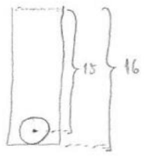
① $\log_3\left(\frac{3^K}{9}\right) = \log_3\left(\frac{3^K}{3^2}\right) = \log_3\left(3^{K-2}\right) = K-2$ (B)

② $\lim f(u_n) = f(\lim u_n) = f(+\infty) = \lim\limits_{x\to+\infty} \frac{1+\ln x}{x} = \lim\limits_{x\to+\infty} \frac{1}{x} + \lim\limits_{x\to+\infty} \underbrace{\left(\frac{\ln x}{x}\right)}_{\text{L.N.}\ 0} = \frac{1}{+\infty} + 0 = 0$ (A)

3.a) $d(0) = 10 + (5-0)e^{-0,05\times 0} = 10 + 5e^0 = 10+5 = 15$ cm



raio $= 1$

$V_{⊕} = \frac{4}{3}\pi \times 1^3 = \frac{4\pi}{3} \approx 4{,}19$

3.b) $d'(t) = 10' + (5-t)'e^{-0,05t} + (5-t)\left(e^{-0,05t}\right)' = 0 - e^{-0,05t} + (5-t)\times(-0{,}05e^{-0,05t}) =$

$= -e^{-0,05t} - 0{,}25e^{-0,05t} + 0{,}05te^{-0,05t} = -1{,}25e^{-0,05t} + 0{,}05te^{-0,05t}$

Zeros de d':

$e^{-0,05t}(0{,}05t - 1{,}25) = 0 \Leftrightarrow \underbrace{e^{-0,05t} = 0}_{\text{imp.}} \lor t = \frac{1{,}25}{0{,}05} \Leftrightarrow t = 25$ s

	0		25	$+\infty$
$e^{-0,05t}$	+	+	+	+
$0{,}05t-1{,}25$	−	−	0	+
d'	−	−	0	+
d	Máx	↘	Mín	↗

R: A distância é mínima para $t = 25$ s.

4.a) f é cont. para $x < \frac{1}{2}$ e para $x > \frac{1}{2}$. Assim, a A.V., a existir, será para $x = \frac{1}{2}$

$\lim\limits_{x\to\frac{1}{2}^-} \frac{e^x - \sqrt{e}}{2x-1} \overset{\left(\frac{0}{0}\right)}{=} \lim\limits_{y\to 0} \frac{e^{y+\frac{1}{2}} - e^{\frac{1}{2}}}{2\left(y+\frac{1}{2}\right)-1} = \lim\limits_{y\to 0} \frac{e^y \times e^{\frac{1}{2}} - e^{\frac{1}{2}}}{2y + \cancel{1} - \cancel{1}} = \lim\limits_{y\to 0} \frac{e^{\frac{1}{2}} \times \overbrace{\frac{(e^y - 1)}{y}}^{\text{L.N.}\ 1}}{2} = \frac{\sqrt{e}}{2}$

$\boxed{\begin{array}{l} y = x - \frac{1}{2} \to 0 \\ x = y + \frac{1}{2} \end{array}}$

$\lim\limits_{x\to\frac{1}{2}^+} \left((x+1)\ln x\right) = \left(\frac{1}{2}+1\right)\times\ln\left(\frac{1}{2}\right) = \frac{3}{2}(\ln 1 - \ln 2) = \frac{3}{2}(0 - \ln 2) = -\frac{3}{2}\ln 2$.

Assim, conclui-se que não há A.V.

4.b) $f(x) = (x+1)\ln x$

$f'(x) = \ln x + (x+1) \times \frac{1}{x} = \ln x + \frac{x+1}{x}$

$f''(x) = \frac{1}{x} + \frac{(x+1)'x - (x+1)x'}{x^2} = \frac{1}{x} + \frac{x - x - 1}{x^2} = \frac{1}{x} - \frac{1}{x^2} = \frac{x-1}{x^2}$

Zeros de f'':

$\frac{x-1}{x^2} = 0 \Leftrightarrow x - 1 = 0 \wedge x^2 \neq 0 \Leftrightarrow x = 1 \wedge x \neq 0 \Leftrightarrow x = 1$

	$\frac{1}{2}$		1	$+\infty$
$x-1$		$-$	0	$+$
x^2		$+$	$+$	$+$
f''		$-$	0	$+$
f		$\cap$	P.I	$\cup$

Concavidade voltada p/ baixo em $]\frac{1}{2}, 1[$

" " p/ cima em $]1, +\infty[$

P.I $\Rightarrow (1, 0)$

$f(1) = (1+1)\ln 1 = 0$

4.c) $f(x) = 3 \Leftrightarrow f(x) - 3 = 0$

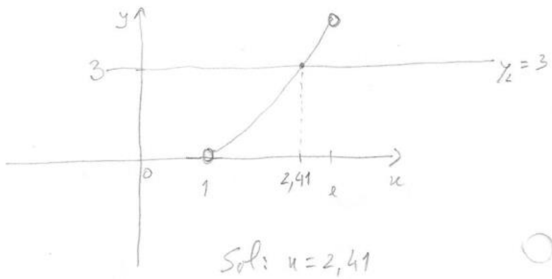
Seja $g(x) = f(x) - 3 = (x+1)\ln x - 3$

g é cont. em $[1, e]$

$g(1) = (1+1)\ln(1) - 3 = -3 < 0$

$g(e) = (e+1)\underbrace{\ln(e)}_{1} - 3 = e + 1 - 3 > 0$

Assim, pelo teorema de Bolzano-Cauchy, $\exists c \in]1, e[: g(c) = 0$, isto é, $f(c) - 3 = 0 \Leftrightarrow f(c) = 3$ c.q.d



Sol: $x = 2{,}41$

⑤ $\log_b a = \frac{1}{3}$

$\log_a(a^2 b) = \log_a a^2 + \log_a b = 2 + \frac{\log_b b}{\log_b a} = 2 + \frac{1}{\frac{1}{3}} = 2 + 3 = 5$ (D)

⑥ $\lim_{x \to 0^-} (2 + e^{x+k}) = 2 + e^{0+k} = 2 + e^k$

$\lim_{x \to 0^+} \frac{2x + \ln(x+1)}{x} = \lim_{x \to 0^+} \frac{2x}{x} + \lim_{x \to 0^+} \frac{\ln(x+1)}{x} = 2 + 1 = 3$ $(*)$

$2 + e^k = 3 \Leftrightarrow e^k = 1 \Leftrightarrow k = 0$ (A)

$(*)$ $\lim_{x \to 0^+} \frac{\ln(x+1)}{x} = \lim_{y \to 0} \frac{y}{e^y - 1} = \lim_{y \to 0} \frac{1}{\underbrace{\frac{e^y - 1}{y}}_{\text{L.N.}\ 1}} = \frac{1}{1} = 1$

$\ln(x+1) = y \to 0$

$x + 1 = e^y$

$x = e^y - 1$

7.a) $\lim_{x\to+\infty}\left(\ln(x-3)-\ln x\right) \overset{(\infty-\infty)}{=} \lim_{x\to+\infty}\left(\ln\left(\frac{x-3}{x}\right)\right) = \lim_{x\to+\infty}\left(\ln\left(1-\frac{3}{x}\right)\right) = \ln(1-0) = \ln 1 = 0$ ⇒ $\underline{y=0}$ A.H

$\lim_{x\to-\infty}\left(1+xe^{x}\right) \overset{(\infty\times 0)}{=} \lim_{y\to+\infty}\left(1-ye^{-y}\right) = \lim_{y\to+\infty}\left(1-\frac{y}{e^{y}}\right) = \lim_{y\to+\infty}\left(1-\frac{1}{\underbrace{\frac{e^{y}}{y}}_{\text{L.N.}\ +\infty}}\right) = 1-\frac{1}{+\infty} = 1-0 = 1$ ⇒ $\underline{y=1}$ A.H

$\boxed{\begin{array}{l} y=-x\to+\infty \\ x=-y \end{array}}$

7.b) $f(x)-2x>1 \Leftrightarrow \cancel{1}+xe^{x}-2x>\cancel{1} \Leftrightarrow x(e^{x}-2)>0$

Zeros:
$x=0 \lor e^{x}-2=0$
$\Leftrightarrow e^{x}=2$
$\Leftrightarrow x=\ln 2$

	$-\infty$		0		$\ln 2$		3
x		$-$	0	$+$	$+$	$+$	$+$
$e^{x}-2$		$-$	$-$	$-$	0	$+$	$+$
$x(e^{x}-2)$		$+$	0	$-$	0	$+$	$+$

$S =]-\infty;0[\ \cup\ ]\ln 2;3]$

7.c) $f'(x) = (\ln(x-3))' - (\ln x)' = \frac{(x-3)'}{x-3} - \frac{x'}{x} = \frac{1}{x-3} - \frac{1}{x}$

$m = f'(4) = \frac{1}{4-3} - \frac{1}{4} = 1 - \frac{1}{4} = \frac{3}{4}$

$f(4) = \ln(4-3) - \ln 4 = \underbrace{\ln 1}_{0} - \ln 4 = -\ln 4$ Ponto de tangência $(4, -\ln 4)$

$y = mx + b$
$y = \frac{3}{4}x + b$
$(4,-\ln 4)$ ⇒ $-\ln 4 = \frac{3}{4}\times 4 + b$
$b = -3 - \ln 4$

R: $y = \frac{3}{4}x - 3 - \ln 4$

8) O gráfico A não pode representar a função f porque se a função f tem derivada finita em todos os pontos do domínio, então é contínua em todo o seu domínio, o que não acontece uma vez que a função representada no gráfico A tem um ponto de descontinuidade.
Tendo em conta que $f''(x)<0,\ \forall x\in\ ]-\infty,0[$, então nesse intervalo o gráfico da função f deveria apresentar a concavidade voltada para baixo, o que não se verifica no gráfico B e, por isso, se exclui.
Finalmente, exclui-se o gráfico C porque se $f'(0)>0$, então a reta tangente ao gráfico no ponto de abcissa zero deveria ter declive positivo, o que não se verifica.

9.a) $N(10) = \dfrac{200}{1+50e^{-0,25\times 10}} = \dfrac{200}{1+50e^{-2,5}}$

$N(20) = \dfrac{200}{1+50e^{-0,25\times 20}} = \dfrac{200}{1+50e^{-5}}$

Taxa média de variação $= \dfrac{N(20)-N(10)}{20-10} = \dfrac{149,6-39,183}{10} \simeq 11$

Entre 1900 e 2000 o número de habitantes, de uma certa região do globo, cresceu em média aproximadamente 11 milhões em cada década.

b) $N = \dfrac{200}{1+50e^{-0,25t}} \Leftrightarrow N\left(1+50e^{-0,25t}\right) = 200 \Leftrightarrow 1+50e^{-0,25t} = \dfrac{200}{N} \Leftrightarrow$

$\Leftrightarrow 50e^{-0,25t} = \dfrac{200}{N} - 1 \Leftrightarrow 50e^{-0,25t} = \dfrac{200-N}{N} \Leftrightarrow e^{-0,25t} = \dfrac{200-N}{50N} \Leftrightarrow -0,25t = \ln\left(\dfrac{200-N}{50N}\right)$

$\Leftrightarrow -\dfrac{1}{4}t = \ln\left(\dfrac{200-N}{50N}\right) \Leftrightarrow -t = 4\ln\left(\dfrac{200-N}{50N}\right) \Leftrightarrow t = -4\ln\left(\dfrac{200-N}{50N}\right) \Leftrightarrow t = \ln\left(\dfrac{200-N}{50N}\right)^{-4} \Leftrightarrow$

$\Leftrightarrow t = \ln\left(\left(\dfrac{200-N}{50N}\right)^{-1}\right)^4 \Leftrightarrow t = \ln\left(\dfrac{50N}{200-N}\right)^4$ c.q.d.

10. a) $\lim\limits_{x\to+\infty}\left(x^2e^{1-x}\right) \overset{(\infty\times 0)}{=} \lim\limits_{x\to+\infty}\left(x^2e^1\times e^{-x}\right) = \lim\limits_{x\to+\infty}\left(\dfrac{x^2}{e^x}\times e\right) = \lim\limits_{x\to+\infty}\left(\dfrac{1}{\underbrace{\frac{e^x}{x^2}}_{\text{L.N.}\ \to +\infty}}\times e\right) = \dfrac{1}{+\infty}\times e = 0\times e = 0$

Logo $y=0$ A.H.

b) $f'(x) = (x^2)'e^{1-x} + x^2\left(e^{1-x}\right)' = 2xe^{1-x} + x^2\left((1-x)'e^{1-x}\right) = 2xe^{1-x} - x^2e^{1-x}$

Zeros de f':

$e^{1-x}(2x-x^2) = 0$

$\underbrace{e^{1-x}=0}_{\text{imp.}} \lor x(2-x) = 0$

$x=0 \lor x=2$

	0		2	$+\infty$
e^{1-x}	+	+	+	+
$2x-x^2$	0	+	0	−
f'	0	+	0	−
f	Mín	↗	Máx	↘

$f(0) = 0$

$f(2) = 2^2e^{1-2} = \dfrac{4}{e}$

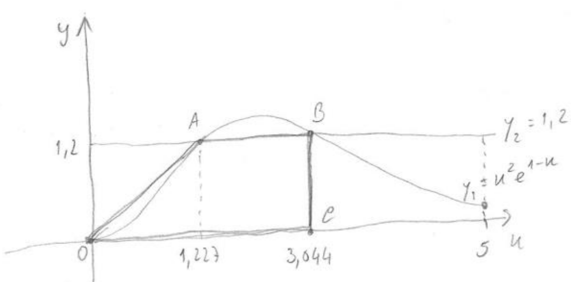
f é crescente em $[0,2]$

f é decrescente em $[2,+\infty[$

0 é mínimo para $x=0$

$\dfrac{4}{e}$ é máximo para $x=2$

c)



$A = \dfrac{3,044+1,817}{2}\times 1,2 \approx 2,92$

$B = 3,044$

$b = 3,044 - 1,227 = 1,817$

(11) $\lim\limits_{x \to 2} \dfrac{f(x)-f(2)}{x^2-2x} = \underbrace{\lim\limits_{x \to 2} \dfrac{f(x)-f(2)}{x-2}}_{f'(2)=6} \times \lim\limits_{x \to 2} \dfrac{1}{x} = 6 \times \dfrac{1}{2} = 3$ (A)

(12)

	$-\infty$	0	$+\infty$
f	$\cap$	PI	$\cup$
f''	$-$	0	$+$

(C)

(13) $f(2)=8 \Leftrightarrow e^{a \ln 2} = 8 \Leftrightarrow e^{\ln 2^a} = 8 \Leftrightarrow 2^a = 8 \Leftrightarrow 2^a = 2^3 \Leftrightarrow a = 3$ (C)

(14) $\lim\limits_{x \to a} \dfrac{a e^{x-a} - a}{x^2 - a^2} = \lim\limits_{x \to a} \dfrac{a(e^{x-a}-1)}{(x+a)(x-a)} = \lim\limits_{x \to a} \dfrac{e^{x-a}-1}{x-a} \times \lim\limits_{x \to a} \dfrac{a}{x+a} =$

$= \underbrace{\lim\limits_{y \to 0} \left(\dfrac{e^y - 1}{y}\right)}_{\text{l.n.}\ \to\ 1} \times \lim\limits_{x \to a} \dfrac{a}{x+a} = 1 \times \dfrac{a}{a+a} = \dfrac{a}{2a} = \dfrac{1}{2}$ (B)

(15) $\lim\limits_{x \to -\infty} \dfrac{f(x) + e^x - x}{x} = 1 \Leftrightarrow \underbrace{\lim\limits_{x \to -\infty} \dfrac{f(x)}{x}}_{m} + \lim\limits_{x \to -\infty} \dfrac{e^x}{x} - \lim\limits_{x \to -\infty} \dfrac{x}{x} = 1 \Leftrightarrow$

$\Leftrightarrow m + \dfrac{e^{-\infty}}{-\infty} - 1 = 1 \Leftrightarrow m + 0 - 1 = 1 \Leftrightarrow m = 2$ (D)

(16) $\lim(u_n) = \lim \dfrac{Kn+3}{2n} = \lim \dfrac{Kn}{2n} = \dfrac{K}{2}$; $\dfrac{K}{2} = 1 \Leftrightarrow K = 2$ (B)

$\lim(v_n) = \lim \left(\ln \underbrace{\left(1+\dfrac{1}{n}\right)^n}_{e}\right) = \ln e = 1$

(17-a) $p = \lim\limits_{x \to -1} \dfrac{f(x)-f(-1)}{x-(-1)} = f'(-1) = e^{-1}\left((-1)^2 - 1 + 1\right) = \dfrac{1}{e}$

$q = -\dfrac{1}{p} = -\dfrac{1}{\frac{1}{e}} = -e$

q é o declive de uma reta perpendicular à reta tangente ao gráfico de f, no ponto de abcissa $x=-1$.

(17.b) $f''(x) = (e^x)'(x^2+x+1) + e^x(x^2+x+1)' = e^x(x^2+x+1) + e^x(2x+1)$

Zeros de f'':

$e^x(x^2+x+1+2x+1) = 0 \Leftrightarrow$

$\Leftrightarrow \underbrace{e^x = 0}_{\text{imp.}} \vee x^2+3x+2 = 0$

$\Leftrightarrow x = \dfrac{-3 \pm \sqrt{9-8}}{2}$

$\Leftrightarrow x = \dfrac{-3 \pm 1}{2}$

$\Leftrightarrow x = -1 \vee x = -2$

	$-\infty$	-2		-1	$+\infty$
x^2+3x+2	+	0	−	0	+
e^x	+	+	+	+	+
f''	+	0	−	0	+
f	$\cup$	PI	$\cap$	PI	$\cup$

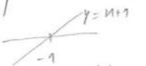
Concav. voltada p/ cima em $]-\infty, -2[$ e em $]-1, +\infty[$

" " " baixo em $]-2, -1[$

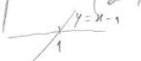
Pontos de inflexão p/ $x = -2$ e p/ $x = -1$

(18.a) f é cont. em $]-\infty, -1[\cup]1, +\infty[$. Assim as A.V., se existirem, serão para $x = -1^-$ e/ou para $x = 1^+$

$\lim\limits_{x \to -1^-} \left(\ln\left(\dfrac{x-1}{x+1}\right)\right) = \ln\left(\dfrac{-2}{0^-}\right) = \ln(+\infty) = +\infty$. Assim, $x = -1$ A.V.



$\lim\limits_{x \to 1^+} \left(\ln\left(\dfrac{x-1}{x+1}\right)\right) = \ln\left(\dfrac{0^+}{2}\right) = \ln(0^+) = -\infty$. Assim, $x = 1$ A.V.



(18.b) Seja $A(a, f(a))$ e $B(-a, f(-a))$. Pretende-se provar que o ponto médio do segmento $[A,B]$ é a origem do referencial.

$f(a) = \ln\left(\dfrac{a-1}{a+1}\right)$

$f(-a) = \ln\left(\dfrac{-a-1}{-a+1}\right) = \ln\left(\dfrac{a+1}{a-1}\right)$

$A\left(a, \ln\left(\dfrac{a-1}{a+1}\right)\right) \quad B\left(-a, \ln\left(\dfrac{a+1}{a-1}\right)\right)$

$M = \left(\dfrac{a-a}{2}, \dfrac{\ln\left(\frac{a-1}{a+1}\right) + \ln\left(\frac{a+1}{a-1}\right)}{2}\right) = (0,0)$ c.q.d.

C.A.:

$\ln\left(\dfrac{a-1}{a+1}\right) + \ln\left(\dfrac{a+1}{a-1}\right) = \ln\left(\dfrac{\cancel{a-1}}{\cancel{a+1}} \times \dfrac{\cancel{a+1}}{\cancel{a-1}}\right) = \ln 1 = 0$

(19) $\log_a(ab^3) = 5 \Leftrightarrow \log_a a + \log_a b^3 = 5 \Leftrightarrow 1 + 3\log_a b = 5 \Leftrightarrow 3\log_a b = 4 \Leftrightarrow \log_a b = \dfrac{4}{3} \Rightarrow$

$\Leftrightarrow \dfrac{\log_b b}{\log_b a} = \dfrac{4}{3} \Leftrightarrow \dfrac{1}{\log_b a} = \dfrac{4}{3} \Leftrightarrow \log_b a = \dfrac{3}{4}$ (B)

(20) $\lim f(u_n) = f\left(\lim \dfrac{n}{e^n}\right) = f\left(\lim \left(\dfrac{1}{\underbrace{\frac{e^n}{n}}_{\text{lim. not. } +\infty}}\right)\right) = f\left(\dfrac{1}{+\infty}\right) = f(0^+) = \ln 0^+ = -\infty$ (A)

21.a) Apenas tem sentido $x \to +\infty$ pois $D = \left]-\frac{\pi}{2}, +\infty\right[$

$$m = \lim_{x\to+\infty} \frac{f(x)}{x} = \lim_{x\to+\infty} \frac{x - \ln x}{x} = \lim_{x\to+\infty} \frac{x}{x} - \lim_{x\to+\infty} \underbrace{\frac{\ln x}{x}}_{\substack{L.N. \\ 0}} = 1 - 0 = 1$$

$$b = \lim_{x\to+\infty} \left(f(x) - mx\right) = \lim_{x\to+\infty} \left(x - \ln x - x\right) = -(+\infty) = -\infty$$

Assim, conclui-se que f não tem Assintota Oblíqua ao gráfico

21.b) $x = \frac{1}{2}$

$$f'(x) = x' - (\ln x)' = 1 - \frac{x'}{x} = 1 - \frac{1}{x}$$

$$m = f'\left(\tfrac{1}{2}\right) = 1 - \frac{1}{\frac{1}{2}} = 1 - 2 = -1 \qquad f\left(\tfrac{1}{2}\right) = \frac{1}{2} - \ln\left(\tfrac{1}{2}\right) = \frac{1}{2} - (\underset{0}{\ln 1} - \ln 2) = \frac{1}{2} + \ln 2$$

Ponto de Tangência $= \left(\frac{1}{2}, f\left(\frac{1}{2}\right)\right) = \left(\frac{1}{2}, \frac{1}{2} + \ln 2\right)$

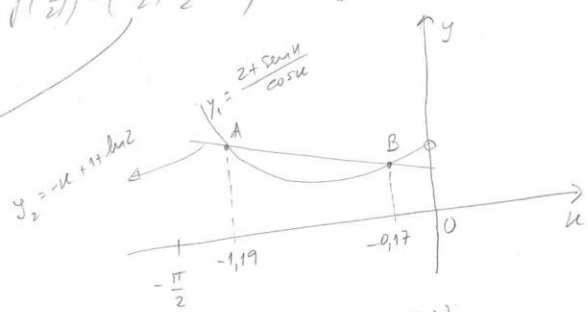
$y = mx + b$

$y = -x + b$

$\frac{1}{2} + \ln 2 = -\frac{1}{2} + b$

$b = 1 + \ln 2$

$$\boxed{y = -x + 1 + \ln 2}$$



R: $x_A \approx -1{,}19$ e $x_B \approx -0{,}17$

22.a) $x = 0{,}003$, $p = 24$, $m = ?$

$$\frac{600 \times 0{,}003}{1 - e^{-0{,}003m}} = 24 \Leftrightarrow \frac{1{,}8}{1 - e^{-0{,}003m}} = 24 \Leftrightarrow 1 - e^{-0{,}003m} = \frac{1{,}8}{24} \Leftrightarrow$$

$$\Leftrightarrow 1 - e^{-0{,}003m} = 0{,}075 \Leftrightarrow e^{-0{,}003m} = 0{,}925 \Leftrightarrow -0{,}003m = \ln(0{,}925) \Leftrightarrow$$

$$\Leftrightarrow m = \frac{\ln(0{,}925)}{-0{,}003} \Leftrightarrow m \approx 26 \text{ meses}$$

22.b)

$$\lim_{x\to 0} \frac{600x}{1 - e^{-mx}} \overset{\left(\frac{0}{0}\right)}{=} \lim_{y\to 0} \frac{-\frac{600y}{m}}{1 - e^{y}} = -\frac{600}{m} \times \lim_{y\to 0} \frac{y}{1 - e^{y}} = -\frac{600}{m} \times \lim_{y\to 0} \frac{1}{\frac{1 - e^{y}}{y}} =$$

$$\boxed{\begin{array}{l} y = -mx \to 0 \\ x = -\frac{y}{m} \end{array}}$$

$$= -\frac{600}{m} \times \lim_{y\to 0} \frac{1}{-\underbrace{\frac{e^{y} - 1}{y}}_{\substack{L.N \\ 1}}} = -\frac{600}{m} \times \frac{1}{-1} = \frac{600}{m}$$

Quando a taxa de juro tende para zero, a prestação mensal tende para o quociente entre o valor do empréstimo e o n.º de prestações mensais.

(23) $g(x) = x+1 \Leftrightarrow \underbrace{g(x)-x-1}_{h(x)} = 0$

Seja $h(x) = g(x) - x - 1$

h é cont. em $[a, g(a)]$

$h(a) = \underbrace{g(a)}_{>a+1} - a - 1 > \underbrace{\cancel{a}+\cancel{1}-\cancel{a}-\cancel{1}}_{0}$, ou seja, $h(a)>0$

$h(g(a)) = \underbrace{g(g(a))}_{a\text{, porque } (g\circ g)(x)=x} - g(a) - 1 = a - g(a) - 1 = \underbrace{-g(a)}_{<-a-1\text{, porque } g(a)>a+1 \Leftrightarrow -g(a)<-a-1} + a - 1 < \underbrace{-\cancel{a}-1+\cancel{a}-1}_{-2}$, ou seja, $h(g(a))<0$

Assim, pelo teor. de Bolzano-Cauchy, $\exists c \in]a, g(a)[: h(c) = 0$, isto é, $g(c)-c-1=0$, ou seja $g(c)=c+1$ c.q.d.

(24)

	$-\infty$	0	$+\infty$
f	$\cap$	PI	$\cup$
f''	$-$	0	$+$

$\underbrace{f''(1)}_{+} \times \underbrace{f''(2)}_{+} > 0$ (D)

(25) $y = -x$ A.O, logo $m = \lim_{x\to+\infty} \frac{f(x)}{x} = -1$ e $m = \lim_{x\to+\infty} \frac{g(x)}{x} = -1$

$\lim_{x\to+\infty} \frac{f(x)\times g(x)}{x} = \underbrace{\lim_{x\to+\infty} \frac{f(x)}{x}}_{-1} \times \underbrace{\lim_{x\to+\infty} \frac{g(x)}{x}}_{-1} \times \lim x = -1\times(-1)\times(+\infty) = +\infty$ (A)

(26.1) $f(0) = 9 - 2.5(e^{1-0} + e^{0-1}) \approx 1,28$

$\sqrt{\underbrace{(f(0))^2+x^2}_{>0}} = 2 \Leftrightarrow 1,28^2 + x^2 = 4 \Leftrightarrow x^2 = 2,3616 \Leftrightarrow x = \pm\sqrt{2,3616} \Leftrightarrow x \approx 1,5$
$x \in [0,7]$

$\left.\begin{matrix} P(0, f(0)) \\ S(x, 0) \end{matrix}\right\}$ $\overline{PS} = \sqrt{(f(0))^2+x^2} = 2$ Assim, 1,5 é a abcissa do ponto da superfície da água do rio que dista dois metros do ponto P.

(26.2) $f'(x) = 0 - 2,5\left(e^{1-0,2x} + e^{0,2x-1}\right)' = -2,5\left((1-0,2x)' e^{1-0,2x} + (0,2x-1)' e^{0,2x-1}\right) =$
$= -2,5\left(-0,2 e^{1-0,2x} + 0,2 e^{0,2x-1}\right)$

Zeros de f':
$-2,5\left(-0,2 e^{1-0,2x} + 0,2 e^{0,2x-1}\right) = 0 \Leftrightarrow -0,2 e^{1-0,2x} + 0,2 e^{0,2x-1} = 0 \Leftrightarrow$
$\Leftrightarrow \cancel{0,2} e^{0,2x-1} = \cancel{0,2} e^{1-0,2x} \Leftrightarrow 0,2x-1 = 1-0,2x \Leftrightarrow 0,4x = 2 \Leftrightarrow x = \frac{2}{0,4} \Leftrightarrow x = 5$

x	0		5		7
f'	$+$	$+$	0	$-$	$-$
f	Mín	$\nearrow$	Máx	$\searrow$	Mín

$$\text{Máx} = f(5) = 9 - 2{,}5\left(e^{1-0{,}2\times 5} + e^{0{,}2\times 5 - 1}\right) =$$
$$= 9 - 2{,}5\left(e^{0} + e^{0}\right) = 9 - 2{,}5 \times 2 = 4$$

Como a altura máxima é 4m, o barco não pode passar

27.1 g é cont para $x=1$ se $\lim\limits_{x\to 1} g(x)$ existir, isto é, $\lim\limits_{x\to 1^+} g(x) = \lim\limits_{x\to 1^-} g(x) = g(1)$

$$\lim_{x\to 1^-} g(x) = \lim_{x\to 1^-} \frac{1-x^2}{1-e^{x-1}} = \lim_{x\to 1^-} \frac{x^2-1}{e^{x-1}-1} = \lim_{x\to 1^-} \frac{(x-1)(x+1)}{e^{x-1}-1} = \lim_{x\to 1^-} \frac{x-1}{e^{x-1}-1} \times \lim_{x\to 1^-}(x+1) =$$

$$= \lim_{y\to 0} \frac{y}{e^{y}-1} \times \underbrace{\lim_{x\to 1^-}(x+1)}_{2} = \lim_{y\to 0} \frac{1}{\underbrace{\frac{e^{y}-1}{y}}_{\text{L.N.}\ 1}} \times 2 = \frac{1}{1}\times 2 = 2$$

$\boxed{\begin{array}{l} y = x-1 \to 0 \\ x = y+1 \end{array}}$

$$\lim_{x\to 1^+}\left(3 + \frac{\operatorname{sen}(x-1)}{1-x}\right) = 3 + \lim_{x\to 1^+}\left(-\frac{\operatorname{sen}(x-1)}{x-1}\right) = 3 + \lim_{y\to 0}\left(-\underbrace{\frac{\operatorname{sen} y}{y}}_{\text{L.N.}\ 1}\right) = 3 + (-1) = 2$$

$\boxed{\begin{array}{l} y = x-1 \to 0 \\ x = y+1 \end{array}}$

$g(1) = 2$

Conclui-se que g é cont. para $x=1$

27.2 $\cancel{3} + \dfrac{\operatorname{sen}(x-1)}{1-x} = \cancel{3} \Leftrightarrow \dfrac{\operatorname{sen}(x-1)}{1-x} = 0 \underset{x\neq 1}{\Leftrightarrow} \operatorname{sen}(x-1) = 0 \Leftrightarrow$

$\Leftrightarrow x - 1 = k\pi \Leftrightarrow x = 1 + k\pi$

No intervalo $]4,5[$:

$k = 0 \to x = 1 \notin]4,5[$

$k = 1 \to x = 1+\pi \in]4,5[$

$k = 2 \to x = 1+2\pi \notin]4,5[$

R: $x = 1+\pi$

27.3 $A = ?$

$g(x) = 0 \Leftrightarrow \dfrac{1-x^2}{1-e^{x-1}} = 0 \underset{x\neq 1}{\Leftrightarrow} 1 - x^2 = 0 \Leftrightarrow x^2 = 1 \Leftrightarrow x = \pm 1$. Como $x < 1$, então $\underline{x = -1}$

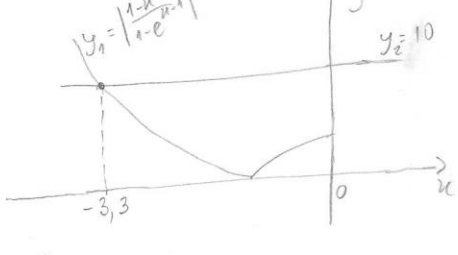
$A(-1, 0)$

$\overline{OA} = 1$

$\dfrac{\overline{OA} \times |g(x)|}{2} = 5 \Leftrightarrow$

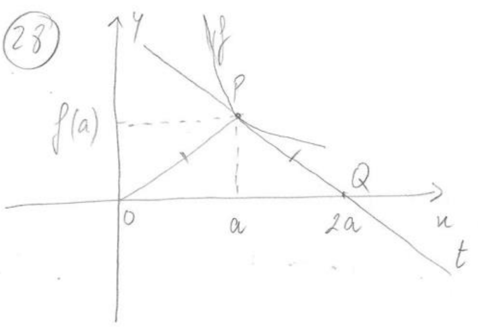
$\Leftrightarrow \dfrac{|g(x)|}{2} = 5 \Leftrightarrow$

$\Leftrightarrow |g(x)| = 10$



R: $x \approx -3{,}3$

(28)



Como $\overline{OP} = \overline{PQ}$, então o $\Delta[OPQ]$ é isósceles e $\overline{OQ} = 2a$.

$$\left.\begin{array}{l} P(a, f(a)) \\ Q(2a, 0) \end{array}\right\} m_{PQ} = \frac{0 - f(a)}{2a - a} = -\frac{f(a)}{a}$$

Assim, $f'(a) = m_{PQ} = -\frac{f(a)}{a}$

$$f'(a) + \frac{f(a)}{a} = -\frac{f(a)}{a} + \frac{f(a)}{a} = 0$$

(29) $\lim\limits_{x \to 2} \frac{x^2 - 2x}{f(x) - f(2)} = \lim\limits_{x \to 2} \frac{x(x-2)}{f(x) - f(2)} = \lim\limits_{x \to 2} \frac{x-2}{f(x) - f(2)} \times \lim\limits_{x \to 2} x =$

$= \lim\limits_{x \to 2} \frac{1}{\frac{f(x) - f(2)}{x - 2}} \times 2 = \frac{2}{f'(2)}$ Assim, $\frac{2}{f'(2)} = 4 \Leftrightarrow f'(2) = \frac{2}{4} = \frac{1}{2}$ (C)

(30) $(g \circ f)(x) = 0 \Leftrightarrow g(\underbrace{f(x)}_{2}) = 0 \Leftrightarrow f(x) = 2 \Leftrightarrow x = 1 \vee x = 5$ (B)

(31) Temos uma translação $(\rightarrow)$ e o sinal de f'' será simétrico

x	$-\infty$		-5		5		15	$+\infty$
f''		+	0	-	0	+	0	-
f		$\cup$		$\cap$		$\cup$		$\cap$

(C)

(32) A.V:

(32.1) f é cont. em $]0, +\infty[$. Assim, a A.V. caso exista será para $x = 0^+$

$\lim\limits_{x \to 0^+} \frac{\ln x}{x} = \frac{\ln 0^+}{0^+} = \frac{-\infty}{0^+} = -\infty$ Logo $x = 0$ A.V.

A.H. (apenas tem sentido $x \to +\infty$, porque $D =]0, +\infty[$)

$\lim\limits_{x \to +\infty} \frac{\ln x}{x} = \underset{(L.N.)}{0}$ Assim, $y = 0$ A.H.

(32.2) $f(x) > 2\ln x \Leftrightarrow \frac{\ln x}{x} > 2\ln x \Leftrightarrow \frac{\ln x}{x} - 2\ln x > 0 \Leftrightarrow \frac{\ln x - 2x\ln x}{x} > 0$

$D = \mathbb{R}^+$ C.A.:

Zeros do numer.:

$\ln x - 2x\ln x = 0$

$\ln x(1-2x) = 0$

$\ln x = 0 \vee 1-2x = 0$

$x = 1 \vee x = \frac{1}{2}$

Zeros do denom.:

$x = 0$

	0		$\frac{1}{2}$		1	$+\infty$
$\ln x$		$-$	$-$	$-$	0	$+$
$1-2x$		$+$	0	$-$	$-$	$-$
$\ln x(1-2x)$		$-$	0	$+$	0	$-$
x		$+$	$+$	$+$	$+$	$+$
$\frac{\ln x - 2x\ln x}{x}$		$-$	0	$+$	0	$-$

$\Leftrightarrow \frac{\ln x(1-2x)}{x} > 0$

$S = \left]\frac{1}{2}, 1\right[$

(32.3) $g'(x) = \left(\frac{K}{x}\right)' + \left(\frac{\ln x}{x}\right)' = -\frac{K}{x^2} + \frac{(\ln x)'x - \ln x \times x'}{x^2} = -\frac{K}{x^2} + \frac{\frac{x'}{\cancel{x}}\times \cancel{x} - \ln x}{x^2} =$

$= -\frac{K}{x^2} + \frac{1 - \ln x}{x^2} = \frac{-K+1-\ln x}{x^2}$

Como g é diferenciável em $\mathbb{R}^+$ e tem extremo relativo para $x = 1$, então:

$g'(1) = 0 \Leftrightarrow \frac{-K+1-\ln 1}{1^2} = 0 \Leftrightarrow \frac{-K+1-0}{1} = 0 \Leftrightarrow -K+1 = 0 \Leftrightarrow K = 1$

(33) $4 + \log_a 5^{\ln a} = 4 + \ln a \times \log_a 5 = 4 + \cancel{\ln a} \times \frac{\ln 5}{\cancel{\ln a}} = \ln(e^4) + \ln 5 = \ln(5e^4)$ (B)

(34.a) $P(2) = \frac{P(1)}{2} \Leftrightarrow 120e^{-2K} = \frac{120e^{-K}}{2} \Leftrightarrow \frac{e^{-2K}}{e^{-K}} = \frac{\cancel{120}}{2\times\cancel{120}} \Leftrightarrow e^{-K} = \frac{1}{2} \Leftrightarrow$

$\Leftrightarrow -K = \ln\left(\frac{1}{2}\right) \Leftrightarrow -K = \ln 1 - \ln 2 \Leftrightarrow -K = -\ln 2 \Leftrightarrow K = \ln 2$

(34.b) $\frac{P(3)-P(0)}{3-0} = \frac{120e^{-2,1} - 120e^0}{3} \approx -35$

Nas primeiras 3 horas, a massa de poluente diminui, em média, 35 g por hora, aproximadamente.

(35) Pretende-se $f' > 0$ e $f'' > 0$ ou $f' < 0$ e $f'' < 0$

$\nearrow$ e $\cup$ ou $\searrow$ e $\cap$

$[2, +\infty[$ ou $[-2, 0]$ (A)

(36) $(f-g)^{-1}(2) = 3 \Leftrightarrow (f-g)(3) = 2 \Leftrightarrow f(3) - g(3) = 2 \Leftrightarrow 4 - g(3) = 2 \Leftrightarrow$

$\Leftrightarrow g(3) = 2$ (B)

37.a) $\lim\limits_{x\to+\infty}(f(x)-x)=\lim\limits_{x\to+\infty}\left[\ln(e^x+x)-x\right]=\lim\limits_{x\to+\infty}\left[\ln(e^x+x)-\ln e^x\right]=$

$=\lim\limits_{x\to+\infty}\left[\ln\left(\frac{e^x-x}{e^x}\right)\right]=\lim\limits_{x\to+\infty}\left[\ln\left(1-\frac{x}{e^x}\right)\right]=\lim\limits_{x\to+\infty}\left[\ln\left(1-\frac{1}{\underbrace{\frac{e^x}{x}}_{\text{L.N.}}}\right)\right]=\ln(1-0)=\ln 1=0$

Como $\lim\limits_{x\to+\infty}(f(x)-x)=0$ então $y=x$ A.O.

37.c) Em $\left]-\frac{3\pi}{2},0\right[$, $f(x)=\frac{1}{4}x^2+\cos x$

$f'(x)=\frac{1}{4}\times 2x-\operatorname{sen}x=\frac{1}{2}x-\operatorname{sen}x$

$f''(x)=\frac{1}{2}-\cos x$

Zeros de f''

$\frac{1}{2}-\cos x=0$

$\cos x=\frac{1}{2}$ em $\left]-\frac{3\pi}{2},0\right[$

$x=-\frac{\pi}{3}$

	$-\frac{3\pi}{2}$		$-\frac{\pi}{3}$		0
f''	///	+	0	−	///
f	///	∪	PI	∩	///

Conc. voltada p/ cima em $\left]-\frac{3\pi}{2},-\frac{\pi}{3}\right]$

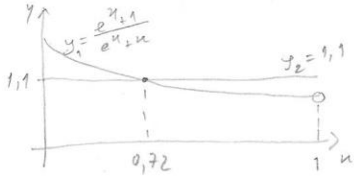
" " " baixo em $\left[-\frac{\pi}{3},0\right[$

Ponto de inflexão para $x=-\frac{\pi}{3}$

37.b) Pretende-se obter $\underline{a}$ de modo que $f'(a)=1,1$

$f'(x)=\left(\ln(e^x+x)\right)'=\frac{(e^x+x)'}{e^x+x}=\frac{e^x+1}{e^x+x}$

$f'(a)=1,1 \Leftrightarrow \frac{e^a+1}{e^a+a}=1,1$



Assim, $a\approx 0,72$

38) $\log_a b+\log_b a=\log_a a^{\frac{1}{3}}+\log_b b^3=\frac{1}{3}+3=\frac{10}{3}$ (C)

$a=b^3 \Leftrightarrow b=\sqrt[3]{a}=a^{\frac{1}{3}}$

39) $(f\circ g)(x)=0 \Leftrightarrow f(g(x))=0 \Leftrightarrow g(x)=-1 \vee g(x)=1 \Leftrightarrow \ln x=-1 \vee \ln x=1 \Leftrightarrow$

$\Leftrightarrow x=e^{-1} \vee x=e^1 \Leftrightarrow x=\frac{1}{e} \vee x=e$ (D)

40) $\lim\left[\left(1+\frac{1}{n}\right)^{3n}\right]=\lim\left(\left(1+\frac{1}{n}\right)^n\right)^3=\left(\lim\left(1+\frac{1}{n}\right)\right)^3=e^3=a$

$\lim\left(\ln(1-2e^{-n})\right)=\lim\left(\ln\left(1-\frac{2}{e^n}\right)\right)=\ln(1-0)=\ln 1=0=b$ (B)

41) $g(0) = \ln K$ $\qquad D =]-K, +\infty[$

$g(K) = \ln(2K)$

$\ln K \times \ln(2K) < 0$

Zeros de $\ln K$: $\ln K = 0 \Leftrightarrow K = e^0 \Leftrightarrow K = 1$

Zeros de $\ln(2K)$: $\ln(2K) = 0 \Leftrightarrow 2K = e^0 \Leftrightarrow 2K = 1 \Leftrightarrow K = \frac{1}{2}$

	$-K$		$\frac{1}{2}$		1	$+\infty$
$\ln K$		$-$	$-$	$-$	0	$+$
$\ln(2K)$		$-$	0	$+$	$+$	$+$
$\ln K \times \ln(2K)$		$+$	0	$-$	0	$+$

$K \in \left]\frac{1}{2}, 1\right[$

42.a) $V(25) = 3\ln\left(\frac{25+300}{25+60}\right) \approx 4{,}02$ Km/s

1 s — 4,02 Km

x — 200 Km

$x = \frac{200}{4{,}02} \Leftrightarrow x \approx 50$ s

42.b) $V(x) = 3 \Leftrightarrow 3\ln\left(\frac{x+300}{x+60}\right) = 3 \Leftrightarrow \ln\left(\frac{x+300}{x+60}\right) = 1 \Leftrightarrow \frac{x+300}{x+60} = e \Leftrightarrow$

$\Leftrightarrow x + 300 = ex + 60e \Leftrightarrow x - ex = 60e - 300 \Leftrightarrow x(1-e) = 60e - 300 \Leftrightarrow$

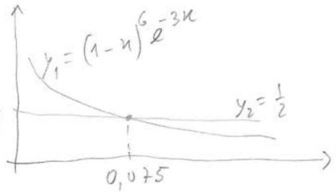
$\Leftrightarrow x = \frac{60e - 300}{1-e} \Leftrightarrow x \approx 80$ $\qquad$ R: 80 milhares de toneladas

43) $f'(x) = \frac{f(2) - f(0)}{2-0} \Leftrightarrow f'(x) = \frac{f(2)-1}{2}$

$0 < f'(x) < 9 \Leftrightarrow 0 < \frac{f(2)-1}{2} < 9 \Leftrightarrow 0 < f(2) - 1 < 18 \Leftrightarrow 1 < f(2) < 19$ (B)

44) $R = d$

$L = \frac{I}{2} \Leftrightarrow I(1-R)^6 e^{-3d} = \frac{I}{2} \Leftrightarrow (1-d)^6 e^{-3d} = \frac{1}{2}$



Assim, $d = R \approx 0{,}075$

45) $\lim \left(\frac{n+K}{n}\right)^n = \lim \left(1 + \frac{K}{n}\right)^n = e^K$

Então $\ln\left(\frac{e^K}{e}\right) = 3 \Leftrightarrow \ln\left(e^{K-1}\right) = 3 \Leftrightarrow K - 1 = 3 \Leftrightarrow K = 4$ (D)

46) $\ln b = 4\ln a \Leftrightarrow \ln b = \ln a^4 \Leftrightarrow b = a^4$

$a^x \geq b^{\frac{1}{x}} \Leftrightarrow a^x \geq (a^4)^{\frac{1}{x}} \Leftrightarrow a^x \geq a^{\frac{4}{x}} \Leftrightarrow x \geq \frac{4}{x} \Leftrightarrow x - \frac{4}{x} \geq 0 \Leftrightarrow \frac{x^2-4}{x} \geq 0$

$x^2 - 4 = 0$, $x = \pm 2$; $\quad x = 0$

	$-\infty$	-2		0		2	$+\infty$
x^2-4	$+$	0	$-$	$-$	$-$	0	$+$
x	$-$	$-$	$-$	0	$+$	$+$	$+$
$\frac{x^2-4}{x}$	$-$	0	$+$	ND	$-$	0	$+$

$S = [-2, 0[\cup [2, +\infty[$

(47)
a) $\frac{e^{2x}-1}{4x} = 0 \wedge x < 0$

$\Leftrightarrow e^{2x}-1 = 0 \wedge x < 0$

$\Leftrightarrow e^{2x} = 1 \wedge x < 0$

$\Leftrightarrow 2x = \ln 1 \wedge x < 0$

$\Leftrightarrow 2x = 0 \wedge x < 0$

$\Leftrightarrow x = 0 \wedge x < 0$

não tem zeros

$\underbrace{\frac{1}{2-\operatorname{sen}(2x)} = 0}_{\text{imp.}}$ $\forall x \in \pi$

não tem zeros

$\Rightarrow$ g não tem zeros (A)

(47.b) g cont para $x=0 \Leftrightarrow \lim_{x\to 0^-} g(x) = \lim_{x\to 0^+} g(x) = g(0)$

$\lim_{x\to 0^-} g(x) = \lim_{x\to 0^-} \frac{e^{2x}-1}{4x} = \lim_{y\to 0} \frac{e^y-1}{2y} = \lim_{y\to 0}\left(\frac{e^y-1}{y}\times\frac{1}{2}\right) = \frac{1}{2}\times\overbrace{\lim\frac{e^y-1}{y}}^{L.N} = \frac{1}{2}\times 1 = \frac{1}{2}$

$\boxed{\begin{array}{c} 2x = y \to 0 \\ x = \frac{y}{2}\end{array}}$

$\lim_{x\to 0^+} g(x) = \lim_{x\to 0^+} \frac{1}{2-\operatorname{sen}(2x)} = \frac{1}{2-\operatorname{sen} 0} = \frac{1}{2-0} = \frac{1}{2}$

$g(0) = \frac{1}{2-\operatorname{sen} 0} = \frac{1}{2}$

Assim, concluímos que g é cont. no ponto 0.

(48) Pretende-se que o declive da reta PQ seja 1

$h(a) = \frac{\ln a}{a}$, logo $P\left(a, \frac{\ln a}{a}\right)$

$h(2a) = \frac{\ln(2a)}{2a}$, logo $Q\left(2a, \frac{\ln(2a)}{2a}\right)$

$m_{PQ} = \frac{\frac{\ln(2a)}{2a} - \frac{\ln a}{a}}{2a-a} = \frac{\frac{\ln 2 + \ln a - 2\ln a}{2a}}{a} = \frac{\ln 2 - \ln a}{2a^2} = \frac{\ln\left(\frac{2}{a}\right)}{2a^2}$

$\frac{\ln\left(\frac{2}{a}\right)}{2a^2} = 1 \Leftrightarrow \ln\left(\frac{2}{a}\right) = 2a^2 \Leftrightarrow \ln\left(\frac{2}{a}\right) - 2a^2 = 0, \quad a \in \left[\frac{1}{2}, 1\right]$

Seja $f(x) = \ln\left(\frac{2}{x}\right) - 2x^2$

$f\left(\frac{1}{2}\right) = \ln 4 - 2\times\frac{1}{4} = \ln 4 - \frac{1}{2} = \ln 4 - \ln e^{\frac{1}{2}} = \ln\left(\frac{4}{e^{\frac{1}{2}}}\right) = \ln\left(\frac{4}{\sqrt{e}}\right) > 0$, porque $4 > \sqrt{e}$

$f(1) = \ln 2 - 2 = \ln 2 - \ln e^2 = \ln\left(\frac{2}{e^2}\right) < 0$, porque $2 < e^2$

f é cont em $\left[\frac{1}{2}, 1\right]$
$f\left(\frac{1}{2}\right)\times f(1) < 0$

Pelo Teorema de Bolzano-Cauchy podemos concluir que existe pelo menos um n.º $a \in \left[\frac{1}{2}, 1\right]$ tal que $f(x) = 0$, isto é, $\ln\left(\frac{2}{a}\right) - 2a^2 = 0$ c.q.d.

(49) $\lim \left[\left(\frac{n+5}{n+1}\right)^{\frac{n}{2}}\right] = \lim \left(\frac{\frac{n}{n}+\frac{5}{n}}{\frac{n}{n}+\frac{1}{n}}\right)^{\frac{n}{2}} = \lim \left[\frac{\left(1+\frac{5}{n}\right)^{n}}{\left(1+\frac{1}{n}\right)^{n}}\right]^{\frac{1}{2}} = \left(\frac{e^5}{e}\right)^{\frac{1}{2}} =$

$= \left(e^4\right)^{\frac{1}{2}} = e^2$ (D)

(50) $\log_2(x+1) \leq 3 - \log_2(8-x) \Leftrightarrow \log_2(x+1) + \log_2(8-x) \leq 3 \Leftrightarrow \log_2\left((x+1)(8-x)\right) \leq 3$

$\Leftrightarrow (x+1)(8-x) \leq 2^3 \Leftrightarrow 8x - x^2 + 8 - x \leq 8 \Leftrightarrow x^2 - 7x \geq 0$

Domínio:

$x+1 > 0 \wedge 8-x > 0$

$x > -1 \wedge x < 8$

$D =]-1, 8[$

Zeros de x^2-7x:

$x^2 - 7x = 0$

$x(x-7) = 0$

$x = 0 \vee x = 7$

	-1		0		7		8
x^2-7x		+	0	-	0	+	

$S =]-1, 0] \cup [7, 8[$

(51.a) $f'(0) = \lim_{x\to 0} \frac{f(x)-f(0)}{x-0} = \lim_{x\to 0} \frac{3+\frac{e^x}{1-x}-4}{x} = \lim_{x\to 0} \frac{\frac{e^x}{1-x}-1}{x} =$

$f(0) = 3 + \frac{e^0}{1-0} = 4$

$= \lim_{x\to 0} \frac{\frac{e^x-1+x}{1-x}}{x} = \lim_{x\to 0} \frac{e^x-1+x}{x(1-x)} = \lim_{x\to 0} \frac{\frac{e^x-1}{x}+\frac{x}{x}}{\frac{x(1-x)}{x}}$

$= \lim_{x\to 0} \frac{\overbrace{\frac{e^x-1}{x}}^{\text{L.N.} \to 1}+1}{1-x} = \frac{1+1}{1-0} = 2$

(51.b) $\boxed{x \to -\infty}$

$\lim_{x\to -\infty} \left(3 + \frac{e^x}{1-x}\right) = 3 + \lim_{x\to -\infty} \frac{e^x}{1-x} = 3 + \frac{0}{1+\infty} = 3+0 = 3$ Assim $y=3$ é a equação da A.H. quando $x \to -\infty$

$\boxed{x \to +\infty}$

$\lim_{x\to +\infty} \frac{\ln(x^2)+2}{x} = \lim_{x\to +\infty} \left(\frac{2\ln x}{x} + \frac{2}{x}\right) = 2\underbrace{\lim_{x\to +\infty} \frac{\ln x}{x}}_{\text{L.N.}} + \lim_{x\to +\infty} \frac{2}{x} = 2\times 0 + \frac{2}{+\infty} =$

$= 0 + 0 = 0$

Assim, $y=0$ é a equação da A.H. quando $x \to +\infty$

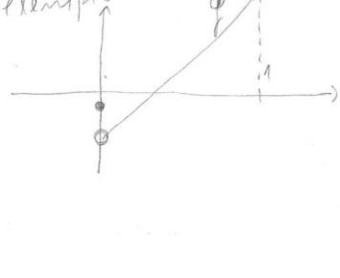
(51.c) $h(x) = x+1$

$(f \circ h^{-1})(2) = f(h^{-1}(2)) = f(1) = \frac{\ln 1 + 2}{1} = \frac{0+2}{1} = 2$ (C)

$h^{-1}(2) = ?$

$x+1 = 2$, $x = 1$ } $h^{-1}(2) = 1$

(52) exemplo



g tem 1 zero

g é limitada

g tem máximo

g não é contínua (D)

(53) $\lim \left(\frac{n+2}{n+1}\right)^{2n} = \lim \left(\frac{\frac{n}{n}+\frac{2}{n}}{\frac{n}{n}+\frac{1}{n}}\right)^{2n} = \lim \left[\frac{\left(1+\frac{2}{n}\right)^n}{\left(1+\frac{1}{n}\right)^n}\right]^2 = \left(\frac{e^2}{e}\right)^2 = e^2$

(C)

(54) $f(x) = x^3 + 6\ln x$

$f'(x) = 3x^2 + 6\,\frac{x'}{x} = 3x^2 + \frac{6}{x}$

$f''(x) = 6x + \frac{6' \times x - 6 \times x'}{x^2} = 6x - \frac{6}{x^2}$

Zeros de f'':

$6x - \frac{6}{x^2} = 0 \Leftrightarrow \frac{6x^3-6}{x^2} = 0 \Leftrightarrow$

$\Leftrightarrow 6x^3 - 6 = 0 \wedge x^2 \neq 0 \Leftrightarrow$

$\Leftrightarrow x^3 = 1 \wedge x \neq 0 \Leftrightarrow x = 1 \wedge x \neq 0$

$D = \mathbb{R}^+$

	0		1	
$6x^3-6$	//	−	0	+
x^2	//	+	+	+
f''	//	−	0	+
f	//	∩	PI	∪

$f(1) = 1^3 + 6\ln 1 = 1 + 6 \times 0 = 1$

Ponto de inflexão $(1,1)$

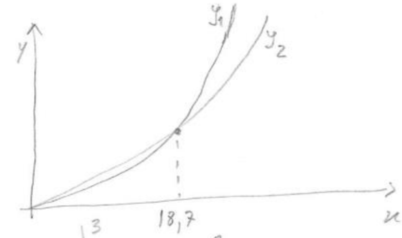
Concavidade voltada p/ baixo em $]0,1]$

" " " cima em $[1,+\infty[$

(55) $t(3x) - t(x) = 3\,t(x) \Leftrightarrow$

$\Leftrightarrow \underbrace{t(3x)}_{y_1} = \underbrace{4\,t(x)}_{y_2}$

R: 18,7 m



(56) $\lim \left(\frac{n-2}{n}\right)^{3n} = \lim \left(\left(1-\frac{2}{n}\right)^n\right)^3 = \left(e^{-2}\right)^3 = e^{-6} = \frac{1}{e^6}$ (C)

(57.a) $g(x) = \frac{e^{-x}}{x}$

$g'(x) = \frac{(e^{-x})' \times x - e^{-x} \times x'}{x^2}$

$= \frac{-e^{-x} \times x - e^{-x}}{x^2}$

Zeros de g':

$-e^{-x}x - e^{-x} = 0 \wedge x^2 \neq 0$

$e^{-x}(-x-1) = 0 \wedge x \neq 0$

$(\underbrace{e^{-x} = 0}_{\text{imp.}} \vee -x-1 = 0) \wedge x \neq 0$

$x = -1 \wedge x \neq 0$

$D = \mathbb{R} \setminus \{0\}$

	$-\infty$		-1		0		$+\infty$
e^{-x}		+	+	+	//	+	
$-x-1$		+	0	−	//	−	
$e^{-x}(-x-1)$		+	0	−	//	−	
x^2		+	+	+	//	+	
g'		+	0	−	//	−	
g		↗	máx	↘	//	↘	

$g(-1) = \frac{e^1}{-1} = -e$

g é crescente em $]-\infty,-1]$

g é decrescente em $[-1,0[$ e em $]0,+\infty[$

$g(-1) = -e$ é máx relativo para $x = -1$

57.b) $h(x) = \frac{e^{-x}}{x} + 2x - \frac{1}{\sqrt{x}}$ em $\mathbb{R}^+$

$$m = \lim_{x\to+\infty} \frac{h(x)}{x} = \lim_{x\to+\infty}\left(\frac{e^{-x}}{x^2} + 2 - \frac{1}{x\sqrt{x}}\right) = \lim_{x\to+\infty} \frac{e^{-x}}{x^2} + 2 - \lim \frac{1}{x\sqrt{x}} =$$

$$= \frac{0}{+\infty} + 2 - \frac{1}{+\infty\times+\infty} = 0+2-0 = 2 \text{ (B)}$$

58) $\ln(a^2-b^2) - 2\ln(a+b) = \ln[(a-b)(a+b)] - 2\ln(a+b) =$
$= \ln(a-b) + \ln(a+b) - 2\ln(a+b) = \ln(a-b) - \ln(a+b) = \ln(a-b) - \ln(2(a-b)) =$
$= \ln\left(\frac{\cancel{a-b}}{2\cancel{(a-b)}}\right) = \ln\left(\frac{1}{2}\right) \approx -0,7$ (C)

59) $f(1) = \log_3 K$

$\lim_{x\to1} \frac{x^2-1}{x-1} = \lim_{x\to1} \frac{\cancel{(x-1)}(x+1)}{\cancel{x-1}} = 1+1 = 2$

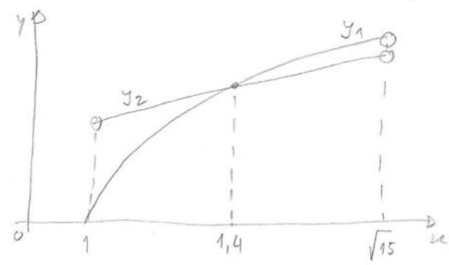
$\log_3 K = 2 \Leftrightarrow K = 3^2 \Leftrightarrow K = 9$ (D)

60) $\lim\left(\frac{n+\ln a}{n}\right)^{n+2} = \lim\left[\underbrace{\left(1+\frac{\ln a}{n}\right)^n}_{\downarrow\, e^{\ln a}} \times \underbrace{\left(1+\frac{\ln a}{n}\right)^2}_{\downarrow\, (1+0)^2=1}\right] = e^{\ln a} \times 1 = a$ (C)

61) $r_1 = 7$, $x \in]8-7, \sqrt{8^2-7^2}[=]1, \sqrt{15}[$
$r_2 = 8$

$d = 9+x$

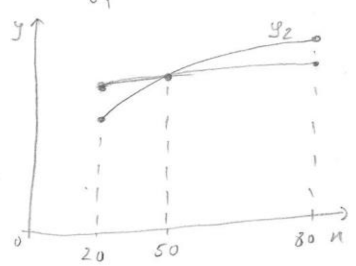
$$\underbrace{\frac{\sqrt{(15^2-x^2)(x^2-1)}}{x}}_{y_1} = \underbrace{9+x}_{y_2}$$



R: $x \approx 1,4$

62) Som inicial: $N = 60 + 10\log I$
Som final: $N = 60 + 10\log(I+150)$

$$\underbrace{60+10\log(I+150)}_{y_1} = \underbrace{0,014(60+10\log I)^2}_{y_2}$$



R: $50\ \mu W/m^2$

(63) $h(x) = \dfrac{e^x}{x-1}$ $\quad D = \mathbb{R}\setminus\{1\}$

a) Como h é cont em $\mathbb{R}\setminus\{1\}$, apenas a reta de equação $x=1$ poderá ser A.V.

$$\lim_{x\to 1^-} h(x) = \lim_{x\to 1^-} \frac{e^x}{x-1} = \frac{e}{0^-} = -\infty$$

$$\lim_{x\to 1^+} h(x) = \lim_{x\to 1^+} \frac{e^x}{x-1} = \frac{e}{0^+} = +\infty$$

Assim $x=1$ é a equação da única A.V.

$$\lim_{x\to -\infty} \frac{e^x}{x-1} = \frac{e^{-\infty}}{-\infty-1} = \frac{0}{-\infty} = 0$$

$$\lim_{x\to +\infty} \frac{e^x}{x-1} = \lim_{x\to +\infty} \frac{\overbrace{\frac{e^x}{x}}^{\text{L.N.} \to +\infty}}{\frac{x}{x}-\frac{1}{x}} = \frac{+\infty}{1-\frac{1}{+\infty}} = \frac{+\infty}{1-0} = +\infty$$

Assim, a reta de equação $y=0$ é A.H.

(63.b) $(x-1)\times\dfrac{e^x}{x-1} + 2e^{-x} = 3 \underset{x\neq 1}{\Leftrightarrow} e^x + \dfrac{2}{e^x} = 3 \Leftrightarrow e^{2x}+2 = 3e^x \Leftrightarrow$

$\Leftrightarrow e^{2x} - 3e^x + 2 = 0 \underset{\boxed{y=e^x}}{\Leftrightarrow} y^2-3y+2=0 \Leftrightarrow y = \dfrac{3\pm\sqrt{9-4\times1\times2}}{2} \Leftrightarrow y=\dfrac{3\pm1}{2}$

$\Leftrightarrow y=2 \vee y=1 \Leftrightarrow e^x=2 \vee e^x=1 \Leftrightarrow x=\ln 2 \vee x=0 \qquad S=\{0;\ln 2\}$

(64) $f'(x)=2x \qquad P(a, f(a)) = (a, a^2)$

$Q(k, f(k)) = (k, k^2) = \left(-\dfrac{1}{4a}, \dfrac{1}{16a^2}\right)$

$m_r = f'(a) = 2a \qquad m_s = -\dfrac{1}{m_r} = -\dfrac{1}{2a}$

$m_s = f'(k) = 2k$

$2k = -\dfrac{1}{2a} \Leftrightarrow \boxed{k = -\dfrac{1}{4a}}$

$y = 2ax + b$

$P: a^2 = 2a\times a + b$

$a^2 = 2a^2 + b$

$-a^2 = b$

$r: y = 2ax - a^2$

$y = -\dfrac{1}{2a}x + b$

$Q: \dfrac{1}{16a^2} = -\dfrac{1}{2a}\times\left(-\dfrac{1}{4a}\right) + b \Leftrightarrow \dfrac{1}{16a^2} = \dfrac{1}{8a^2} + b \Leftrightarrow -\dfrac{1}{16a^2} = b$

$s: y = -\dfrac{1}{2a}x - \dfrac{1}{16a^2}$

Interseção de r com s

$$\begin{cases} y = 2ax - a^2 \\ y = -\frac{1}{2a}x - \frac{1}{16a^2} \end{cases} \quad \begin{cases} 2ax = y + a^2 \\ \text{—} \end{cases} \quad \begin{cases} x = \frac{y+a^2}{2a} \\ y = -\frac{1}{2a}\times\frac{y+a^2}{2a} - \frac{1}{16a^2} \end{cases}$$

$$\begin{cases} \text{—} \\ \underset{(16a^2)}{y} = -\underset{(4)}{\frac{y+a^2}{4a^2}} - \frac{1}{16a^2} \end{cases} \quad \begin{cases} \text{—} \\ 16a^2y = -4y - 4a^2 - 1 \end{cases} \quad \begin{cases} \text{—} \\ 16a^2y + 4y = -1 - 4a^2 \end{cases}$$

$$\begin{cases} \text{—} \\ (16a^2+4)y = -1-4a^2 \end{cases} \quad \begin{cases} \text{—} \\ y = \frac{-1-4a^2}{4+16a^2} \end{cases} \quad \begin{cases} \text{—} \\ y = -\frac{\cancel{1+4a^2}}{4(\cancel{1+4a^2})} \end{cases} \quad \begin{cases} \text{—} \\ y = -\frac{1}{4} \end{cases} \text{ c.q.d.}$$

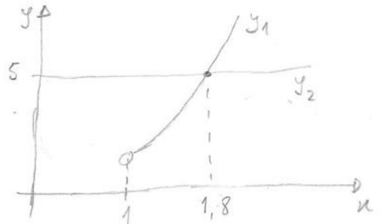
(65) $A\left(a, \frac{\ln a}{a}\right)$ $B(a, e^a)$

base $= \overline{AB} = e^a - \frac{\ln a}{a}$

altura $= a$

$$\frac{a\left(e^a - \frac{\ln a}{a}\right)}{2} = 5 \Leftrightarrow$$

$$\Leftrightarrow \underbrace{\frac{a e^a - \ln a}{2}}_{y_1} = \underbrace{5}_{y_2}$$



$a \approx 1{,}8$

(66) $\lim\limits_{x \to 1^+} \frac{x-1}{x^2+x-2} \overset{\left(\frac{0}{0}\right)}{=} \lim\limits_{x \to 1^-} \frac{\cancel{x-1}}{(\cancel{x-1})(x+2)} = \frac{1}{3}$ Logo $k = \frac{1}{3}$ (C)

	1	1	-2
1		1	2
	1	2	$\boxed{0}$

(67) $(f \circ g)(x) = 7 \Leftrightarrow f(g(x)) = 7 \Leftrightarrow 2g(x) + 1 = 7 \Leftrightarrow 2g(x) = 6 \Leftrightarrow g(x) = 3$ (B)

(68) $\lim \left(\frac{n+2}{n}\right)^{\frac{n}{4}} = \lim \left(\left(1 + \frac{2}{n}\right)^n\right)^{\frac{1}{4}} = (e^2)^{\frac{1}{4}} = e^{\frac{1}{2}}$

$\lim (f(u_n)) = f(\lim u_n) = f(e^{\frac{1}{2}}) = \ln(e^{\frac{1}{2}}) = \frac{1}{2}$ (C)

(69)
a) $m = g'(-1)$

$g'(x) = x' \ln(1-x) + x(\ln(1-x))' = \ln(1-x) + x \times \frac{(1-x)'}{1-x} = \ln(1-x) - \frac{x}{1-x}$

$m = g'(-1) = \ln(1+1) - \frac{-1}{1+1} = \ln 2 + \frac{1}{2}$ (A)

b) $m = \lim\limits_{x \to +\infty} \frac{\frac{1-3x}{1-e^{-x}}}{x} = \lim\limits_{x \to +\infty} \frac{1-3x}{x - xe^{-x}} = \lim\limits_{x \to +\infty} \frac{\frac{1}{x} - 3}{1 - e^{-x}} = \frac{\frac{1}{+\infty} - 3}{1 - e^{-\infty}} = \frac{0-3}{1-0} = -3$

$b = \lim\limits_{x \to +\infty} \left(\frac{1-3x}{1-e^{-x}} + 3x\right) = \lim\limits_{x \to +\infty} \left(\frac{1 - \cancel{3x} + \cancel{3x} - 3xe^{-x}}{1-e^{-x}}\right) = \lim\limits_{x \to +\infty} \frac{1 - \frac{3x}{e^x}}{1 - e^{-x}}$

$= \lim\limits_{x \to +\infty} \frac{1 - \frac{3}{\frac{e^x}{x}}}{1 - e^{-x}} = \frac{1 - \frac{3}{+\infty}}{1 - e^{-\infty}} = \frac{1-0}{1-0} = 1$

L.N. $\frac{e^x}{x} \to +\infty$

Assim, a reta de equação $y = -3x + 1$ é A.O.

(70.a) $f(x)=x+\ln(e^x+1)$ em $]-\infty,2]$

Como f está definida em $]-\infty,2]$, apenas tem sentido $x\to-\infty$

$m=\lim_{x\to-\infty}\frac{f(x)}{x}=\lim_{x\to-\infty}\frac{x+\ln(e^x+1)}{x}=\lim_{x\to-\infty}\left(1+\frac{\ln(e^x+1)}{x}\right)=$

$=1+\lim_{x\to-\infty}\frac{\ln(e^x+1)}{x}=1+\frac{\ln(e^{-\infty}+1)}{-\infty}=1+\frac{\ln(0+1)}{-\infty}=1+\frac{\ln 1}{-\infty}=1+\frac{0}{-\infty}=1+0=1$

$b=\lim_{x\to-\infty}(f(x)-mx)=\lim_{x\to-\infty}(\cancel{x}+\ln(e^x+1)-\cancel{x})=\lim_{x\to-\infty}(\ln(e^x+1))=\ln(e^{-\infty}+1)$

$=\ln(0+1)=\ln 1=0$

Assim, $y=x$ é a equação da assíntota oblíqua.

(70.b) $f(x)=2x+1 \Leftrightarrow x+\ln(e^x+1)=2x+1 \Leftrightarrow \ln(e^x+1)=x+1 \Leftrightarrow e^x+1=e^{x+1} \Leftrightarrow$

$\Leftrightarrow e^{x+1}-e^x-1=0 \Leftrightarrow e^x(e-1)=1 \Leftrightarrow e^x=\frac{1}{e-1} \Leftrightarrow x=\ln\left(\frac{1}{e-1}\right) \Leftrightarrow$

$\Leftrightarrow x=\ln 1-\ln(e-1) \Leftrightarrow x=0-\ln(e-1) \Leftrightarrow x=-\ln(e-1)$

(70.c) $h(x)=f(x)-x=\cancel{x}+\ln(e^x+1)-\cancel{x}=\ln(e^x+1)$

$h(x)=y \Leftrightarrow \ln(e^x+1)=y \Leftrightarrow e^x+1=e^y \Leftrightarrow e^x=e^y-1 \Leftrightarrow x=\ln(e^y-1)$

Assim, $h^{-1}(x)=\ln(e^x-1)$ (C)

(71) $\lim u_n=\lim\frac{8n-4}{n+1}=\lim\frac{8\cancel{n}}{\cancel{n}}=8$

$\lim f(u_n)=f(\lim u_n)\underset{u_n\uparrow}{=}f(8^-)=\log_2(8-8^-)=\log_2(0^+)=-\infty$ (A)

(72) $\log_8 a+\log_8 b=\frac{1}{3} \Leftrightarrow \log_8(a\times b)=\frac{1}{3} \Leftrightarrow a\times b=8^{\frac{1}{3}} \Leftrightarrow a\times b=\sqrt[3]{8} \Leftrightarrow ab=2$ (A)

(73) Como $D=]-\infty,4]$, na determinação de A.H, apenas tem sentido $x\to-\infty$.

$\lim_{x\to-\infty}h(x)=\lim_{x\to-\infty}(1+x\,e^{x-1})=1+\lim_{x\to-\infty}(x\,e^{x-1})\overset{(*)}{=}1+0=1$, logo $y=1$ é a equação da A.H.

(*) Ora, $\lim_{x\to-\infty}(x\,e^{x-1})\overset{(\infty\times 0)}{=}\lim_{-x\to+\infty}\frac{x}{e^{1-x}}=\lim_{y\to+\infty}\frac{-y}{e^{1+y}}=\lim_{y\to+\infty}\frac{-y}{e\times e^y}=$ $\boxed{\begin{array}{l}-x=y\to+\infty\\ x=-y\end{array}}$

$=\lim_{y\to+\infty}\frac{-\frac{y}{y}}{e\times\underbrace{\frac{e^y}{y}}_{\text{L.N.}\to+\infty}}=\frac{-1}{e\times(+\infty)}=\frac{-1}{+\infty}=0$

74.a) $f''(x) = \frac{(2+\ln x)'x - (2+\ln x)x'}{x^2} = \frac{(2' + \frac{1}{x}) \times x - (2+\ln x)\times 1}{x^2} =$

$= \frac{1 - 2 - \ln x}{x^2} = \frac{-1-\ln x}{x^2}$

Zeros de f'':

$-1 - \ln x = 0 \wedge x^2 \neq 0$

$\ln x = -1 \wedge x \neq 0$

$x = e^{-1} \wedge x \neq 0$

	0		$\frac{1}{e}$		$+\infty$
$-1-\ln x$		+	0	−	
x^2		+	+	+	
f''		+	0	−	
f		$\cup$	PI	$\cap$	

Concavidade voltada p/ cima em $]0, \frac{1}{e}]$

" " " baixo em $[\frac{1}{e}, +\infty[$

" Ponto de inflexão de abcissa $\frac{1}{e}$

74.b) $\lim_{x\to 1} \frac{f(x)-f(1)}{1-x^2} = \lim_{x\to 1} \frac{f(x)-f(1)}{(1-x)(1+x)} = \underbrace{\lim_{x\to 1} \frac{f(x)-f(1)}{x-1}}_{f'(1)} \times \underbrace{\lim_{x\to 1}\left(-\frac{1}{1+x}\right)}_{-\frac{1}{1+1} = -\frac{1}{2}}$

$= f'(1) \times \left(-\frac{1}{2}\right) \overset{(*)}{=} 2 \times \left(-\frac{1}{2}\right) = -1$ (B)

(*) $f'(1) = \frac{2+\ln 1}{1} = \frac{2+0}{1} = 2$

75.a) $r = \frac{3}{5}R$

Percentagem pedida $= 50\left(1 - \sqrt{1 - \left(\frac{\frac{3}{5}\cancel{R}}{\cancel{R}}\right)^2}\right) = 50\left(1 - \sqrt{1-\left(\frac{3}{5}\right)^2}\right) =$

$= 50\left(1 - \sqrt{1 - \frac{9}{25}}\right) = 50\left(1 - \sqrt{\frac{16}{25}}\right) =$

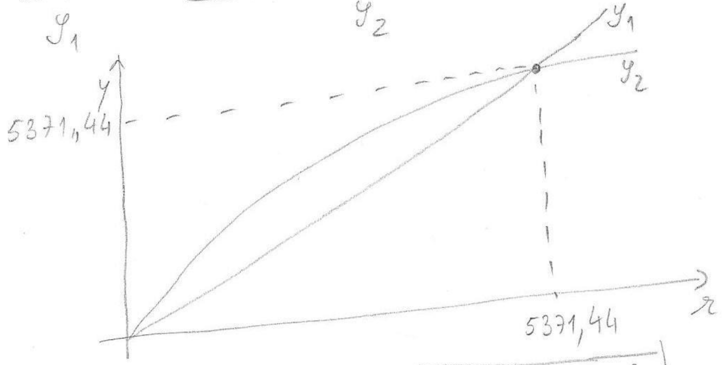
$= 50\left(1 - \frac{4}{5}\right) = 50 \times \frac{1}{5} = 10\%$ (C)

75.b) $R = 6400$

$h = r$

Substituindo na igualdade $r = \frac{R}{h+R}\sqrt{h^2+2hR}$, teremos

$$\underbrace{r}_{y_1} = \underbrace{\frac{6400}{r+6400}\sqrt{r^2+2\times 6400\, r}}_{y_2}$$



$$\text{Percentagem} = 50\left(1-\sqrt{1-\left(\frac{5371{,}44}{6400}\right)^2}\right) \approx 23\%$$

(76)

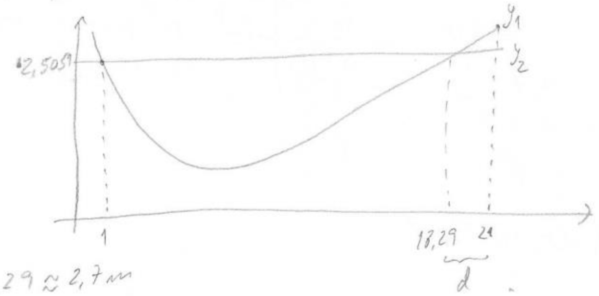
a) $\left| f(0) - f(21) \right| = \left| 3{,}4 - 4{,}0531 \right| \approx 0{,}7$ (B)

$f(0) = 3{,}4$

$f(21) = 0{,}0001 \times 21^4 - 0{,}005 \times 21^3 + 0{,}11 \times 21^2 - 21 + 3{,}4 \approx 4{,}0531$

b) $f(1) = 0{,}0001 \times 1^4 - 0{,}005 \times 1^3 + 0{,}11 \times 1^2 - 1 + 3{,}4 = 2{,}5051$

$\underbrace{f(x)}_{y_1} = \underbrace{2{,}5051}_{y_2}$



$d = 21 - 18{,}29 \approx 2{,}7$ m

(77)

a) g cont em $x = 1 \Leftrightarrow \lim_{x \to 1^-} g(x) = \lim_{x \to 1^+} g(x) = g(1)$

$g(1) = 1^2 - 10 + 8 \ln 1 = -9$

$\lim_{x \to 1^+} (x^2 - 10 + 8 \ln x) = 1^2 - 10 + 8 \ln 1 = -9$

$\lim_{x \to 1^-} \frac{x^2 - x}{k - kx} = \lim_{y \to 0} \frac{(y+1)^2 - (y+1)}{k - k(y+1)} = \lim_{y \to 0} \frac{y^2 + 2y + \cancel{1} - y - \cancel{1}}{\cancel{k} - ky - \cancel{k}} =$

$\boxed{\begin{array}{l} x - 1 = y \to 0 \\ x = y + 1 \end{array}}$

$= \lim_{y \to 0} \frac{\cancel{y}(y+1)}{-k\cancel{y}} = \lim_{y \to 0} \frac{y+1}{-k} = -\frac{1}{k}$ Assim $-\frac{1}{k} = -9 \Leftrightarrow k = \frac{1}{9}$ (D)

b) Em $]1, +\infty[$, $g(x) = x^2 - 10 + 8 \ln x$

$g'(x) = 2x + 8 \times \frac{1}{x}$

$g''(x) = 2 + 8\left(-\frac{1}{x^2}\right) = 2 - \frac{8}{x^2}$

Zeros de g''

$2 - \frac{8}{x^2} = 0$

$\Leftrightarrow \frac{8}{x^2} = 2$

$\Leftrightarrow 2x^2 = 8$

$\Leftrightarrow x^2 = 4$

$\Leftrightarrow x = \pm 2$

em $]1, +\infty[$, $x = 2$

	1		2	$+\infty$
g''		$-$	0	$+$
g		$\cap$	PI	$\cup$

Conc. voltada p/ cima em $[2, +\infty[$

" " baixo em $]1, 2]$

P.I para $x = 2$

77.e g é cont. em $[\sqrt{e}, e]$

$$g(\sqrt{e}) = (\sqrt{e})^2 - 10 + 8\ln\sqrt{e} = e - 10 + 8 \times \frac{1}{2} = e - 6 < 0$$

$$g(e) = e^2 - 10 + 8\ln e = e^2 - 10 + 8 = e^2 - 2 > 0$$

$$g(\sqrt{e}) \times g(e) < 0$$

Pelo teorema de Bolzano-Cauchy, existe pelo menos um valor $x \in]\sqrt{e}, e[$ tal que $g(x) = 0$, isto é, g tem, pelo menos, um zero no intervalo $]\sqrt{e}, e[$