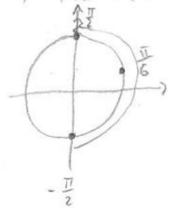
$h\left(\frac{\pi}{2}\right) = 2\cos\frac{\pi}{2} - \frac{\pi}{2}\operatorname{sen}\frac{\pi}{2} = 2\times 0 - \frac{\pi}{2}\times 1 = -\frac{\pi}{2} < -\frac{1}{2}$

$h\left(\frac{3\pi}{2}\right) = 2\cos\frac{3\pi}{2} - \frac{3\pi}{2}\operatorname{sen}\frac{3\pi}{2} = 2\times 0 - \frac{3\pi}{2}\times(-1) = \frac{3\pi}{2} > -\frac{1}{2}$

Pelo Teor. de Bolz-Cauchy, existe pelo menos um valor $x \in \left]\frac{\pi}{2}, \frac{3\pi}{2}\right[$ tal que $h(x) = -\frac{1}{2}$, isto é, $g'(x) = -\frac{1}{2}$ c.q.d.

(47) $f(x) = g(x) \Leftrightarrow K\operatorname{sen}(2x) = K\cos x \overset{K>0}{\Leftrightarrow} \operatorname{sen}(2x) = \cos x \Leftrightarrow 2\operatorname{sen} x\cos x - \cos x = 0$

$\Leftrightarrow \cos x(2\operatorname{sen} x - 1) = 0 \Leftrightarrow \cos x = 0 \vee 2\operatorname{sen} x - 1 = 0 \Leftrightarrow \cos x = 0 \vee \operatorname{sen} x = \frac{1}{2}$



Em $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, $x = -\frac{\pi}{2} \vee x = \frac{\pi}{6} \vee x = \frac{\pi}{2}$

$g\left(-\frac{\pi}{2}\right) = K\underbrace{\cos\left(-\frac{\pi}{2}\right)}_{0} = 0$ $\quad g\left(\frac{\pi}{6}\right) = K\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}K$ $\quad g\left(\frac{\pi}{2}\right) = K\overbrace{\cos\left(\frac{\pi}{2}\right)}^{0} = K\times 0 = 0$

$A\left(-\frac{\pi}{2}, 0\right)$ $\quad B\left(\frac{\pi}{6}, \frac{\sqrt{3}K}{2}\right)$ $\quad C\left(\frac{\pi}{2}, 0\right)$

$\overrightarrow{BA} = A - B = \left(-\frac{\pi}{2} - \frac{\pi}{6}, 0 - \frac{\sqrt{3}K}{2}\right) = \left(-\frac{2\pi}{3}, -\frac{\sqrt{3}K}{2}\right)$

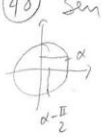
$\overrightarrow{BC} = C - B = \left(\frac{\pi}{2} - \frac{\pi}{6}, 0 - \frac{\sqrt{3}K}{2}\right) = \left(\frac{\pi}{3}, -\frac{\sqrt{3}K}{2}\right)$

Como o $\Delta[ABC]$ é retângulo em B, então $\overrightarrow{BA}\cdot\overrightarrow{BC} = 0 \Leftrightarrow$

$\Leftrightarrow \left(-\frac{2\pi}{3}, -\frac{\sqrt{3}K}{2}\right)\cdot\left(\frac{\pi}{3}, -\frac{\sqrt{3}K}{2}\right) = 0 \Leftrightarrow -\frac{2\pi^2}{9_{(4)}} + \frac{3K^2}{4_{(9)}} = 0 \Leftrightarrow -8\pi^2 + 27K^2 = 0 \Leftrightarrow$

$\Leftrightarrow 27K^2 = 8\pi^2 \Leftrightarrow K^2 = \frac{8}{27}\pi^2 \Leftrightarrow K = \pm\sqrt{\frac{8}{27}}\,\pi$. Como $K > 0$, $K = \sqrt{\frac{8}{27}}\,\pi$

(48) $\operatorname{sen}\left(\alpha - \frac{\pi}{2}\right) = -\frac{1}{5}$

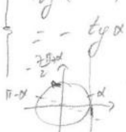


$\Leftrightarrow -\cos\alpha = -\frac{1}{5}$

$\Leftrightarrow \cos\alpha = \frac{1}{5}$

$\operatorname{tg}(\pi - \alpha) + 2\cos\left(-\frac{7\pi}{2} + \alpha\right) =$

$= -\operatorname{tg}\alpha - 2\operatorname{sen}\alpha =$



$= -\sqrt{24} - 2\times\frac{\sqrt{24}}{5} =$

$= -\frac{7\sqrt{24}}{5}$

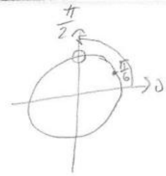
$\operatorname{sen}^2\alpha + \cos^2\alpha = 1 \Leftrightarrow \operatorname{sen}^2\alpha + \left(\frac{1}{5}\right)^2 = 1$

$\Leftrightarrow \operatorname{sen}^2\alpha + \frac{1}{25} = 1 \Leftrightarrow \operatorname{sen}^2\alpha = \frac{24}{25}$

$\Leftrightarrow \operatorname{sen}\alpha = \pm\frac{\sqrt{24}}{5}$. Como $\alpha \in \left]0, \frac{\pi}{2}\right[$ então $\operatorname{sen}\alpha = \frac{\sqrt{24}}{5}$

$\operatorname{tg}\alpha = \frac{\operatorname{sen}\alpha}{\cos\alpha} = \frac{\frac{\sqrt{24}}{5}}{\frac{1}{5}} = \sqrt{24}$

(49) $h'(x) = \cos x + 2\cos x(-\operatorname{sen} x) = \cos x - 2\operatorname{sen} x \cos x$



$h'(x) = 0 \Leftrightarrow \cos x(1 - 2\operatorname{sen} x) = 0 \Leftrightarrow \cos x = 0 \vee \operatorname{sen} x = \frac{1}{2}$

$\Leftrightarrow x = \frac{\pi}{6}$ em $[0, \frac{\pi}{2}[$

	0		$\frac{\pi}{6}$		$\frac{\pi}{2}$
h'	$+$	$+$	0	$-$	
h	mín	$\nearrow$	máx	$\searrow$	

$h(0) = \operatorname{sen} 0 + \cos^2 0 = 0 + 1^2 = 1$

$h(\frac{\pi}{6}) = \operatorname{sen}(\frac{\pi}{6}) + \cos^2(\frac{\pi}{6}) = \frac{1}{2} + \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{2} + \frac{3}{4} = \frac{5}{4}$

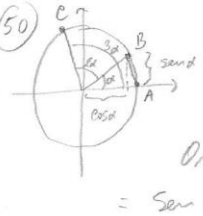
h é crescente em $[0, \frac{\pi}{6}]$

h é decrescente em $[\frac{\pi}{6}, \frac{\pi}{2}[$

1 é mínimo relat. para $x = 0$

$\frac{5}{4}$ é máximo relat. para $x = \frac{\pi}{6}$

(50)



$A = \frac{b \times h}{2} = \frac{1 \times \operatorname{sen}\alpha}{2} = \frac{\operatorname{sen}\alpha}{2}$

$\frac{\operatorname{sen}\alpha}{2} = K \Leftrightarrow \boxed{\operatorname{sen}\alpha = 2K}$

Ordenada de $C = \operatorname{sen}(3\alpha) = \operatorname{sen}(\alpha + 2\alpha) =$

$= \operatorname{sen}\alpha \cos(2\alpha) + \cos\alpha \operatorname{sen}(2\alpha) =$

$= \operatorname{sen}\alpha(\cos^2\alpha - \operatorname{sen}^2\alpha) + \cos\alpha \times 2\operatorname{sen}\alpha\cos\alpha =$

$= \operatorname{sen}\alpha(1 - \operatorname{sen}^2\alpha - \operatorname{sen}^2\alpha) + 2\operatorname{sen}\alpha\cos^2\alpha =$

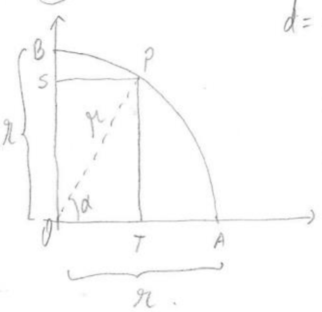
$= \operatorname{sen}\alpha(1 - 2\operatorname{sen}^2\alpha) + 2\operatorname{sen}\alpha(1 - \operatorname{sen}^2\alpha) =$

$= \operatorname{sen}\alpha - 2\operatorname{sen}^3\alpha + 2\operatorname{sen}\alpha - 2\operatorname{sen}^3\alpha =$

$= 3\operatorname{sen}\alpha - 4\operatorname{sen}^3\alpha =$

$= 3 \times (2K) - 4(2K)^3 = 6K - 4 \times 8K^3 = 6K = 32K^3$

(51)



$d = \alpha r \Leftrightarrow \boxed{\alpha = \frac{d}{r}}$

$P(r\cos\alpha, r\operatorname{sen}\alpha)$

$P(\underbrace{r\cos(\frac{d}{r})}_{\overline{OT}}, \underbrace{r\operatorname{sen}(\frac{d}{r})}_{\overline{OS}})$

$\overline{TA} = r - \overline{OT} = r - r\cos(\frac{d}{r})$

$\overline{BS} = r - \overline{OS} = r - r\operatorname{sen}(\frac{d}{r})$

$\overline{BS} + \overline{TA} = r - r\operatorname{sen}(\frac{d}{r}) + r - r\cos(\frac{d}{r}) = 2r - r\operatorname{sen}(\frac{d}{r}) - r\cos(\frac{d}{r}) =$

$= r\left(2 - \operatorname{sen}(\frac{d}{r}) - \cos(\frac{d}{r})\right)$ c.q.d.

(53) $g(x) = \underbrace{\log_2(1-\cos x) + \log_2(1+\cos x)} + \underbrace{2\log_2(2\cos x)} =$

$= \log_2\big((1-\cos x)(1+\cos x)\big) + \log_2(2\cos x)^2 =$

$= \log_2(1-\cos^2 x) + \log_2(4\cos^2 x) =$

$= \log_2(\operatorname{sen}^2 x) + \log_2(4\cos^2 x) = \log_2(4\operatorname{sen}^2 x\cos^2 x) = \log_2(\underbrace{2\operatorname{sen} x\cos x}_{\operatorname{sen}(2x)})^2 =$

$= 2\log_2(\operatorname{sen}(2x))$ c.q.d.

(52) $\lim\limits_{x\to 1^-} f(x) = f(1) = 1-2+\ln(3-2) = 1-2+\ln 1 = -1$

$\lim\limits_{x\to 1^+} f(x) = \lim\limits_{x\to 1^+}\left(\frac{\operatorname{sen}(x-1)}{1-x^2}+K\right) = \lim\limits_{x\to 1^+}\frac{\operatorname{sen}(x-1)}{1-x^2} + \underbrace{\lim\limits_{x\to 1^+} K}_{K} =$

$= K + \lim\limits_{y\to 0^+}\frac{\operatorname{sen} y}{1-(y+1)^2} = K + \lim\limits_{y\to 0^+}\frac{\operatorname{sen} y}{\cancel{1}-y^2-2y-\cancel{1}} = K + \lim\limits_{y\to 0^+}\frac{\operatorname{sen} y}{y(-y-2)} =$

$\boxed{\begin{array}{l} x-1=y\to 0^+ \\ x=y+1\end{array}}$

$= K + \underbrace{\lim\limits_{y\to 0^+}\frac{\operatorname{sen} y}{y}}_{L.N.} \times \lim\limits_{y\to 0^+}\frac{1}{-y-2} = K + 1\times\left(-\frac{1}{2}\right) = K - \frac{1}{2}$

Então $K-\frac{1}{2} = -1 \Leftrightarrow K = \frac{1}{2}-1 \Leftrightarrow K = -\frac{1}{2}$

(54) f é cont em $x=2 \Leftrightarrow \lim\limits_{x\to 2^-} f(x) = \lim\limits_{x\to 2^+} f(x) = f(2)$

$f(2) = \frac{e^{2-2}}{2+2} = \frac{1}{4}$

$\lim\limits_{x\to 2^+}\frac{e^{2-x}}{x+2} = \frac{e^{2-2}}{2+2} = \frac{1}{4}$

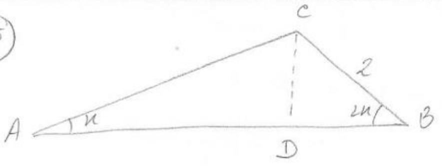
$\lim\limits_{x\to 2^-}\frac{\operatorname{sen}(x-2)}{x^2-4} = \lim\limits_{y\to 0}\frac{\operatorname{sen} y}{(y+2)^2-4} = \lim\limits_{y\to 0}\frac{\operatorname{sen} y}{y^2+4y+\cancel{4}-\cancel{4}} = \lim\limits_{y\to 0}\frac{\operatorname{sen} y}{y(y+4)} =$

$\boxed{\begin{array}{l} x-2=y\to 0 \\ x=y+2\end{array}}$

$= \underbrace{\lim\limits_{y\to 0}\frac{\operatorname{sen} y}{y}}_{L.N.} \times \lim\limits_{y\to 0}\frac{1}{y+4} = 1\times\frac{1}{0+4} = \frac{1}{4}$

Assim, concluímos que f é cont. em $x=2$

55



$\cos(2x) = \frac{\overline{BD}}{2} \Leftrightarrow \overline{BD} = 2\cos(2x)$

$\text{sen}(2x) = \frac{\overline{CD}}{2} \Leftrightarrow \overline{CD} = 2\,\text{sen}(2x)$

$\text{tg}\, x = \frac{\overline{CD}}{\overline{AD}} \Leftrightarrow \text{tg}\, x = \frac{2\,\text{sen}(2x)}{\overline{AD}} \Leftrightarrow$

$\Leftrightarrow \overline{AD} = \frac{2\,\text{sen}(2x)}{\text{tg}\, x}$

$\overline{AB} = \overline{AD} + \overline{DB} = \frac{2\,\text{sen}(2x)}{\text{tg}\, x} + 2\cos(2x) = \frac{2\,\text{sen}(2x)}{\frac{\text{sen}\, x}{\cos x}} + 2\cos(2x) =$

$= \frac{2\cos x\,\text{sen}(2x)}{\text{sen}\, x} + 2(\cos^2 x - \text{sen}^2 x) = \frac{2\cos x \times 2\,\text{sen}\, x \cos x}{\text{sen}\, x} + 2(\cos^2 x - 1 + \cos^2 x)$

$= 4\cos^2 x + 2\cos^2 x - 2 + 2\cos^2 x = 8\cos^2 x - 2$

56 f cont em $x=0 \Leftrightarrow \lim_{x\to 0^-} f(x) = \lim_{x \to 0^+} f(x) = f(0)$

$f(0) = \ln\sqrt{e+0} = \ln e^{\frac{1}{2}} = \frac{1}{2}\ln e = \frac{1}{2}$

$\lim_{x\to 0^+} \ln\sqrt{e+x} = \ln\sqrt{e+0} = \frac{1}{2}$

$\lim_{x\to 0^-} \frac{1-\cos x}{x} = \lim_{x\to 0^-} \frac{(1-\cos x)(1+\cos x)}{x(1+\cos x)} = \lim_{x\to 0^-} \frac{1-\cos^2 x}{x(1+\cos x)} = \lim_{x\to 0^-} \frac{\text{sen}^2 x}{x(1+\cos x)}$

$= \underbrace{\lim_{x\to 0^-} \frac{\text{sen}\, x}{x}}_{L.N.} \times \lim_{x\to 0^-} \frac{\text{sen}\, x}{1+\cos x} = 1 \times \frac{\text{sen}\, 0}{1+\cos 0} = 1 \times \frac{0}{1+1} = 0$

Assim, concluímos que f não é cont. em $x=0$

57 Em $]0,\pi[$, $g'(x) = x + 2\cos^2 x$

$g''(x) = 1 + 2\times 2\cos x(-\text{sen}\, x) = 1 - 4\,\text{sen}\, x\cos x = 1 - 2\times 2\,\text{sen}\, x \cos x = 1 - 2\,\text{sen}(2x)$

Zeros de g''

$1 - 2\,\text{sen}(2x) = 0 \Leftrightarrow \text{sen}(2x) = \frac{1}{2} \Leftrightarrow 2x = \frac{\pi}{6} + 2k\pi \vee 2x = \pi - \frac{\pi}{6} + 2k\pi, k \in \mathbb{Z}$

$\Leftrightarrow x = \frac{\pi}{12} + k\pi \vee x = \frac{5\pi}{12} + k\pi, k\in\mathbb{Z}$

$k=0 \hookrightarrow x = \frac{\pi}{12}$ ✓ $\vee\ x = \frac{5\pi}{12}$ ✓

$k=1 \hookrightarrow x = \frac{13\pi}{12}$ ✗ $\vee\ x = \frac{17\pi}{12}$ ✗

$k=-1 \hookrightarrow x = -\frac{11\pi}{12}$ ✗ $\vee\ x = -\frac{7\pi}{12}$ ✗

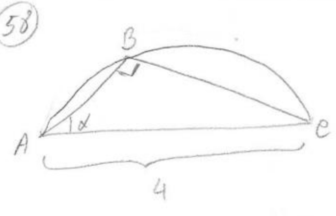
	0		$\frac{\pi}{12}$		$\frac{5\pi}{12}$		π
g''		+	0	−	0	+	
g		∪	PI	∩	PI	∪	

Conc. voltada para cima em $]0, \frac{\pi}{12}]$ e em $[\frac{5\pi}{12}, \pi[$

" " " baixo em $[\frac{\pi}{12}, \frac{5\pi}{12}]$

P.I para $x = \frac{\pi}{12}$ e para $x = \frac{5\pi}{12}$

(58)



$\cos\alpha = \frac{\overline{AB}}{4} \Leftrightarrow \overline{AB} = 4\cos\alpha$

$\operatorname{sen}\alpha = \frac{\overline{BC}}{4} \Leftrightarrow \overline{BC} = 4\operatorname{sen}\alpha$

$A_{\triangle} = \frac{b \times h}{2} = \frac{\overline{AB} \times \overline{BC}}{2} = \frac{4\cos\alpha \times 4\operatorname{sen}\alpha}{2} = 8\operatorname{sen}\alpha\cos\alpha$

$= 4 \times 2\operatorname{sen}\alpha\cos\alpha = 4\operatorname{sen}(2\alpha)$

$A_{\bigcirc} = \frac{\pi \times 2^2}{2} = 2\pi$

$A_{\text{sombreada}} = A_{\bigcirc} - A_{\triangle} = 2\pi - 4\operatorname{sen}(2\alpha)$

(59) $\left.\begin{array}{l} Q(2\cos\alpha;\ 2\operatorname{sen}\alpha) \\ P(2\cos\alpha;\ -2\operatorname{sen}\alpha) \end{array}\right\}$ $\overrightarrow{OP}\cdot\overrightarrow{OQ} = 3 \Leftrightarrow (2\cos\alpha, -2\operatorname{sen}\alpha)\cdot(2\cos\alpha, 2\operatorname{sen}\alpha) = 3$

$\Leftrightarrow 4\cos^2\alpha - 4\operatorname{sen}^2\alpha = 3 \Leftrightarrow 4(\underbrace{\cos^2\alpha - \operatorname{sen}^2\alpha}_{\cos(2\alpha)}) = 3 \Leftrightarrow$

$\Leftrightarrow 4\cos(2\alpha) = 3 \Leftrightarrow \cos(2\alpha) = \frac{3}{4}$

(60)

a) $f(x) = \operatorname{sen}(2x) + x$

$f'(x) = 2\cos(2x) + 1 = 2(\cos^2 x - \operatorname{sen}^2 x) + x = 2\cos^2 x - 2\operatorname{sen}^2 x + 1 =$

$= 2(1 - \operatorname{sen}^2 x) - 2\operatorname{sen}^2 x + 1 = 2 - 2\operatorname{sen}^2 x - 2\operatorname{sen}^2 x + 1 = 3 - 4\operatorname{sen}^2 x$ (D)

b) $\exists x \in \left]\frac{\pi}{6}, \frac{\pi}{3}\right[: f(x) = -x + 2$

Seja $h(x) = f(x) + x - 2 = \operatorname{sen}(2x) + x + x - 2 = \operatorname{sen}(2x) + 2x - 2$

h cont em $\left[\frac{\pi}{6}, \frac{\pi}{3}\right]$

$h\left(\frac{\pi}{6}\right) = \operatorname{sen}\left(\frac{\pi}{3}\right) + \frac{\pi}{3} - 2 = \frac{\sqrt{3}}{2} + \frac{\pi}{3} - 2 \approx -0{,}09 < 0$

$h\left(\frac{\pi}{3}\right) = \operatorname{sen}\left(\frac{2\pi}{3}\right) + \frac{2\pi}{3} - 2 = \frac{\sqrt{3}}{2} + \frac{2\pi}{3} - 2 \approx 0{,}96 > 0$

Assim, pelo Teorema de Bolzano-Cauchy, $\exists x \in \left]\frac{\pi}{6}, \frac{\pi}{3}\right[: h(x) = 0$,

isto é, $\exists x \in \left]\frac{\pi}{6}, \frac{\pi}{3}\right[: f(x) = -x + 2$ c.q.d.

(61) $\lim\limits_{x\to 0} \frac{\operatorname{sen}(2x)}{x} \overset{\left(\frac{0}{0}\right)}{=}$ $\boxed{\begin{array}{l} 2x = y \to 0 \\ x = \frac{y}{2} \end{array}}$ $\lim\limits_{y\to 0} \frac{\operatorname{sen} y}{\frac{y}{2}} = 2\underbrace{\lim\limits_{y\to 0} \frac{\operatorname{sen} y}{y}}_{L.N.} = 2 \times 1 = 2$ (D)

(62) $\cos\left(\frac{\pi}{2} - \alpha\right) = \frac{2}{\overline{OB}}$

$\Leftrightarrow \operatorname{sen}\alpha = \frac{2}{\overline{OB}}$

$\Leftrightarrow \boxed{\overline{OB} = \frac{2}{\operatorname{sen}\alpha}}$

$\cos\alpha = \frac{2}{\overline{OA}}$

$\Leftrightarrow \boxed{\overline{OA} = \frac{2}{\cos\alpha}}$

$A = \frac{2\,\overline{OA} \times \overline{OB}}{2} = \frac{2 \times \frac{2}{\cos\alpha} \times \frac{2}{\operatorname{sen}\alpha}}{2} =$

$= \frac{8}{2\operatorname{sen}\alpha\cos\alpha} = \frac{8}{\operatorname{sen}(2\alpha)}$ c.q.d.

(63) $\overline{AB} = tg\alpha$

$$A_{[ABC]} = A_{[OAB]} - A_{[OAC]} = \frac{1 \times tg\alpha}{2} - \frac{1 \times sen\alpha}{2} = \frac{tg\alpha - sen\alpha}{2} = \frac{2\sqrt{2} - \frac{2\sqrt{2}}{3}}{2} =$$

$$= \frac{4\sqrt{2}}{6} = \frac{2\sqrt{2}}{3}$$

C.A

$sen^2\alpha + cos^2\alpha = 1$

$\Leftrightarrow sen^2\alpha + \left(\frac{1}{3}\right)^2 = 1$

$\Leftrightarrow sen^2\alpha = \frac{8}{9}$

$\Leftrightarrow sen\alpha = \pm\frac{2\sqrt{2}}{3}$

Como $0 < \alpha < \frac{\pi}{2}$, $sen\alpha = \frac{2\sqrt{2}}{3}$

$tg\alpha = \frac{sen\alpha}{cos\alpha} = \frac{\frac{2\sqrt{2}}{3}}{\frac{1}{3}} = 2\sqrt{2}$