(1.a) f cont em $x=1 \Leftrightarrow \lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = f(1)$

$f(1) = -1^2(1+2\ln 1) = -1$

$\lim_{x\to 1^-}\left(-x^2(1+2\ln x)\right) = -1^2(1+2\ln 1) = -1$

$\lim_{x\to 1^+} \frac{5-5e^{x-1}}{x^2+3x-4} \overset{\frac{0}{0}}{=} \lim_{y\to 0} \frac{5-5e^{y}}{(y+1)^2+3(y+1)-4} = \lim_{y\to 0} \frac{5-5e^{y}}{y^2+2y+\cancel{1}+3y+\cancel{3}-\cancel{4}} =$

$\boxed{\begin{array}{l} x-1=y\to 0 \\ x=y+1 \end{array}}$

$= \lim_{y\to 0} \frac{5-5e^y}{y^2+5y} = \lim_{y\to 0} \frac{5(1-e^y)}{y(y+5)} = -\underbrace{\lim_{y\to 0}\frac{e^y-1}{y}}_{L.N.} \times \lim_{y\to 0}\frac{5}{y+5} = -1\times\frac{5}{0+5} = -1$

Assim, concluímos que f é cont em $x=1$

(1.b) Em $]0,1[$, $f(x) = -x^2(1+2\ln x)$

$f'(x) = -2x(1+2\ln x) + (-x^2)\left(2\times\frac{1}{x}\right) = -2x-4x\ln x-2x = -4x-4x\ln x$

Zeros de f'

$-4x-4x\ln x = 0$

$\Leftrightarrow x(-4-4\ln x)=0$

$\Leftrightarrow x=0 \vee \ln x = -1$

$\Leftrightarrow x=0 \vee x = e^{-1} = \frac{1}{e}$

em $]0,1[$, $x=\frac{1}{e}$

	0		$\frac{1}{e}$		1
f'		$+$	0	$-$	
f		$\nearrow$	Máx	$\searrow$	

$f\left(\frac{1}{e}\right) = -\left(\frac{1}{e}\right)^2\left(1+2\ln\left(\frac{1}{e}\right)\right) = -\frac{1}{e^2}\left(1+2\times(-1)\right) = \frac{1}{e^2}$

f crescente em $]0,\frac{1}{e}]$

f decrescente em $[\frac{1}{e},1[$

$\frac{1}{e^2}$ é máx relat para $x=\frac{1}{e}$

(2) Como $D=\mathbb{R}^+$, apenas tem sentido $x\to+\infty$

$m = \lim_{x\to+\infty}\frac{h(x)}{x} = \lim_{x\to+\infty}\frac{x^2}{2x^2-\ln x} = \lim_{x\to+\infty}\frac{\frac{x^2}{x^2}}{\frac{2x^2}{x^2}-\frac{\ln x}{x^2}} = \lim_{x\to+\infty}\frac{1}{2-\frac{1}{x}\underbrace{\left(\frac{\ln x}{x}\right)}_{L.N.\ \to 0}} = \frac{1}{2-\frac{1}{+\infty}\times 0} = \frac{1}{2}$

$b = \lim_{x\to+\infty}(h(x)-mx) = \lim_{x\to+\infty}\left(\frac{x^3}{2x^2-\ln x}-\frac{x}{2}\right) = \lim_{x\to+\infty}\frac{\cancel{2x^3}-\cancel{2x^3}+x\ln x}{4x^2-2\ln x} = \lim_{x\to+\infty}\frac{\frac{x\ln x}{x^2}}{\frac{4x^2}{x^2}-\frac{2\ln x}{x^2}}$

$= \lim_{x\to+\infty}\frac{\overbrace{\left(\frac{\ln x}{x}\right)}^{L.N.\ \to 0}}{4-\frac{2}{x}\times\underbrace{\left(\frac{\ln x}{x}\right)}_{L.N.\ \to 0}} = \frac{0}{4-\frac{2}{+\infty}\times 0} = \frac{0}{4-0\times 0} = \frac{0}{4} = 0$

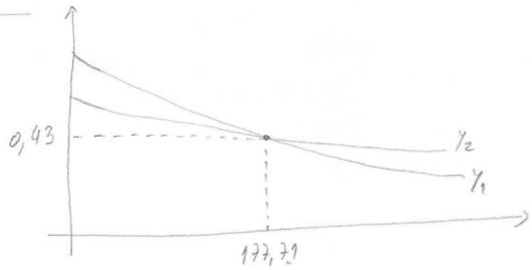
$\therefore y = \frac{1}{2}x$

③ a) início: $a(0) = 1,8 - (0,216 + 0,0039 \times 0)^{\frac{2}{3}} = 1,44$

meio $= \frac{1,8}{2} = 0,9$

$1,44 - 0,9 = 0,54$ (B)

b)
$$\begin{array}{ccc} 0 & t_1 & 2t_1 \\ a(0) & a(t_1) & \underbrace{a(2t_1)}_{y_1} = \underbrace{\frac{a(t_1)}{2}}_{y_2} \end{array}$$



$177,71 \approx 178$ minutos

$\Downarrow$

$t_1 = 2h\,58m$

⑤ $v_n = 2 - \frac{5}{n+3} \to 2^-$

$\lim g(v_n) = g(\lim v_n) = g(2^-) = 1$ (B)

④ $\ln\left((1-x)e^{x-1}\right) = x$

Domínio:

$(1-x)e^{x-1} > 0$

	$-\infty$	1	$+\infty$
$1-x$	+	0	-
e^{x-1}	+	+	+
$(1-x)e^{x-1}$	+	0	-

ou $(1-x)\overset{+}{e^{x-1}} > 0$

$\Leftrightarrow 1-x > 0$

$\Leftrightarrow x < 1$

$D =]-\infty, 1[$

$\Leftrightarrow (1-x)e^{x-1} = e^x$ (*)

$\Leftrightarrow (1-x)e^{x-1} - e^x = 0$

$\Leftrightarrow e^x\left((1-x)e^{-1} - 1\right) = 0$

$\Leftrightarrow e^x = 0 \vee \frac{1-x}{e} - 1 = 0$

$\Leftrightarrow e^x = 0 \vee \frac{1-x}{e} = 1$

$\Leftrightarrow e^x = 0 \vee 1-x = e$

$\Leftrightarrow \underbrace{e^x = 0}_{\text{imp.}} \vee x = 1-e \in D$, logo $\underline{\underline{x = 1-e}}$

(*) ou

$\overset{e^x \neq 0}{\Leftrightarrow} \frac{(1-x)e^{x-1}}{e^x} = 1 \Leftrightarrow (1-x)e^{x-1-x} = 1 \Leftrightarrow (1-x)e^{-1} = 1 \Leftrightarrow \frac{1-x}{e} = 1 \Leftrightarrow 1-x = e$

$\Leftrightarrow x = 1-e$

(6.a) $\lim_{x\to-\infty} \frac{x-e^{-x}}{x} = \lim_{x\to-\infty}\left(\frac{x}{x}-\frac{e^{-x}}{x}\right) = \lim_{x\to-\infty}\left(1-\frac{e^{-x}}{x}\right) = 1 - \lim_{y\to+\infty}\frac{e^{y}}{-y} =$ $\boxed{\begin{array}{l} -x = y \to +\infty \\ x = -y \end{array}}$

$= 1 + \underbrace{\lim_{y\to+\infty}\frac{e^{y}}{y}}_{L.N.} = 1 + (+\infty) = +\infty$

$\lim_{x\to+\infty}\left(\frac{\sqrt{x^2+1}}{x+1}-3\right) = \lim_{x\to+\infty}\left(\frac{\sqrt{\frac{x^2}{x^2}+\frac{1}{x^2}}}{\frac{x}{x}+\frac{1}{x}}-3\right) = \lim_{x\to+\infty}\left(\frac{\sqrt{1+\frac{1}{x^2}}}{1+\frac{1}{x}}-3\right) =$

$= \frac{\sqrt{1+\frac{1}{(+\infty)^2}}}{1+\frac{1}{+\infty}} - 3 = \frac{\sqrt{1+0}}{1+0} - 3 = \frac{1}{1} - 3 = -2$

Assim, $y=-2$ é a equação da única A.H

(6.b) Para $x<0$, $f(x) = \frac{x-e^{-x}}{x}$

$f'(x) = \frac{(x-e^{-x})'x-(x-e^{-x})x'}{x^2} = \frac{(1+e^{-x})x-x+e^{-x}}{x^2} = \frac{\cancel{x}+xe^{-x}-\cancel{x}+e^{-x}}{x^2}$

$= \frac{xe^{-x}+e^{-x}}{x^2}$

$m = f'(-2) = \frac{-2e^{2}+e^{2}}{4} = -\frac{e^2}{4}$

$f(-2) = \frac{-2-e^{2}}{-2} = \frac{2+e^2}{2}$

Ponto de tangência $P(-2, f(-2)) = (-2, \frac{2+e^2}{2}) = (-2, 1+\frac{e^2}{2})$

$y = mx + b$

$y = -\frac{e^2}{4}x + b$

$(-2, 1+\frac{e^2}{2}) \Leftrightarrow 1+\frac{e^2}{2} = -\frac{e^2}{4}\times(-2)+b$

$1+\cancel{\frac{e^2}{2}} = \cancel{\frac{e^2}{2}} + b$

$1 = b$

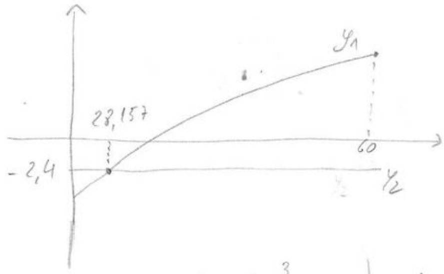
$\therefore y = -\frac{e^2}{4}x + 1$

(7.a) $T(t) = 30 \Leftrightarrow 20+100e^{-Kt} = 30 \Leftrightarrow 100e^{-Kt} = 10 \Leftrightarrow e^{-Kt} = \frac{1\cancel{0}}{10\cancel{0}} \Leftrightarrow e^{-Kt} = \frac{1}{10}$

$\Leftrightarrow -Kt = \ln\left(\frac{1}{10}\right) \Leftrightarrow -Kt = \ln(10^{-1}) \Leftrightarrow -Kt = -\ln 10 \Leftrightarrow Kt = \ln 10 \Leftrightarrow K = \frac{\ln 10}{t}$ (c)

7.b) $T(t) = 20 + 100\, e^{-0,04t}$; $T(0) = 20 + 100\, e^{0} = 120$ $\quad 0 \le t \le 60$

$\underset{[0,t_2]}{t.m.v} = -2,4 \Leftrightarrow \dfrac{T(t_2) - T(0)}{t_2 - 0} = -2,4 \Leftrightarrow \underbrace{\dfrac{T(t_2) - 120}{t_2}}_{y_1} = \underbrace{-2,4}_{y_2}$



$0,157 \times 60 \approx 9$

$t_2 \approx 28m\,9s$

8) $\lim\limits_{x \to 0^-} g(x) = \lim\limits_{x \to 0^-} \left(\dfrac{x^3 - x}{x^2 - x} + K \right) = K + \lim\limits_{x \to 0^-} \dfrac{\cancel{x}(x^2 - 1)}{\cancel{x}(x-1)} = K + \lim\limits_{x \to 0^-} \dfrac{\cancel{(x-1)}(x+1)}{\cancel{x-1}} =$

$= K + \lim\limits_{x \to 0^-} (x+1) = K + (0+1) = K+1$

$\lim\limits_{x \to 0^+} g(x) = \lim\limits_{x \to 0^+} (2 + x \ln x) = 2 + \lim\limits_{x \to 0^+} \dfrac{\ln x}{\frac{1}{x}} = 2 + \lim\limits_{y \to +\infty} \dfrac{\ln\left(\frac{1}{y}\right)}{y} = 2 + \lim\limits_{y \to +\infty} \dfrac{-\ln y}{y} =$

$\boxed{\begin{array}{l} \frac{1}{x} = y \to +\infty \\ x = \frac{1}{y} \end{array}}$

$= 2 - \underbrace{\lim\limits_{y \to +\infty} \dfrac{\ln y}{y}}_{L.N.} = 2 - 0 = 2$ $\quad$ Assim, $K + 1 = 2 \Leftrightarrow K = 1$

9) $x \ln(1-x) - \ln(1-x) = (1-x) \ln(3-2x) \Leftrightarrow$

$\Leftrightarrow (x-1) \ln(1-x) = (1-x) \ln(3-2x)$

$\Leftrightarrow (x-1) \ln(1-x) - (1-x) \ln(3-2x) = 0$

$\Leftrightarrow (x-1) \ln(1-x) + (x-1) \ln(3-2x) = 0$

$\Leftrightarrow (x-1) \left(\ln(1-x) + \ln(3-2x) \right) = 0$

$\Leftrightarrow (x-1) \ln\left((1-x)(3-2x) \right) = 0$

$\Leftrightarrow (x-1) \ln\left((1-x)(3-2x) \right) = 0$

$\Leftrightarrow x - 1 = 0 \vee \ln\left((1-x)(3-2x) \right) = 0$

$\Leftrightarrow x = 1 \vee (1-x)(3-2x) = e^0$

$\Leftrightarrow x = 1 \vee 3 - 2x - 3x + 2x^2 = 1$

$\Leftrightarrow x = 1 \vee 2x^2 - 5x + 2 = 0$

$\Leftrightarrow x = 1 \vee x = \dfrac{5 \pm \sqrt{(-5)^2 - 4 \times 2 \times 2}}{2 \times 2}$

$\Leftrightarrow x = 1 \vee x = 2 \vee x = \frac{1}{2}$ e $D =]-\infty, 1[$

Logo $x = \frac{1}{2}$

Domínio

$1 - x > 0 \wedge 3 - 2x > 0$

$\Leftrightarrow x < 1 \wedge 2x < 3$

$\Leftrightarrow x < 1 \wedge x < \frac{3}{2}$

$D =]-\infty, 1[$

(10) $u_n = 2n^2 - n$

$\lim u_n = \lim (2n^2 - n) = \lim (2n^2) = 2 \times (+\infty)^2 = +\infty$

$\lim f\left(\frac{1}{u_n}\right) = +\infty \Leftrightarrow f\left(\lim \frac{1}{u_n}\right) = +\infty \Leftrightarrow f\left(\frac{1}{+\infty}\right) = +\infty \Leftrightarrow f(0^+) = +\infty$

(A)

(11) Em $]-\infty, 1[$, $f(x) = x - 2 + \ln(3-2x)$

$f'(x) = 1 - 0 + \frac{-2}{3-2x} = 1 - \frac{2}{3-2x}$

Zeros

$1 - \frac{2}{3-2x} = 0 \Leftrightarrow \frac{2}{3-2x} = 1 \Leftrightarrow 3 - 2x = 2 \Leftrightarrow 2x = 1 \Leftrightarrow x = \frac{1}{2}$

	$-\infty$		$\frac{1}{2}$		1
f'		+	0	−	
f		↗	Máx	↘	

f crescente em $]-\infty, \frac{1}{2}]$

f decrescente em $[\frac{1}{2}, 1[$

Máx relat $= -\frac{3}{2} + \ln 2$, para $x = \frac{1}{2}$

$f\left(\frac{1}{2}\right) = \frac{1}{2} - 2 + \ln(3-1) =$

$= -\frac{3}{2} + \ln 2$

(12)

a) $\lim_{t \to +\infty} N(t) = \lim_{t \to +\infty} \left(N_0 e^{1{,}08t - 0{,}3t^2}\right) = N_0 e^{-\infty} = N_0 \times 0 = 0$ (D)

C.A.

$\lim_{t \to +\infty} (1{,}08t - 0{,}3t^2) =$

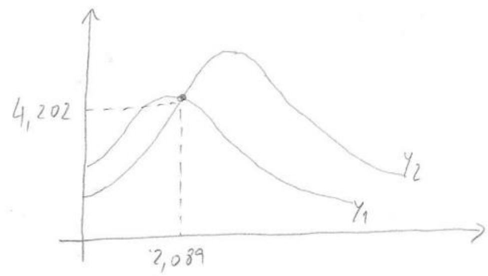
$= \lim_{t \to +\infty} (-0{,}3t^2) =$

$= -0{,}3 \times (+\infty)^2 = -\infty$

b) $N(t) = 1{,}63 e^{1{,}08t - 0{,}3t^2}$

$t_1 - 1$ t_1

$N(t_1 - 1)$ $\underbrace{N(t_1)}_{y_1} = \underbrace{0{,}5 + N(t_1 - 1)}_{y_2}$



$0{,}089 \times 60 \approx 5$

$t_1 \approx 2h05m$

(13) $\lim\limits_{x\to-\infty} \dfrac{x^2-4}{x^2-5x+6} \overset{\left(\frac{\infty}{\infty}\right)}{=} \lim\limits_{x\to-\infty} \dfrac{x^2}{x^2} = 1 \to$ Equação $y=1$

$\lim\limits_{x\to+\infty} \dfrac{x-1}{e^{x-2}} \overset{\left(\frac{\infty}{\infty}\right)}{=} \lim\limits_{x\to+\infty} \dfrac{\frac{x}{x}-\frac{1}{x}}{\frac{e^x}{x}\times e^{-2}} = \lim\limits_{x\to+\infty} \dfrac{1-\frac{1}{x}}{\underbrace{\frac{e^x}{x}}_{+\infty \text{ L.N.}}\times e^{-2}} = \dfrac{1-\frac{1}{+\infty}}{+\infty\times e^{-2}} = \dfrac{1-0}{+\infty} = \dfrac{1}{+\infty} = 0$

Equação $y=0$

$\therefore\ y=1$ e $y=0$

(14) $e^{-x}(4+e^{2x}) \geq 5$

$\Leftrightarrow 4e^{-x}+e^{x}-5 \geq 0$

$\Leftrightarrow \dfrac{4}{e^x}+e^x-5 \geq 0$

$\boxed{e^x = y}$

$\Leftrightarrow \dfrac{4}{y}+y-5 \geq 0$

$\Leftrightarrow 4+y^2-5y \geq 0$

$\Leftrightarrow y^2-5y+4 \geq 0$

$\Leftrightarrow y \leq 1 \vee y \geq 4$

$\Leftrightarrow e^x \leq 1 \vee e^x \geq 4$

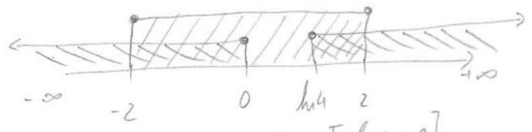
$\Leftrightarrow x \leq \ln 1 \vee x \geq \ln 4$

$\Leftrightarrow x \leq 0 \vee x \geq \ln 4$

C.A.

Zeros: $y = \dfrac{5 \pm \sqrt{(-5)^2-4\times1\times4}}{2\times1}$

$\Leftrightarrow y=1 \vee y=4$



$S = [-2, 0] \cup [\ln 4; 2]$

(15) $f'(x) = 4x$

$g'(x) = -2(x-1) = -2x+2$

Seja $A(a, f(a))$ e $B(b, g(b))$, isto é, $A(a, 2a^2)$ e $B(b, -(b-1)^2)$

$f'(a) = g'(b) \Leftrightarrow 4a = -2b+2 \Leftrightarrow 2b = 2-4a \Leftrightarrow \boxed{b = 1-2a}$

$\overrightarrow{BA} = A-B = (a, 2a^2)-(b, -(b-1)^2) = (a-b, 2a^2+(b-1)^2) = (a-1+2a, 2a^2+(1-2a-1)^2)$
$= (3a-1, 2a^2+4a^2) =$
$= (3a-1, 6a^2)$

$m_{\overrightarrow{BA}} = f'(a)$

$\Leftrightarrow \dfrac{6a^2}{3a-1} = 4a \Leftrightarrow 6a^2 = 12a^2-4a \Leftrightarrow 6a^2-12a^2+4a = 0$

$\Leftrightarrow -6a^2+4a = 0 \Leftrightarrow a(-6a+4) = 0 \Leftrightarrow a = 0 \vee -6a+4 = 0$

$\Leftrightarrow \underbrace{a=0}_{\text{imp.}} \vee a = \dfrac{2}{3}$

Como $a = \dfrac{2}{3}$ então $b = 1-2\times\dfrac{2}{3} = 1-\dfrac{4}{3} = -\dfrac{1}{3}$

$\therefore\ x_A = \dfrac{2}{3}$ e $x_B = -\dfrac{1}{3}$

(16) Em $]-\infty,-2[$, $f(x) = \dfrac{e^{2-x}}{x+2}$

$$f'(x) = \frac{(e^{2-x})'(x+2) - e^{2-x}(x+2)'}{(x+2)^2} = \frac{-e^{2-x}(x+2) - e^{2-x}}{(x+2)^2}$$

Zeros de f'

$$-e^{2-x}(x+2) - e^{2-x} = 0 \wedge (x+2)^2 \neq 0$$

$$\Leftrightarrow e^{2-x}(-x-2-1) = 0 \wedge x+2 \neq 0$$

$$\Leftrightarrow \underbrace{e^{2-x} = 0}_{\text{Eq. imp.}} \vee -x-3 = 0 \wedge x \neq -2$$

$$\Leftrightarrow x = -3 \wedge x \neq -2$$

x	$-\infty$		-3		-2
f'		+	0	−	
f		↗	Máx	↘	

f crescente em $]-\infty,-3]$
f decrescente em $[-3,-2[$
máx. relat. $= -e^5$ para $x=-3$

$$f(-3) = \frac{e^{2+3}}{-3+2} = -e^5$$

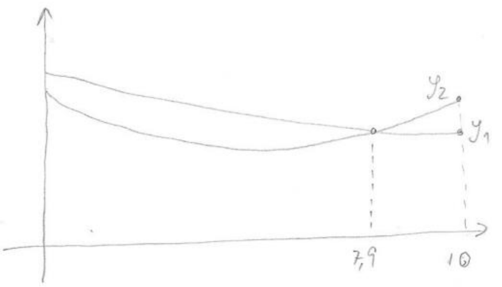
(17) a)

$h(5) = 6,3\left(e^{\frac{5-5}{12,6}} + e^{\frac{5-5}{12,6}}\right) - 7,6$
$= 5$

$y^2 = 5^2 + 5^2$
$\Leftrightarrow y^2 = 50$
$\Leftrightarrow y = \pm\sqrt{50}$, $y>0$
$\Leftrightarrow y = \sqrt{50} \approx 7,1$ (A)

b) $0,5d$ d

$\underbrace{h(0,5d)}_{y_1} = \underbrace{h(d) - 0,3}_{y_2}$ $h(d)$



$\therefore d \approx 7,9$ m

(18) A.V.

g cont para $x > 1$. Assim a única A.V., se existir, será para $x = 1^+$

$$\lim_{x \to 1^+} (5x - 3\ln(x-1)) = 5 \times 1 - 3\ln(0^+) = 5 - 3 \times (-\infty) = 5 + \infty = +\infty$$

equação da A.V: $x = 1$

A.O.

Como $D =]1, +\infty[$, apenas tem sentido $x \to +\infty$

$$m = \lim_{x \to +\infty} \frac{5x - 3\ln(x-1)}{x} = \lim_{x \to +\infty} \left(5 - \frac{3\ln(x-1)}{x}\right) = 5 - 3\lim_{x \to +\infty} \frac{\ln(x-1)}{x} = 5 - 3 \times 0 = 5$$

C.A.

$$\lim_{x \to +\infty} \frac{\ln(x-1)}{x} = \lim_{x \to +\infty} \frac{\ln\left(x\left(1 - \frac{1}{x}\right)\right)}{x} = \lim_{x \to +\infty} \left(\overset{\overset{0}{\uparrow \text{ L.N.}}}{\frac{\ln x}{x}} + \frac{\ln\left(1 - \frac{1}{x}\right)}{x}\right) = 0 + \frac{\ln\left(1 - \frac{1}{+\infty}\right)}{+\infty} = \frac{\ln(1-0)}{+\infty} = \frac{0}{+\infty} = 0$$

ou, usando mudança de variável $y = x - 1$ ou $y = \ln(x-1)$

$$b = \lim_{x \to +\infty} (5x - 3\ln(x-1) - 5x) = -3\lim_{x \to +\infty} (\ln(x-1)) = -3 \times \ln(+\infty - 1) = -3 \times \ln(+\infty) = -3 \times (+\infty) = -\infty$$

Assim, não há A.O.

(19) $(e^x - 1)\ln(5-2x) + e^x\ln(3-x) = \ln(3-x)$

$\Leftrightarrow (e^x - 1)\ln(5-2x) + e^x\ln(3-x) - \ln(3-x) = 0$

$\Leftrightarrow (e^x - 1)\ln(5-2x) + (e^x - 1)\ln(3-x) = 0$

$\Leftrightarrow (e^x - 1)(\ln(5-2x) + \ln(3-x)) = 0$

$\Leftrightarrow (e^x - 1)\ln((5-2x)(3-x)) = 0$

$\Leftrightarrow e^x - 1 = 0 \vee (5-2x)(3-x) = 1$

$\Leftrightarrow e^x = 1 \vee 15 - 5x - 6x + 2x^2 = 1$

$\Leftrightarrow x = 0 \vee 2x^2 - 11x + 14 = 0$

$\Leftrightarrow x = 0 \vee x = \frac{11 \pm \sqrt{(-11)^2 - 4 \times 2 \times 14}}{2 \times 2}$

$\Leftrightarrow x = 0 \vee x = 2 \vee x = \frac{7}{2}$

Assim, $x = 0 \vee x = 2$

Domínio

$5 - 2x > 0 \wedge 3 - x > 0$

$\Leftrightarrow 2x < 5 \wedge x < 3$

$\Leftrightarrow x < \frac{5}{2} \wedge x < 3$

$D = \left]-\infty, \frac{5}{2}\right[$

(20) $A(a, f(a))$ e $B(b, f(b))$

$A\left(a, \frac{K}{a}\right) \quad B\left(b, \frac{K}{b}\right)$

$\vec{AB} = B - A = \left(b-a, \frac{K}{b} - \frac{K}{a}\right)$

$$\begin{cases} f'(x) = -\frac{K}{x^2} \\ f'(c) = -\frac{K}{c^2} \end{cases}$$

$$f'(c) = m_{\vec{AB}} \Leftrightarrow -\frac{K}{c^2} = \frac{\frac{K}{b} - \frac{K}{a}}{b-a} \Leftrightarrow -\frac{K}{c^2} = \frac{\frac{aK-bK}{ab}}{b-a} \Leftrightarrow$$

$$\Leftrightarrow -\frac{K}{c^2} = \frac{aK-bK}{ab(b-a)} \Leftrightarrow -\frac{K}{c^2} = \frac{(a-b)K}{ab(b-a)} \Leftrightarrow -\frac{K}{c^2} = -\frac{\cancel{(b-a)}K}{ab\cancel{(b-a)}} \Leftrightarrow$$

$$\Leftrightarrow \frac{K}{c^2} = \frac{K}{ab} \Leftrightarrow c^2 = ab \Leftrightarrow c = \pm\sqrt{ab},\ c>0 \Leftrightarrow c = \sqrt{ab}$$

a, $\sqrt{ab}$ e b são termos consecutivos de uma P.G

se $\frac{b}{\sqrt{ab}} = \frac{\sqrt{ab}}{a}$

Ora $\frac{b}{\sqrt{ab}} = \frac{\sqrt{ab}}{a} \Leftrightarrow ab = (\sqrt{ab})^2 \Leftrightarrow ab = ab$ Verdade c.q.d.

(21) C

(22) Em $]-\infty, 0[$, $g'(x) = 3e^{2x} - 7e^x$

$$g'(x) = -2 \Leftrightarrow 3e^{2x} - 7e^x + 2 = 0 \Leftrightarrow 3y^2 - 7y + 2 = 0 \Leftrightarrow y = \frac{7 \pm \sqrt{(-7)^2 - 4\times3\times2}}{2\times3}$$

$\boxed{y = e^x}$

$$\Leftrightarrow y = 2 \vee y = \frac{1}{3} \Leftrightarrow e^x = 2 \vee e^x = \frac{1}{3} \Leftrightarrow x = \ln 2 \vee x = \ln\left(\frac{1}{3}\right) \Leftrightarrow x = \ln 2 \vee x = -\ln 3$$

$\therefore$ Em $]-\infty, 0[$, $x = -\ln 3$

(23) A.V

h cont para $x>0$, logo a única A.V., caso existe, será para $x = 0^+$

$$\lim_{x\to 0^+} \frac{e^x + \ln x}{e^x - 1} = \frac{e^0 + \ln(0^+)}{e^0 - 1} = \frac{1 + (-\infty)}{0^+} = \frac{-\infty}{0^+} = -\infty$$

Assim $x = 0$ é equação de uma A.V.

A.H

Como $D =]0, +\infty[$, apenas tem sentido $x \to +\infty$

$$\lim_{x\to+\infty} \frac{e^x + \ln x}{e^x - 1} = \lim_{x\to+\infty} \frac{\frac{e^x}{e^x} + \frac{\ln x}{e^x}}{\frac{e^x}{e^x} - \frac{1}{e^x}} = \lim_{x\to+\infty} \frac{1 + \frac{\ln x}{e^x}}{1 - \frac{1}{e^x}} = \frac{1+0}{1 - \frac{1}{+\infty}} = \frac{1}{1-0} = 1$$

c.A. $\lim_{x\to+\infty} \frac{\ln x}{e^x} = \lim_{x\to+\infty} \frac{\underbrace{\frac{\ln x}{x}}_{\to 0 \text{ L.N.}}}{\underbrace{\frac{e^x}{x}}_{\to +\infty \text{ L.N.}}} = \frac{0}{+\infty} = 0$

Assim, $y = 1$ é equação de uma A.H.

(24) a) $m(0) = \dfrac{30(1+0{,}006\times 0)^3 - 29}{(1+0{,}006\times 0)^2} = 1$

$m(1) = \dfrac{30(1+0{,}006\times 1)^3 - 29}{(1+0{,}006\times 1)^2} \simeq 1{,}52$

1 —— 100%

$1{,}52$ —— x

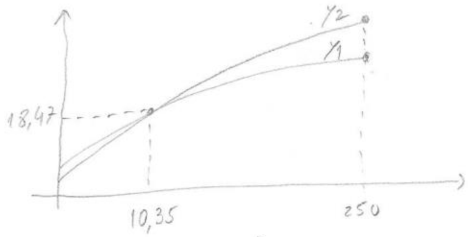
$x = \dfrac{1{,}52 \times 100}{1} = 152\%$

Aumento de 52% (B)

b) $t \quad ; \quad t+30$

$m(t)$

$\underbrace{m(t+30)}_{y_1} = \underbrace{3m(t)}_{y_2}$



$0{,}35 \times 60 = 21$

$\therefore t = 10m\,21s$

(25) $f'(x) = x^2 + 2ax + a^2$

Zeros de f'

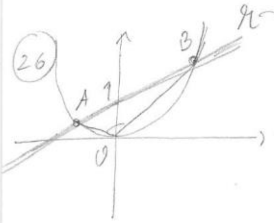
$x = \dfrac{-2a \pm \sqrt{(2a)^2 - 4\times 1 \times a^2}}{2\times 1}$

$\Leftrightarrow x = \dfrac{-2a \pm 0}{2} = -a$

x	$-\infty$		$-a$		$+\infty$
f'		$+$	0	$+$	
f		$\nearrow$		$\nearrow$	

Assim, para qualquer valor de a, f não tem extremos

(26)



$A(a, a^2)$, $B(b, b^2)$ $\quad \overrightarrow{AB} = B - A = (b-a,\ b^2 - a^2)$

$m_{\overrightarrow{AB}} = \dfrac{b^2 - a^2}{b-a} = \dfrac{\cancel{(b-a)}(b+a)}{\cancel{b-a}} = b+a$

$r: y = (b+a)x + 1$

$A(a, a^2) \in r$ logo $a^2 = (b+a)\times a + 1 \Leftrightarrow \cancel{a^2} = ab + \cancel{a^2} + 1 \Leftrightarrow \boxed{ab = -1}$

$A\hat{O}B = 90°$ sse $\overrightarrow{OA} \cdot \overrightarrow{OB} = 0$

Ora $\overrightarrow{OA} \cdot \overrightarrow{OB} = (a, a^2)\cdot(b, b^2) = ab + a^2b^2 = ab + (ab)^2 = -1 + (-1)^2 = 0$ c.q.d.

(27) $\lim\left(\left(\dfrac{n+k}{n}\right)^n\right) = \lim\left(\left(\dfrac{n}{n} + \dfrac{k}{n}\right)^n\right) = \lim\left(\left(1 + \dfrac{k}{n}\right)^n\right) = e^k$ (C)

(28) f cont em $x = 0 \Leftrightarrow \lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0^+} f(x) = f(0)$

$\therefore$ Assim, f é cont em $x = 0$

$f(0) = \dfrac{3}{5}$

$\lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0} \dfrac{3x}{e^{5x} - 1} = \lim\limits_{x\to 0} \dfrac{\frac{3x}{x}}{\frac{e^{5x}-1}{x}} = \lim\limits_{x\to 0} \dfrac{3}{\frac{e^{5x}-1}{x}} = \dfrac{3}{5}$

C.A. $\lim\limits_{x\to 0} \dfrac{e^{5x}-1}{x} \underset{(*)}{=} \lim\limits_{y\to 0} \dfrac{e^y - 1}{\frac{y}{5}} = 5 \underbrace{\lim\limits_{y\to 0} \dfrac{e^y - 1}{y}}_{L.N.} = 5 \times 1 = 5$

(*) $\boxed{5x = y \to 0 \quad x = \frac{y}{5}}$

(29) A.O.

a) $m = \lim_{x\to-\infty} \frac{\ln(1+e^x)-x}{x} = \lim_{x\to-\infty}\left(\frac{\ln(1+e^x)}{x} - 1\right) = \frac{\ln(1+e^{-\infty})}{-\infty} - 1 =$

$= \frac{\ln(1+0)}{-\infty} - 1 = \frac{\ln 1}{-\infty} - 1 = \frac{0}{-\infty} - 1 = 0 - 1 = -1$

$b = \lim_{x\to-\infty}\left(\ln(1+e^x) - \cancel{x} + \cancel{x}\right) = \lim_{x\to-\infty} \ln(1+e^x) =$

$= \ln(1+e^{-\infty}) = \ln(1+0) = \ln 1 = 0$

$y = -x$ é a equação da A.O.

A.H

$\lim_{x\to+\infty}\left(\ln(1+e^x) - x\right) = \lim_{x\to+\infty}\left(\ln\left(e^x\left(\frac{1}{e^x}+1\right)\right) - x\right) = \lim_{x\to+\infty}\left(\ln e^x + \ln\left(\frac{1}{e^x}+1\right) - x\right)$

$= \lim_{x\to+\infty}\left(\cancel{x} + \ln\left(\frac{1}{e^x}+1\right) - \cancel{x}\right) = \lim_{x\to+\infty} \ln\left(\frac{1}{e^x}+1\right) = \ln\left(\frac{1}{e^{+\infty}}+1\right) = \ln\left(\frac{1}{+\infty}+1\right) = \ln(0+1) = 0$

$y = 0$ é a equação da A.H.

Outras formas...

- $\lim_{x\to+\infty}\left(\ln(1+e^x) - x\right) = \lim_{x\to+\infty}\left(\ln(1+e^x) - \ln(e^x)\right) = \lim_{x\to+\infty} \ln \frac{1+e^x}{e^x} = \ldots$

- $\boxed{\begin{array}{l} 1+e^x = y \to +\infty \\ e^x = y-1 \\ x = \ln(y-1) \end{array}}$ $\lim_{y\to+\infty}\left(\ln y - \ln(y-1)\right) = \lim_{y\to+\infty}\left(\ln \frac{y}{y-1}\right) = \ldots$

b) Reta r

$g'(x) = \frac{(1+e^x)'}{1+e^x} - 1 = \frac{e^x}{1+e^x} - 1$

$m_r = g'(0) = \frac{e^0}{1+e^0} - 1 = \frac{1}{2} - 1 = -\frac{1}{2}$

$g(0) = \ln(1+e^0) - 0 = \ln 2$

$y = -\frac{1}{2}x + b$

$(0, \ln 2) \to \ln 2 = -\frac{1}{2}\times 0 + b$

$b = \ln 2$

$r: y = -\frac{1}{2}x + \ln 2$

Intersecção de r com xx $(y=0)$

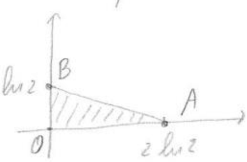
$-\frac{1}{2}x + \ln 2 = 0$

$-x + 2\ln 2 = 0$

$x = 2\ln 2$

Intersecção de r com yy $(x=0)$

$y = \ln 2$

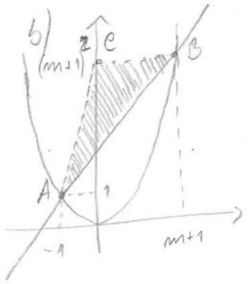


$A = \frac{\cancel{2}\ln 2 \times \ln 2}{\cancel{2}} = (\ln 2)^2$ c.q.d.

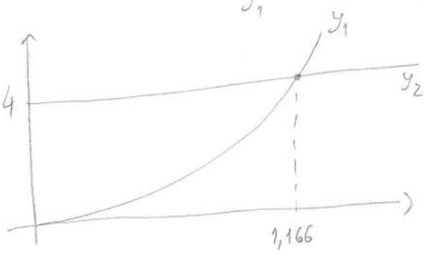
(30) a) $y = mx + b$

$(-1,1) \rightarrow 1 = m\times(-1) + b \Leftrightarrow 1 = -m + b \Leftrightarrow b = 1 + m$ (A)

b)



$$A_{\triangle} = \frac{b\times h}{2} = \underbrace{\frac{(m+1)\times\left((m+1)^2 - 1\right)}{2}}_{y_1} = \underbrace{4}_{y_2}$$



Assim, $m \approx 1,17$

(31) $$g''(x) = \frac{(x - e^{3x})'x - (x - e^{3x})x'}{x^2} =$$

$$= \frac{(1 - 3e^{3x})x - x + e^{3x}}{x^2} = \frac{\cancel{x} - 3xe^{3x} - \cancel{x} + e^{3x}}{x^2}$$

Zeros de g''

$-3xe^{3x} + e^{3x} = 0$

$\Leftrightarrow e^{3x}(-3x + 1) = 0$

$\Leftrightarrow e^{3x} = 0 \vee -3x + 1 = 0$

$\Leftrightarrow \underbrace{e^{3x} = 0}_{\text{cond. imp.}} \vee x = \frac{1}{3}$

	0		$\frac{1}{3}$	$+\infty$
g''		+	0	-
g		∪	PI	∩

Concav. voltada para cima em $]0, \frac{1}{3}]$

" " " baixo em $[\frac{1}{3}, +\infty[$

P.I para $x = \frac{1}{3}$

(32) $\frac{1}{2}\log_{\sqrt{2}}(9x+1) = \log_{\sqrt{2}}(6x)$

$\Leftrightarrow \log_{\sqrt{2}}(9x+1) = 2\log_{\sqrt{2}}(6x)$

$\Leftrightarrow \log_{\sqrt{2}}(9x+1) = \log_{\sqrt{2}}(6x)^2$

$\Leftrightarrow 9x + 1 = 36x^2$

$\Leftrightarrow 36x^2 - 9x - 1 = 0$

$\Leftrightarrow x = \frac{9 \pm \sqrt{(-9)^2 - 4\times 36\times(-1)}}{2\times 36}$

$\Leftrightarrow x = -\frac{1}{12} \vee x = \frac{1}{3}$

Domínio

$9x + 1 > 0 \wedge 6x > 0$

$\Leftrightarrow 9x > -1 \wedge x > 0$

$\Leftrightarrow x > -\frac{1}{9} \wedge x > 0$

$D =]0, +\infty[$

Assim, $S = \{\frac{1}{3}\}$

(33) Estudemos a monotonia de f

$f'(x) = \left((kx)^{\frac{1}{2}} - \ln(kx)\right)' =$

$= \frac{1}{2}(kx)^{-\frac{1}{2}} \times k - \frac{k}{kx} =$

$= \frac{k}{2\sqrt{kx}} - \frac{1}{x}$

<u>Zeros de f'</u>

$\frac{k}{2\sqrt{kx}} - \frac{1}{x} = 0$

$\Leftrightarrow \frac{kx - 2\sqrt{kx}}{2x\sqrt{kx}} = 0$

$\Leftrightarrow kx - 2\sqrt{kx} = 0 \wedge 2x\sqrt{kx} \neq 0$

$\Leftrightarrow kx = 2\sqrt{kx} \wedge 2x \neq 0 \wedge \sqrt{kx} \neq 0$

$\underset{k>0, x>0}{\Leftrightarrow} k^2x^2 = 4kx \wedge \underbrace{x \neq 0}_{\text{cond. univ.}} \wedge \underbrace{k \neq 0}_{\text{cond. univ.}} \wedge \underbrace{x \neq 0}_{\text{cond. univ.}}$

$\Leftrightarrow k^2x^2 - 4kx = 0$

$\Leftrightarrow kx(kx-4) = 0$

$\Leftrightarrow kx = 0 \vee kx = 4$

$\Leftrightarrow \underbrace{k = 0 \vee x = 0}_{\text{cond. imp.}} \vee x = \frac{4}{k}$

	0		$\frac{4}{k}$	$+\infty$
f'		$-$	0	$+$
f		$\searrow$	mín	$\nearrow$

$f\left(\frac{4}{k}\right) = \sqrt{k \times \frac{4}{k}} - \ln\left(k \times \frac{4}{k}\right)$

$= \sqrt{4} - \ln 4$

$= 2 - \ln 4$

$\lim_{x \to 0^+} \left(\sqrt{kx} - \ln(kx)\right) = 0 - \ln(0^+) = -(-\infty) = +\infty$

Assim, como f é cont, $D' = [2 - \ln 4 ; +\infty[$

(34) $\lim\left(\left(1 + \frac{1}{n}\right)^{2n}\right) = \lim\left(\left(1+\frac{1}{n}\right)^n\right)^2 = \left(\lim\left(1+\frac{1}{n}\right)^n\right)^2 = e^2$ (C)

(35) $f'(x) = -2x e^{1-x^2}$

$f''(x) = (-2x)' e^{1-x^2} + (-2x)\left(e^{1-x^2}\right)' = -2e^{1-x^2} - 2x(1-x^2)' e^{1-x^2} =$

$= -2e^{1-x^2} - 2x(-2x)e^{1-x^2} = -2e^{1-x^2} + 4x^2 e^{1-x^2}$

<u>Zeros de f''</u>

$e^{1-x^2}(-2+4x^2) = 0 \Leftrightarrow \underbrace{e^{1-x^2} = 0}_{\text{cond. imp.}} \vee -2+4x^2 = 0 \Leftrightarrow x^2 = \frac{1}{2} \Leftrightarrow x = \pm\frac{\sqrt{2}}{2}$

	$-\infty$	$-\frac{\sqrt{2}}{2}$		$\frac{\sqrt{2}}{2}$	$+\infty$
e^{1-x^2}	$+$	$+$	$+$	$+$	$+$
$-2+4x^2$	$+$	0	$-$	0	$+$
f''	$+$	0	$-$	0	$+$
f	$\cup$	PI	$\cap$	PI	$\cup$

Conc. voltada para cima em $\left]-\infty, -\frac{\sqrt{2}}{2}\right]$ e em $\left[\frac{\sqrt{2}}{2}, +\infty\right[$

Conc. voltada para baixo em $\left[-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right]$

PI para $x = -\frac{\sqrt{2}}{2}$ e para $x = \frac{\sqrt{2}}{2}$

36 a) g cont. em $x=1$ sse $\lim\limits_{x\to1^-} g(x) = \lim\limits_{x\to1^+} g(x) = g(1)$

- $\lim\limits_{x\to1^-} \dfrac{4x-4}{e^{x-1}-1} \underset{\boxed{\begin{array}{l} x-1=y\to0^- \\ x=y+1\end{array}}}{=} \lim\limits_{y\to0^-} \dfrac{4(y+1)-4}{e^{y}-1} = \lim\limits_{y\to0^-} \dfrac{4y+4-4}{e^{y}-1} = 4\lim\limits_{y\to0^-} \dfrac{y}{e^{y}-1} =$

$= 4\times \dfrac{1}{\underbrace{\lim\limits_{y\to0^-} \frac{e^{y}-1}{y}}_{L.N.}} = 4\times\dfrac{1}{1} = 4$

- $\lim\limits_{x\to1^+} \left(7\times3^{x-1}-3\right) = 7\times3^{1-1}-3 = 7-3 = 4$

- $g(1) = 7\times3^{1-1}-3 = 4$

Assim, concluímos que g é cont em $x=1$

b) $g(x)>0$ em $[1,+\infty[$

$\log_3\left(7\times3^{x-1}-3\right) = x + \log_3 2 \Leftrightarrow$

$\Leftrightarrow \log_3\left(7\times3^{x-1}-3\right) = \log_3 3^{x} + \log_3 2 \Leftrightarrow$

$\Leftrightarrow \log_3\left(7\times3^{x-1}-3\right) = \log_3\left(2\times3^{x}\right)$

$\Leftrightarrow 7\times3^{x-1}-3 = 2\times3^{x} \Leftrightarrow$

$\Leftrightarrow 7\times3^{x}\times3^{-1}-3-2\times3^{x}=0$

$\boxed{y=3^{x}}$

$\Leftrightarrow \dfrac{7}{3}y - \underset{(3)}{3} - \underset{(3)}{2y} = \underset{(3)}{0} \Leftrightarrow$

$\Leftrightarrow 7y-9-6y=0$

$\Leftrightarrow y=9$

$\Leftrightarrow 3^{x}=9 \Leftrightarrow x=2$

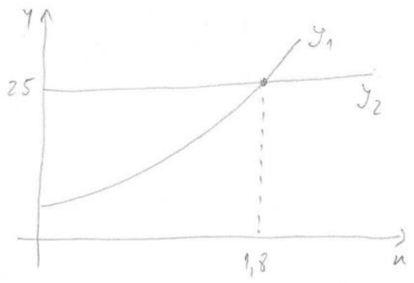
$S=\{2\}$

37

$t+3$	$a(t+3)$
t	$a(t)$

$\left.\begin{array}{l}a(t+3)\\ \\ a(t)\end{array}\right\}25$

$\underbrace{a(t+3)-a(t)}_{y_1} = \underbrace{25}_{y_2}$



R: 1,8 s

(38) $\lim\limits_{x\to+\infty} \left(f(x)-2x+1\right)=0 \Leftrightarrow \lim\limits_{x\to+\infty}\left(f(x)-(2x-1)\right)=0$ logo $y=2x-1$

é A.O. ao gráfico de f em $+\infty$, pelo que não existe A.H ao gráfico de f em $+\infty$. Assim, I é Falsa

Como g é diferenciável, então g é cont, em particular, g é cont em $x=1$, logo $\lim\limits_{x\to1} g(x)=g(1)$

Como a reta r é tangente ao gráfico de g no ponto de abcissa 1, então $g(1)=2\times1-1=1$, isto é, $g(1)=1\neq2$

Assim, II é Falsa

f tem conc. voltada p/ cima, logo $\forall x\in\mathbb{R}$, $f''(x)\geq0$

g " " " " baixo, " $\forall x\in]0,+\infty[$, $g''(x)<0$

Assim, $\forall x\in]0,+\infty[$, $f''(x)>g''(x)$ e portanto, III é Falsa

(39) $f'(x)=2ax+b$

Ora $f'(x)=a \Leftrightarrow$

$\Leftrightarrow 2ax+b=a \Leftrightarrow$

$\Leftrightarrow 2ax=a-b$

$\Leftrightarrow x=\dfrac{a-b}{2a}$

$\begin{cases} y=ax+b \\ y=ax^2+bx\end{cases}$ $\Leftrightarrow \begin{cases} 2ax=a-b \\ ax^2+bx=ax+b\end{cases} \Leftrightarrow$

$\Leftrightarrow \begin{cases} 2ax+b=a \\ x(ax+b)=ax+b\end{cases} \Leftrightarrow \begin{cases} 2a+b=a \\ \boxed{x=1}\end{cases}$

$\Leftrightarrow \begin{cases} 2a-a=-b \\ -\end{cases} \Leftrightarrow \begin{cases} a=-b \\ y=ax+b\end{cases} \Leftrightarrow$

$\Leftrightarrow \begin{cases} y=-b\times1+b\end{cases} \Leftrightarrow \begin{cases}\boxed{y=0}\end{cases}$

Ponto de tangência $(1,0)$

ou Ponto de tangência $(x, f(x))$

$f(x)=f\left(\dfrac{a-b}{2a}\right)=a\left(\dfrac{a-b}{2a}\right)^2+b\left(\dfrac{a-b}{2a}\right)=$

$=\dfrac{\cancel{a}(a^2-2ab+b^2)}{4a^{\cancel{2}}}+\dfrac{ba-b^2}{\underset{(2)}{2a}}=\dfrac{a^2-\cancel{2ab}+b^2+\cancel{2ab}-2b^2}{4a}=\dfrac{a^2-b^2}{4a}$

Ponto de tangência $\left(\dfrac{a-b}{2a}, \dfrac{a^2-b^2}{4a}\right)$ pertence à reta de equação $y=ax+b$, logo

$\dfrac{a^2-b^2}{4a}=\cancel{a}\dfrac{a-b}{2\cancel{a}}+b \Leftrightarrow \dfrac{a^2-b^2}{4a}=\underset{(2a)}{\dfrac{a-b}{2}}+\underset{(4a)}{b} \overset{a\neq0}{\Leftrightarrow}$

$\Leftrightarrow a^2-b^2=2a^2-2ab+4ab \Leftrightarrow -a^2-b^2-2ab=0 \Leftrightarrow$

$\Leftrightarrow a^2+2ab+b^2=0 \Leftrightarrow (a+b)^2=0 \Leftrightarrow a+b=0 \Leftrightarrow \boxed{a=-b}$

Ponto de tangência $\left(\dfrac{-b-b}{-2b}, \dfrac{(-b)^2-b^2}{-4b}\right)=(1,0)$

Outra forma...

$\begin{cases} y=ax+b \\ y=ax^2+bx\end{cases} \Leftrightarrow ax^2+bx=ax+b \Leftrightarrow ax^2+(b-a)x-b=0 \Leftrightarrow x=\dfrac{a-b\pm\sqrt{(b-a)^2+4ab}}{2a}$

Esta equação tem 1 única solução, logo $(b-a)^2+4ab=0 \Leftrightarrow a^2-2ab+b^2+4ab=0$.

$\Leftrightarrow a^2+2ab+b^2=0 \Leftrightarrow (a+b)^2=0 \Leftrightarrow a+b=0 \Leftrightarrow a=-b$

Então $x=\dfrac{-b-b\pm0}{-2b}=\dfrac{-2b}{-2b}=1$ e $f(1)=a\times1^2+b=a+b=-b+b=0$

Assim, ponto de tangência $(1,0)$

(40) $\lim \left(1-\frac{2}{n}\right)^n = e^{-2}$

$\lim\left(-\frac{n^2+1}{n}\right) = \lim\left(-\frac{n^2}{n}-\frac{1}{n}\right) = \lim\left(-n-\frac{1}{n}\right) = -\infty - 0 = -\infty$

$\lim \frac{4n+3}{3n+4} = \lim \frac{4n}{3n} = \frac{4}{3}$

$\lim \frac{(-1)^n}{n}$ $\begin{cases} n \text{ par: } \lim \frac{1}{n} = 0 \\ n \text{ ímpar: } \lim\left(-\frac{1}{n}\right) = 0 \end{cases}$ logo $\lim \frac{(-1)^n}{n} = 0$ (D)

(41) $a + e^b = 5 \Leftrightarrow \boxed{e^b = 5 - a}$

$a + e^{2b} = 7 \Leftrightarrow a + (e^b)^2 = 7 \Leftrightarrow a + (5-a)^2 = 7 \Leftrightarrow a + 25 - 10a + a^2 = 7$

$\Leftrightarrow a^2 - 9a + 18 = 0 \Leftrightarrow a = \frac{9 \pm \sqrt{81 - 4\times 1\times 18}}{2} \Leftrightarrow a = 6 \vee a = 3$

se $a = 6$ então $e^b = 5 - 6 \Leftrightarrow e^b = -1$ imp.

se $a = 3$ então $e^b = 5 - 3 \Leftrightarrow e^b = 2 \Leftrightarrow b = \ln 2$

Assim, $a = 3$ e $b = \ln 2$

(42) • Como g é diferenciável, g é contínua. Assim, $\lim_{x\to 0^-} g(x) = g(0) < 0$

Tal não acontece no referencial I

• Como $g'(x) < 0$, $\forall x \in]-\infty, -1[$, então g é decrescente em $]-\infty, -1[$

Tal não acontece no referencial II

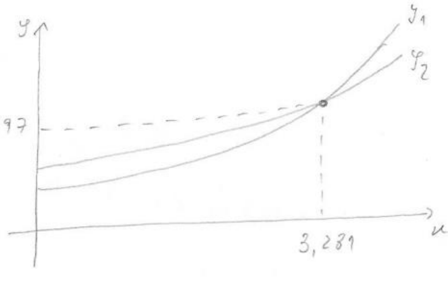
• Como g é par e $\lim_{x\to 1^-} g(x) = +\infty$, então $\lim_{x\to -1^+} g(x) = +\infty$

Tal não acontece no referencial III

(43) $\underbrace{p(2j)}_{y_1} = \underbrace{p(j) + 120}_{y_2}$

$y_1 = p(2x)$

$y_2 = p(x) + 120$



R: $j \approx 3{,}281\%$

44) **A.H.** Como $D =]0,+\infty[$, apenas tem sentido $x \to +\infty$

a) $\lim\limits_{x\to+\infty} \frac{\ln x + 2x}{x} = \underbrace{\lim\limits_{x\to+\infty} \frac{\ln x}{x}}_{L.N.} + \lim\limits_{x\to+\infty} \frac{2x}{x} = 0 + 2 = 2$. Assim, $y = 2$ é uma equação de A.H.

A.V. Como f é cont. no seu domínio, a única A.V., caso exista, será para $x = 0$

$\lim\limits_{x\to0^+} \frac{\ln x + 2x}{x} = \frac{\ln(0^+) + 0}{0^+} = \frac{-\infty}{0^+} = -\infty$. Assim $x = 0$ é a equação de A.V.

b) $f'(x) = \frac{\left(\frac{1}{x}+2\right)x - (\ln x - 2x)\times 1}{x^2} = \frac{1 + 2x - \ln x - 2x}{x^2} = \frac{1 - \ln x}{x^2}$

Zeros de f':

$1 - \ln x = 0 \wedge x^2 \neq 0$

$\Leftrightarrow \ln x = 1 \wedge x \neq 0$

$\Leftrightarrow x = e$

	0		e	+∞
$1-\ln x$	///	+	0	−
x^2	///	+	+	+
f'	///	+	0	−
f	///	↗	máx	↘

$f(e) = \frac{\ln e + 2e}{e} = \frac{1+2e}{e} = 2 + \frac{1}{e}$

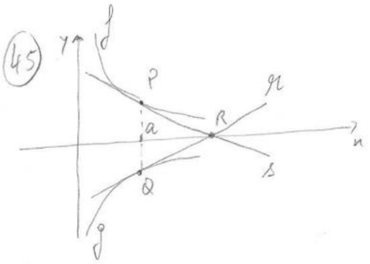
Conclusão:

f crescente em $]0, e]$

f decrescente em $[e, +\infty[$

máx relat $= 2 + \frac{1}{e}$ para $x = e$

45)



$P\left(a, \frac{K}{a}\right)$ $Q\left(a, -\frac{K}{a}\right)$

$\overline{PQ} = \frac{K}{a} - \left(-\frac{K}{a}\right) = \frac{2K}{a}$

reta s:

$f'(x) = -\frac{K}{x^2}$ $\quad y = -\frac{K}{a^2}x + b$

$m_s = f'(a) = -\frac{K}{a^2}$ $\quad P\left(a, \frac{K}{a}\right) \in$ $\frac{K}{a} = -\frac{K}{a^2}\times a + b$

$\Leftrightarrow \frac{K}{a} = -\frac{K}{a} + b$

$\Leftrightarrow b = \frac{2K}{a}$

$$\boxed{s: y = -\frac{K}{a^2}x + \frac{2K}{a}}$$

De modo semelhante, para a reta r (...) "os cálculos devem ser efetuados!

$$\boxed{r: y = \frac{K}{a^2}x - \frac{2K}{a}}$$

interseção das retas r e s:

$\frac{K}{a^2}x - \frac{2K}{a} = -\frac{K}{a^2}x + \frac{2K}{a} \Leftrightarrow Kx - 2Ka = -Kx + 2Ka \overset{K\neq0}{\Leftrightarrow} x - 2a = -x + 2a \Leftrightarrow$

$\Leftrightarrow 2x = 4a \Leftrightarrow x = 2a$

Triângulo [PQR]:

base $= \overline{PQ} = \frac{2K}{a}$

altura $= 2a - a = a$

$A = \frac{\frac{2K}{a}\times a}{2} = \frac{2K}{2} = K$ c.q.d.

(46) Graficamente, verifica-se, com facilidade, que apenas $u_n = \frac{1}{n}$ é monótona (decrescente) (D)

(47)

a) • $\lim_{x\to 0^-} \frac{\text{sen}(ax)}{e^x-1} \overset{\left(\frac{0}{0}\right)}{=} \lim_{x\to 0^-} \frac{\frac{\text{sen}(ax)}{x}}{\frac{e^x-1}{x}} = \frac{\lim_{x\to 0^-} \frac{\text{sen}(ax)}{x}}{\lim_{x\to 0^-} \frac{e^x-1}{x}} = \frac{a}{1} = a$

C.A.

$\lim_{x\to 0^-} \frac{\text{sen}(ax)}{x} \overset{\left(\frac{0}{0}\right)}{=} \boxed{\begin{array}{l} ax = y \to 0^- \\ x = \frac{y}{a} \end{array}} \lim_{y\to 0^-} \frac{\text{sen}\, y}{\frac{y}{a}} = a \underbrace{\lim_{y\to 0^-} \frac{\text{sen}\, y}{y}}_{L.N.} = a \times 1 = a$

$\underbrace{\lim_{x\to 0^-} \frac{e^x-1}{x}}_{L.N.} = 1$

• $\lim_{x\to 0^+} \left(\ln(2-e^{-x})+x+2\right) = \ln(2-e^0)+0+2 = \ln 1 + 2 = 0+2 = 2$

• Para que f seja cont., $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x)$. Logo $a = 2$

b) $m = \lim_{x\to +\infty} \frac{\ln(2-e^{-x})+x+2}{x} \overset{\left(\frac{\infty}{\infty}\right)}{=} \lim_{x\to +\infty} \frac{\ln(2-e^{-x})}{x} + \lim_{x\to +\infty} \frac{x}{x} + \lim_{x\to +\infty} \frac{2}{x} =$

$= \frac{\ln(2-e^{-\infty})}{+\infty} + 1 + 0 =$

$= \frac{\ln 2}{+\infty} + 1 = 0 + 1 = 1$

$b = \lim_{x\to +\infty} \left(\ln(2-e^{-x})+\cancel{x}+2-\cancel{x}\right) = \lim_{x\to +\infty} \left(\ln(2-e^{-x})\right) + \lim_{x\to +\infty} 2 =$

$= \ln(2-e^{-\infty}) + 2 =$

$= \ln(2-0) + 2 = \ln 2 + 2$

Assim, $y = x + 2 + \ln 2$

(48)

$\log_a\left(\frac{b}{a}\right) = 2$

$\Leftrightarrow \log_a b - \log_a a = 2$

$\Leftrightarrow \log_a b - 1 = 2$

$\Leftrightarrow \log_a b = 3$

$\log_a\left(\sqrt{a^3} \times b^2\right) = \log_a a^{\frac{3}{2}} + \log_a b^2 =$

$= \frac{3}{2} + 2\log_a b =$

$= \frac{3}{2} + 2\times 3 =$

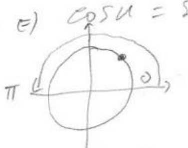
$= \frac{3}{2} + 6 =$

$= \frac{15}{2}$ (B)

49 a) $g'(x) = e^x\cos x + e^x(-\operatorname{sen} x) = e^x\cos x - e^x\operatorname{sen} x = e^x(\cos x - \operatorname{sen} x)$

Zeros de g'

$\underbrace{e^x = 0}_{\text{imp.}} \vee \cos x - \operatorname{sen} x = 0 \Leftrightarrow \cos x = \operatorname{sen} x$



$\Leftrightarrow x = \frac{\pi}{4}$ em $[0, \pi[$

	0		$\frac{\pi}{4}$		π
e^x	+	+	+	+	
$\cos x - \operatorname{sen} x$	+	+	0	−	
g'	+	+	0	−	
g	mín	↗	máx	↘	

$g(0) = e^0\cos 0 = 1\times 1 = 1$

$g(\frac{\pi}{4}) = e^{\frac{\pi}{4}}\cos(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}e^{\frac{\pi}{4}}$

Assim:

g crescente em $[0, \frac{\pi}{4}]$

g decrescente em $[\frac{\pi}{4}, \pi[$

mín relat $= 1$ para $x = 0$

máx relat $= \frac{\sqrt{2}}{2}e^{\frac{\pi}{4}}$ para $x = \frac{\pi}{4}$

b) Pretende-se mostrar que

$\exists x \in]\frac{\pi}{3}, \frac{\pi}{2}[: g(x) = x$

Seja $h(x) = g(x) - x$

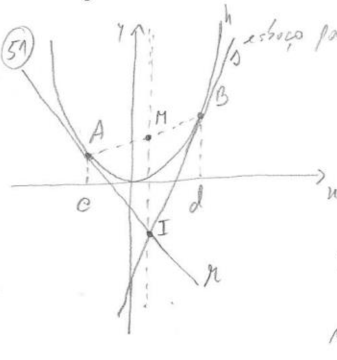
h é cont em $[\frac{\pi}{3}, \frac{\pi}{2}]$

$$\left.\begin{array}{l} h(\frac{\pi}{3}) = e^{\frac{\pi}{3}} \times \cos(\frac{\pi}{3}) - \frac{\pi}{3} = \frac{1}{2}e^{\frac{\pi}{3}} - \frac{\pi}{3} \approx 0{,}38 > 0 \\ h(\frac{\pi}{2}) = e^{\frac{\pi}{2}} \times \cos(\frac{\pi}{2}) - \frac{\pi}{2} = -\frac{\pi}{2} < 0 \end{array}\right\} h(\tfrac{\pi}{3}) \times h(\tfrac{\pi}{2}) < 0$$

Pelo Corolário do Teor. de Bolzano-Cauchy, $\exists x \in]\frac{\pi}{3}, \frac{\pi}{2}[: h(x) = 0$, isto é, $\exists x \in]\frac{\pi}{3}, \frac{\pi}{2}[: g(x) - x = 0$, ou seja, $\exists x \in]\frac{\pi}{3}, \frac{\pi}{2}[: g(x) = x$

50 $\frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{1}{3} \Leftrightarrow 3e^x - 3e^{-x} = e^x + e^{-x} \Leftrightarrow 2e^x = 4e^{-x} \Leftrightarrow e^x = 2e^{-x} \underset{e^{-x}\neq 0}{\Leftrightarrow} \frac{e^x}{e^{-x}} = 2 \Leftrightarrow e^{2x} = 2 \Leftrightarrow 2x = \ln 2 \Leftrightarrow x = \frac{\ln 2}{2}$ $\quad S = \{\frac{\ln 2}{2}\}$

51



esboço para $a > 0$ (para $a < 0$ é semelhante)

$A(c, ac^2)$ $B(d, ad^2)$

$M\left(\frac{c+d}{2}, \frac{ac^2 + ad^2}{2}\right)$

Pretende-se mostrar que a abcissa de I é $\frac{c+d}{2}$

reta r

$h'(x) = 2ax$ $\quad y = 2acx + b$

$m_r = h'(c) = 2ac$ $\quad A: ac^2 = 2ac^2 + b$

$\Leftrightarrow b = -ac^2$

$\boxed{r: y = 2acx - ac^2}$

Para a reta s, semelhante!... efetuar os cálculos!!

$$\boxed{s \curvearrowright y = 2adx - ad^2}$$

<u>intersecção de r e s:</u>

$$2aex - ae^2 = 2adx - ad^2$$

$$\overset{a \neq 0}{\Leftrightarrow} 2ex - e^2 = 2dx - d^2$$

$$\Leftrightarrow 2ex - 2dx = e^2 - d^2 \Leftrightarrow 2x(e-d) = (e-d)(e+d) \Leftrightarrow$$

$$\Leftrightarrow x = \frac{\cancel{(e-d)}(e+d)}{2\cdot\cancel{(e-d)}} \Leftrightarrow x = \frac{e+d}{2} \quad \text{c.q.d.}$$