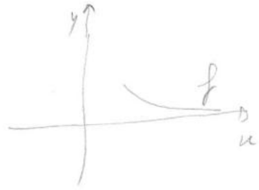
(1) $D = \mathbb{R}^+$

Como $D = \mathbb{R}^+$ apenas tem sentido $x \to +\infty$

$$\lim_{x \to +\infty} \frac{1}{f(x)} = \frac{1}{\lim\limits_{x \to +\infty} f(x)} = \frac{1}{0^+} = +\infty$$. Logo o gráfico de $\frac{1}{f}$ não tem A.H.



(2) $\lim\limits_{x \to +\infty} \big(f(x) - g(x) \big) = 0$. Logo $g(x)$ é a equação da assíntota de f. (A)

Definição de assíntota

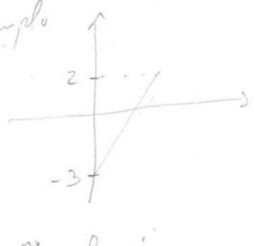
(3)

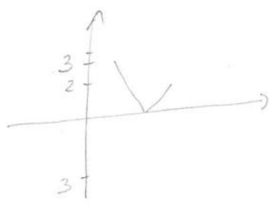
$$\lim_{x \to 4^-} f(x) = f(4)$$

$$\lim_{x \to 4^+} f(x) \neq f(4)$$

(D)

(4) exemplo





$D' = [0, 3]$ (A)

(5)

$D' = \,]0, +\infty[$ (C)

(6) Sendo $D = \mathbb{R}^+$ apenas tem sentido $x \to +\infty$ para estudar A.H.

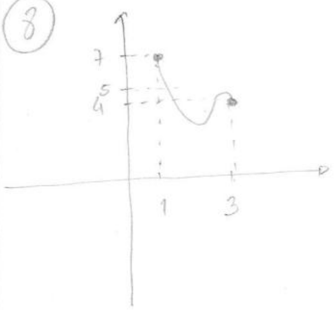
$$\lim_{x\to+\infty} h(x) = \lim_{x\to+\infty} \frac{g(x)}{x^2} = \lim_{x\to+\infty}\left(\frac{g(x)}{x} \times \frac{1}{x}\right) = \underbrace{\lim_{x\to+\infty} \frac{g(x)}{x}}_{1,\ \text{porque } y=x \text{ é A.O de } g} \times \lim_{x\to+\infty} \frac{1}{x} =$$

$$= 1 \times \frac{1}{+\infty} = 1 \times 0 = 0.$$ Assim $y = 0$ é A.H de h c.q.d.

(7)
$$\lim_{x\to 3} \frac{f(x)-f(3)}{x^2-9} = \lim_{x\to 3} \frac{f(x)-f(3)}{(x-3)(x+3)} = \underbrace{\lim_{x\to 3} \frac{f(x)-f(3)}{x-3}}_{4,\ \text{porque } f'(3)=4} \times \lim_{x\to 3} \frac{1}{x+3}$$

$$= 4 \times \frac{1}{3+3} = 4 \times \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$$ (D)

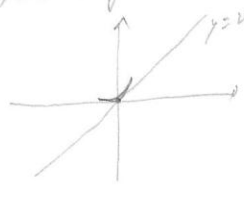
(8)



(C)

(9) Se $y = x$ é tangente, então $f'(0) = 1$

Em qual das opções se verifica $f(0) = 0$ e $f'(0) = 1$?



(A) $x^2 + x \to f(0) = 0^2 + 0 = 0$

$f'(x) = 2x+1$

$f'(0) = 2\times0+1 = 1$

(10) $h(x)=0 \Leftrightarrow g(x)\cdot(x+3)^2=0 \Leftrightarrow g(x)=0 \vee (x+3)^2=0 \Leftrightarrow g(x)=0 \vee x+3=0$

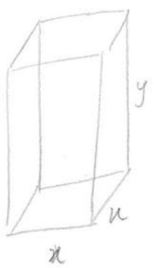
$\Leftrightarrow g(x)=0 \vee x=-3$

2 zeros

(B) $\{-3, 1, 4\}$

(11)

a)



$V=2$

$\Leftrightarrow x\times x\times y=2$

$\Leftrightarrow y=\frac{2}{x^2}$

$A_{base}=x^2$

$A_{lateral}=x\times y=x\times\frac{2}{x^2}=\frac{2x}{x^2}=\frac{2}{x}$

$A_{total}=2x^2+4\times\frac{2}{x}=2x^2+\frac{8}{x}=$

$=\frac{2x^3+8}{x}$

b) $A'(x)=\frac{(2x^3+8)'\times x-(2x^3+8)\times x'}{x^2}=\frac{6x^2\times x-(2x^3+8)\times 1}{x^2}=$

$=\frac{6x^3-2x^3-8}{x^2}=\frac{4x^3-8}{x^2}$

Zeros de A':

$4x^3-8=0 \wedge x^2\neq 0$

$\Leftrightarrow 4x^3=8 \wedge x\neq 0$

$\Leftrightarrow x^3=2$

$\Leftrightarrow x=\sqrt[3]{2}$

	0		$\sqrt[3]{2}$	
$4x^3-8$	ND	$-$	0	$+$
x^2	ND	$+$	$+$	$+$
A'	ND	$-$	0	$+$
A	ND	$\searrow$	m	$\nearrow$

m: $x=\sqrt[3]{2}$

(12) $\lim_{x\to-\infty}\frac{1}{f(x)}=\frac{1}{\lim_{x\to-\infty}f(x)}=\frac{1}{0^+}=+\infty$ (C)

(13) Como em $[0,3]$, $f'<0$ logo f é decrescente.
Sendo decrescente, $f(3)<f(0)=2$. Logo $f(3)=1$ (A)

(14)

h = f×g	-	0	+	0	-	0	+
	$-\infty$	-2		-1		2	$+\infty$
$h = f \times g$	$-$	0	$+$	0	$-$	0	$+$
g	$+$	0	$-$	$-$	$-$	0	$+$
f	$-$		$-$	0	$+$		$+$

→ Apenas opção (C) está de acordo

(15)

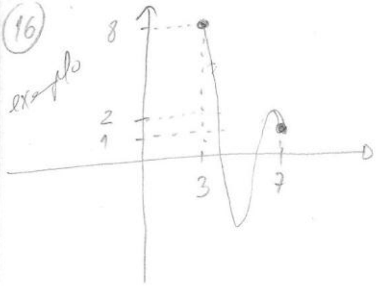
$g(x) = 1 + f(x+1)$ → deslocamento p/ a esquerda 1 unidade

deslocamento p/ cima 1 unidade

(D)

(16)

exemplo



(D)

(17)

$g(x) = \sqrt{f(x)}$ (D)

$D_g = \{x \in \mathbb{R} : f(x) \geq 0\} = [-3, +\infty[$

(18)

$\lim_{x \to 5} f(x) = -3$

$\lim_{x \to +\infty} f(x) = 2$ → $y = 2$ A.H.

$\lim_{x \to -\infty} (f(x) - x) = 0$ → $y = x$ A.O. (A)

(19) $\lim_{x \to 1^+} \frac{f(x)}{g(x)} = \frac{\oplus}{-\infty} = 0$ (A)

(20) Como $f>0$ e $g>0$ então $f(x)+g(x)>0$. Logo $f(x)+g(x)=0$ é impossível (A)

(21) Como $D=]0,+\infty[$ apenas tem sentido $x\to+\infty$

$$m=\lim_{x\to+\infty}\frac{h(x)}{x}=\lim_{x\to+\infty}\frac{\frac{x^2}{g(x)}}{x}=\lim_{x\to+\infty}\frac{x^2}{x\,g(x)}=\lim_{x\to+\infty}\frac{x}{g(x)}=$$

$$=\lim_{x\to+\infty}\frac{1}{\frac{g(x)}{x}}=\frac{1}{\underbrace{\lim_{x\to+\infty}\frac{g(x)}{x}}_{1\text{, porque } y=x+2 \text{ é A.O de } g}}=\frac{1}{1}=1$$

$$b=\lim_{x\to+\infty}(h(x)-x)=\lim_{x\to+\infty}\left(\frac{x^2}{g(x)}-x\right)=\lim_{x\to+\infty}\frac{x^2-x\,g(x)}{g(x)}=$$

$$=\lim_{x\to+\infty}\frac{x(x-g(x))}{g(x)}=\underbrace{\lim_{x\to+\infty}\frac{x}{g(x)}}_{1\text{ (calculado acima)}}\times\lim_{x\to+\infty}[x-g(x)]=$$

$$=\lim_{x\to+\infty}[x-g(x)]=-\underbrace{\lim_{x\to+\infty}[g(x)-x]}_{2\text{, porque } y=x+2 \text{ é A.O. de } g}=-2$$

Como $m=1$ e $b=-2$ então $y=x-2$ A.O de h

(22) $\lim_{x\to1^-}\frac{1}{f(x)}=\frac{1}{+\infty}=0$ → exclui opção A

$\lim_{x\to-\infty}\frac{1}{f(x)}=\frac{1}{0^+}=+\infty$ → exclui opção C

$\lim_{x\to0^-}\frac{1}{f(x)}=\frac{1}{0^+}=+\infty$ → exclui opção D

R: B

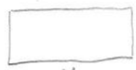
(23) Sendo $y = 2x + 3$ assíntota de g, então

$$\lim_{x \to +\infty} \frac{g(x)}{x} \qquad \lim_{x \to +\infty} \left(g(x) - 2x \right)$$

Assim, $\lim_{x \to +\infty} \left[\frac{g(x)}{x} \times \left(g(x) - 2x \right) \right] = 2 \times 3 = 6$ (C)

(24) Como $\lim_{x \to 3} f(x)$ não existe, então $\lim_{x \to 3} \frac{1}{f(x)}$ tb não existe (D)

(25)



y

x

$A = 5$
$\Leftrightarrow xy = 5$
$\Leftrightarrow y = \frac{5}{x}$

$$P = 2x + 2y = 2x + 2 \times \frac{5}{x} = 2x + \frac{10}{x} = \frac{2x^2 + 10}{x}$$

(A)

(26) $\lim_{x \to 1^+} \frac{h(x)}{g(x)} = \frac{\lim_{x \to 1^+} h(x)}{\lim_{x \to 1^+} g(x)} = \frac{1-1}{+\infty} = 0$ Se $x \to 1^-$, obtém-se $\frac{1-1}{+\infty} = 0$

(C)

(27)

x		-2		2	
$4-x^2$	$-$	0	$+$	0	$-$

$\lim_{x \to 2^+} \frac{1}{4-x^2} = \frac{1}{0^-} = -\infty$ (D)

(28) $g(4) = f(4-1) + 4 = f(3) + 4 = 0 + 4 = 4 \Rightarrow (4, 4)$ (B)

(29) $y = -x - 1$ assíntota de f, logo $\lim_{x \to -\infty} \left[f(x) - (-x-1) \right] = 0 \Leftrightarrow$

$\Leftrightarrow \lim_{x \to -\infty} \left(f(x) + x + 1 \right) = 0$

(B)

(30)

	$-\infty$		0		$*$		$+\infty$
f'		$+$	ND	$-$	0	$+$	
f		$\nearrow$	máx	$\searrow$	mín	$\nearrow$	

(C)

(31) (B) $x_n = -2 - \frac{1}{n} \to -2^-$ porque $\lim\limits_{n\to+\infty} g(x_n) = g\left(\lim x_n\right) = g(-2^-) = +\infty$

(32) $\lim\limits_{x\to-\infty} \frac{3}{f(x)} = \frac{3}{\lim\limits_{x\to-\infty} f(x)} = \frac{3}{-1} = -3$ (B)

(33) $\lim\limits_{x\to+\infty} h(x) = \lim\limits_{x\to+\infty} \frac{x}{f(x)} = \lim\limits_{x\to+\infty} \frac{1}{\frac{f(x)}{x}} = \frac{1}{\frac{1}{3}} = 3$ logo $y = 3$ A.H (D)

$y = \frac{1}{3}x + 2$ A.O de f

$m = \lim\limits_{x\to+\infty} \frac{f(x)}{x}$

(34) Como $\lim\limits_{x\to+\infty} (f(x) - 2x) = 0$, então $y = 2x$ A.O do gráfico de f.

$m = \lim\limits_{x\to+\infty} \frac{f(x)}{x}$

A.O do gráfico de g:

$m = \lim\limits_{x\to+\infty} \frac{g(x)}{x} = \lim\limits_{x\to+\infty} \frac{f(x)+x^2}{x} = \lim\limits_{x\to+\infty} \left(\frac{f(x)}{x} + \frac{x^2}{x}\right) = \lim\limits_{x\to+\infty} \frac{f(x)}{x} + \lim\limits_{x\to+\infty} x$

$= 2 + (+\infty) = +\infty$. Assim, conclui-se que não há A.O.

NOTA: Como $D = \mathbb{R}^+$ apenas tem sentido $x \to +\infty$

(35) $\lim\limits_{x\to+\infty} \frac{f(x)}{x} = m = 1$ (C)

(36) $h = f + g$

$h' = f' + g' =$

$= \underset{m>0}{2x} + \underset{b<0}{\ominus}$ (B)

(37) Como $\lim_{x \to +\infty} [g(x) - x] = 0$, então $y = x$ A.O. de g

$\Downarrow$

(A) e (C) eliminadas

Como $\lim_{x \to 0} g(x) = -\infty$ a opção correta será (D)

(38)

$\lim_{x \to a^+} f(x) = a^2 - a + 3$

$\lim_{x \to a^-} f(x) = a^2 - 2a$

$f(a) = a^2 - a + 3$

$\cancel{a^2} - a + 3 = \cancel{a^2} - 2a$

$2a - a = -3$

$a = -3$ (A).

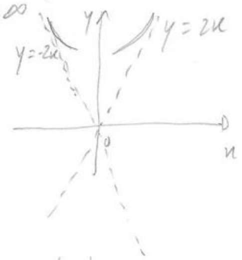
(39) $g(x) = f(x) + x$

$g'(x) = f'(x) + 1 \to$ o gráfico de f' desloca na vertical $\uparrow +1$ (D)

(40) $\lim_{x \to 1^-} \frac{3x}{f(x)} = \frac{3}{+\infty} = 0$ (C)

(41) Como $\lim_{x \to +\infty} (h(x) - 2x) = 0$, então $y = 2x$ A.O

h é par $\to$



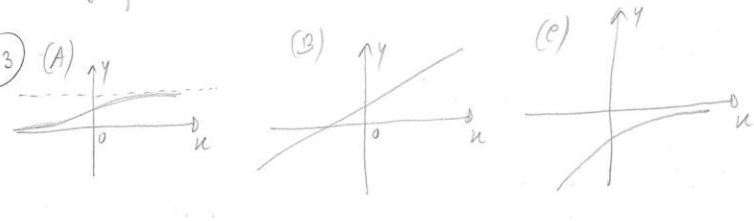
$\lim_{x \to -\infty} h(x) = +\infty$ (A)

(42)

	$-\infty$	a	$+\infty$
f'	+	0	−
f	↗	máx	↘

(C)

(43)



Resta a opção (D)

(44) $\lim(u_n) = \lim h\left(4-\frac{1000}{n}\right) = h\left[\lim\left(4-\frac{1000}{n}\right)\right] = h(4^-) = 1$ (B)

(45) Como $y = 2x-4$ é A.O então

$\lim\limits_{x\to+\infty}\left[g(x)-(2x-4)\right]=0 \Leftrightarrow \lim\limits_{x\to+\infty}\left(g(x)-2x+4\right)=0$ (C)

(46)

(A) $\underbrace{f'(0)}_{\ominus}\times\underbrace{f'(6)}_{\oplus} > 0$ (C) $\underbrace{f'(-3)}_{\oplus}\times\underbrace{f'(0)}_{\ominus} < 0$

(B) $\underbrace{f'(-3)}_{\oplus}\times\underbrace{f'(6)}_{\oplus} > 0$ (D) $\underbrace{f'(0)}_{\ominus}\times\underbrace{f'(6)}_{\oplus} < 0$ Opção D

(47) $\lim f(x_n) = f\left(\lim x_n\right) = f(1^-) = +\infty$ (A)

$x_n \to 1$
$x_n \in]-1,1[$

(48) D

(49) $\lim\limits_{x\to 0^+}\frac{1}{f(x)} = \frac{1}{\lim\limits_{x\to 0^+} f(x)} = \frac{1}{-\infty} = 0$ (exclui opções B e D)

$\lim\limits_{x\to+\infty}\frac{1}{f(x)} = \frac{1}{\lim\limits_{x\to+\infty} f(x)} = \frac{1}{+\infty} = 0$ R: Opção C

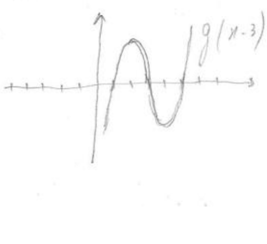
$y = x$ A.O de f

(50) Se $\lim\limits_{x\to-\infty} f(x) = 3$, então $y=3$ A.H

Se $\lim\limits_{x\to 1^+} f(x) = +\infty$, então $x = 1$ A.V. Opção B

$\lim\limits_{x\to+\infty}\left(f(x)-2x\right)=1 \Leftrightarrow \lim\limits_{x\to+\infty}\left(f(x)-2x-1\right)=0 \Leftrightarrow \lim\limits_{x\to+\infty}\left[f(x)-(2x+1)\right]=0$, logo $y = 2x+1$ A.O

(51)



$g(x-3) = f(x)$

	$-\infty$		2		4		$+\infty$
f'		+	0	−	0	+	
f		↗	M	↘	m	↗	

(A)

(52) Apenas tem sentido $x \to -\infty$ porque $D =]-\infty, 4]$

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{3x+3}{\sqrt{x^2+9}} \overset{\frac{\infty}{\infty}}{=} \lim_{x\to-\infty} \frac{x\left(3+\frac{3}{x}\right)}{\sqrt{x^2\left(1+\frac{9}{x^2}\right)}} = \lim_{x\to-\infty} \frac{x\left(3+\frac{3}{x}\right)}{\sqrt{x^2} \times \sqrt{1-\frac{9}{x^2}}} =$$

$$= \lim_{x\to-\infty} \frac{x\left(3+\frac{3}{x}\right)}{|x| \times \sqrt{1-\frac{3}{x^2}}} = \lim_{x\to-\infty} \frac{\cancel{x}\left(3+\frac{3}{x}\right)}{-\cancel{x} \times \sqrt{1-\frac{3}{x^2}}} = \frac{3+0}{-\sqrt{1-0}} = -3 \quad \therefore y = -3 \text{ A.H}$$

(53) $u_n = 2 + \frac{1}{n} \to 2^+$

$\lim f(u_n) = +\infty \Leftrightarrow f(\lim u_n) = +\infty \Leftrightarrow f(2^+) = +\infty$ (C)

(54) Se $y = 2x - 5$ A.O de f

então $\lim_{x\to+\infty} \frac{f(x)}{x} = 2$ e $\lim_{x\to+\infty} f(x) = +\infty$

$$\lim_{x\to+\infty} \frac{6x-1}{f(x)} = \lim_{x\to+\infty} \left(\frac{6x}{f(x)} - \frac{1}{f(x)}\right) =$$

$$= \lim_{x\to+\infty} \frac{6x}{f(x)} - \lim_{x\to+\infty} \frac{1}{f(x)} = \lim_{x\to+\infty} \left(6 \times \frac{x}{f(x)}\right) - \lim_{x\to+\infty} \frac{1}{f(x)}$$

$$= \lim_{x\to+\infty} \left(6 \times \frac{1}{\underbrace{\frac{f(x)}{x}}_{2}}\right) - \lim_{x\to+\infty} \frac{1}{f(x)} = 6 \times \frac{1}{2} - \frac{1}{+\infty} = 3 - 0 = 3$$

(C)

(55)

$u_1 = a$

$u_2 = -3u_1 + 2 = -3a + 2$

$u_3 = -3u_2 + 2 = -3(-3a+2) + 2 = 9a - 6 + 2 = 9a - 4$ (B)

(56) (C) porque $u_n = -\frac{1}{n}$: $-1, -\frac{1}{2}, -\frac{1}{3}, -\frac{1}{4}, \dots \to 0$

Sucessão crescente

" limitada $-1 \leq u_n < 0$

(57) a_n geométrica, $a_3 = \frac{1}{4}$, $a_6 = 2$

$\frac{a_6}{a_3} = r^3 \Leftrightarrow \frac{2}{\frac{1}{4}} = r^3 \Leftrightarrow 8 = r^3 \Leftrightarrow r = 2$

Assim, $a_{20} = a_6 \times r^{14} = 2 \times 2^{14} = 32768$ (C)

(59) $u_4 = 32$, $u_8 = 8192$

$\frac{u_8}{u_4} = r^4 \Leftrightarrow \frac{8192}{32} = r^4 \Leftrightarrow 256 = r^4 \Leftrightarrow r = \pm\sqrt[4]{256} \Leftrightarrow r = \cancel{\pm} 4$

u_n crescente e termos positivos

Assim $u_5 = u_4 \times r = 32 \times 4 = 128$ (B)

(58) $\lim_{x\to 2} \frac{f(x)-f(2)}{x^2-2x} = \lim_{x\to 2} \frac{f(x)-f(2)}{x(x-2)} = \underbrace{\lim_{x\to 2} \frac{f(x)-f(2)}{x-2}}_{f'(2)=6} \times \lim_{x\to 2} \frac{1}{x} =$

$= 6 \times \frac{1}{2} = 3$ (A)