1) $f(x) = x + \cos x \Leftrightarrow \cancel{x} + \sin x = \cancel{x} + \cos x \Leftrightarrow$

$\Leftrightarrow \sin x = \cos x \Leftrightarrow \sin x = \sin\left(\frac{\pi}{2} - x\right) \Leftrightarrow$

$\Leftrightarrow x = \frac{\pi}{2} - x + 2k\pi \ \vee\ \cancel{x} = \pi - \frac{\pi}{2} + \cancel{x} + 2k\pi \ ; k \in \mathbb{Z} \Leftrightarrow$

$\Leftrightarrow 2x = \frac{\pi}{2} + 2k\pi \ \vee\ \text{imp} \Leftrightarrow$

$\Leftrightarrow x = \frac{\pi}{4} + k\pi \ ; \ k \in \mathbb{Z}$

$k = 0 \to x = \frac{\pi}{4}$ ✓

$k = 1 \to x = \frac{5\pi}{4}$ ✓

$k = 2 \to x = \frac{9\pi}{4}$ ✗

$k = -1 \to x = -\frac{3\pi}{4}$ ✗

$C.S. = \left\{ \frac{\pi}{4} ; \frac{5\pi}{4} \right\}$

(2) $f(x) = a + b\sin^2 x$

a) $\left.\begin{matrix} a = 2 \\ b = -5 \end{matrix}\right\} \quad f(x) = 2 - 5\sin^2 x$

$\operatorname{tg}\theta = \frac{1}{2}$ $\quad 1 + \operatorname{tg}^2\theta = \frac{1}{\cos^2\theta} \Leftrightarrow 1 + \frac{1}{4} = \frac{1}{\cos^2\theta} \Leftrightarrow$

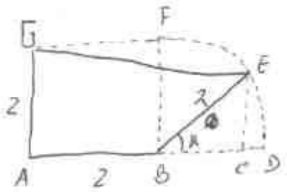
$\Leftrightarrow \frac{5}{4} = \frac{1}{\cos^2\theta} \Leftrightarrow \boxed{\cos^2\theta = \frac{4}{5}}$

$\sin^2\theta + \cos^2\theta = 1 \Leftrightarrow \sin^2\theta = 1 - \frac{4}{5} \Leftrightarrow \boxed{\sin^2\theta = \frac{1}{5}}$

$f(\theta) = 2 - 5\sin^2\theta = 2 - \cancel{5} \times \frac{1}{\cancel{5}} = 2 - 1 = 1$

b) $\left\{\begin{matrix} f(0) = 1 \\ f\left(\frac{\pi}{2}\right) = -3 \end{matrix}\right. \quad \begin{matrix} a + b \times 0 = 1 \\ a + b = -3 \end{matrix} \quad \left\{\begin{matrix} \boxed{a = 1} \\ \boxed{b = -4} \end{matrix}\right.$

(3)



$\cos x = \frac{\overline{BC}}{2} \Rightarrow \overline{BC} = 2\cos x$

$\sin x = \frac{\overline{EC}}{2} \Rightarrow \overline{EC} = 2\sin x$

a) $A_{trapézio} = \frac{B + b}{2} \times h = \frac{2 + \overline{EC}}{2} \times (2 + \overline{BC}) =$

$= \frac{2 + 2\sin x}{2} \times (2 + 2\cos x) = (1 + \sin x)(2 + 2\cos x)$

$A_{triângulo} = \frac{\overline{BC} \times \overline{EC}}{2} = \frac{\cancel{2}\cos x \, 2\sin x}{\cancel{2}} = 2\sin x\cos x$

$A_{sombreada} = A_{trapézio} - A_{triângulo} = (1 + \sin x)(2 + 2\cos x) - 2\sin x\cos x =$

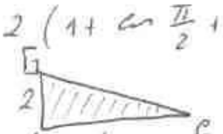
$= 2 + 2\cos x + 2\sin x + \cancel{2\sin x\cos x} - \cancel{2\sin x\cos x} =$

$= 2(1 + \cos x + \sin x)$ c.q.d.

b) $A(0) = 2(1 + \cos 0 + \sin 0) = 2(1 + 1 + 0) = 4$

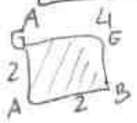
$A\left(\frac{\pi}{2}\right) = 2\left(1 + \cos\frac{\pi}{2} + \sin\frac{\pi}{2}\right) = 2(1 + 0 + 1) = 4$

$A(0) \rightarrow$



$A = \frac{2 \times 4}{2} = \frac{8}{2} = 4$ ✓ (triângulo)

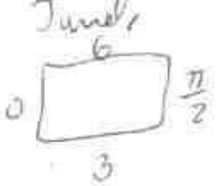
$A\left(\frac{\pi}{2}\right) \rightarrow$

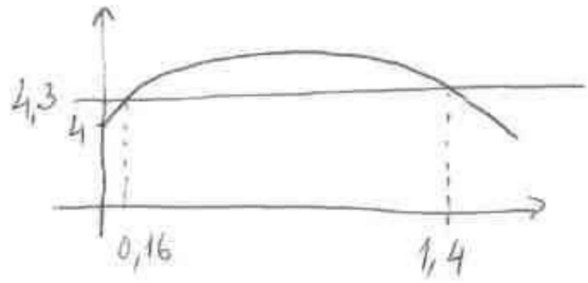


$A = 2 \times 2 = 4$ ✓ (quadrado)

c) $A(x) = 2(1 + \sin x + \cos x)$

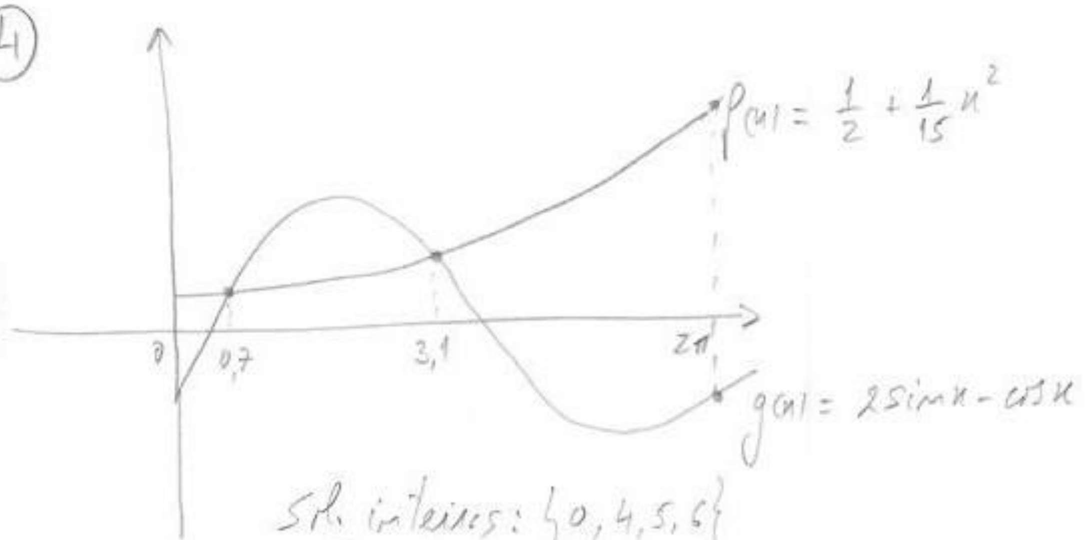
$A(x) = 4{,}3$?





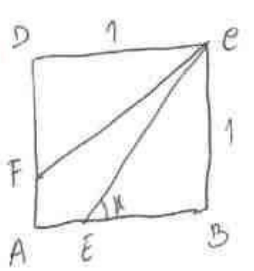
R: A área é 4,3 para $x \simeq 0{,}2$ ou $x \simeq 1{,}4$.

(4)



Sol. inteiras: $\{0, 4, 5, 6\}$
$[0, 2\pi]$

5)

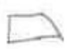


$\sin x = \frac{1}{\overline{EC}} \Rightarrow \boxed{\overline{EC} = \frac{1}{\sin x}}$

logo $\boxed{\overline{FC} = \frac{1}{\sin x}}$

$\operatorname{tg} x = \frac{1}{\overline{EB}} \Rightarrow \overline{EB} = \frac{1}{\operatorname{tg} x} \Rightarrow \boxed{\overline{AE} = 1 - \frac{1}{\operatorname{tg} x}}$ e $\boxed{\overline{AF} = 1 - \frac{1}{\operatorname{tg} x}}$

Perímetro $= 2 \times \frac{1}{\sin x} + 2\left(1 - \frac{1}{\operatorname{tg} x}\right) = \frac{2}{\sin x} + 2 - \frac{2}{\operatorname{tg} x}$ c.q.d.

6)  $= \frac{B+b}{2}\times h$

$h=\frac{1}{3}$

$B=?$ $\quad f(x)=0 \Leftrightarrow \cos x = 0 \Leftrightarrow x=\frac{\pi}{2}$ em $]-\pi,\pi[$, logo $B=\frac{\pi}{2}$

$b=?$ $\quad f(x)=\frac{1}{3} \Leftrightarrow \frac{\cos x}{1+\cos x}=\frac{1}{3} \Leftrightarrow 3\cos x = 1+\cos x \Leftrightarrow 2\cos x = 1 \Leftrightarrow \cos x=\frac{1}{2} \Leftrightarrow$

$\Leftrightarrow x=\frac{\pi}{3}$ em $]-\pi,\pi[$, logo $b=\frac{\pi}{3}$

$\therefore A = \frac{\frac{\pi}{2}+\frac{\pi}{3}}{2}\times\frac{1}{3} = \frac{5\pi}{12}\times\frac{1}{3}=\frac{5\pi}{36}$

7) 

$\cos x = \frac{1}{h} \Leftrightarrow h=\frac{1}{\cos x}$

$A_{pirâmide} = 4 + 4\times\frac{\cancel{2}\times\frac{1}{\cos x}}{\cancel{2}} = 4+\frac{4}{\cos x} = \frac{4\cos x + 4}{\cos x}$

Com $x\in\left]0,\frac{\pi}{2}\right[$

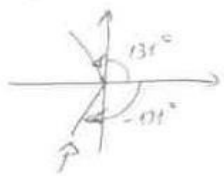
8) $d=\frac{7820}{1+0{,}07\cos x}$

a) $x=180° \to d=\frac{7820}{1+0{,}07(-1)}\approx 8409$ Km

Altitude $= 8409-6378 = 2031$ Km

b) $d(x)=8200 \Leftrightarrow \frac{782\cancel{0}}{1+0{,}07\cos x}=820\cancel{0} \Leftrightarrow 1+0{,}07\cos x=\frac{782}{820} \Leftrightarrow$

$\Leftrightarrow 0{,}07\cos x = \frac{782}{820}-1 \Leftrightarrow x\approx 131° \vee x\approx -131°$



Como $x\in 3.°Q$ então $x\approx 180°+(180°-131°)=229°$

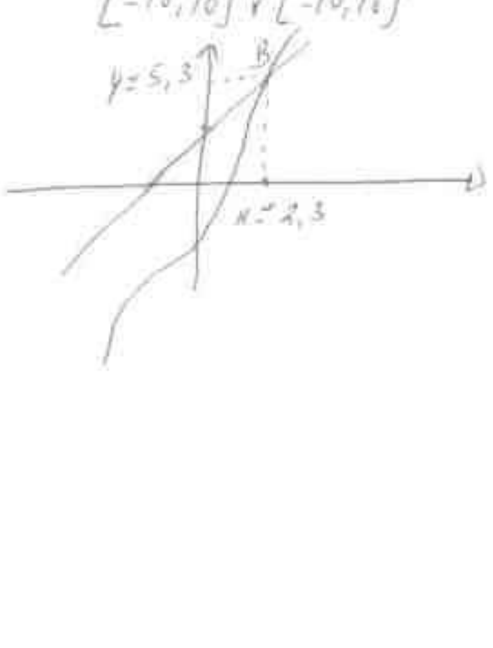
9) Coordenadas de B?

Calculadora

$y=x+3 \quad (tg\,45°=1)$

$y=2x-\cos x$

$[-10,10]\times[-10,10]$



$A_\Delta = \frac{b\times h}{2}=\frac{3\times5{,}3}{2}\approx 8.$

10) $f(n) = 12,2 + 2,64 \sin \frac{\pi(n-81)}{183}$

a) 24 Março $\rightarrow$ $31+29+24 = 84$

$f(84) \simeq 12,336 \rightarrow 12h\,20\,min$

$$\begin{array}{r} 6h\,30\,min \\ +\,12h\,20\,min \\ \hline 18h\,50\,min \end{array}$$

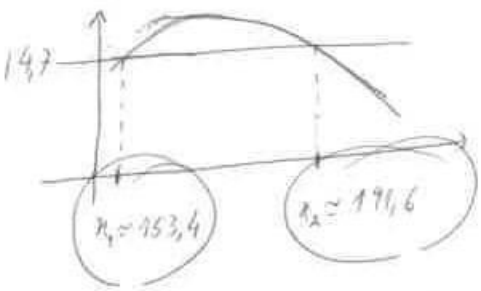
O pôr-do-sol ocorreu às 18h50min.

b) $f(n) > 14,7 \Leftrightarrow n \in ?$

$[150, 250] \times [12, 15]$

$y_1 = 12,2 + 2,64 \sin \frac{\pi(n-81)}{183}$

$y_2 = 14,7$

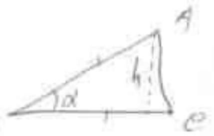


$191,6 - 153,4 \cong 38$ dias

11)

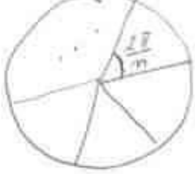
a) $A_\Delta = \frac{b \times h}{2} = \frac{\overline{BC} \times \overline{BC} \sin\alpha}{2} = \frac{\overline{BC}}{2} \sin\alpha$

$(\alpha \in]0, \pi[$



$\sin\alpha = \frac{h}{\overline{AB}} = \frac{h}{\overline{BC}}$

b)

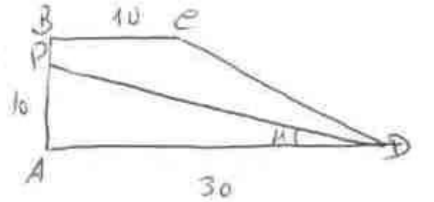


$A_\Delta = \frac{r}{2} \sin\left(\frac{2\pi}{n}\right) = \frac{1}{2} \sin\left(\frac{2\pi}{n}\right) \Leftrightarrow h = \overline{BC} \sin\alpha$

Para n Δs, temos

$A_{polígono} = n \times \frac{1}{2} \sin\left(\frac{2\pi}{n}\right) = \frac{n}{2} \sin\left(\frac{2\pi}{n}\right) = A_n$

12)



$A_{\square} = \frac{B+b}{2} \times h = \frac{30+10}{2} \times 10 =$

$= 200$ //

Metade $= 100$

$tg\,x = \frac{\overline{AP}}{30} \Rightarrow \overline{AP} = 30\,tg\,x$, logo $A_{\triangle} = \frac{b \times h}{2} =$

$= \frac{30 \times 30\,tg\,x}{2} = \frac{30^2\,tg\,x}{2}$

Então $\frac{30^2\,tg\,x}{2} = 100$. Resposta (B)

13)

$\begin{cases} d(0) = 2 \\ d\left(\frac{\pi}{2}\right) = 1 \\ d(\pi) = 0 \end{cases}$

$\boxed{(A)\ d(\alpha) = 1 + \cos\alpha}$

(B) $d(\alpha) = 2 + \sin\alpha$

$d\left(\frac{\pi}{2}\right) = 2 + 1 = 3$ X

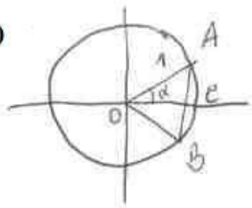
(C) $d(\alpha) = 1 - \cos\alpha$

$d(0) = 1 - 1 = 0$ X

(D) $d(\alpha) = 2 - \sin\alpha$

$d(\pi) = 2 - 0 = 2$ X

14)



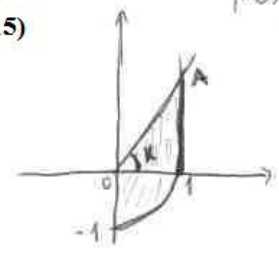
$\sin\alpha = \frac{\overline{AC}}{1} \Rightarrow \boxed{\overline{AC} = \sin\alpha}$

$\boxed{\cos\alpha = \overline{OC}}$

$A_{\triangledown} = \frac{b \times h}{2} = \frac{\cancel{2}\sin\alpha\cos\alpha}{\cancel{2}} = \sin\alpha\cos\alpha$

Resposta (A)

15)



$A_{total} = A_{\nabla} + A_{\triangle}$

$A_{\nabla} = \frac{1}{4}\pi r^2 = \frac{1}{4}\pi$

$A_{\triangle} = \frac{b \times h}{2} = \frac{1 \times tg\,x}{2}$

$tg\,x = \frac{h}{1} \Rightarrow h = tg\,x$

$A_{total} = \frac{\pi}{4} + \frac{tg\,x}{2}$ Resposta (C)

16) $V(x) = 80(x - \text{sen}\, x)$

$[0; 2\pi]$

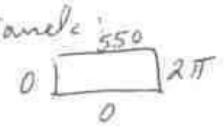
a) $V(2\pi) = 80(2\pi - \text{sen}\, 2\pi) = 80 \times 2\pi = 160\pi \simeq 503\ m^3$

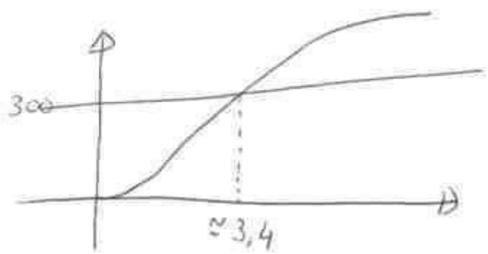
b) $V(x) = 300$

$y_1 = 80(x - \text{sen}\, x)$

$y_2 = 300$

Janela:





R: 3,4 rad

c)

$\cos\alpha = \dfrac{\frac{1}{4}}{\frac{1}{2}} = \dfrac{2}{4} = \dfrac{1}{2}$

$\Downarrow$

$\alpha = \dfrac{\pi}{3} \Rightarrow 2\alpha = \dfrac{2\pi}{3}$

$V\left(\dfrac{2\pi}{3}\right) = 80\left(\dfrac{2\pi}{3} - \text{sen}\, \dfrac{2\pi}{3}\right) \simeq 98\ m^3$

d) D X → a altura não cresce linearmente

C X → " " " decresce nunca

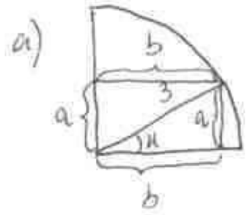
A X → a " cresce + rápido no início e no fim e não o contrário.

Resposta: B

17) A área é máx. para $\frac{\pi}{2}$ e $\frac{3\pi}{2}$ logo A

18) $P = \dfrac{14\pi}{9} - \dfrac{2\pi}{9} = \dfrac{12\pi}{9} = \dfrac{4\pi}{3}$ D

19)

a) 

$\sin x = \frac{a}{3} \Leftrightarrow a = 3\sin x$

$\cos x = \frac{b}{3} \Leftrightarrow b = 3\cos x$

 $A = \frac{3\sin x \; 3\cos x}{2} = \frac{9\sin x \cos x}{2}$

 $2\pi - \pi r^2$

$x - A$

$A = \frac{\cancel{\pi} r^2 x}{2\cancel{\pi}} = \frac{9x}{2}$

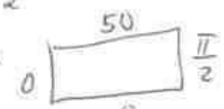
Área Total $= 4 \times \frac{9\sin x\cos x}{2} + 4 \times \frac{9x}{2} = \frac{36\sin x\cos x}{2} + \frac{36x}{2} =$

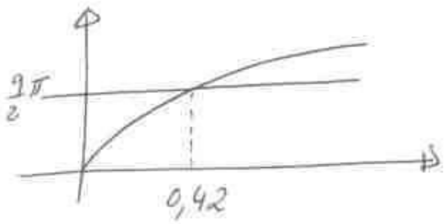
$= 18\sin x\cos x + 18x = 18(x + \sin x\cos x)$

b) $A(x) = \frac{1}{2} A_{\circ} \Leftrightarrow A(x) = \frac{1}{2}\pi 9 \Leftrightarrow A(x) = \frac{9}{2}\pi$

$y_1 = 18(x + \sin x \times \cos x)$

$y_2 = 9\pi : 2$

Janela: 



R: $x = 0{,}42$ rad

20)

$a = A\sqrt{\cos\theta}$ e $a = \pi r^2$ e $A = \pi R^2$ $\quad R = \sqrt[4]{2}\, r$

Substituindo a e A, temos $\quad r^2 = (\sqrt[4]{2}\, r)^2 \sqrt{\cos\theta}$

$\cancel{\pi} r^2 = \cancel{\pi} R^2 \sqrt{\cos\theta} \Leftrightarrow r^2 = R^2\sqrt{\cos\theta} \Leftrightarrow$

$r^2 = (2^{\frac{1}{4}})^2 r^2 \sqrt{\cos\theta} \Leftrightarrow$

$\Leftrightarrow r^2 = (2^{\frac{1}{4}} \times r)^2 \sqrt{\cos\theta} \Leftrightarrow r^2 = \sqrt{2}\, \cancel{r^2} \sqrt{\cos\theta} \Leftrightarrow 1 = \sqrt{2}\sqrt{\cos\theta} \Leftrightarrow$

$\Leftrightarrow r^2 = 2^{\frac{1}{2}} r^2 \sqrt{\cos\theta} \Leftrightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$

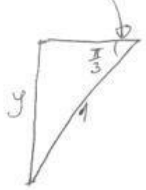
$\Leftrightarrow \sqrt{\cos\theta} = \frac{1}{\sqrt{2}} \Leftrightarrow \cos\theta = \left(\frac{1}{\sqrt{2}}\right)^2 \Leftrightarrow$

$\theta \in]0; \frac{\pi}{2}[$

21)

$\alpha(0) = \frac{\pi}{2} - \frac{\pi}{6}\cos(0) = \frac{\pi}{2} - \frac{\pi}{6} = \frac{2\pi}{6} = \frac{\pi}{3}$

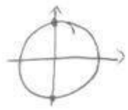
a)



$\operatorname{sen}\left(\frac{\pi}{3}\right) = \frac{y}{1} \Leftrightarrow \frac{\sqrt{3}}{2} = \frac{y}{1} \Leftrightarrow y = \frac{\sqrt{3}}{2}$

b) $\alpha(t) = \frac{\pi}{2} \Leftrightarrow \cancel{\frac{\pi}{2}} - \frac{\pi}{6}\cos(\sqrt{9{,}8}\,t) = \cancel{\frac{\pi}{2}} \Leftrightarrow \cos(\sqrt{9{,}8}\,t) = 0 \Leftrightarrow$

$\Leftrightarrow \sqrt{9{,}8}\,t = \frac{\pi}{2} \Leftrightarrow t = \frac{\frac{\pi}{2}}{\sqrt{9{,}8}} \Leftrightarrow t \simeq 0{,}5$



22)

a) distância mínima $= d(0) = 149{,}6(1 - 0{,}0167\cos 0) \simeq 147{,}1$

" máxima $= d(\pi) = 149{,}6(1 - 0{,}0167\cos\pi) \simeq 152{,}1$

b.1) $x = \pi$ $\frac{2\pi t}{T} = \pi - 0{,}0167\sin\pi \Leftrightarrow \frac{2\pi t}{T} = \pi \Leftrightarrow 2\cancel{\pi} t = \cancel{\pi} T \Leftrightarrow$

$\Leftrightarrow t = \frac{T}{2}$ c.q.d.

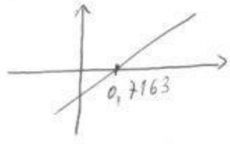
Interpretação: o tempo que decorre entre a passagem da Terra pelo Periélio até ao ponto da órbita da Terra mais afastado do Sol é metade do tempo que a Terra demora a descrever uma órbita completa.

b.2) Entre 4 Jan e 14 Fev. decorrem 41 dias

$t = 41$ e $T = 365{,}24$

$\frac{2\pi \times 41}{365{,}24} = x - 0{,}0167\sin x \Leftrightarrow 0{,}7053 = x - 0{,}0167\sin x \Leftrightarrow$

$\Leftrightarrow x - 0{,}0167\sin x - 0{,}7053 = 0$



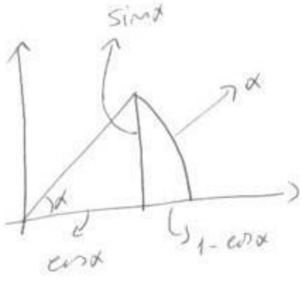
$d(0{,}7163) =$

$= 149{,}6(1 - 0{,}0167\cos 0{,}7163) =$

$= 147{,}7$ milhões

23)

$P_{\Delta} = \alpha \pi = \alpha \times 1 = \alpha$



Perímetro pretendido $= 1+\alpha+\sin\alpha-\cos\alpha$

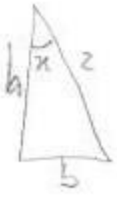
(D)

24) Resumidamente:

- Excluo gráfico 1 porque x varia de 0 a π
- " " 4 " " " de $]0;\pi[$
- " " 3 " a área é constante (base e altura permanecem constantes).

<u>OBS</u>: A composição deve ser mais detalhada e justificada.

25) $\sin x = \frac{b}{2} \Rightarrow b = 2\sin x$



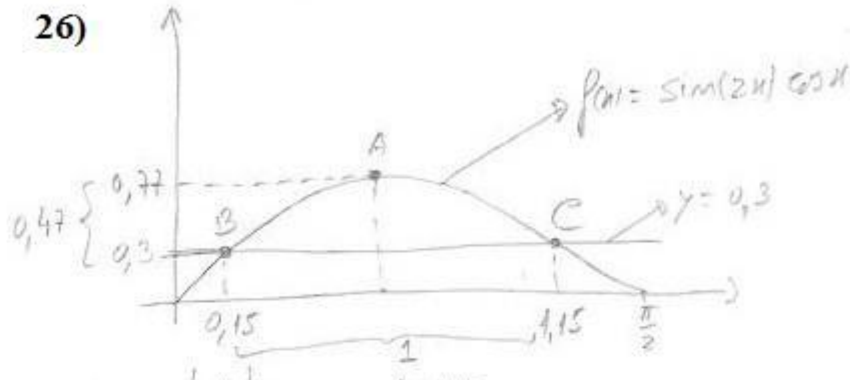
$\cos x = \frac{h}{2} \Rightarrow h = 2\cos x$

$A_{\Delta} = \frac{b \times h}{2} = \frac{\cancel{2}\sin x \times 2\cos x}{\cancel{2}}$

$= 2\sin x\cos x = \sin(2x)$

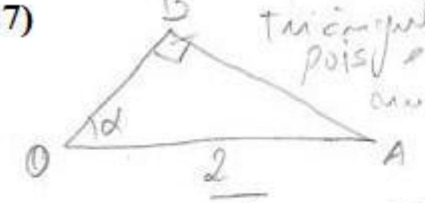
$A_{pedida} = A_{\bigcirc} - A_{\Delta} = \pi - \sin(2x)$

26)



$A = \frac{b \times h}{2} = \frac{1 \times 0,47}{2} \simeq 0,2$

27)



Triângulo retângulo, pois está inscrito numa circunf.

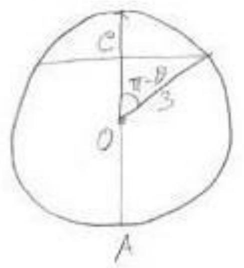
$\sin\alpha = \frac{\overline{AB}}{2} \Leftrightarrow \overline{AB} = 2\sin\alpha$

$\cos\alpha = \frac{\overline{OB}}{2} \Leftrightarrow \overline{OB} = 2\cos\alpha$

$f(\alpha) = 2 + 2\sin\alpha + 2\cos\alpha = 2(1+\cos\alpha+\sin\alpha)$ c.q.d.

28)

a)



$\cos(\pi-\theta) = \frac{\overline{OC}}{3}$

$\overline{OC} = 3\cos(\pi-\theta)$

$\overline{OC} = -3\cos\theta$

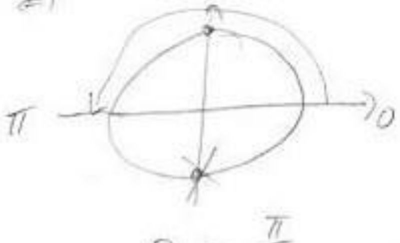
$\overline{AC} = \overline{AO} + \overline{OC} = 3 - 3\cos\theta$ c.q.d.

b) $h(\theta) = 3 \Leftrightarrow$

$\Leftrightarrow \cancel{3} - 3\cos(\theta) = \cancel{3} \Leftrightarrow$

$\Leftrightarrow -3\cos(\theta) = 0$

$\Leftrightarrow \cos(\theta) = 0$



$\theta = \frac{\pi}{2}$

Para que a altura do combustível seja 3 cm, o depósito tem que estar meio e o ângulo θ formado é $\frac{\pi}{2}$ rad.

(29) $f(x) = 4\cos(2x)$

$f\left(-\frac{\pi}{6}\right) = 4\cos\left(-\frac{2\pi}{6}\right) = 4 \times \frac{1}{2} = 2$, logo $h = 2$

$f(x) = 0 \Leftrightarrow 4\cos(2x) = 0 \Leftrightarrow \cos(2x) = 0$

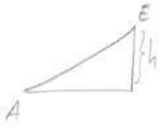
$\Leftrightarrow 2x = \frac{\pi}{2} + k\pi \, ; k \in \mathbb{Z}$

$\Leftrightarrow x = \frac{\pi}{4} + \frac{k\pi}{2} \, ; k \in \mathbb{Z}$

Se $k = 0$ então $x = \frac{\pi}{4}$

Assim $B = \frac{\pi}{6} + \frac{\pi}{4} = \frac{10\pi}{24} = \frac{5\pi}{12}$

$A = \frac{B + b}{2} \times h = \frac{\frac{5\pi}{12} + \frac{\pi}{6}}{\cancel{2}} \times \cancel{2} = \frac{7\pi}{12}$

(32) 

$tg\, x = \frac{h}{2}$

$h = 2\, tg\, x$

$A_{\triangle ABE} = \frac{4 \times 2\, tg\, x}{2} = 4\, tg\, x$

$A_{sombreada} = 4 \times 4 - 4 \times 4\, tg\, x = 16 - 16\, tg\, x$

$= 16(1 - tg\, x)$ c.q.d.

(30)

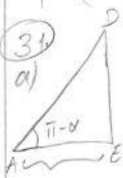
$\overline{EO} = \sin\theta$

$\overline{EA} = \cos\theta$

$\overline{OA} = 1$

$P = (1 + \sin\theta + \cos\theta) \times 2$ (C)

(31)

a) 

$\sin(\pi - \alpha) = \frac{1}{\overline{AD}} \Leftrightarrow \overline{AD} = \frac{1}{\sin\alpha}$

$tg(\pi - \alpha) = \frac{1}{\overline{AE}} \Leftrightarrow \overline{AE} = -\frac{1}{tg\,\alpha}$

$P = 3 + \frac{1}{\sin\alpha} - \frac{1}{tg\,\alpha} = 3 + \frac{1}{\sin\alpha} - \frac{\cos\alpha}{\sin\alpha}$

$= 3 + \frac{1 - \cos\alpha}{\sin\alpha}$ c.q.d.

b) $tg^2\theta + 1 = \frac{1}{\cos^2\theta}$

$8 + 1 = \frac{1}{\cos^2\theta}$

$9 = \frac{1}{\cos^2\theta}$

$\cos^2\theta = \frac{1}{9}$

$\cos\theta = \cancel{\pm} \frac{1}{3}$

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta + \frac{1}{9} = 1$

$\sin^2\theta = \frac{8}{9}$

Assim

$\frac{1 - \cos\theta}{\sin^2\theta} = \frac{1 + \frac{1}{3}}{\frac{8}{9}}$

$= \frac{\frac{4}{3}}{\frac{8}{9}} = \frac{36}{24} = \frac{3}{2}$