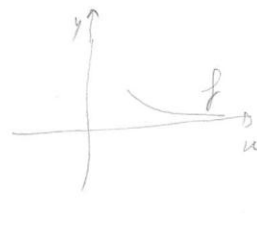


① $\mathcal{D} = \mathbb{R}^+$

Como $\mathcal{D} = \mathbb{R}^+$ apenas tem sentido $x \rightarrow +\infty$

$$\lim_{x \rightarrow +\infty} \frac{1}{f(x)} = \frac{1}{\lim_{x \rightarrow +\infty} f(x)} = \frac{1}{0^+} = +\infty. \text{ Logo } 0$$

gráfico de $\frac{1}{f}$ não tem A.H.



② $\lim_{x \rightarrow +\infty} (f(x) - g(x)) = 0$. Logo $g(x)$ é quociente de assíntota def. (A)

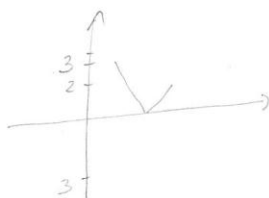
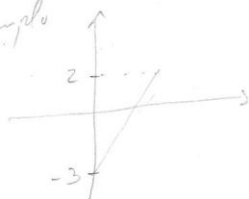
Definição de assíntota

③ $\lim_{x \rightarrow 4^-} f(x) = f(4)$

$\lim_{x \rightarrow 4^+} f(x) \neq f(4)$

(D)

④ exemplo



$\mathcal{D}' = [0, 3]$ (A)

⑤



$\mathcal{D}' =]0, +\infty[$ (C)

⑥ Sendo $D = \mathbb{R}^+$ e para todo $x \rightarrow +\infty$ por o.s.t.a A.H.

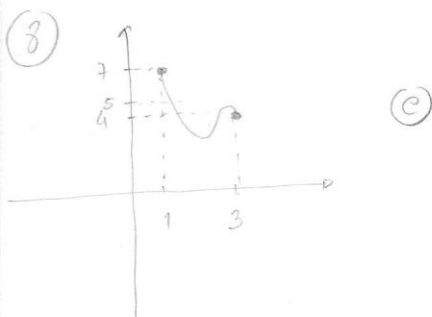
$$\lim_{x \rightarrow +\infty} h(x) = \lim_{x \rightarrow +\infty} \frac{g(x)}{x^2} = \lim_{x \rightarrow +\infty} \left(\frac{g(x)}{x} \times \frac{1}{x} \right) = \lim_{x \rightarrow +\infty} \frac{g(x)}{x} \times \lim_{x \rightarrow +\infty} \frac{1}{x} =$$

$$= 1 \times \frac{1}{+\infty} = 1 \times 0 = 0. \text{ Assim } y=0 \text{ é A.H. de } h \left\{ \begin{array}{l} 1, \text{ porque} \\ y=x \text{ é A.O. de } g \end{array} \right. \text{ c.g.d.}$$

$$\textcircled{7} \lim_{x \rightarrow 3} \frac{f(x) - f(3)}{x^2 - 9} = \lim_{x \rightarrow 3} \frac{f(x) - f(3)}{(x-3)(x+3)} = \lim_{x \rightarrow 3} \frac{f(x) - f(3)}{x-3} \times \lim_{x \rightarrow 3} \frac{1}{x+3}$$

4, porque $f'(3) = 4$

$$= 4 \times \frac{1}{3+3} = 4 \times \frac{1}{6} = \frac{4}{6} = \frac{2}{3} \quad \textcircled{D}$$



⑨ Se $y=x$ é tangente, então $f'(0)=1$
 Em qual dos casos se verifica $f(0)=0$ e $f'(0)=1$?

(A) $x^2 + x$

$f(0) = 0^2 + 0 = 0$

$f'(x) = 2x + 1$

$f'(0) = 2 \cdot 0 + 1 = 1$



(10) $h(x)=0 \Leftrightarrow g(x) \cdot (x+3)^2 = 0 \Leftrightarrow g(x)=0 \vee (x+3)^2=0 \Leftrightarrow g(x)=0 \vee x+3=0$

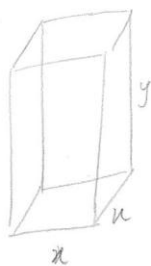
$\Leftrightarrow g(x)=0 \vee x=-3$

2 zeros

(B) $\{-3, 1, 4\}$

(11)

a)



$V=2$
 $\Leftrightarrow x \times x \times y = 2$
 $\Leftrightarrow y = \frac{2}{x^2}$

$A_{\text{base}} = x^2$

$A_{\text{total}} = x \times y = x \times \frac{2}{x^2} = \frac{2x}{x^2} = \frac{2}{x}$

$A_{\text{total}} = 2x^2 + 4x \cdot \frac{2}{x} = 2x^2 + \frac{8}{x} = \frac{2x^3 + 8}{x}$

b) $A'(x) = \frac{(2x^3+8)' \cdot x - (2x^3+8) \cdot x'}{x^2} = \frac{6x^2 \cdot x - (2x^3+8) \cdot 1}{x^2} =$

$= \frac{6x^3 - 2x^3 - 8}{x^2} = \frac{4x^3 - 8}{x^2}$

zeros de A' :

$4x^3 - 8 = 0 \wedge x^2 \neq 0$

$\Leftrightarrow 4x^3 = 8 \wedge x \neq 0$

$\Leftrightarrow x^3 = 2$

$\Leftrightarrow x = \sqrt[3]{2}$

	0	$\sqrt[3]{2}$	
$4x^3 - 8$	-	0	+
x^2	+	+	+
A'	-	0	+
A	$\searrow$	m	$\uparrow$

$m: x = \sqrt[3]{2}$

(12) $\lim_{x \rightarrow -\infty} \frac{1}{f(x)} = \frac{1}{\lim_{x \rightarrow -\infty} f(x)} = \frac{1}{0^+} = +\infty$ (c)

(13) Como em $[0, 3]$, $f' < 0$ logo f é decrescente.
 Sendo decrescente, $f(3) < f(0) = 2$. Logo $f(3) = 1$ (A)

(14)

	$-\infty$	-2	-1	2	$+\infty$
$h = f \times g$	$-$	0	$+$	0	$+$
g	$+$	0	$-$	$-$	0
f	$-$	$-$	0	$+$	$+$

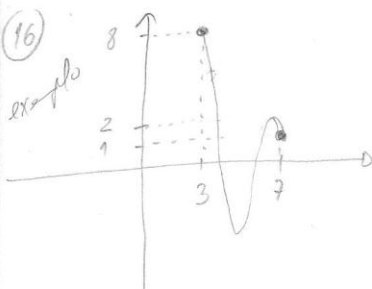
→ Apenas op, p, s @ está de acordo

(15)

$g(x) = 1 + f(u+1)$
 deslocamento p/c seguinte 1 unidade
 deslocamento
 p/c acima 1 unidade

(D)

(16)



(D)

(17)

$$g(x) = \sqrt{f(x)}$$

$$D_g = \{x \in \mathbb{R} : f(x) \geq 0\} = [-3, +\infty[$$

(D)

(18)

$$\lim_{x \rightarrow 5} f(x) = -3$$

$$\lim_{x \rightarrow +\infty} f(x) = 2$$

$$\lim_{x \rightarrow -\infty} (f(x) - x) = 0$$

$$y = 2 \text{ A.H.}$$

$$y = x \text{ A.O.}$$

(A)

(19)

$$\lim_{x \rightarrow 1^+} \frac{f(x)}{g(x)} = \frac{+}{-\infty} = 0$$

(A)

(20) Como $f > 0$ e $g > 0$ então $f(x) + g(x) > 0$. Logo $f(x) + g(x) = 0$ é impossível (A)

(21) Como $D =]0, +\infty[$ sempre tem sentido $x \rightarrow +\infty$

$$m = \lim_{x \rightarrow +\infty} \frac{h(x)}{x} = \lim_{x \rightarrow +\infty} \frac{\frac{x^2}{g(x)}}{x} = \lim_{x \rightarrow +\infty} \frac{x^2}{x g(x)} = \lim_{x \rightarrow +\infty} \frac{x}{g(x)} =$$

$$= \lim_{x \rightarrow +\infty} \frac{1}{\frac{g(x)}{x}} = \frac{1}{\lim_{x \rightarrow +\infty} \frac{g(x)}{x}} = \frac{1}{1} = 1$$

1, porque $y = x+2$ é A.O. de g

$$b = \lim_{x \rightarrow +\infty} (h(x) - x) = \lim_{x \rightarrow +\infty} \left(\frac{x^2}{g(x)} - x \right) = \lim_{x \rightarrow +\infty} \frac{x^2 - x g(x)}{g(x)} =$$

$$= \lim_{x \rightarrow +\infty} \frac{x(x - g(x))}{g(x)} = \lim_{x \rightarrow +\infty} \frac{x}{g(x)} \times \lim_{x \rightarrow +\infty} [x - g(x)] =$$

1 (calculado acima)

$$= \lim_{x \rightarrow +\infty} [x - g(x)] = - \lim_{x \rightarrow +\infty} [g(x) - x] = -2$$

2, porque $y = x+2$ é A.O. de g

Como $m = 1$ e $b = -2$ então $y = x - 2$ A.O. de h

(22) $\lim_{x \rightarrow 1^-} \frac{1}{f(x)} = \frac{1}{+\infty} = 0 \rightarrow$ ordeni opeas A

$\lim_{x \rightarrow 0^+} \frac{1}{f(x)} = \frac{1}{0^+} = +\infty \rightarrow$ ordeni opeas C

$\lim_{x \rightarrow 0^-} \frac{1}{f(x)} = \frac{1}{0^+} = +\infty \rightarrow$ ordeni opeas D

R: B

(23) Sendo $y = 2x + 3$ assíntota de g , então

$$\lim_{x \rightarrow +\infty} \frac{g(x)}{x} \quad \lim_{x \rightarrow +\infty} (g(x) - 2x)$$

Assim, $\lim_{x \rightarrow +\infty} \left[\frac{g(x)}{x} \times (g(x) - 2x) \right] = 2 \times 3 = 6$ (C)

(24) Como $\lim_{x \rightarrow 3} f(x)$ não existe, então $\lim_{x \rightarrow 3} \frac{1}{f(x)}$ também não existe (D)

(25)



$$A = 5$$

$$\Rightarrow xy = 5$$

$$\Rightarrow y = \frac{5}{x}$$

$$p = 2x + 2y =$$

$$= 2x + 2 \times \frac{5}{x} =$$

$$= 2x + \frac{10}{x} =$$

$$= \frac{2x^2 + 10}{x}$$

(A)

(26) $\lim_{x \rightarrow 1^+} \frac{h(x)}{g(x)} = \frac{\lim_{x \rightarrow 1^+} h(x)}{\lim_{x \rightarrow 1^+} g(x)} = \frac{1-1}{+\infty} = 0$ Se $x \rightarrow 1^-$, obtenemos $\frac{1-1}{+\infty} = 0$ (C)

(27)

$$\frac{4-x^2}{4-x^2} = \frac{-2}{0} = \frac{2}{0} = -$$

$$\lim_{x \rightarrow 2^+} \frac{1}{4-x^2} = \frac{1}{0^-} = -\infty$$

(D)

(28) $g(4) = f(4-1) + 4 = f(3) + 4 = 0 + 4 = 4$ (4, 4) (B)

(29) $y = -x - 1$ assíntota de f , logo $\lim_{x \rightarrow -\infty} [f(x) - (-x-1)] = 0$ (C)

$$\Rightarrow \lim_{x \rightarrow -\infty} (f(x) + x + 1) = 0$$

(B)

30

	$-\infty$	0	*	$+\infty$
f'	+	ND	-	0
f	$\nearrow$	$\searrow$	$\searrow$	$\nearrow$

(c)

31. (B) $x_n = -2 - \frac{1}{n} \rightarrow -2$ porque $\lim_{n \rightarrow +\infty} g(x_n) = g(\lim_{n \rightarrow +\infty} x_n) = g(-2) = +\infty$

32. $\lim_{x \rightarrow -\infty} \frac{3}{f(x)} = \frac{3}{\lim_{x \rightarrow -\infty} f(x)} = \frac{3}{-1} = -3$ (B)

33. $\lim_{x \rightarrow +\infty} h(x) = \lim_{x \rightarrow +\infty} \frac{x}{f(x)} = \lim_{x \rightarrow +\infty} \frac{1}{\frac{f(x)}{x}} = \frac{1}{\frac{1}{3}} = 3$ logo $y = 3$ A.H. (D)

$y = \frac{1}{3}x + 2$ A.O. de f
 $m = \lim_{x \rightarrow +\infty} \frac{f(x)}{x}$

34. Como $\lim_{x \rightarrow +\infty} (f(x) - 2x) = 0$, então $y = 2x$ A.O. do gráfico de f .
 $m = \lim_{x \rightarrow +\infty} \frac{f(x)}{x}$

A.O. do gráfico de g :

$m = \lim_{x \rightarrow +\infty} \frac{g(x)}{x} = \lim_{x \rightarrow +\infty} \frac{f(x) + x^2}{x} = \lim_{x \rightarrow +\infty} \left(\frac{f(x)}{x} + \frac{x^2}{x} \right) = \lim_{x \rightarrow +\infty} \frac{f(x)}{x} + \lim_{x \rightarrow +\infty} x$

$= 2 + (+\infty) = +\infty$. Assim, conclui-se que não há A.O.

NOTA: Como $D = \mathbb{R}^+$ e pedem sem sentido $x \rightarrow +\infty$

35. $\lim_{x \rightarrow +\infty} \frac{f(x)}{x} = m = 1$ (C)

36. $h = f + g$
 $h' = f' + g' =$
 $= 2x + 0$
 $\uparrow \quad \uparrow$
 $m > 0 \quad b < 0$ (B)

(37) Como $\lim_{n \rightarrow +\infty} (g(n) - u) = 0$, então $y = u$ A.D. de g

Como $\lim_{n \rightarrow 0} g(n) = -\infty$ a opção correta será (D)

(38) $\lim_{n \rightarrow a^+} f(n) = a^2 - a + 3$
 $\lim_{n \rightarrow a^-} f(n) = a^2 - 2a$
 $f(a) = a^2 - a + 3$

$a^2 - a + 3 = a^2 - 2a$
 $2a - a = -3$
 $a = -3$ (A)

(39) $g(n) = f(n) + n$
 $g'(n) = f'(n) + 1 \rightarrow$ o gráfico de f' desloca no vertical $\uparrow +1$ (D)

(40) $\lim_{n \rightarrow 1^-} \frac{3n}{f(n)} = \frac{3}{+\infty} = 0$ (C)

(41) Como $\lim_{n \rightarrow +\infty} (h(n) - 2n) = 0$, então $y = 2n$ A.D.

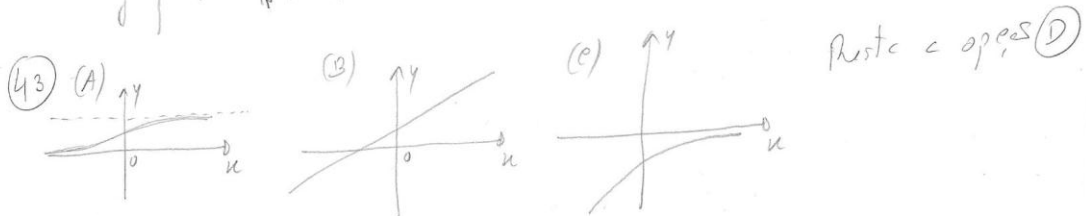
h é par $\rightarrow$

$\lim_{n \rightarrow -\infty} h(n) = +\infty$ (A)

(42)

	$-\infty$	a	$+\infty$
f'	$+$	0	$-$
f	$\nearrow$	$\searrow$	

(C)



44) $\lim (u_n) = \lim h(4 - \frac{1000}{n}) = h[\lim (4 - \frac{1000}{n})] = h(4^-) = 1$ (B)

45) Como $y = 2x - 4$ é A.O. então

$\lim_{x \rightarrow +\infty} [g(x) - (2x - 4)] = 0 \Leftrightarrow \lim_{x \rightarrow +\infty} (g(x) - 2x + 4) = 0$ (C)

46) (A) $\frac{f'(0)}{+} \times \frac{f'(6)}{+} > 0$ (C) $\frac{f'(-3)}{+} \times \frac{f'(0)}{-} < 0$

(B) $\frac{f'(-3)}{+} \times \frac{f'(6)}{+} > 0$ (D) $\frac{f'(0)}{-} \times \frac{f'(6)}{+} < 0$ Opção D

47) $\lim_{n \rightarrow +\infty} f(x_n) = f(\lim_{n \rightarrow +\infty} x_n) = f(1^-) = +\infty$ (A)
 $x_n \rightarrow 1$
 $x_n \in]-1, 1[$

48) D

49) $\lim_{x \rightarrow 0^+} \frac{1}{f(x)} = \frac{1}{\lim_{x \rightarrow 0^+} f(x)} = \frac{1}{-\infty} = 0$ (exclui opções B e D)

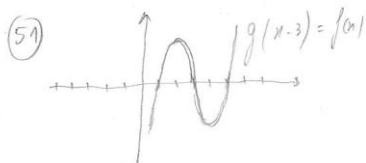
$\lim_{x \rightarrow +\infty} \frac{1}{f(x)} = \frac{1}{\lim_{x \rightarrow +\infty} f(x)} = \frac{1}{+\infty} = 0$ Res. Opção C
 $y = x$ A.O. de f

50) Se $\lim_{x \rightarrow -\infty} f(x) = 3$, então $y = 3$ A.H.

Se $\lim_{x \rightarrow 1^+} f(x) = +\infty$, então $x = 1$ A.V.

Opção B

$\lim_{x \rightarrow +\infty} (f(x) - 2x) = 1 \Leftrightarrow \lim_{x \rightarrow +\infty} (f(x) - 2x - 1) = 0 \Leftrightarrow \lim_{x \rightarrow +\infty} [f(x) - (2x + 1)] = 0$, logo $y = 2x + 1$ A.O.



	$-\infty$	2	4	$+\infty$
f'	+	0	-	0
f	$\nearrow$	m	b	m

(A)

52) Apenas tem sentido $x \rightarrow -\infty$ porque $D =]-\infty, 4]$

$$\lim_{x \rightarrow -\infty} f(x) = \lim_{x \rightarrow -\infty} \frac{3x+3}{\sqrt{x^2+9}} \stackrel{\frac{-\infty}{\infty}}{=} \lim_{x \rightarrow -\infty} \frac{x(3+\frac{3}{x})}{\sqrt{x^2(1+\frac{9}{x^2})}} = \lim_{x \rightarrow -\infty} \frac{x(3+\frac{3}{x})}{\sqrt{x^2} \times \sqrt{1+\frac{9}{x^2}}} =$$

$$= \lim_{x \rightarrow -\infty} \frac{x(3+\frac{3}{x})}{|x| \times \sqrt{1+\frac{9}{x^2}}} = \lim_{x \rightarrow -\infty} \frac{x(3+\frac{3}{x})}{-x \times \sqrt{1+\frac{9}{x^2}}} = \frac{3+0}{-\sqrt{1+0}} = -3 \therefore y = -3 A.H$$

53) $u_n = 2 + \frac{1}{n} \rightarrow 2^+$

$$\lim_{n \rightarrow \infty} f(u_n) = +\infty \Leftrightarrow f(\lim u_n) = +\infty \Leftrightarrow f(2^+) = +\infty \quad (C)$$

54) Se $y = 2x - 5$ A.O do f

$$\lim_{x \rightarrow +\infty} \frac{f(x)}{x} = 2 \text{ e } \lim_{x \rightarrow +\infty} f(x) = +\infty \quad \left| \begin{aligned} \lim_{x \rightarrow +\infty} \frac{6x-1}{f(x)} &= \lim_{x \rightarrow +\infty} \left(\frac{6x}{f(x)} - \frac{1}{f(x)} \right) = \\ &= \lim_{x \rightarrow +\infty} \frac{6x}{f(x)} - \lim_{x \rightarrow +\infty} \frac{1}{f(x)} = \lim_{x \rightarrow +\infty} \left(6 \times \frac{x}{f(x)} \right) - \lim_{x \rightarrow +\infty} \frac{1}{f(x)} \\ &= \lim_{x \rightarrow +\infty} \left(6 \times \frac{1}{\frac{f(x)}{x}} \right) - \lim_{x \rightarrow +\infty} \frac{1}{f(x)} = 6 \times \frac{1}{2} - \frac{1}{+\infty} = 3 - 0 = 3 \end{aligned} \right. \quad (C)$$

55) $u_1 = a$

$$u_2 = -3u_1 + 2 = -3a + 2$$

$$u_3 = -3u_2 + 2 = -3(-3a + 2) + 2 = 9a - 6 + 2 = 9a - 4 \quad (B)$$

56) (C) porque $u_n = -\frac{1}{n} : -1, -\frac{1}{2}, -\frac{1}{3}, -\frac{1}{4}, \dots \rightarrow 0$

suasas crescente
" limitada $-1 < u_n < 0$

57) a_n geométrica

$$\left. \begin{aligned} a_3 &= \frac{1}{4} \\ a_6 &= 2 \end{aligned} \right\} \begin{aligned} \frac{a_6}{a_3} &= r^3 \Leftrightarrow \frac{2}{\frac{1}{4}} = r^3 \Leftrightarrow 8 = r^3 \Leftrightarrow r = 2 \\ \text{Assim, } a_{20} &= a_6 \times r^{14} = 2 \times 2^{14} = 32768 \end{aligned} \quad (C)$$

59) $u_4 = 32$
 $u_8 = 8192$

$$\left. \begin{aligned} u_8 &= r^4 \Leftrightarrow \frac{8192}{32} = r^4 \Leftrightarrow 256 = r^4 \Leftrightarrow r = \sqrt[4]{256} \Leftrightarrow r = \frac{4}{4} \end{aligned} \right\}$$

u_n crescente e termos positivos

Assim $u_5 = u_4 \times r = 32 \times 4 = 128 \quad (B)$

$$\textcircled{58} \quad \lim_{x \rightarrow 2} \frac{f(x) - f(2)}{x^2 - 2x} = \lim_{x \rightarrow 2} \frac{f(x) - f(2)}{x(x-2)} = \lim_{x \rightarrow 2} \underbrace{\frac{f(x) - f(2)}{x-2}}_{f'(2)=6} \times \lim_{x \rightarrow 2} \frac{1}{x} =$$

$$= 6 \times \frac{1}{2} = 3 \quad \textcircled{A}$$